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REAL INTEREST AND CONSUMPTION

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ABSTRACT

One of the important determinants of the response of saving and consumption to the real interest rate is the elasticity of intertemporal substitution. That elasticity can be measured by the response of the rate of change of consumption to changes in the expected real interest rate. A detailed study of data for the twentieth-century United States shows no strong evidence that the elasticity of intertemporal substitution is positive. Earlier studies finding substantially positive elasticities are shown to suffer from a bias related to the timing of instrumental variables.

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Introduction

A higher expected real interest rate makes consumers defer consumption, everything else held constant. The magnitude of this intertemporal substitution effect is one of the central questions of macroeconomics. If consumers can be induced to postpone consumption by modest increases in interest rates, then

(1) movements of interest rates will make consumption decline whenever other components of aggregate demand rise--total output will not be much influenced by changes in those components,

(2) the dead-weight loss from the taxation of interest is important,

(3) the burden of the national debt or unfunded social security is relatively unimportant,

(4) consumption will move along with real interest rates over the business cycle

to name four of the many issues that rest on the intertemporal substitutability of consumption.

This paper attempts to estimate parameters of the representative individual's utility function, rather than parameters of the consumption function or savings function. As Robert Lucas (1976) has pointed out, there may not be anything that could properly be called a consumption or savings function--the relation between consumption, income, and interest rates depends on the wider macroeconomic context and may not be stable over time, even though consumers are always trying to maximize the same utility function. The techniques of this paper are more robust with respect to this kind of instability than are standard econometric models of consumption and savings.

The essential idea of the paper is the following: Consumers plan to change their consumption from one year to the next by an amount that depends on their expectations of real interest rates. Actual movements of consumption differ from planned movements by a completely unpredictable random variable that indexes all of the information available next year that was not incorporated in the planning process the year before. If expectations of real interest rates shift, then there should be a corresponding shift in the rate of change of consumption. The magnitude of the response of consumption to a change in real interest expectations measures the intertemporal elasticity of substitution. All of this is set up in a formal econometric model where the assumptions are formalized and the estimation techniques rigorously justified.

Over the postwar period, there have been downward and upward shifts in the expected real return from common stocks, Treasury bills, and savings accounts, the investments that presumably set the relevant real interest rate for most consumers. Over the same period, there has been only small shifts in the rate of growth of consumption. Consequently, all of the estimates presented in this paper of the intertemporal elasticity of substitution are small. Most of them are also quite precise, supporting the strong conclusion that the elasticity is unlikely to be much above 0.1, and may well be zero.

1. *Theory of the consumer under uncertain real interest rates*

Finance theory has examined the role of the consumer in an economy with one or more securities with stochastic returns. Douglas Breeden (1977,1979) was the pioneer in what has become known as the consumption capital asset pricing model. Hansen and Singleton (1983) provide an application of the model to macroeconomic consumption data. Mankiw, Rotemberg, and Summers (1985) have extended the model to include labor supply. The literature on consumption and stochastic asset returns has suffered from a serious problem from the point of view of the macroeconomist interested specifically in the issue of intertemporal substitution. In the literature, consumers are viewed as maximizing the expected value of an intertemporal utility function,

$$\Sigma e^{-\delta t} c_t^{1-\alpha} \qquad (1.1)$$

The parameter α is identified as the coefficient of relative risk aversion. However, from the theory of the consumer under certainty, it is clear that α controls intertemporal substitution as well. The intertemporal elasticity of substitution, σ, is just the reciprocal of α. If consumers are highly risk averse, they must have low intertemporal substitution as well.

Larry Selden (1978) has argued that expected utility models are inherently ill-suited to the characterization of intertemporal choice under uncertainty. The logic of expected utility precludes the needed flexibility to describe a consumer's views about uncertainty and about consumption in different time periods (or, for that matter, about consumption of different commodities). Selden has proposed a more general framework, based on what he calls the ordinal certainty equivalent (OCE) representation hypothesis .

