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Central bank's announcements about future monetary policy make economic agents to react before the announced policy takes place. We evaluate the anticipation effects of such announcements in the context of a realistic dynamic economic model of central banking. In our experiments, we consider temporary and permanent anticipated changes in policy rules including changes in inflation target, natural rate of interest and Taylor-rule coefficients, as well as anticipated switches from inflation targeting to price-level targeting and average inflation targeting. We show that the studied nonrecurrent news shocks about future policies have sizable anticipation effects on the economy. Our methodological contribution is to develop a novel perturbation-based framework for constructing nonstationary solutions to economic models with nonrecurrent news shocks.

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The Power of Open-Mouth Policies*

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Abstract

Central banks' announcements about future monetary policy make economic agents to react before the announced policy takes place. We evaluate the anticipation effects of such announcements in the context of a realistic dynamic economic model of central banking. In our experiments, we consider temporary and permanent anticipated changes in policy rules including changes in inflation target, natural rate of interest and Taylor-rule coefficients, as well as anticipated switches from inflation targeting to price-level targeting and average inflation targeting. We show that the studied nonrecurrent news shocks about future policies have sizable anticipation effects on the economy. Our methodological contribution is to develop a novel perturbation-based framework for constructing nonstationary solutions to economic models with nonrecurrent news shocks.

JEL classification: C61, C63, C68, E31, E52

Key Words: news shocks; turnpike theorem; time-dependent models; nonstationary models; unbalanced growth; time-varying parameters; regime switches; monetary policies; price-level targeting; average inflation targeting

1 Introduction

Central banks increasingly rely on communication to implement their monetary policy. Through their communication to the public, the monetary authorities indicate their future intentions as well as their views of the future states of the economy. For example, a central bank may promise to fix its interest rate for a certain number of periods before normalizing its policy (forward guidance) or it may announce a future change in the inflation target. Understanding the effects of communication is essential from the policy evaluation perspective.

In this paper, we assess the effects of central bank communication by analyzing several monetary policies that are conjectured in the literature to be welfare improving. The studied policies include one-time and gradual anticipated changes in economic policies, as well as more complex scenarios in which various monetary policies happen with some probabilities. A distinctive feature of our model with communication is that the agents react to the news of a policy change even when the new policy will not take effect until a later date. We find that such anticipation effects can be very large.

Our analysis is carried out in a realistically calibrated prototypical central banking model, a smaller replica of the Terms of Trade Model (ToTEM) used by the Bank of Canada for projection and policy analysis. Importantly, our "baby" ToTEM follows the full size ToTEM model as close as possible and generates very similar impulse response functions; see Dorich et al. (2013), and Lepetyuk, Maliar and Maliar (2020, henceforth, LMM) for a description of the ToTEM and "baby" ToTEM models, respectively.

The policy experiments we consider include: (1) a gradual decline in the natural rate of interest; (2) a gradual change in the inflation target that happens in the future either with certainty or with some probability; (3) normalization of monetary policy regarding future nominal interest rates, when the economy is initially at a zero lower bound (ZLB) on nominal interest rates; (4) a switch to a more aggressive Taylor rule; (5) a switch to price-level targeting instead of inflation targeting; (6) a switch to average inflation targeting instead of inflation targeting.

Our findings are as follows: (1) Being fully anticipated by agents, a gradual decline in the natural rate of interest of one percent over consecutive five years leads to a substantial (more than 1 percent) expansion of output over the whole period. (2) Postponing an increase in the inflation target by one year has a large (nearly 50%) increase in output over the transition to a new steady state. Even if the announced policy is implemented with some probability, there are still substantial anticipation effects both before and after uncertainty is resolved. (3) When an economy is at ZLB on nominal interest rates, the central bank uses policy announcements (forward guidance) about its future return to the standard interest rate rule to direct the economy's transition out of ZLB. The more it postpones such a return to the standard rule, the larger is output expansion over the transition; an initial jump in output is however invariant to the horizon of this forward guidance policy. Therefore, this experiment informs policy makers on optimal horizons of monetary policy normalizations after ZLB periods. (4) Nevertheless, a more aggressive (but realistic) behavior of the central bank toward targeting inflation and output is not translated into important anticipatory effects on the side of economic agents. (5) Switching from inflation-level targeting to price-level targeting has smaller impacts with larger implementation lags. Price-level targeting was argued in the literature to be welfare improving. Therefore, a central bank that waits to implement the new policy in practice loses time, and the economy does not get earlier benefits from higher output. (6) Finally, a switch to average inflation targeting also has modest anticipation effects; this is because average inflation targeting is in a middle ground between inflation targeting and price-level targeting. In sum, our analysis shows that the model's implications about the importance of anticipation effects depend on a specific experiment considered: there are substantial policy anticipation effects present in our experiments (1)–(3), but such effects are relatively small in experiments (4)–(6).

On the methodological side, we argue that announcements about future economic conditions and policies can be modeled as non-recurrent news shocks. Such news shocks represent a challenge to economic dynamics because they result in time-dependent optimal decision rules that change from one period to another. Conventional perturbation methods cannot be used for analyzing the anticipatory effects since they

are designed for constructing time-invariant (stationary) solutions. We modify the conventional perturbation framework to facilitate the construction of a sequence of time-dependent decision functions following the turnpike analysis; see Maliar, Maliar, Taylor and Tsener (2020, henceforth, MMTT) for a discussion and review of related literature. Our perturbation method is comparable in accuracy to a global projection method developed in MMTT (2020), but it is tractable in problems with much higher dimensionality, such as large-scale central-banking models. Our ubiquitous software is written using the popular Dynare platform combined with user-friendly MATLAB interface and can be easily adapted to other applications the reader might be interested in.

There is a literature that studies economic models with recurrent news shocks; in particular, there is a perturbation-based method by Schmitt-Grohé and Uribe (2012) that can analyze the anticipatory effects by augmenting the state space to include future shocks. In their analysis, news shocks follow a stationary Markov process and, hence, the resulting solution is stationary (time-invariant). In contrast, our future shocks are nonrecurrent, which leads to time-dependent decision rules. We view the two approaches as complementary: for example, our method can be used to study a pre-determined one-time accession or exit of a new member to the EU, while Schmitt-Grohé and Uribe's (2012) method can be used to study countries that access and exit the EU with some stationary probability distribution. Quantitatively, we find that the two methods lead to very different solutions in some of the examples considered.

The rest of the paper is organized as follows: Section 2 describes the large-scale central banking model and presents our perturbation methodology for analyzing anticipatory effects. Section 3 analyzes our six policy experiments and presents comparison results with news shocks framework, and, finally, Section 4 concludes.

2   Methodology

In this section, we present a central banking model and outline the methodology of our numerical analysis.

2.1   The model

Nowadays, the central banks, leading international organizations and government agencies, use large-scale macroeconomic models for projection and policy analysis. A prominent example is the Terms of Trade Economic Model (ToTEM) of the Bank of Canada. That model includes several types of utility-maximizing consumers, several profit-maximizing production sectors, monetary and fiscal authorities, as well as a foreign sector. ToTEM is huge: it contains 356 equations and unknowns, including 215 state variables; see Dorich et al. (2013).

In the paper, we consider a scaled-down version of ToTEM developed by LMM (2020). Like the full-scale model, the "baby" ToTEM (in short, bToTEM) is a small open-economy model. It features multiple new-Keynesian Phillips curves – one due to sticky prices in domestic production, one due to sticky wages and one due to sticky import prices. We incorporate the rule-of-thumb price settlers in line with Galí and Gertler (1999). We assume quadratic adjustment costs of investment and convex costs of capital utilization to generate more realistic model's performance, in particular, with respect to monetary-policy transmission. The international trade consists of exporting domestic consumption goods and commodities and importing foreign goods for domestic production. Even though bToTEM is much smaller than ToTEM (it has only 47 equations and unknowns, including 21 state variables) it generates realistic impulse-responses of the Canadian economy to shocks, which are very similar to those produced by the full scale ToTEM model; see LMM (2020) for comparison results.

Final-good production.   Final consumption goods are produced in two stages. In the first stage, intermediate goods are produced competitively using labor, capital, commodities and imports. In the second stage, final goods are aggregated from differentiated goods that are each produced by a monopolistically competitive firm from the intermediate goods and from the final goods. The final goods can be consumed

by households. They can also be transformed using linear technologies into other types of goods, namely, investment goods and noncommodity exports goods.

In the first production stage, a representative perfectly competitive firm produces an intermediate good by solving the following profit maximization problem:

$$\max_{\left\{Z_t^g, Z_t^n, L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t\right\}} E_0 \sum_{t=0}^{\infty} \mathcal{R}_{0,t} \left(P_t^z Z_t^n - W_t L_t - P_t^i I_t - P_t^{com} COM_t^d - P_t^m M_t \right)$$

$$\text{s.t. } Z_t^g = \left[\delta_l \left(A_t L_t \right)^{\frac{\sigma-1}{\sigma}} + \delta_k \left(u_t K_{t-1} \right)^{\frac{\sigma-1}{\sigma}} + \delta_{com} \left(COM_t^d \right)^{\frac{\sigma-1}{\sigma}} + \delta_m \left(M_t \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

$$\log A_t = \varphi_a \log A_{t-1} + (1 - \varphi_a) \log \bar{A} + \xi_t^a, \tag{2}$$

$$K_t = (1 - d_t) K_{t-1} + I_t, \tag{3}$$

$$d_t = d_0 + \bar{d} e^{\rho(u_t - 1)}, \tag{4}$$

$$Z_t^n = Z_t^g - \frac{\chi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t, \tag{5}$$

where Z_t^g and Z_t^n are gross production and net (of adjustment costs) production of final goods; L_t, K_t, I_t COM_t^d, M_t, u_t and d_t are labor, capital, investment, commodity inputs, imports, capital utilization and depreciation rate, respectively; A_t is the level of labor-augmenting technology; ξ_t^a is a normally distributed variable, and φ_a is an autocorrelation coefficient. The firm discounts nominal payoffs according to household's stochastic discount factor $\mathcal{R}_{t,t+j} = \beta^j \left(\lambda_{t+j}/\lambda_t \right) \left(P_t/P_{t+j} \right)$, where λ_t is the household's marginal utility of consumption, and P_t is the final good price. Investment goods and noncommodity exports are assumed to be produced from the final goods according to linear technology, $P_t^i = \iota_i P_t$ and $P_t^{nc} = \iota_x P_t$, where P_t^i and P_t^{nc} are the price of investment goods and noncommodity exports goods, respectively.

In the second stage of production, monopolistically competitive firms produce a continuum of differentiated good. Then, these differentiated goods are aggregated into the final good by an aggregating firm that solves the following cost minimization problem

$$\min_{\{Z_{it}\}} \int_0^1 P_{it} Z_{it} di$$

$$\text{s.t. } Z_t = \left(\int_0^1 Z_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where Z_t and P_{it} are given; Z_{it} is a differentiated good i, The cost minimization implies the following demand function for the differentiated good i:

$$Z_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Z_t, \quad \text{with } P_t \equiv \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

Each differentiated good is produced from the intermediate goods and from the final goods using technology featuring perfect complementarity,

$$Z_{it} = \min \left(\frac{Z_{it}^n}{1 - s_m}, \frac{Z_{it}^{mi}}{s_m} \right), \tag{6}$$

where Z_{it}^n is an intermediate good and Z_{it}^{mi} is a final good input, and s_m is a Leontief parameter.

