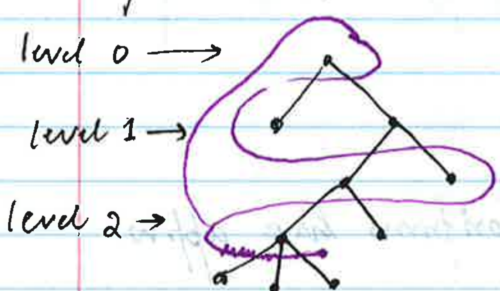
Lecture-8 Approximation Algorithms. 1/29/2015

→ Greedy: Max-Cut.
→ Relaxation: Vertex Cover.
→ Combinatorial: The Traveling Salesman.

Don't Give up:
1. Find instances that are doable in polynomial time.
2. Approximate.

→ Trees.
→ Bipartite Graphs.
→ Sparse Graphs. → use probabilistic method to prove there is an independent set.

→ Every tree is a bipartite graph.

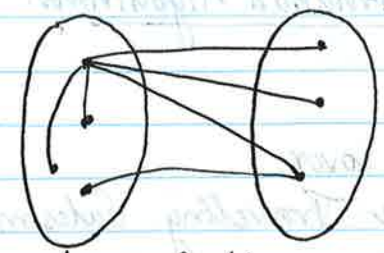


Clues: NP hardproblem

→ MAX CUT deterministic

Abdul's algorithm: sort by degree isn't half approximation.

- $V = \{x_1\}$
  while $\exists\, x_0$ s.t moving x_0 to other side increases cut size, move x_0 to other side
  end



Upon termination

"Key insight: every vertex has atleast half of its degree associated with cut edges.

X: $X \geq \frac{1}{2} \sum_{i=1}^{n} \frac{deg(i)}{2} = \frac{1}{2} \cdot \frac{1}{2} \cdot 2m = \frac{m}{2}$ OPT $\leq m$.

hand shake lemma.

true because of termination condition.

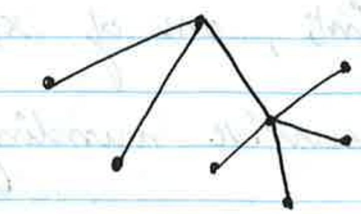
$\Rightarrow \frac{X}{OPT} \geq \frac{1}{2}$

- { Maximization approx algorithms have approx ratio $0 < \text{ratio} < 1$.
  ... minimization ...
  $1 < \text{ratio}$.

- 0.878 [Goemans-Williamson]

CONVEX RELAXATION for vertex cover.

→ find min. no. of nodes such that all edges are covered.



We are going to

let $x_i \in \{0,1\}$ indicator variable. if node i is in the vertex cover.

$$\left[\begin{array}{ll} \min \sum_{i=1}^{n} x_i & \\ \text{s.t.}\ \ x_i + x_j \geq 1 & \forall (i,j) \in E \\ \forall i \ \ x_i \in \{0,1\} & \end{array}\right.$$

can't handle this constraint

$\forall i \quad 0 \leq x_i \leq 1$ (made # of feasible points much larger).

→ LpOPT $\leq$ OPT

- Fixing solution to relaxation is called "Rounding".

II. • Solve L.P

Rounding if $x_i \geq \frac{1}{2}$ set $x_i = 1$
else set to 0.

→ Because of Lp constraint, one of x_i, x_j for $(i,j) \in E$ will survive rounding

$$\text{Total Cost} \le 2 \sum x_i \le 2\,\text{LPOPT} \le 2\,\text{OPT}$$

⇒ 2 approx. ratio.

Best approx we have: $2 - \frac{1}{\sqrt{\log(n)}}$

UGC: Unique Games Conjecture.

Travelling Salesman Problem

→ Find hamiltonian cycle of min. weight

→ hopeless: because hamiltonian cycle is NP hard to decide.

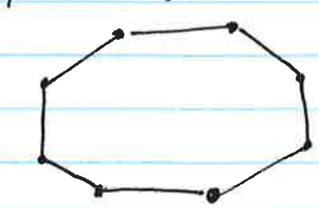
→ Have weighted complete graph instead.

NP hard to approximate

→ Weights. Metric Inequality

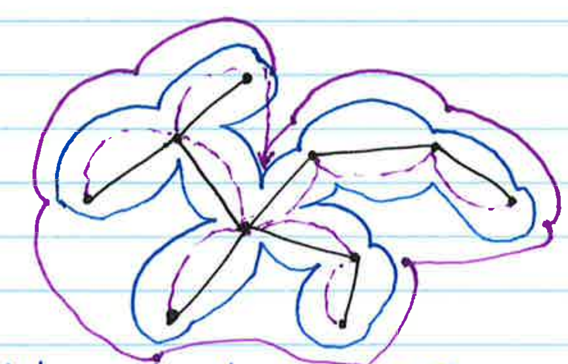
• Let's find an approximation algorithm.

•→ The handle is going to be the minimum spanning tree.



Hamiltonian Cycle is more expensive than MST because it is a spanning tree + one edge.

MST ≤ OPT

→ 

Eulerian ~~de~~ tour only exists if you have even degree nodes.

≤ OPT	step 1.	Compute MST T
≤ 2OPT	2.	Double edges of T
≤ 2OPT	3.	Compute Eulerian Tour $x_1, x_2, \ldots, x_1$
≤ 2OPT	4.	Shortcut repeat visits.

appx. 2

Teaser for 1.5 :