In essence, the OCE approach characterizes the consumer's views about uncertainty through a one-period utility function, $V(c_t)$. The more concave is V, the more risk averse is the consumer. But the curvature of V has no bearing on the consumer's willingness to substitute consumption from one period to another. Rather, its role is to convert the stochastic c_t into its certainty equivalent, $\tilde{c}_t$. Specifically, $\tilde{c}_t$ is defined by

(1.2) $$V(\tilde{c}_t) = E[V(c_t)]$$

The intertemporal aspects of preferences are captured by another utility function, U. Though Selden deals with the general case where U is not separable across time periods, this paper will deal exclusively with the special case of separability. In that case, the consumer's intertemporal preferences are described by the utility function,

$$\Sigma\, e^{-\delta t}\, u(\tilde{c}_t) \tag{1.3}$$

A consumer with a sharply concave u will avoid intertemporal substitution and will strongly prefer a consumption path that is approximately equal in all time periods.

For the case of two periods, with period 1 consumption certain, Selden obtains results that support his claim to a substantial advance over the expected utility approach. First, as long as the consumer's preferences about second-period uncertainty admit an expected utility representation, conditional on the value of first-period consumption, there exists an OCE representation of his complete preferences. Second, any expected utility preference ordering has an OCE representation, but many interesting OCE orderings do not have an expected utility representation. Third, and most relevant for this paper, for the OCE ordering based on a V function with constant relative risk aversion and a u function with constant intertemporal elasticity of substitution, the only member that admits an expected utility representation is one where the elasticity of substitution is the reciprocal of the coefficient of relative risk aversion.

Selden's OCE approach lends itself to empirical work along the lines of Hansen and Singleton (1983). In fact, the OCE procedure for estimating the elasticity of substitution is exactly the same as the procedure employed by Hansen and Singleton to estimate the reciprocal of the coefficient of relative risk aversion. From the standpoint of the OCE representation, Hansen and Singleton have estimated the elasticity of intertemporal substitution alone, not some average value of the intertemporal and risk aversion parameters.

For the purposes of what follows, it is not necessary to make specific assumptions about the market setting of the maximization. At one extreme, the consumer could face a full set of markets in contingent commodities, and then the budget constraint would say that the sum of all the consumer's demands for the contingent claims valued at market prices would equal his endowment. At the other extreme, the consumer could be Robinson Crusoe, with a single risky investment in a real asset. Then the budget constraint would say that his holdings of the real asset could never be negative. For a further discussion of this point, see Sanford Grossman and Robert Shiller (1982).

In any case, one of the many choices facing the consumer is to spend a little less in year t-1, invest the savings in one asset, and spend the stochastic proceeds in year t. Suppose that a unit investment in year t-1 has the stochastic return $e^{r_{t-1}}$ in year t. At the point of maximum expected utility, the consumer will have thought through all possibilities of this kind. In particular, the consumer will have solved the maximization problem,

$$\max_{c_{t-1}} [u(c_{t-1}) + e^{-\delta t} u(\tilde{c}_t)] \tag{1.4}$$

subject to

(i) certainty equivalence,

$$V(\tilde{c}_t) = E[V(c_t)] \tag{1.5}$$

and

(ii) the budget constraint,

$$c_t = c_t^* + e^{r_{t-1}}(c_{t-1}^* - c_{t-1}) \tag{1.6}$$

Here, $c^{\star}_{t-1}$ and $c^{\star}_{t}$ are given levels of consumption and c_{t-1} and c_t are feasible variations. Taking the derivative with respect to c_{t-1} (evaluated at $c_{t-1} = c^{\star}_{t-1}$ and $c_t = c^{\star}_t$) and substituting the certainty equivalence definition and the budget constraint gives the first-order condition,

$$(1.7) \qquad \frac{E[e^{r_{t-1}} V'(c_t)]}{V'(\tilde{c}_t)} u'(\tilde{c}_t) = e^{\delta} u'(c_{t-1})$$

This formula is the precise mathematical formulation of the principle that the marginal rate of substitution should equal the ratio of the prices of present and future consumption. Under uncertainty, it is not true that the expected marginal rate of substitution should equal the expected price ratio (the discount function). Rather, the appropriate discount rate is the risk-adjusted one described by the first factor in equation 1.7; it is the expectation of the product of the discount function and the marginal stochastic utility next period. This expression is related to the "consumption beta" of modern finance theory.