There are two types of the monopolistically competitive firms producing differentiated goods: rule-of-thumb firms of measure ω and forward-looking firms of measure $1 - \omega$. Both rule-of-thumb firms and forward-looking firms index their price to the inflation target $\bar{\pi}_t$ with probability θ as $P_{it} = \bar{\pi}_t P_{i,t-1}$. The

rule-of-thumb firms partially index their price to lagged inflation and target inflation with probability $1-\theta$,

$$P_{it} = (\pi_{t-1})^\gamma (\bar{\pi}_t)^{1-\gamma} P_{i,t-1}. \tag{7}$$

Forward-looking firms choose their prices P_t^* with probability $1 - \theta$ to maximize profits generated when the price remains effective

$$\max_{P_t^*} E_t \sum_{j=0}^\infty \theta^j \mathcal{R}_{t,t+j} \left(\prod_{k=1}^j \bar{\pi}_{t+k} P_t^* Z_{i,t+j} - (1 - s_m) P_{t+j}^z Z_{i,t+j} - s_m P_{t+j} Z_{i,t+j} \right) \tag{8}$$

$$\text{s.t. } Z_{i,t+j} = \left(\frac{\prod_{k=1}^j \bar{\pi}_{t+k} P_t^*}{P_{t+j}} \right)^{-\varepsilon} Z_{t+j}.$$

The production in the first stage Z_t^n and that in the second stages Z_t are related via price dispersion Δ_t,

$$Z_t^n = \int_0^1 Z_{it}^n di = (1 - s_m) \int_0^1 Z_{it} di = (1 - s_m) \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Z_t di = (1 - s_m) \Delta_t Z_t, \tag{9}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$.

Production of commodities. Commodities are produced by a domestic firm using final goods and land as inputs. They are sold domestically or exported to the rest of the world. The domestic firm solves

$$\max_{Z_t^{com}, COM_t} \{ P_t^{com} COM_t - P_t Z_t^{com} \}$$

$$\text{s.t. } COM_t = (Z_t^{com})^{s_z} (A_t F)^{1-s_z} - \frac{\chi_{com}}{2} \left(\frac{Z_t^{com}}{Z_{t-1}^{com}} - 1 \right)^2 Z_t^{com}, \tag{10}$$

where Z_t^{com} is the final good input, and F is a fixed production factor, which may be considered as land. Similar to production of final goods, the commodity producers incur quadratic adjustment costs when they adjust the level of final good input. The commodities are sold domestically or exported to the rest of the world, $COM_t = COM_t^d + X_t^{com}$. They are sold at the world price adjusted by the nominal exchange rate, $P_t^{com} = e_t P_t^{comf}$, where e_t is the nominal exchange rate (i.e., domestic price of a unit of foreign currency), and P_t^{comf} is the world commodity price; in real terms, the latter price is given by $p_t^{com} = s_t p_t^{comf}$, where $p_t^{com} \equiv P_t^{com}/P_t$ and $p_t^{comf} \equiv P_t^{comf}/P_t^f$ are domestic and foreign relative prices of commodities, respectively, P_t^f is the foreign consumption price level, and $s_t = e_t P_t^f/P_t$ is the real exchange rate.

Production of imports. The representative perfectly competitive firm produces the final imported good M_t from a continuum of intermediate imported goods M_{it} and solves the following cost-minimization problem,

$$\min_{\{M_{it}\}} \int_0^1 P_{it}^m M_{it} di$$

$$\text{s.t. } M_t = \left(\int_0^1 M_{it}^{\frac{\varepsilon_m-1}{\varepsilon_m}} di \right)^{\frac{\varepsilon_m}{\varepsilon_m-1}},$$

where M_{it} is an intermediate imported good i. The demand for an intermediate imported good i is given by

$$M_{it} = \left(\frac{P_{it}^m}{P_t^m} \right)^{-\varepsilon_m} M_t, \quad \text{with } P_t^m \equiv \left(\int_0^1 (P_{it}^m)^{1-\varepsilon_m} di \right)^{\frac{1}{1-\varepsilon_m}}.$$

Prices of the intermediate imported goods are sticky in a similar way as the prices of the differentiated final goods. A measure ω_m of the importers follows the rule-of-thumb pricing, and the others are forward looking. The optimizing forward-looking importers choose the price P_t^{m*} in order to maximize profits generated when the price remains effective

$$\max_{P_t^{m*}} E_t \sum_{j=0}^{\infty} (\theta_m)^j \, \mathcal{R}_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*} M_{i,t+j} - e_{t+j} P_{t+j}^{mf} M_{i,t+j} \right)$$

$$M_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*}}{P_{t+j}^m} \right)^{-\varepsilon_m} M_{t+j},$$

where P_t^{mf} is the price of imports in the foreign currency. All importers face the same marginal cost given by the foreign price of imports.

Households. Households maximize the lifetime utility by choosing holdings of domestic and foreign-currency denominated bonds, labor and consumption, and they are subject to habits in consumption. Each household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive. The representative household of type h solves the following utility-maximization problem:

$$\max_{C_t, L_{ht}, B_t, B_t^f} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\mu}{\mu-1} \left(C_t - \xi \bar{C}_{t-1} \right)^{\frac{\mu-1}{\mu}} \exp \left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 (L_{ht})^{\frac{\eta+1}{\eta}} \, dh \right) \eta_t^c \right\},$$

$$\text{s.t. } P_t C_t + \frac{B_t}{R_t} + \frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f \right)} = B_{t-1} + e_t B_{t-1}^f + \int_0^1 W_{ht} L_{ht} dh + \Pi_t, \tag{11}$$

$$\log \eta_t^c = \varphi_c \log \eta_{t-1}^c + \xi_t^c, \tag{12}$$

where C_t, L_{ht}, B_t, B_t^f are consumption of final goods, labor service of type h, holdings of domestic and foreign-currency denominated bonds, respectively; $\bar{C}_t$ is the aggregate consumption, taken by the household as given; $\beta \in (0,1)$ is a subjective discount factor; μ and η are the utility-function parameters; η_t^c is a consumption demand shock, ξ_t^c is a normally distributed variable, and φ_c is an autocorrelation coefficient; R_t and R_t^f are domestic and foreign nominal interest rate, respectively; κ_t^f is the risk premium on the foreign interest rate; W_{ht} is the nominal wage of labor of type h; Π_t is profits paid by the firms. The representative household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive.

Labor packer. A labor packer aggregates differentiated labor services by solving

$$\min_{\{L_{ht}\}} \int_0^1 W_{ht} L_{ht} dh$$

$$\text{s.t. } L_t = \left(\int_0^1 L_{ht}^{\frac{\varepsilon_w-1}{\varepsilon_w}} \, dh \right)^{\frac{\varepsilon_w}{\varepsilon_w-1}},$$

where L_t is aggregated labor demanded by firms in the first stage of production. Cost minimization of the labor packer implies the following demand for individual labor:

$$L_{ht} = \left(\frac{W_{ht}}{W_t} \right)^{-\varepsilon_w} L_t, \quad \text{with } W_t \equiv \left(\int_0^1 W_{ht}^{1-\varepsilon_w} dh \right)^{\frac{1}{1-\varepsilon_w}}. \tag{13}$$

Labor unions.   Labor unions set wages. There are two types of labor unions: rule-of-thumb unions of measure ω_w and forward-looking unions of measure $1 - \omega_w$. Within each type, with probability θ_w the labor unions index their wage to the inflation target $\bar{\pi}_t$ as follows $W_{it} = \bar{\pi} W_{i,t-1}$. The rule-of-thumb unions that do not index their wage in the current period follow the rule

$$W_{it} = \left(\pi_{t-1}^w \right)^{\gamma_w} \left(\bar{\pi}_t \right)^{1-\gamma_w} W_{i,t-1}. \tag{14}$$

A forward-looking unions that do not index its wage solves

$$\max_{W_t^*} E_t \sum_{j=0}^{\infty} (\beta \theta_w)^j \left\{ \frac{\mu}{\mu - 1} \left(C_{t+j} - \xi \bar{C}_{t+j-1} \right)^{\frac{\mu-1}{\mu}} \exp \left(\frac{\eta (1 - \mu)}{\mu (1 + \eta)} \int_0^1 \left(L_{ht+j} \right)^{\frac{\eta+1}{\eta}} dh \right) \eta_{t+j}^c \right\} \tag{15}$$

$$\text{s.t. } L_{h,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^*}{W_{t+j}} \right)^{-\varepsilon_w} L_{t+j}, \tag{16}$$

$$P_{t+j} C_{t+j} = \prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^* L_{h,t+j} dh + \Psi_{t+j}, \tag{17}$$

where Ψ_{t+j} includes terms in budget constraints (11) other than C_{t+j} and $L_{h,t+j}$.

Monetary authority.   The central bank uses a Taylor rule to set the short-term nominal interest rate,

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) \left[\bar{R} + \rho_\pi (\pi_t - \bar{\pi}_t) + \rho_Y \left(\log Y_t - \log \bar{Y}_t \right) \right] + \eta_t^r, \tag{18}$$

where ρ_r measures the degree of smoothing of the interest rate; $\bar{R}$ is the nominal neutral interest rate; ρ_π measures a response to the inflation gap; $\bar{\pi}_t$ is the inflation target; ρ_Y measures a response to the output gap; $\bar{Y}_t$ is potential output; η_t^r is an interest rate shock following a process

$$\eta_t^r = \varphi_r \eta_{t-1}^r + \xi_t^r,$$

where ξ_t^r is a normally distributed variable, and φ_r is an autocorrelation coefficient. Potential output changes with productivity according to

$$\log \bar{Y}_t = \varphi_z \log \bar{Y}_{t-1} + (1 - \varphi_z) \log \left(\frac{A_t \bar{Y}}{\bar{A}} \right).$$

Foreign demand for noncommodity exports.   By analogy with the demand for imports, the foreign demand function for noncommodity exports is assumed to be

$$X_t^{nc} = \gamma^f \left(\frac{P_t^{nc}}{e_t P_t^f} \right)^{-\phi} Z_t^f, \tag{19}$$

where P_t^{nc} is a domestic price of noncommodity exports; γ^f is the demand-function parameter. In real terms, we have

$$X_t^{nc} = \gamma^f \left(\frac{s_t}{p_t^{nc}} \right)^{\phi} Z_t^f. \tag{20}$$

Balance of payments.   The balance of payments in nominal terms is given by

$$\frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f \right)} - e_t B_{t-1}^f = P_t^{nc} X_t^{nc} + P_t^{com} X_t^{com} - P_t^m M_t, \tag{21}$$

where B_t^f is domestic holdings of foreign-currency denominated bonds, and R_t^f is the nominal interest rate on the bonds. By normalizing the bonds holdings as $b_t^f \equiv \frac{e_t B_t^f}{\pi_{t+1}^f P_t \bar{Y}}$, the balance of payments in real terms becomes

$$\frac{b_t^f}{r_t^f \left(1 + \kappa_t^f\right)} - b_{t-1}^f \frac{s_t}{s_{t-1}} = \frac{1}{\bar{Y}} \left(p_t^{nc} X_t^{nc} + p_t^{com} X_t^{com} - p_t^m M_t\right), \tag{22}$$

where r_t^f is the real interest rate on the foreign-currency denominated bonds.

Rest-of-the-world economy. The rest of the world is specified by three exogenous processes that, respectively, describe the evolution of foreign output Z_t^f, the foreign real interest rate r_t^f, and the foreign commodity price p_t^{comf},

$$\log Z_t^f = \varphi_{zf} \log Z_{t-1}^f + \left(1 - \varphi_{zf}\right) \log \bar{Z}^f + \xi_t^{zf}, \tag{23}$$

$$\log r_t^f = \varphi_{rf} \log r_{t-1}^f + \left(1 - \varphi_{rf}\right) \log \bar{r} + \xi_t^{rf}, \tag{24}$$

$$\log p_t^{comf} = \varphi_{comf} \log p_{t-1}^{comf} + \left(1 - \varphi_{comf}\right) \log \bar{p}^{comf} + \xi_t^{comf}, \tag{25}$$

where ξ_t^{zf}, ξ_t^{rf} and ξ_t^{comf} are normally distributed random variables, and φ_{Zf}, φ_{rf} and φ_{comf} are autocorrelation coefficients.

Uncovered interest rate parity. We impose an augmented uncovered interest rate parity condition

$$e_t = E_t \left[(e_{t-1})^\varkappa \left(e_{t+1} \frac{R_t^f \left(1 + \kappa_t^f\right)}{R_t} \right)^{1-\varkappa} \right]. \tag{26}$$

Stationarity condition for the open-economy model. The risk premium κ_t^f is a decreasing function of foreign assets

$$\kappa_t^f = \varsigma \left(\bar{b}^f - b_t^f\right), \tag{27}$$

where $\bar{b}^f$ is the steady state level of the normalized bond holdings. This assumption ensures a decreasing rate of return to foreign assets.

