CME 305 1/6/2015

→ Graph Theory

↳ Algebraic Graph Theory
Minimum Spanning Trees to Matroids

→ Maximum Flow to Submodularity

→ NP hardness
→ Approximation algorithms.
→ The probabilistic Method

→ APPLICATIONS : Spectral Sparsification

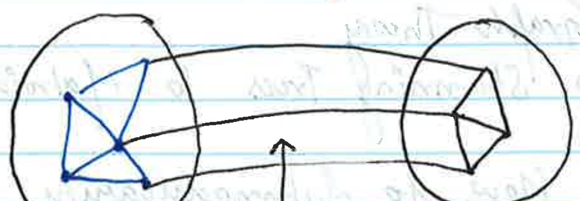
Grade : 50% Final
30% midterm
20% Assignments

Graph $G(V,E)$ → undirected
unweighted
simple graphs (not multiple edges)

$|V| = n$, (no. of nodes)
$|E| = m$
$E \subset V \times U$? Cross product something

How many roads need to be destroyed ?

→ A cut is a subset of vertices $S \subseteq V$



cut size

edges that go from S to V/S

Just one cut

GLOBAL MINIMUM CUT

Naive approach: Take a random cut

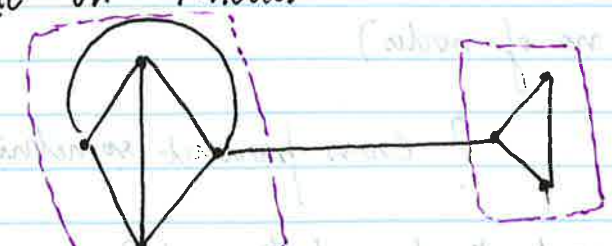
Total cuts $2^{n/2} - 1$ ← how do we get that.

how m $2^n - 2$

all subsets

not empty, not everything

Clique on 4 nodes:



Global minimum cut 2

Prob. of finding global min. cut in the naive scenario is

$$\frac{2}{2^n - 2}$$

Global min cut degree?

Popular Algorithm.

→ KARGER'S GLOBAL MIN CUT ALGORITHM:

size of node set

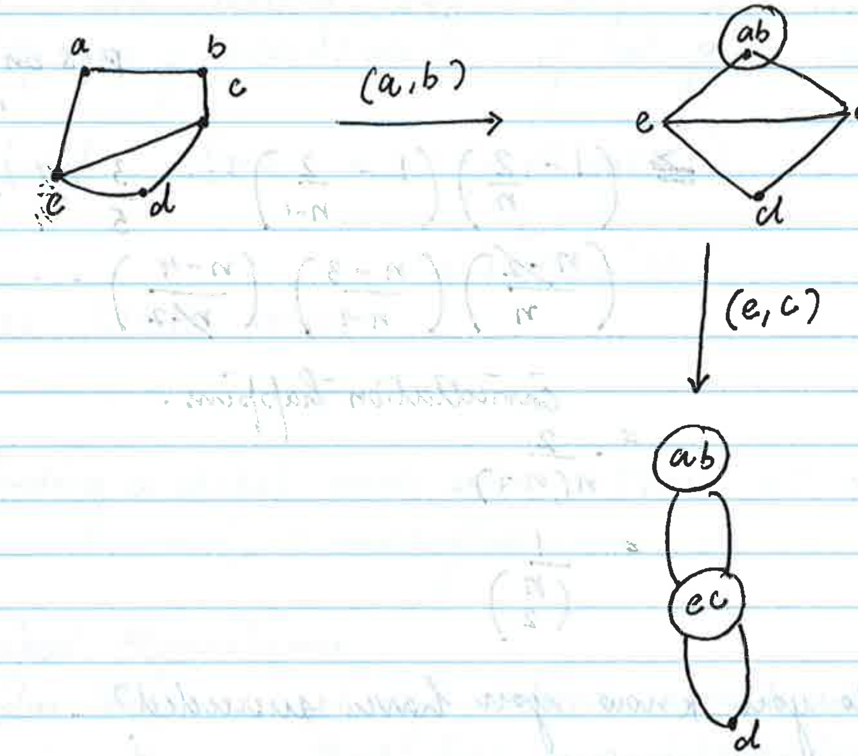
while $|V| > 2$

uniformly pick $e = (u, v)$

$G \leftarrow$ contract u and v in G

end.

Eg.



Prob. of killing a global min. cut?

→ Let's say global min. cut has k edges.

$$P(\text{Failure on step 1}) = \frac{k}{|E|} \leq \frac{k}{\frac{n \cdot k}{2}} = \frac{2}{n}$$

Show: no. of edges in graph $\geq \frac{n \cdot k}{2}$.

know this by heart.

→ $|E| = \frac{1}{2}\sum_{i=1}^{n} \deg(i) \geq \frac{1}{2}\sum_{i=1}^{n} k = \frac{nk}{2}$

→ $P(\text{Failure on step 2} / \text{non failure on first}) \leq \frac{2}{n-1}$

→ $P(\text{success}) = P(\text{success on step 1}) \cdot P(\text{su on step 2} \mid \text{son step 1})$

$\vdots$

$P(\text{s on step } n-2 \mid \text{s on } n-3)$

$$\geq \left(1-\frac{2}{n}\right)\left(1-\frac{2}{n-1}\right)\cdots \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}$$

$$\left(\frac{n-2}{n}\right)\left(\frac{n-3}{n-1}\right)\left(\frac{n-4}{n-2}\right)\cdots \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}$$

cancellation happens.

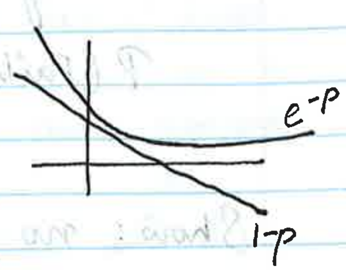
$$= \frac{2}{n(n-1)}$$

$$= \frac{1}{\binom{n}{2}}$$

How do you know you have succeeded?

→ Prob. of failure of finding the best cut after t times.

$$P(\text{failure after } t \text{ times}) \leq (1-p)^t \leq e^{-tp}$$



Let's set $t = \frac{1}{p}\log(n)$ ↳ $= \binom{n}{2}\log(n) = O(n^2 \log n)$

Now, $e^{-\log(n)} = \frac{1}{n}$

→ One run of Karger takes $O(m)$ time via cutting largest edge in minimum spanning tree (MST) of shuffled edges.

total $O(mt) = O(m n^2 \log(n))$

↓

$O(n^2 \log^3(n))$.

→ KARGER STEIN insight?

→ finding a global min cut. Then ST min cut. (s-t max flow)

Randomized Algorithms:

→ Las Vegas : Always returns correct answer But has non-deterministic running time.

→ Monte-carlo Algorithms : some probability of success but deterministic running time.