The reallocation condition of equation 1.7 is the generalization of the proposition investigated in my earlier paper (Hall (1978)) that marginal utility should be a trended random walk when real interest rates are constant over time. Further progress in translating the reallocation condition into consequences for observed variables requires assumptions about the distributions of the random influences. A set of assumptions related to those introduced by Breeden (1977) seems a natural approach. First, assume that the real interest rate, r_{t-1}, conditional on information available in year t-1, obeys the normal distribution with mean $\bar{r}_{t-1}$ and variance v_r. Because interest rates as they are defined in this paper can be indefinitely negative, the normal distribution is a natural assumption. Second, assume that the consumer's rule for processing new information about income and interest rates makes the distribution of consumption log-normal, conditional on information available last year; that is, $\log c_t$ is normal with mean $\bar{c}_t$ and variance v_c. Because the new information arriving in year t has a bearing on both the actual return to investments maturing in t and on the consumer's long-term well being estimated in that year, the two random variables r_{t-1} and $\log c_t$ will be correlated; I will let $v_{r,c}$ stand for their covariance.

Breeden assumed, along with all other workers in this area, that the coefficient of relative risk aversion and the intertemporal elasticity of substitution were the reciprocals of one another. Following Selden, I will assume that the two are constant, but are independent parameters. The coefficient of relative risk aversion is α:

$$V(c) = \frac{c^{1-\alpha}}{1-\alpha} \tag{1.8}$$

and the intertemporal elasticity of substitution is σ:

$$u(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma} \tag{1.9}$$

Then the random variable in the definition of the consumption beta, $e^{r_{t-1}}V'(c_t)$, is log-normal. The rule that the expectation of the exponential of a normal random variable with mean μ and variance v is $e^{\mu+v/2}$ makes it straightforward to evaluate the consumption beta. To finish the derivation of the econometrically useful form of the allocation condition, I will redefine c to be the log of consumption in the rest of the paper. The certainty-equivalent consumption is

$$\tilde{c}_t = \bar{c}_t + \frac{1-\alpha}{2}v_c \tag{1.10}$$

Note that the certainty equivalent log-consumption exceeds or falls short of the mean of the log of consumption as α is below or above one. However, the certainty equivalent is always less than the log of the mean of consumption, which is

$$(1.11) \qquad \bar{c}_t + \frac{1}{2} v_c$$

Now applying the intertemporal allocation condition and substituting the value of the certainty-equivalent of consumption gives the relation between the expected value of log-consumption in period t given consumption in period t-1 and the mean of the distribution of the real interest rate:

$$(1.12) \qquad \bar{c}_t = \sigma \bar{r}_{t-1} + c_{t-1} + k$$

Here k is a constant that depends on the variances and covariance of r and c. I will assume that it does not change significantly over time.

The condition just derived says that the mean level of consumption in period t generated by the consumer's choice as of period t-1 is the level of consumption chosen for period t-1 plus an adjustment positively related to the mean of the real interest rate plus a constant. The coefficient that governs the influence of the expected real interest rate is precisely the elasticity of intertemporal substitution. A high value of σ means that, when the real interest rate is expected to be high, the consumer will actively defer consumption to the later period.

The condition is a constraint on the consumption rule. It says that an optimal rule will wind up choosing a level of consumption in period t, after the new information becomes available, whose mean obeys this restriction. The condition is not a complete description of consumption behavior under uncertainty. It does not describe the actual amount by which consumption changes when new information about income or asset returns becomes available.

The actual log of consumption in period t, c_t, differs from the mean, $\bar{c}_t$, by a completely unpredictable surprise, which I will call ϵ_t. By the hypotheses already stated, ϵ_t is a normal random variable. The two equations of interest can be put in the form of a bivariate regression,

(1.13) $$c_t = \sigma \bar{r}_{t-1} + c_{t-1} + k + \epsilon_t$$

(1.14) $$r_{t-1} = \bar{r}_{t-1} + \nu_t$$

The random variable ν_t also has the normal distribution.