Summary of the model's variables. For each period t, there are the following four types of variables in this model: 47 endogenous (or non-predetermined) variables,

$$y_t \equiv \left\{ \begin{array}{c} F_{1t}, F_{2t}, F_{1t}^w, F_{2t}^w, F_{1t}^m, F_{2t}^m, q_t, \lambda_t, s_t, \\ L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t, Z_t^g, Z_t^n, Z_t, C_t, Y_t, \pi_t, rmc_t, \Delta_t, \pi_t^m, \bar{\pi}_t, p_t^m, R_t, p_t^z, w_t, \\ MPK_t, R_t^k, p_t^i, \kappa_t^f, b_t^f, X_t^{nc}, X_t^{com}, COM_t, Z_t^{com}, \pi_t^w, w_t^*, \Delta_t^w, \bar{Y}_t, p_t^{com}, p_t^{nc}, p_t^{mf}, p_t^y, \end{array} \right\},$$

where $\{F_{1t}, F_{2t}\}$, $\{F_{1t}^w, F_{2t}^w\}$, $\{F_{1t}^m, F_{2t}^m\}$ are supplementary variables in Phillips curves for prices, wages and imports, respectively; q_t is Tobin's q; rmc_t and MPK_t are real marginal cost and marginal productivity of capital, respectively; p_t^{nc} and p_t^y are prices of noncommodity goods and output, respectively; 15 endogenous state variables

$$y_{t-1}^- \equiv \left\{ C_{t-1}, R_{t-1}, s_{t-1}, \pi_{t-1}, \Delta_{t-1}, w_{t-1}, \pi_{t-1}^w, \Delta_{t-1}^w, p_{t-1}^m, \pi_{t-1}^m, I_{t-1}, Z_{t-1}^{com}, b_{t-1}^f, \bar{Y}_{t-1}, K_{t-1} \right\},$$

where π_{t-1}^w, Δ_{t-1}^w, p_{t-1}^m, π_{t-1}^m are wage inflation, wage dispersion, and price and inflation of imports; 19 endogenous forward variables

$$y_{t+1}^+ \equiv \left\{ \begin{array}{c} F_{1t+1}, F_{2t+1}, F_{1t+1}^w, F_{2t+1}^w, F_{1t+1}^m, F_{2t+1}^m, \lambda_{t+1}, q_{t+1}, u_{t+1}, I_{t+1}, \\ Z_{t+1}^{com}, \pi_{t+1}, \bar{\pi}_{t+1}, \pi_{t+1}^m, \pi_{t+1}^w, p_{t+1}^{com}, p_{t+1}^z, MPK_{t+1}, s_{t+1} \end{array} \right\};$$

and 6 exogenous state variables

$$z_t \equiv \left\{ A_t, \eta_t^R, \eta_t^c, p_t^{comf}, r_t^f, Z_t^f \right\}.$$

In Appendix A, we describe our calibration procedure, which closely follows the calibration of the full scale ToTEM model.

2.2 Methodology of our numerical analysis

LMM (2020) analyze conventional time-invariant (stationary) solutions to our model.[1] Here, we construct novel time-dependent (nonstationary) solutions to the model. We consider announcements about economic policies that will be implemented at some future dates (non-recurrent news shock) and analyze a reaction of economic agents to such announcements. Assuming that we are at $t = 0$ and that a given policy will be implemented at $T > 0$, we obtain a sequence of optimal decision functions for periods $t = 0, 1, ..., T$ that characterize anticipatory effects (obviously, the optimal decision functions will depend on how far the economy is from the moment the policy is introduced). Below, we outline our perturbation-based framework for analyzing economies with non-recurrent news shocks.

Let us consider an infinite-horizon nonstationary equilibrium problem in which a solution is characterized by a set of equilibrium conditions for $t = 0, 1, \ldots$,

$$E_t \left[G_t \left(y_{t-1}^-, y_t, y_{t+1}^+, z_t, z_{t+1} \right) \right] = 0, \tag{28}$$

$$z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right), \tag{29}$$

where $\left(z_0, y_{-1}^- \right)$ is given; E_t denotes the expectations operator conditional on information available at t; $z_t \in \mathbb{R}^{d_z}$ is a vector of exogenous state variables at t; Z_t is a time-dependent law of motion for z_t; $y_t \in \mathbb{R}^{d_y}$ is a vector of endogenous variables; $y_{t-1}^- \in \mathbb{R}^{d_{ym}}$ is a vector of endogenous (random) state variables at t; $y_{t+1}^+ \in \mathbb{R}^{d_{yp}}$ is a vector of endogenous forward variables at t; $\epsilon_{t+1} \in \mathbb{R}^{d_\epsilon}$ is a vector of shocks; G_t is a continuously differentiable vector function. Note that the latter function is time-dependent because the model is nonstationary (due to, for example, time-dependent parameters in policy rules, production function, utility function). A solution is given by a set of time-dependent equilibrium functions $y_t = Y_t \left(z_t, y_{t-1}^- \right)$ that satisfy (28), (29) in the relevant area of the state space.

Our perturbation analysis proceeds in the following two steps:

Step I: solving a T-period stationary economy. Assume that in a very remote period T, the economy becomes stationary, i.e., $G_t (\cdot) = G (\cdot)$ and $Z_t (\cdot) = Z (\cdot)$ for all $t \geq T$. Therefore, the system (28), (29) becomes

$$E_t \left[G \left(y_{t-1}^-, y_t, y_{t+1}^+, z_t, z_{t+1} \right) \right] = 0, \tag{30}$$

$$z_{t+1} = Z \left(z_t, \epsilon_{t+1} \right). \tag{31}$$

Solving (30), (31) allows us to find the solution $y_T = \widehat{Y}_T \left(z_T, y_{T-1}^- \right)$.

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[1] LLM (2020) compare perturbation solutions with more accurate global projection solutions constructed using deep learning analysis. That paper finds that high order perturbation solutions are sufficiently accurate in the bToTEM model. Since our nonstationary analysis is more costly, we limit attention to perturbation solutions only.

Step II: constructing a function path. Using a T-period solution $y_T = \widehat{Y}_T \left(z_T, y_{T-1}^- \right)$ as a terminal condition, iterate backward for $T-1, ..., 1$ on the corresponding equilibrium conditions to construct a sequence (path) of time-dependent value and decision functions $\{Y_{T-1}(\cdot), ..., Y_1(\cdot)\}$. For example, for period t, the system on which we iterate backward is

$$E_t \left[G_t \left(y_{t-1}^-, y_t, Y_{t+1}^+ \left(z_{t+1}, y_t^- \right), z_t, z_{t+1} \right) \right] = 0,$$
$$z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right),$$

Here, we solve for today's endogenous variables y_t, given tomorrow's functions $z_{t+1} = Z_t \left(z_t, \epsilon_{t+1} \right)$ and $y_{t+1}^+ = Y_{t+1}^+ \left(z_{t+1}, y_t^- \right)$, where $Y_{t+1}^+ \left(z_{t+1}, y_t^- \right)$ is a subset of functions $Y_{t+1} \left(z_{t+1}, y_t^- \right)$.

In both steps, we use perturbation to find numerical approximations of the decision functions. For our central banking model, the variables z_t, y_t^-, y_t, y_t^+ are defined in Section 2.1.

In general, nonstationary models like ours have no deterministic steady state, so it is unclear around what point(s) decision rules must be approximated. To deal with this issue, we can augment the model's equations with a time-varying growth rates γ_{xt} that capture how much endogenous state variables grow from period t to $t+1$ due to the time trend or the parameter change. We can first assume that growth rates $\{\gamma_{xt}\}_{t=1}^T$ are given and then find those growth rates iteratively. (Note that growth rates should not be the same for all variables).

One objective of this paper is to make the proposed perturbation framework ubiquitous and portable to other applications. To this purpose, we show how to construct time-dependent decision functions using Dynare. To explain how to solve a nonstationary model in Dynare, let's first consider a standard second-order perturbation solution to a stationary model around a deterministic steady state $(\bar{v}; 0)$,

$$g(v; \sigma) \approx \underbrace{g(\bar{v}; 0) + g_x(\bar{v}; 0)(v - \bar{v})}_{\text{1st-order perturbation solution}} + \underbrace{\frac{1}{2} g_{xx}(\bar{v}; 0)(v - \bar{v})^2 + \frac{1}{2} g_{\sigma\sigma}(\bar{v}; 0) \sigma^2}_{\text{2nd-order terms}}, \tag{32}$$

where $g(v; \sigma)$ is a decision function to be approximated; $v = (x, z)$ is a vector of endogenous and exogenous state variables; σ is a perturbation parameter that scales volatility of shocks; $(\bar{v}; 0)$ is a deterministic steady state; $g(\bar{v}; 0)$, $g_x(\bar{v}; 0)$ and $g_{xx}(\bar{v}; 0)$ are, respectively, steady state values, Jacobian and Hessian matrices of g; $(v - \bar{v})$ is a deviation from a steady state; $(v - \bar{v})^2 \equiv (v - \bar{v}) \otimes (v - \bar{v})$ is a tensor product of the deviations. Three observations are in order: First, a constant term of the policy function is given by $g(\bar{v}; 0) + \frac{1}{2} g_{\sigma\sigma}(\bar{v}; 0) \sigma^2$ and hence, is affected by variances of shocks. Second, the first-order perturbation solution does not depend on the degree of volatility σ, i.e., $g_\sigma(\bar{v}; 0) = 0$. Finally, the term $g_{\sigma x}(\bar{v}; 0)$ is omitted as well because it is equal to zero; see Schmitt-Grohé and Uribe (2004).

In Step I, a Taylor expansion of the policy functions in a stationary model is found around the deterministic steady state $\bar{v}$ of the model. In Step II, we consider two alternative options. The first option is to find solutions for v_{t+1} and v_t around v_t and v_{t-1}, respectively, such that $v_t = v_{t-1} \equiv \bar{v}_t$; in Dynare, it can be implemented by coding v_t and v_{t+1} using the same variable names. The other option is to consider v_{t+1} and v_t perturbed around $\bar{v}_t$ and $\bar{v}_{t-1}$, respectively, such that $\bar{v}_t = \bar{v}_{t-1}\gamma_{v,t-1}$, where $\gamma_{v,t-1}$ is a time-dependent growth rate; in Dynare, it can be implemented by coding v_t and v_{t+1} with different variable names.

In Appendix A, we illustrate our methodology of constructing time-dependent perturbation solutions by using a toy example of a neoclassical stochastic growth model with labor augmenting technological progress. A useful property of that model is balanced growth which allows us to construct an accurate reference solution. We use such a solution for accessing accuracy of our perturbation solutions which we obtain without relying on the property of balanced growth. In particular, we show how to iteratively construct time-dependent growth rates. In the main text, we apply the proposed methodology for the analysis of our large-scale open-economy model.

3 Analyzing nonstationary news shocks

In this section, we show a series of policy experiments in which we consider anticipated changes in one or several model's parameters. In all the figures, the variables are shown in percentage deviations from the initial risky steady state, except for the interest rate and the inflation rate, which are both shown in percentage point deviations from the risky steady state and expressed in annualized terms.[2]

3.1 A decline in the real neutral interest rate

Some future changes in economic environment could be envisioned by the public. A central bank's announcements could play a significant role by revealing information not just about its policy, but also about a central bank's assessment of the economic outlook. In particular, Nakamura and Steinsson (2018) argue that Fed announcements contain information about the path of the natural rate. In this experiment, we model an anticipated gradual decline in the natural rate of interest. Namely, we assume that initial value of the real neutral interest rate is 3 percent and that it starts to go down to 2 percent gradually over 20 quarters. To model a decrease in the neutral rate, we exploit the fact that in steady state, this rate is equal to the inverse of the discount factor, and we translate the assumed decrease in the neutral interest rate into an increase in the discount factor.

