Lecture - 13 2/19/2015

→ APPROXIMATION ALGORITHMS.
1. Bin Packing
2. Asymmetric TSP

→ Given n items $\in [0,1]$, pack into smallest number of unit size bins.

Gabriel's algorithm (Any - Fit algorithm)

1. order items arbitrarily

while (the current item fits into any already opened bin, put item into that bin).

otherwise open a new bin.

→ At any point during the algorithm, there is at most one bin that is not strictly more than half full.

Proof. Consider an item 2 cases:
1. Heavy ($\geq 1/2$)
2. Light ($< 1/2$)

$$\# \text{ bins} < 2\sum_{i=1}^{n} a_i \quad + \quad 1 \quad \leq 2 \sum_{i=1}^{n} a_i$$

OPT is atleast $\geq \left\lceil \sum_{i=1}^{n} a_i \right\rceil$

$$\frac{\#\ \text{bins}}{OPT} \le \frac{2\sum a_i}{\lceil \sum a_i \rceil} \le \frac{2\sum a_i}{\sum a_i} = 2$$

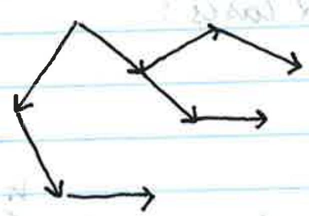
Asymptotic polynomial time approx. scheme.

$0 < \varepsilon < 1 \qquad (1+\varepsilon)\,OPT + 1$

→ Complete Graph
→ Triangle Inequality
→ No symmetry (not necessarily)

For the rest of the lecture, graph is directed.

ARBORESCENCE : we can compute in polynomial time, minimum spanning arborescences.



→ Let's construct a graph. Consider a directed graph G where ~~∅~~ $dist_{ij}$ = shortest path from i to j in another graph H.

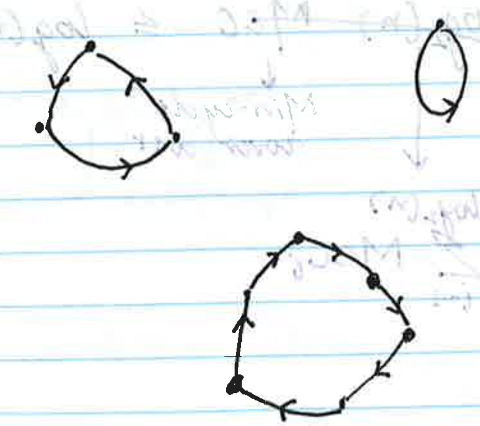
✓ • G is complete
Why does G have Δ inequality?

~~This is a family~~

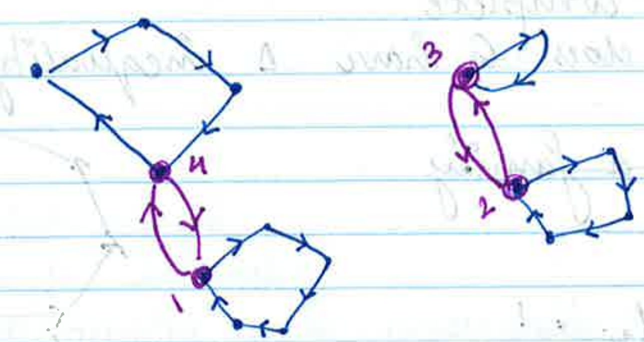
$d_{ji} = 1$
$d_{ij} = n-1$

CYCLE COVERS

HW4: A collection of (directed) cycles that cover all nodes (each node is part of exactly one cycle).



Key 1. Hamiltonian cycles are cycle cover.
2. Minimum length cycle cover can be computed in poly. time.



→ Every time we add a cycle cover, we group together at least half the nodes.

⇒ at most $\log_2(n)$ cycle covers are needed.

→ All other operations (shortcutting) reduces cost.

$$\text{cost} \le \underbrace{\log_2(n)\, MCC}_{\sum_{i=1}^{\log_2(n)} MCC_i} \le \log(n)\, OPT.$$

MCC: Min. cycle cover cost

→ How do you write a linear program for TSP?

Indicator variable per edge.. X_e

$$\min \sum_{(i,j)\in E} x_{ij} c_{ij}$$

How to write it with linear constraints?

✓ $\forall i$ indegree$(i) = 1$

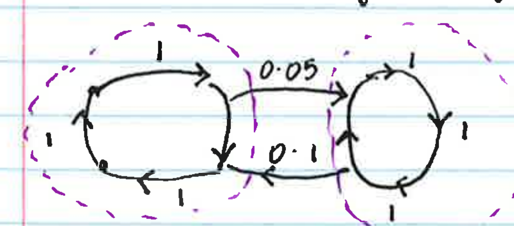
✓ out degree $(i) = 1$ $\forall i$

} only constraints for cycle cover.

→ Held / Karp 82.

- LP's with exponentially many constraints can be solved as long as ∃ black box to find violated constraints (or to correctly claim there are none).

→ Outdegree$(S) \ge 1$ $\forall S \subseteq V$.
outdegree out of every single cut is atleast 1.



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