If the expected real interest rate, $\bar{r}_{t-1}$ is observed directly, then the key parameter σ, the intertemporal elasticity of substitution, can be estimated simply by regressing the log-change in consumption on the expected real rate. That regression also has the property that no other variable known in period t-1 belongs in the regression. The strong testable implication of the theory is that the mean of the rate of growth of consumption is shifted only by the mean of the real interest rate.

information available in year t-1 is helpful in predicting the rate of growth of consumption only to the extent that it predicts the real interest rate. This testable implication is the logical extension of the one derived in my earlier paper (Hall (1978)) under constancy of real interest rates. In that case, no variable known in year t-1 should help predict the rate of growth of consumption.

In the OCE framework, the bivariate relation between consumption and real interest rates does not reveal anything about risk aversion. In order to infer anything about risk aversion, more than one asset must be considered. Estimation of the risk aversion parameter would be possible in a multivariate system with the real returns to two or more assets. Then the magnitudes of the risk premiums together with the correlations of the returns with consumption would provide estimates of the coefficient of relative risk aversion.

Hansen and Singleton (1983) studied the joint distribution of the rate of growth of consumption and asset returns in the conventional expected intertemporal utility framework. They do not mention intertemporal substitution in their discussion at all. They identify the single critical parameter they estimate as the coefficient of relataive risk aversion. Their statistical model is the same as the one derived here. Their estimation technique involves, in effect, regressing the rate of change of consumption on expected real asset returns and interpreting the coefficient as the reciprocal of the coefficient of relative risk aversion. In their framework, as I mentioned earlier, the coefficient is also the intertemporal elasticity of substitution.

It appears that a better interpretation of Hansen and Singleton's estimated coefficient is the intertemporal elasticity of substitution, not the coefficient of relative risk aversion. The OCE framework provides the simplest means for distinguishing the two, and the coefficient is unambiguously the intertemporal one. The earlier version of this paper (Hall (1981)) constructed an argument within the expected utility framework that reached the same conclusion. Moreover, simple economic intuition suggests that the rate of change of consumption over time is more likely to reveal something about intertemporal substitution than about risk aversion.

Hansen and Singleton, and Grossman and Shiller (1982) before them, are on firm ground in treating the differences in returns among assets as revealing something about risk aversion. Indeed, Hansen and Singleton's rejection of the cross-equation restrictions in a model combining consumption growth with returns on multiple assets may well occur because the intertemporal elasticity of substitution is different from the reciprocal of the coefficient of relative risk aversion.

2. *Expectations of the real interest rate*

The measurement of the expected real interest rate is one of the empirical issues investigated in this paper. Two basic approaches are taken. First, I study the change in consumption over a period for which survey data on expected price changes are available. The expected real interest rate is the market nominal rate for an instrument of suitable term, adjusted for taxes, less the expected rate of change of the price level. Real returns from the stock market can also be used in this framework, because survey data on expected nominal stock prices are available.

The second approach relates the conditional mean of the real interest rate, $\bar{r}_{t-1}$, to observed variables known to consumers at the time that they choose c_{t-1}. Recall that $\bar{r}_{t-1}$ is the mean of the subjective distribution for the real interest rate held by the typical consumer at the time consumption decisions are made for year t-1. What I will call the "conventional specification" for expectations has been employed frequently in macroeconomic models derived from rational expectations and, in particular, underlies the recent work of Hansen and Singleton. The conventional specification says that the mean of the subjective distribution is a linear combination of observed variables:

(2.1) $$\bar{r}_{t-1} = x_{t-1}\beta$$

and the coefficients, β, are known in advance. Under this specification, the complete model of expectations and consumption becomes a simple application of bivariate regression with parameter constraints across the equations. Alternatively, the same estimation technique can be thought of as instrumental variables applied to the consumption equation, with the determinants of the expected real rate as the instruments. The alternative is the interpretation offered by Hansen and Singleton.