Empirical evidence indicates that long-term rates declined from the early 1960s through the mid-1970s, increased until the late 1980s, and declined again from that point on; see, e.g., Yi and Zhang (2019). Moreover, such a decline is not related to the Great Recession. The factors that are responsible for declining long-run rates include lower TFP growth, lower working-age population growth, long-run trends in marginal productivity of capital and risk premium. With this experiment, we investigate how a gradual reduction in the real interest rate captured by an increase in the discount factor affects the economy.

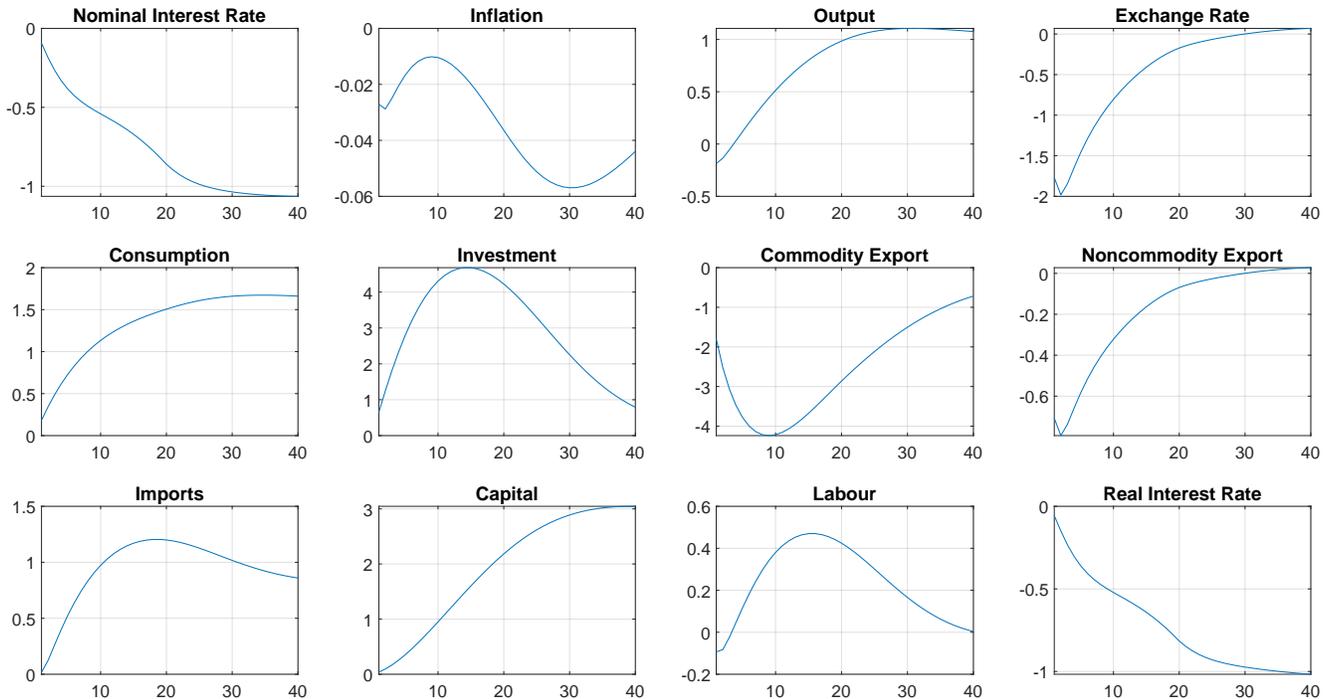Figure 1: A gradual decline in the real neutral interest rate

As it is seen from Figure 1, a long-run gradual decrease in the real interest rate results in the corresponding gradual increase in consumption, investment, labor, capital, and imports. For example, investment

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[2] By risky steady state, we mean a state to which a stochastic economy converges in the absence of exogenous shocks.

increases by 5 percent at the peak. Because of significantly higher investment and labor, output grows by more than 1 percent. The changes in inflation are so small that the nominal and real interest rates behave almost identically. The anticipation effects are the largest in the commodity export and exchange rate which fall by 2 percent and 1.8 percent, respectively, when it became known that the real neutral interest rate will gradually decrease. The results of this experiment indicate that recent trends in the real rate of interest are beneficial for growth. Note that we arrive at this result by assuming that the decline in the natural rate is caused by people becoming more impatient. However, if the decline is caused by other factors, our conclusion might not go through.

3.2   A change in the inflation target

In this experiment, we consider a change in the inflation target that appears in the Taylor rule (18). In particular, we assume that the central bank announces in advance that it will increase the inflation target $\bar{\pi}_t$ and that everyone considers the announcement to be fully credible. Why is it a relevant policy experiment? During the Great Recession of 2007–2009, central bank's nominal policy rates across a number of countries fell to a ZLB on nominal interest rates. There is ample literature arguing that the inflation target is a good policy instrument for dealing with ZLB episodes. For example, Summers (1991) and Fischer (1996) suggest to keep an inflation target as high as 2 or 3 percent if the economy hits ZLB. Krugman (1998) proposes to use a 4 percent inflation target in the Japanese economy to deal with persisting deflation. Furthermore, Blanchard, Dell'Arriccia and Mauro (2010), Williams (2009) and Ball (2013) argue that a higher inflation target would have prevented the interest rate from falling to the ZLB.

In Canada, inflation-targeting framework was adopted in 1991, and since 1995, the inflation target was maintained at the level of 2 percent. The inflation target is reviewed and renewed every five years. In particular, the last review was in October of 2016, when the Bank of Canada decided to keep the target at the same level; this renewal covers the period from January 1st, 2017 to December 31st, 2021. There are two types of possible anticipation effects here. First, we would have had a policy implementation lag leading to anticipation effects if the Bank of Canada decided to change the target in October 2016. Second, in spite of the fact that the inflation target was not changed in 2016, anticipation effects were still present as there were some chances that it would be changed given that Canada was close to the ZLB at that time and policymakers were seriously discussing this possibility.

Figure 2 displays dynamics of the main model's variables. We present the results for the method that finds a perturbation solution obtained around a deterministic steady state (labeled as Method 2 in Appendix B; our sensitivity results for other methods predict similar patterns of behavior).

We consider two cases: first, at $t = 1$, the central bank makes an announcement that starting from $t = 1$, it will gradually increase the inflation target $\bar{\pi}_t$ from 2 percent to 3 percent during a period of 8 quarters, and second, the same change takes place but starting from $t = 5$ (i.e., in one year); the inflation target remains at the new (higher) level after it is reached.

When the inflation-target change begins at $t = 1$, inflation follows the same pattern as the target. What is the reason for such behavior of inflation? In our experiment, we assume full credibility of the inflation-target policy. Inflation repeats the pattern of the inflation target because agents determining the behavior of inflation are mainly non-optimizers who index their price by inflation target. As a result, the nominal interest rate gradually increases over the first fifteen periods by 1 percent, and it stays at the new level forever (see Figure 2; note that the real neutral rate is the same as before). Following the announcement, output, investment and commodity exports jump up, and over the transition, the economy experiences an investment- and export-driven growth with the peak increase of output of 0.2 percent. Output begins to descend toward its original level after one year. Consequently, there is only a temporary expansionary effect on the economy due to a higher inflation target.

When the inflation-target change is delayed for one year, the variables behave qualitatively similar. One visible difference from the previous case is that most variables in the figure experience larger increases at the peak (the exchange rate and noncommodity export are exceptions). Therefore, it pays for the central bank to announce this type of policy in advance as output increases more during the transition. The
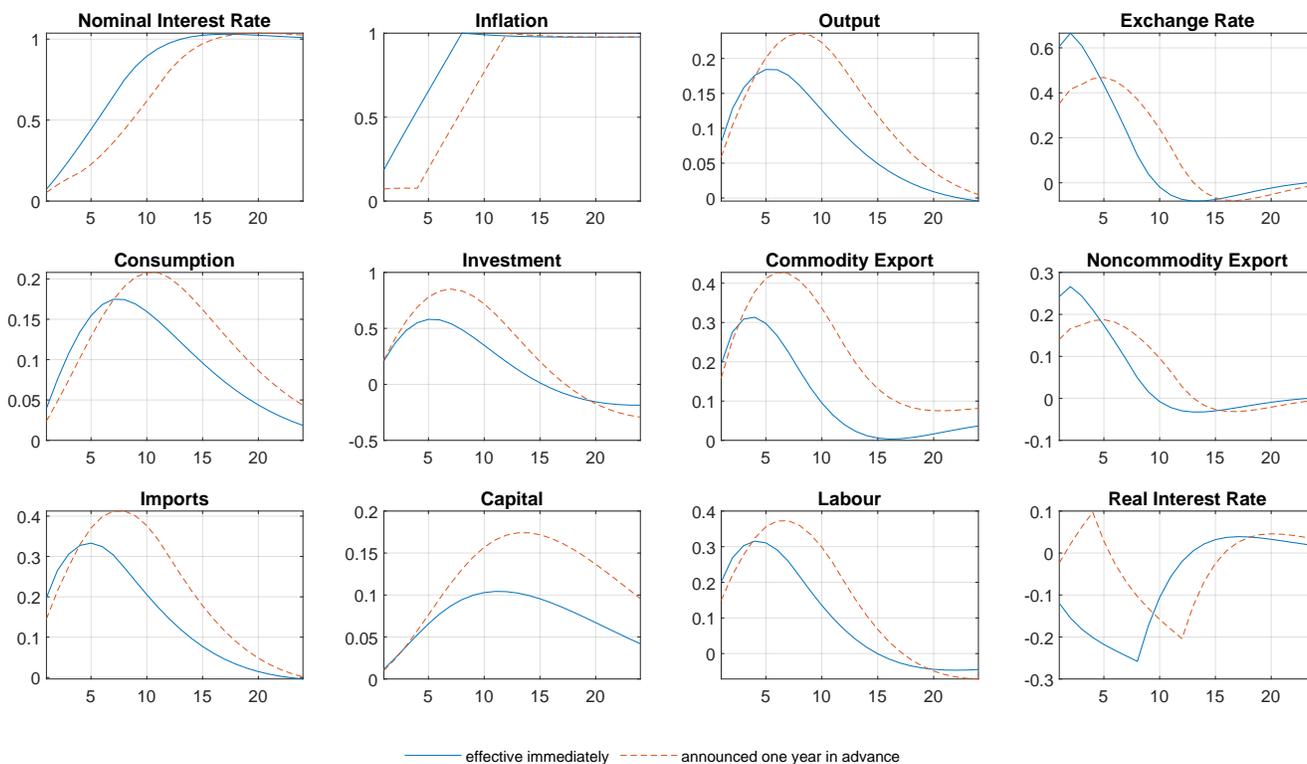
Figure 2: A gradual increase in the inflation target

	R_t	π_t	Y_t	C_t	I_t	X_t^{nc}	X_t^{com}	M_t	L_t	K_t
Average	-4.66	-5.25	-4.09	-4.47	-5.11	-4.10	-3.17	-5.10	-4.42	-5.91
Maximum	-4.30	-5.17	-4.03	-4.38	-5.03	-4.08	-3.13	-4.98	-4.37	-5.84

Table 1: Residuals in the model's equations on the simulated path, $\log_{10}$ units. R_t, π_t, Y_t, C_t, I_t, X_t^{nc}, X_t^{com}, M_t, L_t, K_t are the nominal interest rate, inflation, output, consumption, investment, noncommodity export, commodity export, imports, labor and capital, respectively.

larger jumps in such variables as output, consumption, investment, capital are entirely due to anticipatory effects. That is, agents expect the real interest rate to be lower in the near future, and they accumulate more capital in advance of the more favorable environment which has positive effects on the economy's output today.

Table 1 contains the mean and maximum residuals in the model's equations used for computing the corresponding variables in the table. As we can see, the maximum residuals range between $10^{-3.13}$ and $10^{-5.84}$, i.e., between .07 percent and .0001 percent, which are very low.[3] For the remaining experiments, the residuals in equations are of similar size so our solutions are very accurate (to save on space, these residuals are not reported).