3. *Time aggregation*

The basic equation for the rate of change of consumption,

$$(3.1) \qquad \Delta c_t = \sigma \bar{r}_{t-1} + k + \epsilon_t$$

refers to consumption in discrete time. From the derivation in section 1, it is also apparent that it applies to observations on the instantaneous flow of consumption measured at two points of time in a setup where time is measured continuously. However, it does not correctly characterize the behavior of time averages of consumption. If c_t is the average flow of consumption over an interval of continuous time, then the relation of its rate of change to the real interest rate is more complex.

As with other aggregation problems in econometrics, time aggregation for the left-hand variable causes only mild problems. If the right-hand variable is observed continuously, or at least quite frequently, then the aggregation of the left-hand variable in effect defines an appropriate way to aggregate the righthand variable. The problem of time aggregation becomes much more difficult if only a time average of the right-hand variable is available (see Grossman, Melino, and Shiller (1985)). However, in the present case, interest rates and rates of inflation are measured monthly or more frequently over the whole time span for which any data at all are available for consumption, so the time aggregation problem is readily soluble.

Suppose that only a time average of consumption is observed, say once a year. Each month, the expected real interest rate is known; call it $\bar{r}_{t,m}$ with t the year and m the month. There is an unobserved $c_{t,m}$ each month, and it evolves as

(3.2) $$c_{t,m} = \sigma\bar{r}_{t,m-1} + \epsilon_{t,m}$$

Now write out $c_{t-1,m}$ and $c_{t,m}$ as increments over the initial value $c_{t-1,1}$. Note that

(3.3) $$c_t = \Sigma\, c_{t,m}$$

Then a little manipulation shows that the change in aggregate consumption is

$$\Delta c_t = \sum_{m=1}^{12} (m-1)(\sigma\bar{r}_{t-1,m-1} + \epsilon_{t-1,m}) + \sum_{m=1}^{12} (12-m+1)(\sigma\bar{r}_{t,m-1} + \epsilon_{t,m}) \tag{3.4}$$

Define the time aggregates of the expected real interest rate and tbe random element:

$$\bar{r}_{t-1} = \Sigma (m-1)\bar{r}_{t-1,m-1} + \Sigma (12-m+1)\bar{r}_{t,m-1} \tag{3.5}$$

$$\epsilon_t = \Sigma (m-1)\epsilon_{t-1,m} + \Sigma (12-m+1)\epsilon_{t,m} \tag{3.6}$$

Then the relation among the time aggregates is

$$\Delta c_t = \sigma\bar{r}_{t-1} + \epsilon_t \tag{3.7}$$

Two properties of the aggregate random element ϵ_t call for note. First, as Holbrook Working derived in a famous paper (1960), ϵ_t is not white noise; rather, it obeys a first-order moving average process with serial correlation

0.25. Second, ϵ_t is likely to be correlated with r_{t-1} or with its determinants or instruments, even if these variables are uncorrelated at the monthly level.

4. *Data*

Following are brief definitions of the data series used in this study:

c_t: log of real consumption of nondurables (not including services) in year, quarter, or month t, from the U.S. national income and product accounts. Available monthly from 1959, quarterly from 1947, and annually from 1919. For derivation before 1929, see Hall (1985).

r_t: realized real return after taxes on a investment in the Standard and Poor's 500 stock portfolio, liquidated at a later date corresponding to the consumption variable,

OR

realized real return after taxes from a savings account earning the regulated passbook interest rate,

OR

realized real return after taxes from holding a sequence of four 90-day Treasury bills over the year.

h_t: log of the S&P 500 index of share prices, deflated.

d_t: dividend yield of the S&P 500.

z_t: nominal yield of Treasury bills, discount basis

q_t: nominal passbook interest rate in the third quarter

p_t: log of the implicit deflator for consumption of nondurables (used as deflator for all deflated variables).

After-tax magnitudes were calculated using the effective marginal rate under the federal personal income tax from Barro and Sahasakul (1983). The full nominal amount of dividends and interest was assumed to be taxed at this effective marginal rate. Capital gains and losses were assumed to be untaxed, on the grounds that the combination of low statutory rates, taxation only at realization, and forgiveness of accrued gains at death make the effective rate close to zero. All data for the study are listed in an appendix available from the author.

5. *Summary of results*

Following is a brief summary of the various attempts I have made to estimate the intertemporal elasticity of substitution by regressing the rate of change of consumption on expected real interest rates.

The first set of results uses inflation and stock price expectations recorded in the Livingston survey. In this work, the expected real return is measured directly and the elasticity of substitution estimated by simple regression. For real returns in the stock market, the results are informative--the elasticity of substitution is close to zero and the estimate has a small standard error. For savings accounts and Treasury bills, the estimates are almost useless because of large standard errors. In these cases, the lack of variation in the expected real return makes it difficult to estimate the elasiticity.

A second set of results uses annual changes in consumption starting in 1923. The real return on Treasury bills is aggregated from monthly data as suggested in section 3. Because this technique uses a longer span of data and uses all of the data for each year, the standard error of the estimate of the intertemporal elasticity is much smaller. The point estimate of the elasticity is negative. All positive values lie outside the 95 percent confidence interval.

A third set of results reconciles the findings of this paper—that the intertemporal elasticity is around zero--with Hansen and Singleton's finding of large positive elasticities. Some of the more obvious explanations are rejected: The discrepancy is not caused by their failure to consider the problem of time aggregation. Rather, almost all of the difference comes from their use of instruments that are correlated with the innovation in the real return.

A fourth set of results examines Lawrence Summers' (1982) findings of intertemporal elasticities of around one, using quarterly postwar data. Again, use of properly timed instruments reverses his conclusion.

My overall conclusion from all four sets of results is that the evidence points in the direction of a low value for the intertemporal elasticity. The value may even be zero and is probably not above 0.2.

Before plunging into formal econometric results, I think it is useful to indicate why the data point toward the answer that pervades of the the results of this paper, namely that the intertemporal elasticity of substitution is small. Some simple facts about the data are apparent just by taking averages over five-year intervals. The averaging removes most of the random expectation errors but turns out to leave a good deal of variation in the real interest rate. Figure 1 shows the real after-tax return on Treasury bills and the rate of change of consumption for intervals from 1921 through 1940 and 1946-83 (the last interval is only three years long).

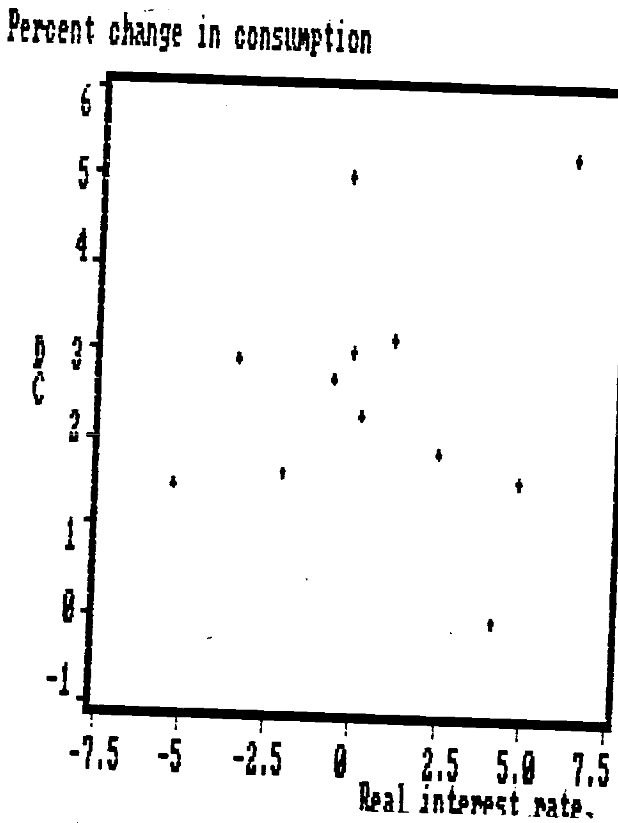

Figure 1. Five-year averages of the real return on Treasury bills (horizontal axis) and the rate of change of consumption (vertical axis), 1921-40 and 1946-83.