In the second experiment, we model a probabilistic setting in which agents rationally expect that the inflation target might change to two possible levels with some probabilities. Specifically, we assume that there is a 50-percent chance that starting from $t = 5$ the inflation target $\bar{\pi}_t$ gradually increases from 2 to 3 percent during 8 quarters; otherwise, the inflation target remains the same. Our computational method is easy to adapt to modeling more sophisticated anticipation scenarios like one considered in that experiment. When computing policies in period $t = 4$, we explicitly use the Dynare macro language to set the period 4

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[3] In LMM (2020), we reported larger residuals when the economy was hit by a large negative demand shock and the ELB was reached.

expectation functions to be equal to the weighted sums of expectations over the two possible realizations in period $t = 5$.
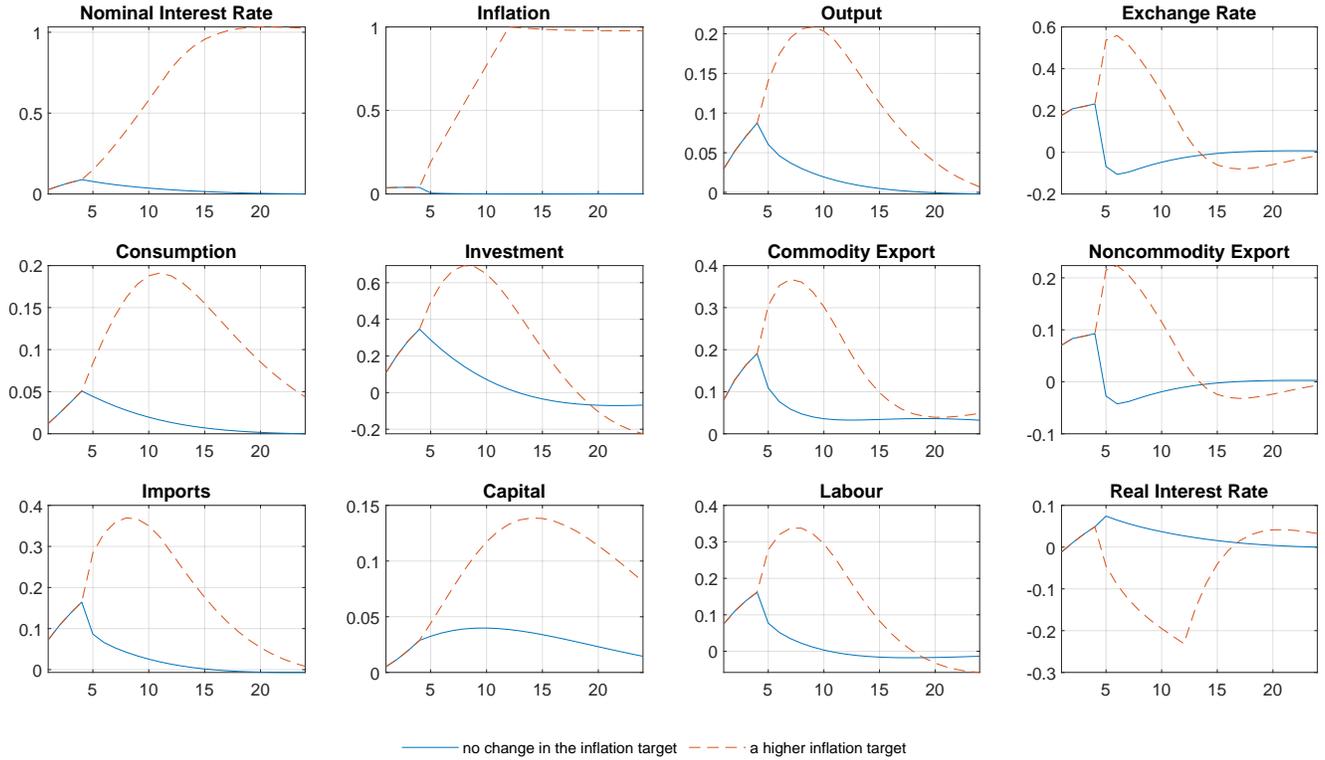

Figure 3: A gradual increase in the inflation target (50% probability)

This experiment is plotted in Figure 3. There are two alternative transition paths differing from period 5 onwards, one per each scenario, i.e., with and without an increase in $\bar{\pi}_t$. Similar to the previous experiment, inflation mimics the behavior of the inflation target: it gradually rises to the new steady state level. Starting from the risky steady state at $t = 1$, all the variables experience mild increases, which are due to anticipatory effects on the side of economic agents. Once it becomes known whether the target will go up or not, all the variables quickly return to the original steady state if the target does not increase, and they experience a more pronounced hump-shape behavior and return to a new steady state if the target increases. In Appendix B, we extend the latter experiment to vary the probability of switching to a higher inflation target at $t = 5$, namely, it is either 25 percent or 75 percent (instead of 50 percent). As those figures show, in case of no inflation-target change, the transition back to the old steady state is significantly faster for the 25-percent case than for the 75-percent case.

3.3 Monetary policy normalization

During the Great Recession of 2007–2009, the nominal interest rate hit the ZLB. As a result, central banks could not rely on Taylor rules to conduct their monetary policy and resorted to forward guidance – an unconventional monetary policy consisting in announcing future interest-rate changes. As emphasized by the literature, central bank's communication of the policy-rate's future path is the main channel through which forward guidance policy affects the economy. Eggertsson and Woodford (2003) demonstrate that a central bank's promises to keep low interest rates for longer periods helps alleviate negative consequences of binding ZLB. As agents expect future interest rates to be lower than in the absence of forward guidance, they increase today's investment and consumption, which stimulates today's economy. Campbell et al. (2012) name this form of forward guidance Odyssean. Another form is Delphic forward guidance: a central

bank may have better information about the state of the shocks that hit the economy, and it communicates a forecasted path of policy rates.[4] In the Odyssean case, future intentions are known, while in the Delphic case, forward guidance is implied – agents do not know its exact duration.

In this paper, we assume that the central bank uses forward guidance to convey a policy change when lifting off from an effective lower bound (ELB) on the bank's policy rates. In particular, we assume that initially the economy is at ELB and at $t = 1$ the central bank announces that it will keep the interest rate at that level for T periods and afterwards it will return to the standard Taylor rule (18). To model the central bank's policy at the ELB periods, we assume that the nominal interest rate is given by

$$R_t = R^{elb},$$

where R^{elb} is the ELB. When the interest-rate policy is normalized after T periods, the Taylor rule's coefficients return back to normal values, and the policy is described by the rule (18).

Figure 4 presents the results for this experiment when the solutions are approximated around a deterministic steady state. The change in the interest rate rule announced at $t = 1$ is anticipated by agents. We compare three cases, depending on whether the interest-rate policy returns to normal (i) in one quarter ($T = 1$), (ii) in one year ($T = 4$), (iii) in two years ($T = 8$). In all the cases, the initial interest rate is below its risky steady state, however, it eventually returns to the steady state.
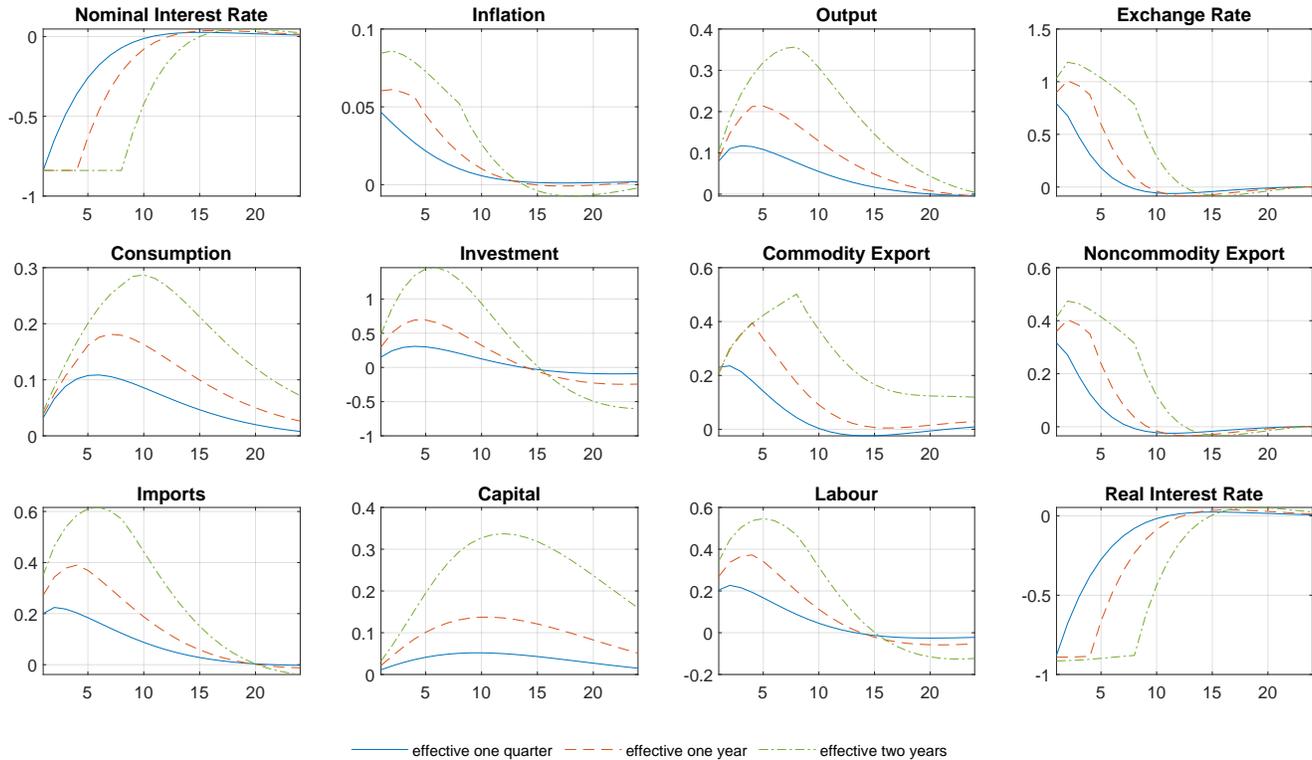
Figure 4: Forward guidance

When the policy is announced, the exchange rate, inflation, and the real variables jump up above the steady state. Local currency depreciation makes domestic exports more competitive, which leads to an increase in exports of both commodities and noncommodity goods. Domestic firms benefit from increased sales, which leads to immediate increases in output, labor, investment and capital. On the other hand, as households work more, they demand more of imported goods, so that imports go up as well. Evidently, an

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[4] Marinkov (2020) argues that during the ZLB period, agents may misjudge a central bank's reaction function and bias their expectations. In this case, the central bank may want to use forward guidance as a guiding tool to correct agents' beliefs.

output increase is the largest when the announced policy is kept for the longest horizon of eight periods. The peak increase in output is 70 percent higher than the one for the forward guidance horizon of four quarters. The differences in output dynamics across the considered cases are only present over the transition but not in the initial period – in all three cases, an initial output jump is of equal size. The dependence of the initial reaction in output on the horizon of the forward guidance policy is known in the literature as a forward guidance puzzle. Even though there is no such dependence in the figure, we do still see that the policy horizon matters for the total effect on output: it reacts more if the policy change is postponed further away in the future.

Our above experiment adds to the discussion on central bank's communication strategy. After the Great Recession of 2007–2009, when the economic conditions improved, an important policy question was how and when to normalize the monetary policy, where normalizing means switching back to some Taylor rule; see Yellen (2015). In particular, the following questions arose after the end of the crisis: (1) Should the central bank normalize policy now or later? (2) Should the central bank do it gradually or all at once? (3) Should the regime shift be announced in advance? (4) Should the policy normalization be time or state dependent? All these questions are hard to address in the context of conventional stationary new Keynesian framework because by definition a monetary policy normalization is a nonstationary change. Nevertheless, the technique developed in this paper enables us to study these questions easily. In our above experiment, we compare the economy's behavior under policies that differ in horizon of return to normal values, which corresponds to questions (1) and (3). We conclude that there are gains from announcing a future lift-off in advance and we quantify these gains for different durations of forward guidance. Similarly, questions (2) and (4) can be answered using our techniques; we leave them for future research.