Except for three of the observations, the rate of change of consumption is close to its average value of a little below three percent per year. When consumption was near average, however, the real interest rate varied from -5 percent to +5 percent.

The only observation combining a high real interest rate and rapid consumption growth was for 1921-25, in the upper right-hand corner. The other observation with high consumption growth was for 1936-40, when the real interest rate was almost exactly zero. The period 1931-35 had a high real interest rate and slightly negative consumption change. As a general matter, Figure 1 makes a fairly strong case that periods of high real interest rates have not typically been periods of high consumption growth. Rather, consumption growth has generally stuck fairly close to its average value no matter what has happened to real interest rates.

6. *Results based on the Livingston survey*

Each November, Joseph Livingston asks a panel of economists to predict the values of a long list of economic variables for the following June. Among the variables are the Consumer Price Index and the S&P 400 stock price index. From these, it is possible to construct three measures of expected real returns that are relevant for consumers:

Treasury bills. The starting point is the market value of a bill maturing in June as reported in November. All elements of the expected real rate are known except for the marginal tax rate, which is highly predictable. I computed the expected real return at as

$$r = \log\left[\frac{2-mz}{2-z}\frac{p_N}{p_J}\right] \tag{6.1}$$

Here z is the nominal return measured in discount form at an annual rate (as a decimal), m is the marginal tax rate, p_N is the known price level in November, and p_J is the expected price level in June.

Savings accounts. Nominal bank rates, q, not entirely known in advance, but are highly predictable. I compute the expected real after-tax return as

$$\log\left[\left(e^{7/12q} - m(1 - e^{7/12q})\right)\frac{p_N}{p_J}\right] \tag{6.2}$$

Stocks. I treat the dividend yield, d, as known and use the survey data for the expected share price. The expected real after-tax return is

$$(6.3) \qquad \log[(\tfrac{7}{12}(1-m)d + \frac{h_J}{h_N})\frac{p_N}{p_J}]$$

Here h_N is the known stock price index in November and h_J is the expected index for the following June. The results from regressing the log-change in consumption on these three measures of the expected real return are:

Security	Estimate of σ (standard error)
Treasury bills	0.346 (0.337)
Savings accounts	0.271 (0.330)
Stocks	0.066 (0.050)

The results for treasury bills and savings accounts are hardly conclusive. The variation in the expected real returns over the 24 7-month periods in the data is inadequate to provide any useful information about the elasticity of substitution, σ. But for the stock market, the results are conclusive. The estimate of σ is close to zero and the standard error is small as well. The confidence interval for σ excludes all values that correspond to strong intertemporal substitution.

7. *Results from annual data with consistent time aggregation*

Annual averages of consumption are available starting just after World War I. Monthly data on the realized return on treasury bills can be calculated for the same period. Aggregation of the real return data to annual rates, as described in section 3 makes it possible to estimate the intertemporal elasticity of substitution from a much longer historical record with much more variance in expected real returns. Though there are good reasons to doubt their accuracy, ordinary least squares results are a good starting point. The regression coefficient for the annual average of real returns with the annual log-change in consumption as the dependent variable is, for the years 1924 to 1940 and 1950 to 1983:

Estimate of σ:	-0.339
	(0.104)

Recall that the disturbance in this regression has a theoretical serial correlation of 0.25. The Durbin-Watson statistic is 1.64, close to its theoretical value. After making the appropriate correction for the first-order moving average process, the standard error of the regression coefficient rose by about 10 percent and the value of the estimate was hardly changed. For this reason, I did not try to make any further corrections for serial correlation in the other results.

Ordinary least squares is not a suitable estimator for two reasons. First, after time aggregation, the disturbance is correlated with the right-hand variable. The correlation would exist even if the expected real interest rate were measured correctly; nothing in the theory rules out the possibility that an event that brings an upward jump in consumption early in the year will not also cause an expected real interest rate to rise later in the year. Second, when the realied real rate is used in place of the expected real rate, the difference, ν, is probably correlated with the surprise in consumption. For both reasons, an instrumental variables estimator is required.