An effective commitment to keep the interest rate at the ELB implies that the rate should be kept at this low level longer than a Taylor rule would imply. In particular, during the COVID-19 pandemic, the interest rates reached ZLB across a number of developed economics. In the U.S., the Fed has already announced that it expects to keep its benchmark interest rate pinned near zero through 2023.[5] Taylor (2021) argues that the Taylor rules considered by the Fed in the February 2021 Monetary Policy Report imply that the federal fund rate should be higher than the actual zero level and that the Fed "should now engage in a strategy or rule in which people and markets understand that it would raise the policy interest rate if economic growth increases and inflation rises as they are now forecast to do." Our above analysis, however, plays up the importance of commitment to the announced policy on which hinges the desired monetary expansion.

3.4 Switching to a more aggressive Taylor rule

In this experiment, we consider a one-time change in the sensitivity of the policy rate to inflation and the output gap in the Taylor rule (18), as measured by ρ_π and ρ_Y, respectively.[6] It differs from previous experiments, in which the anticipated changes in the model's parameters are gradual. Figure 5 plots the economy's responses to two-time increases in either ρ_π or ρ_Y or both, relative to benchmark parameterization. Note that this change in the coefficient values is quite large relative to what a central bank would typically consider. Switching to more aggressive Taylor rules is anticipated at $t = 1$ but occurs at $t = 2$, so that there are immediate anticipatory effects in all the model's variables.

As we can see, both policies – a higher ρ_π and a higher ρ_Y – are inflationary. However, a double increase in the sensitivity to inflation ρ_π is more effective in expanding the economy: output, consumption, investment, capital, labor are visibly higher both at peak and in the new steady state than in the old steady state; commodity production slightly drops, which is related a lower commodity exports. A double increase in the sensitivity to the output gap has more modest effects however. When there is a stronger response to both inflation and the output gap, the quantitative expressions of the effects are roughly in between the other two cases. That is, given that there is a trade off between inflation and the output gap in

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[5] See https://apnews.com/9b9a335a1ce05d69fc97a1d6197371ab
[6] For example, Taylor (1999) argues that the Taylor rule with $\rho_\pi = 0.5$ and $\rho_Y = 1$ is more reasonable than the one advocated in Taylor (1993) when $\rho_\pi = 0.5$ and $\rho_Y = 0.5$.
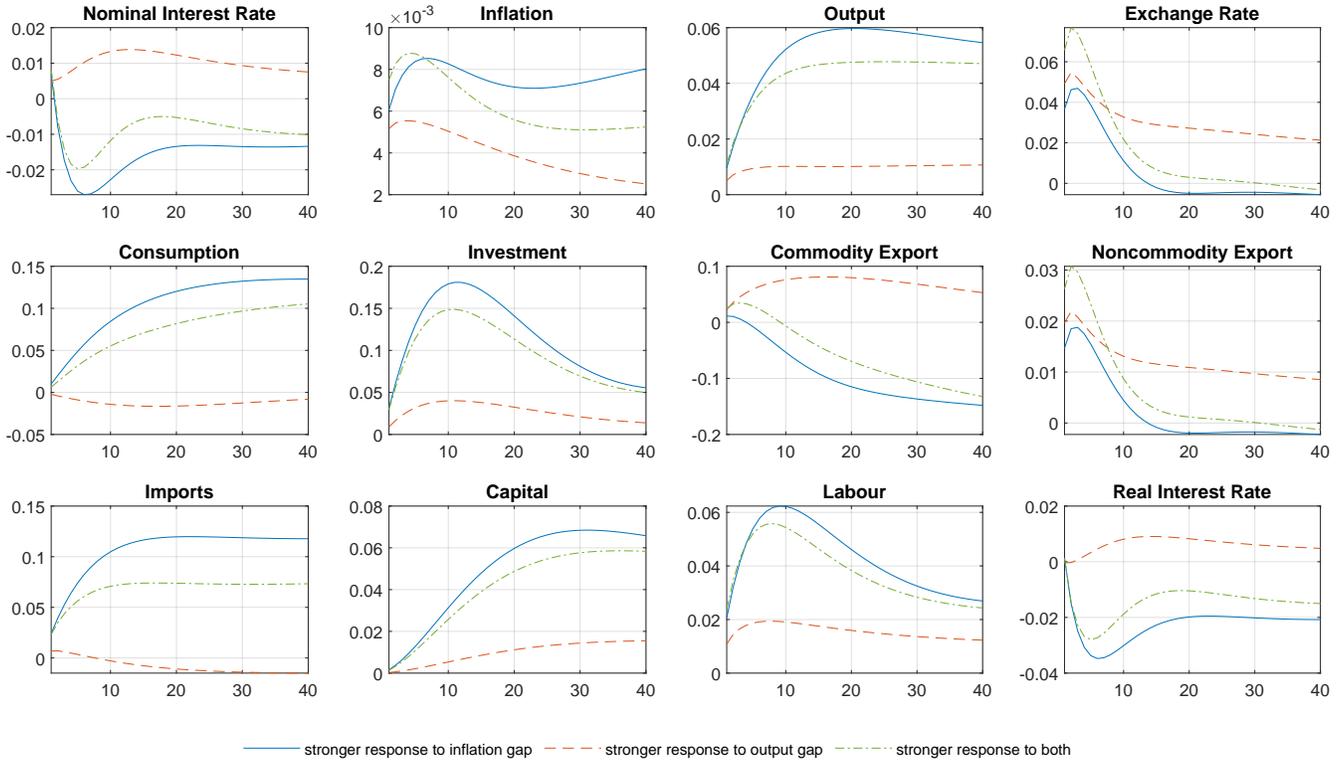
Figure 5: A switch to more aggressive Taylor rules

the policy rule, responding stronger to the output gap undoes the effects of stronger responses to inflation. Overall, total effects are not quantitatively important in our experiment: switching to a significantly more aggressive Taylor rule has only minor effects on the economy's behavior when the economy is not hit by any shocks.

3.5 Switching from inflation targeting to price-level targeting

Since the seminal paper of Svensson (1999), the literature argues that price-level targeting is a "free lunch" in a sense that it positively affects a short-run trade off between inflation and output variability (namely, it reduces inflation variability without an increase in output variability); see Hatcher and Minford (2016) and Ambler (2009) for surveys. Bernanke (2017) proposes to use a temporary price-level target when short-term interest rates are at (or near) ZLB. When ZLB prevents policymakers from providing adequate stimulus, inflation is below target. Price-level-targeting policymakers compensate for periods of low inflation below target by following a temporary surge in inflation The Bank of Canada has seriously considered the use of price-level targeting; see Kahn (2009) and Bank of Canada (2011).

We first consider a scenario, in which central bank switches from the standard Taylor rule (18) targeting inflation to the one targeting a price-level gap,

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) \left\{ \bar{R} + \rho_\pi \left(\log P_t - \log \bar{P}_t \right) + \rho_Y \left(\log Y_t - \log \bar{Y}_t \right) \right\} + \eta_t^r, \tag{33}$$

where P_t is the actual price level, and $\bar{P}_t$ is the target price level that grows at the rate of inflation target $\bar{P}_t = \bar{P}_{t-1} \bar{\pi}_t$. Therefore, price-level targeting does not suggest that policymakers pursue a constant price level but set a target for the price level that rises over time.

An inflation-targeting central bank does not pay attention to temporary changes in inflation as long as inflation comes back to target after some time ("lets bygones to be bygones"). In contrast, price-level-targeting central bank aims at reversing temporary deviations of inflation from target each time it misses

it (e.g., a central bank increases inflation when inflation falls below target). As a result, under inflation targeting, an inflation shock permanently shifts price path to a different level, while under price-level-targeting, any movement in inflation above target is matched with an equal and opposite movement in inflation below target, so that the economy goes along a predetermined price path. Consequentially, with inflation targeting, agents will face a considerable amount of uncertainty about the future price level (the central bank treats past target misses as bygones and returns inflation to the target level gradually, without taking into account any impact on the price level), while with price-level targeting, agents will be much more confident on where the prices will be in the future, even with a positive average inflation.
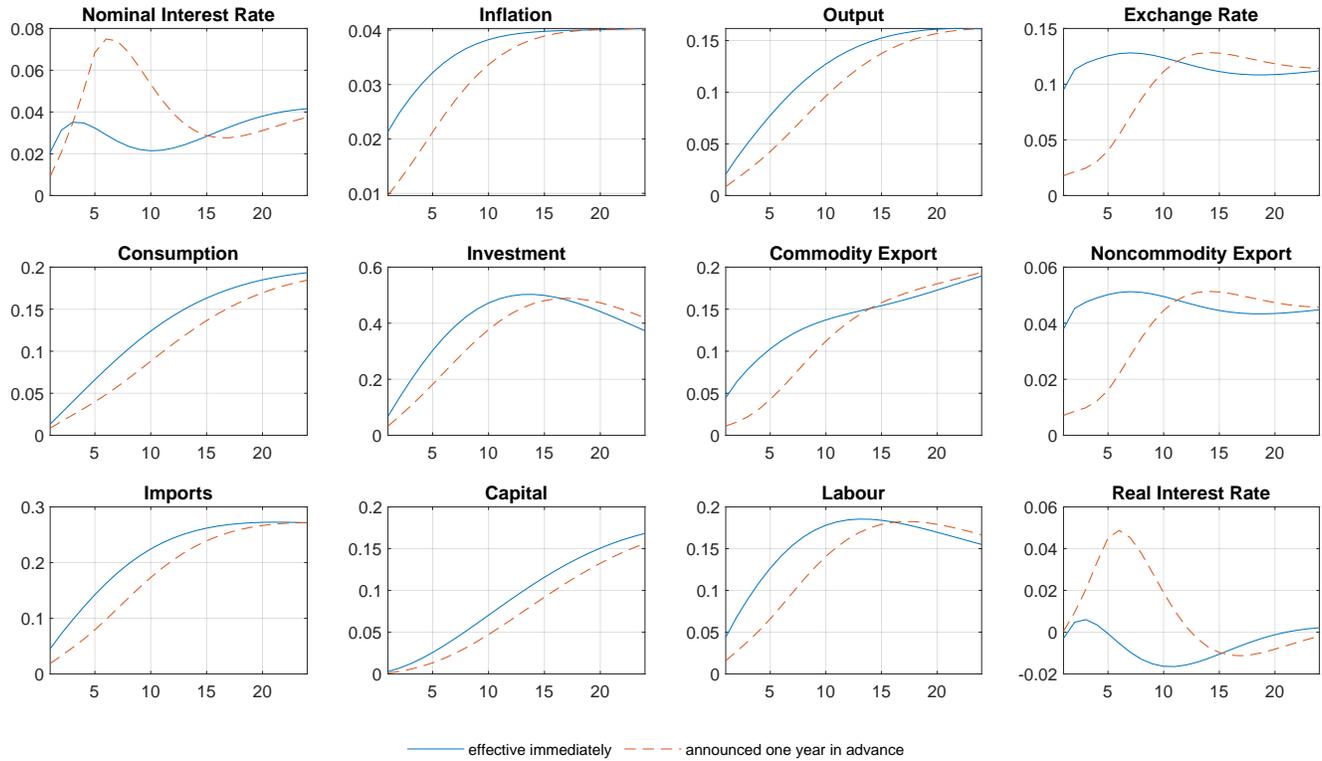

Figure 6: A switch to price-level targeting

In Figure 6, we present the results for two policy experiments in which the policy change becomes effective either immediately (at $t = 1$) or in one year after being announced (at $t = 5$). The new interest-rate rule is associated with higher steady state levels for all the model's variables in the figure. Therefore, switching to price-level targeting has expansionary effects on the economy. Moreover, for all of the variables (except of the nominal interest rate), the immediately implemented policy gives larger benefits than the policy announced one year in advance. That is, if the central bank postpones to implement the switch, the economy reaches the new steady state almost at the same time as the immediate policy, but over the transition the effects are smaller.

In the next experiment, we shock the economy, so that there appears a large output gap. In particular, we consider a permanent negative demand shock – a decrease in foreign demand, modeled as a negative innovation in the random-walk process for this shock. (A version of this experiment with a permanent decrease in productivity is presented in Appendix B.) In response, the central bank can either continue using a policy rule with inflation targeting or can switch to price-level targeting, which, as we saw, leads to higher steady state output. As a result, switching to price-level targeting can be viewed as an attempt to revive the economy.

Figure 7 presents the results of this experiment when the switch is either implemented immediately or is delayed for one year (but it is still announced today). As we see, there is barely any difference between
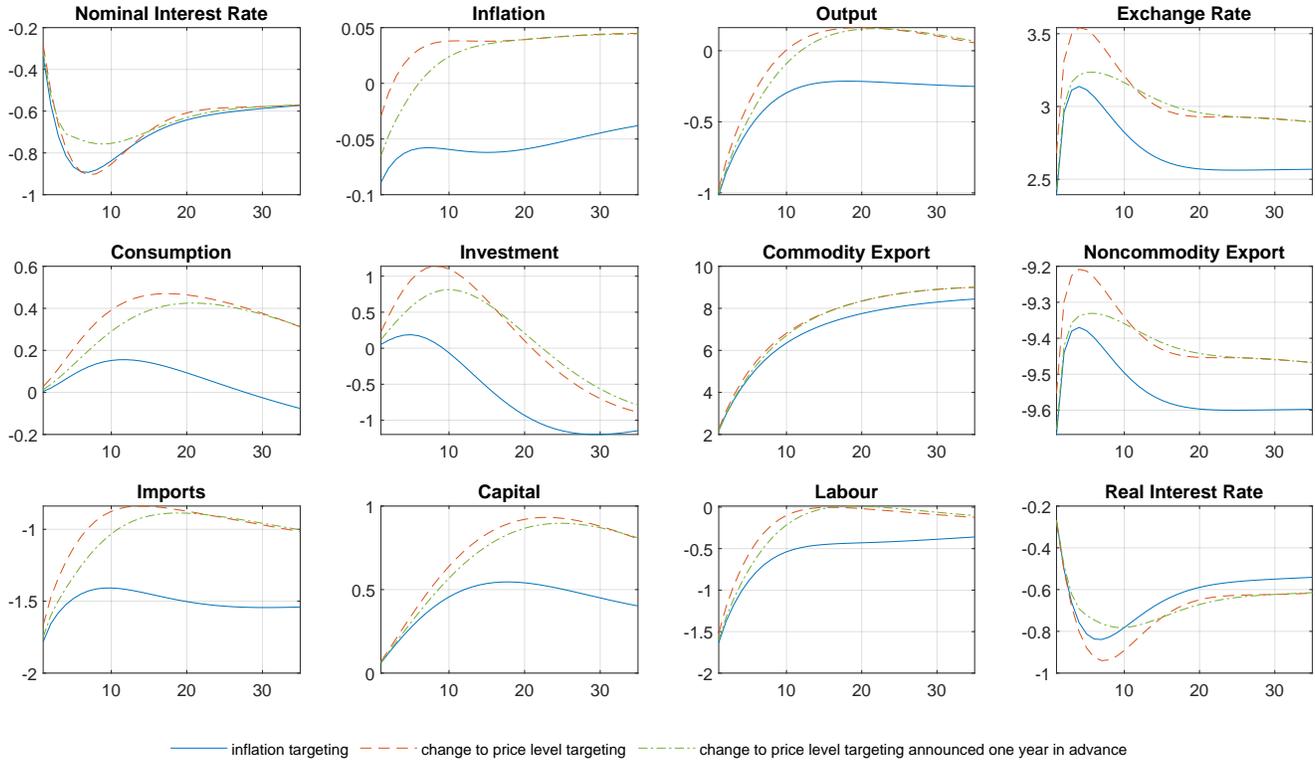
Figure 7: A negative foreign demand shock and a switch to price-level targeting

the immediate and delayed changes in monetary policy for such variables as commodities and commodity export. For all other variables, the immediate policy change has larger impacts than the delayed policy change, which is in line with the previous experiment in Figure 6. Therefore, the anticipation effects work in the direction of softening the effects of the negative demand shock, with impulse responses lying between the cases of no-change and immediate change. In particular, the dynamics of the nominal interest rate is smoother in case of anticipated policy, which leads to smoother behavior of the remaining variables. The initial impact on the economy is significant, e.g., output and labor fall by 1 and 1.5 percent in the three scenarios considered. With no switch in monetary policy, output recovers a bit but its new steady state is still below old steady state. With the policy switch, output is nearly the same as before the shock, and consumption is even higher. Note that each considered policy implies that the central bank tightens monetary policy, even in the economic downturn.

3.6   Switching from inflation targeting to average inflation targeting

On August 27th 2020, the Fed's Chair Jeromy Powell announced that Fed will switch from inflation targeting to average inflation targeting; see Powell (2020). However, as was stated by Richard Clarida, during his presentation at the Hoover Economic Policy Working Group on January 13, 2021, one month prior to that, there was evidence that Fed would introduce that framework, and as a result, there were substantial anticipatory price moves in the U.S. economy.

In our experiment, we consider a switch from the inflation-targeting Taylor rule (18) to a rule that incorporates an average of the past inflation (including the actual inflation),

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) \left[\bar{R} + \rho_\pi \left(\frac{1}{M+1} \sum_{j=0}^{M} \pi_{t-j} - \bar{\pi}_t \right) + \rho_Y \left(\log Y_t - \log \bar{Y}_t \right) \right] + \eta_t^r. \qquad (34)$$

The policy of average inflation targeting shares many of the properties of price-level targeting. As was suggested by the previous literature, average inflation targeting is a middle ground between price-level targeting and inflation targeting; see Nessén and Vestin (2005). Under average inflation targeting, a central bank reacts to a deviation of today's inflation averaged with previous inflation from target inflation. For example, if the inflation target is 2 percent, the averaging window is 3 years, and after consistently archiving 2 percent inflation in the past, in the most recent year, inflation deviates to 3 percent, the central bank will aim to achieve policy-induced inflation of 1 percent in the next year. As a result, inflation will oscillate around average inflation target and the average inflation target is achieved on average. The price level will stay close to its trend, even though the level will sometimes deviate from the fixed trend.
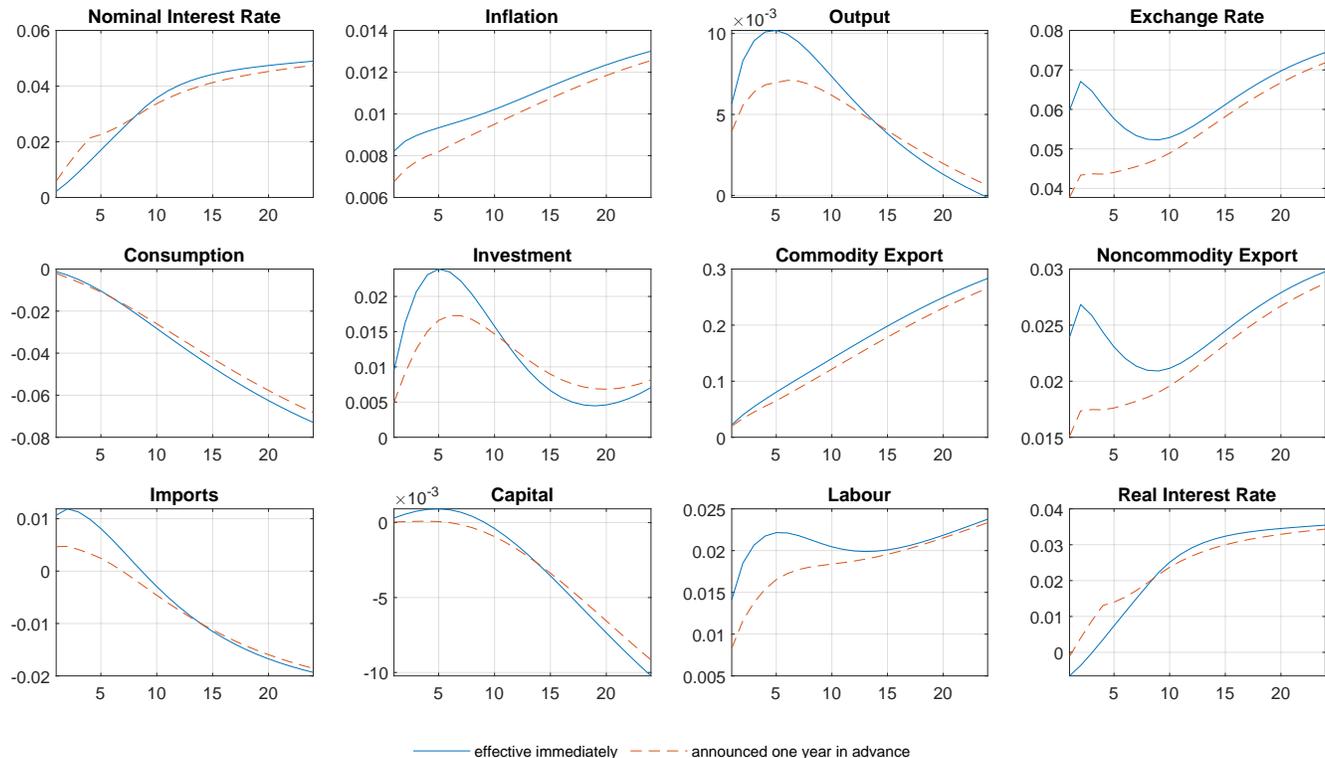
Figure 8: A switch to average inflation targeting

Figure 8 displays the results for two cases: one is when the switch happens immediately and the other when it is implemented with a lag of one year after it was announced. Amano et al. (2020) study optimal history dependence under average inflation targeting in the context of the standard new Keynesian model accounting for the ELB, and they find that optimal M ranges from 2 to 8. We assume $M = 8$ which is the largest number of lags found by Amano et al. (2020).

It turns out that this policy change has very modest anticipation effects on the economy in the absence of any shocks. In fact, when the policy becomes effective immediately, there are larger responses in such variables as output, labor, imports, and noncommodity exports. That is, reacting to average inflation rather than inflation smooths out dynamics to a new steady state. Therefore, we would not expect the economy to experience any drastic changes in the course of transition to average inflation targeting.

4   Recurrent versus non-recurrent news shocks

News shocks had been analyzed in several papers, including Barro and King (1984), Beaudry and Portier (2006, 2007), Jaimovich and Rebelo (2009). In particular, Schmitt-Grohé and Uribe (2012) introduce a tractable perturbation-based framework for solving models with recurrent news shocks. In their analysis,

news shocks follow a stationary Markov process and happen with a fixed periodicity and time horizon. As a result, models with recurrent news shocks have stationary time-invariant Markov solutions. An interesting question is how our nonstationary solutions constructed under the assumption of non-recurrent news shocks compare to stationary solutions constructed by Schmitt-Grohé and Uribe's (2012) method under the assumption of recurrent shocks.

To study this question, we consider a version of the experiment of Section 3.4 in which the central bank switches to a more aggressive Taylor rule; namely, we assume that the sensitivity to inflation ρ_π in the Taylor rule (18) is doubled relative to its benchmark value: the change is announced at $t = 1$ and implemented at $t = 2$. The assumption of non-recurrent shocks provides a natural way of modeling this scenario. Namely, we construct a stationary solution for period $t = 2$ and we find a solution for period $t = 1$ that matches a given terminal condition (decision rule) constructed for period $t = 2$.