The timing of the instruments turns out to be critical. If the data measured the instantaneous flow of consumption at two isolated points, any variable known at the time that c_{t-1} was chosen would be eligible as an instrument. However, when c_{t-1} is an annual average, it is apparent that any variable measured during calendar year t-1 can be correlated with the disturbance, ϵ_t. The most recent permissable instrument is one measured in December of year t-2. Annual aggregates for year t-2 and earlier are usable, but not those for year t-1. Accordingly, I used the following as instruments: the change in annual log-consumption in year t-2, the level of the average real return over year t-2, and the nominal return in December of year t-2

The result from two-stage least squares is

Estimate of σ: -0.455
(0.186)

The finding of a negative value of the intertemporal elasticity of substitution was not sensitive to the choice of instruments, as long as endogenous variables from year t-1 were excluded. Separate estimates for the pre- and post-war periods showed that the estimate was somewhat negative in the earlier period and positive for the later period. However, the pooled estimate clearly rejects all positive values of σ. It simply cannot be said that the relation between the real return and the rate of change of consumption supports strong intertemporal substitution.

8. *Results based on recent monthly data*

Hansen and Singleton (1983) obtain results which can be interpreted as evidence of large values for σ. Although I have not attempted to reproduce their results exactly, simple instrumental estimates do give high estimates of σ, espcially over the particular time period they studied. For example, for data from July 1959 through December 1978, with the real rate lagged two, three, and four months and the rate of change of consumption lagged one, two, and three months as instruments, I obtain:

Estimate of σ: 1.342
(0.361)

Incorporating data through December of 1983, I still get a fairly high value:

Estimate of σ: 0.668
(0.235)

However, the use of the immediately lagged change in log-consumption as an instrument, following Hansen-Singleton, is not permitted in the framework of this paper. Section 3 showed that last year's change in consumption depends on some of the same random disturbances as this year's change. The most recent change in consumption admissable as an instrument is the one lagged two years. Dropping Δc_{t-1} from the list of instruments reduces the estimate of σ dramatically:

Estimate of σ: 0.207
(0.370)

For the stock market, use of recent monthly data does not change the conclusion that the estimate of the elasticity is reliably low:

Estimate of σ: -0.060
(0.051)

Large fluctuations occurred over the period from 1959 through 1983 in the expected real return from the stock market, not matched by corresponding changes in the rate of change of consumption. The monthly results for the stock market strongly confirm the results from 7-month changes in the earlier study with the Livingston data.

9. *Results based on postwar quarterly data*

Lawrence Summers (1982) presents results to support the view that the intertemporal elasticity of consumption is substantial. In a subsequent paper (Summers (1984)), he has cited his findings in making a case for the interest-elasticity of saving: "...available evidence tends to suggest that savings are likely to be interest elastic. I find in the more reliable estimates in my working paper [Summers (1982)] values of the intertemporal elasticity of substitution which cluster at the high end of the range Evans and I considered [above one]. Similar estimates are found...by Hansen-Singleton. Where investigators find low estimates of intertemporal elasticty of substitution, it is usually because of the difficulty in modelling *ex ante* rates of return on corporate stock."

I have not tried to duplicate Summers' findings exactly. With postwar quarterly data on consumption and real after-tax yields on Treasury bills, I have obtained the following estimate of σ using the same inappropriate instruments as Summers, namely the real yield, the inflation rate, and the rate of change of consumption dated t-1 and t-2:

Estimate of σ 0.234

(.120)

However, deletion of the instruments known to be correlated with the disturbance reverses the finding of an unambiguously positive σ:

Estimate of σ -.151

(0.170)

10. *Conclusions*

My investigation has shown little basis for a conclusion that the behavior of aggregate consumption in the United States in the twentieth century reveals an important positive value of the intertemporal elasticity of substitution. All investigators have agreed that the covariation of stock market returns and consumption did not suggest that consumption rises more rapidly in times of high expected real returns in the stock market. Earlier evidence basied on interest-bearing securities such as Treasury bills had suggested values of σ as high as one. However, use of appropriate instruments reverses this finding. Moreover, extension of the investigation to pre-war years strengthens the evidence that periods of high expected real interest rates have not been periods of rapid growth of consumption.

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