Schmitt-Grohé and Uribe (2012) does not specify how their perturbation method can be used for analyzing non-recurrent anticipated shocks like the one we describe above. We tried out two ways of adapting periodic news shocks to our experiment: First, we consider a unit-root process for $\rho_{\pi,t}$, i.e., $\rho_{\pi,t} = \rho_{\pi,t-1} + \varepsilon_{t-1}$, in which initially $\rho_{\pi,0} = \rho_{\bar\pi}$. In this specification, the shock innovation, ε_t captures news that become known at period t and that have a direct impact at $t+1$. In our experiment, $\varepsilon_1 = \rho_{\bar\pi}$ at period $t = 1$, and at all other periods the shock innovation is zero. It implies that $\rho_{\pi,1} = \rho_{\bar\pi}$ and $\rho_{\pi,t} = 2\rho_{\bar\pi}$ for all $t \geq 2$. Second, we consider the news shock to be temporary, i.e., $\rho_{\pi,t} = \rho_{\bar\pi} + \varepsilon_{t-1}$. In that case, we get $\rho_{\pi,2} = 2\rho_{\bar\pi}$ in period $t = 2$ and $\rho_{\pi,t} = \rho_{\bar\pi}$ in all other periods.

Figure 9 compares the second-order perturbation solutions constructed by our method with those produced by Schmitt-Grohé and Uribe's (2012) method. The volatility of the news shock is assumed to be zero, so the initial risky steady state is the same for all three solutions. The following observations are in order: First, it appears that our solution with non-recurrent shocks is situated in between the two recurrent news-shock solutions. Second, both our solution and permanent recurrent news-shock solution converge to new (although different) risky steady states, while the temporary recurrent news-shock solution converges to the old steady state, given the temporary nature of the shock. Third, in the three cases, all the variables behave in qualitatively similar manner: a more aggressive central bank leads to an increase in inflation and a decrease in nominal and real interest rates, which raises output, investment, capital and imports. Fourth, the gap between our solution and permanent news shock solution depends on the initial condition: we observe in our sensitivity experiments (these experiments are not reported) that the gap is smaller if we start below steady state. This is because the anticipation effects are mixed up with upward-sloping transition dynamics. Finally, the difference between the two solutions with recurrent news shocks and our second-order perturbation solution comes from the differences in slopes of the decision rules (like the term $g_{xx}(\bar{v}; 0)$ in (32)). If we were to consider the first-order perturbation, the economy would remain at the deterministic steady state in the two recurrent news-shock solutions but not in our solution.

Furthermore, it is important to emphasize that the permanent recurrent news-shock approach predicts dramatically larger effects associated with the switch to a more aggressive Taylor rule than our approach (except for consumption). This is true both for the anticipation effects and for differences in steady states. For example, anticipation effects in investment are five times larger at peak for the recurrent news shocks than for our perturbation solutions. We conclude that, the two approaches may lead to very different results: the recurrent news-shock approach significantly overstates the importance of anticipated effects. This is because we assume a unit-root process for $\rho_{\pi,t}$, which implies that once a news shock happens, its effects will persist forever. With an autoregressive process for $\rho_{\pi,t}$, the effect of news shocks critically depend on the stochastic process assumed.

There is a simple intuition on why the solutions with non-recurrent news shock differ from those produced by assuming recurrent Markov news shocks. In the former case, the agent's decision rule is constructed to be the best response to a given deterministic sequence of news shocks and in the latter case, it is constructed to be the best response to the given Markov stationary process. With recurrent shock, the response to news is determined not only by the news itself but also by the properties of the Markov stationary process that is assumed for constructing the solution, which is not the case in our analysis with
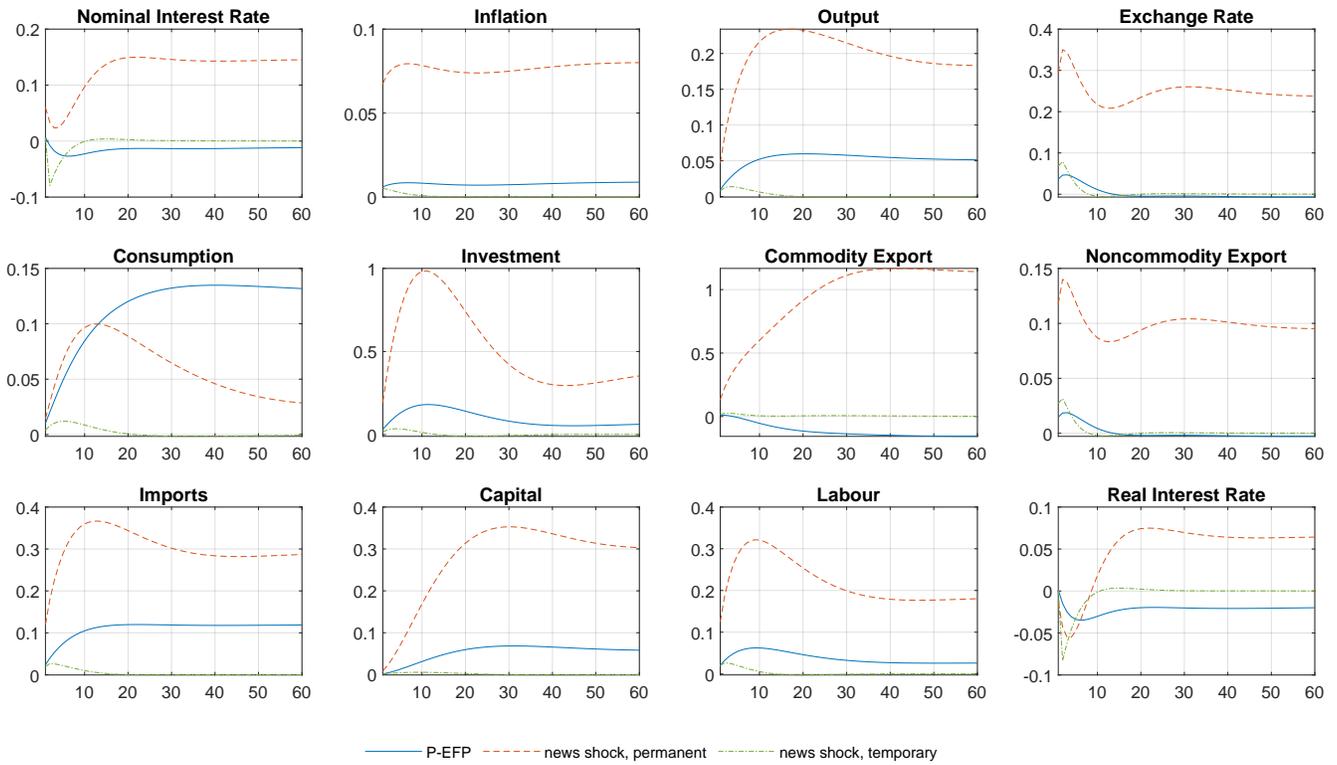
Figure 9: A switch to a more aggressive Taylor rule at $t = 2$ announced at $t = 1$.

non-recurrent shocks.

5 Conclusion

The literature recognizes that private sector's expectations are important for policy outcomes and central banks use the open mouth policies to anchor the expectations. However, little work has been done on evaluating the effects of the open mouth policy within a DSGE framework. This paper fills in this gap. We find that the anticipation effects are the strongest for such time dependent economic policies as a gradual change in the natural rate of interest, policy-rate normalization in the aftermath of the ZLB crisis, and a gradual change in the inflation target level. The other time dependent policy changes like a switch to a more aggressive policy rate rule, a switch to price-level or average inflation targeting lead to more modest anticipation effects.

Our methodology is not limited to central-banking models. Many economic policies are announced ahead of being implemented. For instance, changes to taxes, tariffs, minimum wage, pension reforms, Social Security are frequently signed into law well before they are put in practice. Other notable examples include an announcement about a new member state's accession to the European Union (EU) or a member state's exit from the EU (i.e., Brexit), an announcement of the outcome of presidential elections before the new elected president comes to power. Our perturbation-based framework for solving, calibrating, simulating and estimating of parameters provides a simple and tractable way of analyzing nonrecurrent transitions associated with such policy changes. Literally, our analysis makes it possible to construct a model-consistent path of real-world economies.

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Appendix A

In this section, we illustrate the implementation of perturbation-based method on a toy example – a neoclassical stochastic growth model with labor augmenting technological progress. We consider a version of the model that allows for balanced growth. To solve this model, we proceed as if growth was unbalanced, and then compare our solutions to those obtained by an accurate projection method that solves detrended (stationary) model.

The growth model with labor-augmenting technological progress. We consider the following neoclassical stochastic growth model with labor augmenting technological progress:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{35}$$

$$\text{s.t. } c_t + k_{t+1} = (1 - \delta) k_t + z_t f(k_t, A_t), \tag{36}$$

$$\ln z_{t+1} = \rho \ln z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, 1), \tag{37}$$

where (k_0, z_0) is given; E_t is an operator of conditional expectation; $c_t \geq 0$ and $k_t \geq 0$ are consumption and capital, respectively; $A_t = A_0 \gamma_A^t$ is labor augmenting technological progress with the rate $\gamma_A \geq 1$; u and f are utility and production functions, respectively; and $\beta \in (0, 1)$; $\delta \in [0, 1]$; $\rho \in (-1, 1)$; $\sigma \in (0, \infty)$.

Discussion. Why cannot we solve a nonstationary model with conventional solution methods? For (35)–(37), the Euler equations is given by

$$u(c_t) = \beta E_t \left[u'(c_{t+1}) (1 - \delta + z_{t+1} f_k(k_{t+1}, A_{t+1})) \right].$$

The mainstream of economic literature considers stationary models. We can make our model stationary by setting at $A_t = A$ for all t. The solution to such model is characterized by time-invariant (stationary) decision functions. Conventional solution methods iterate on the Euler equation until a fixed-point decision function for consumption $c_t = C(k_t, z_t)$ is found.[7] However, if A_t grows over time, then the optimal decision function changes over time $C_t(\cdot) \neq C_{t+1}(\cdot)$, then there is no fixed-point solution $C(\cdot)$ so that the conventional methods are not applicable. Under additional restrictions on preferences and technology, the model with labor augmenting progress has balanced growth and can be converted into stationary; see King et al. (1988). We will not focus on this special case but will approximate a sequence (path) of time-dependent functions $\{C_0(\cdot), C_1(\cdot), ...\}$.

In doing this, we exploit a turnpike theorem; see Majumdar and Zilcha (1987) and Mitra and Nyarko (1991) for examples of turnpike theorems for nonstationary models. A turnpike theorem studies the convergence of finite-horizon economies to infinite horizon economies as the time horizon increases. For this model, the turnpike theorem is proven in MMTT (2020). Two important consequences of the turnpike theorem help us compute solutions in the nonstationary economy: First, the infinite- and finite-horizon solutions follow closely one another for a long time and diverge only when the economy approaches to a terminal condition. Second, two terminal conditions $k_T = k'$ and $k_T = k''$ that are close to the solution to nonstationary model make the finite-horizon path closer to the infinite-horizon path. The former allows us to approximate infinite-horizon solutions by finite-horizon solutions, while the latter tells us that it is important to select a good terminal condition, the one close to the infinite-horizon equilibrium path.

To implement the perturbation procedure described in the main text, in Step I, we construct a stationary (time-invariant) model of period T and construct the corresponding Markov decision rule for consumption $C_T(\cdot)$, and in Step II, we use C_T to iterate backward on Euler equations in order to construct a sequence (path) of time-dependent value and decision functions $\{C_{T-1}(\cdot), C_{T-2}(\cdot), ..., C_0(\cdot)\}$, respectively. As a final step, we check the turnpike theorem by verifying that the constructed finite-horizon solution for initial τ periods converges periodwise to a limiting $\{C_0^*(\cdot), C_1^*(\cdot), ..., C_\tau^*(\cdot)\}$ as time horizon T increases, where τ is the final time period in which we want the solution to be accurate. We elaborate on Steps I and II in details below.

First of all note that the model with growth has no natural steady state. To deal with this issue, we introduce time-varying growth rates of capital γ_{kt} that capture how much this state variable grows from period t to $t + 1$ due to the time trend or the parameter change.

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[7] Solution to the growth model could equally well be expressed by a decision function for next period capital $k_{t+1} = K(k_t, z_t)$.

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Step I: Solving for terminal decision functions. In Step I, we aim to construct stationary Markov terminal condition in the form of a decision function for consumption $c_T = C_T(k_T, z_T)$ which is as close as possible to unknown decision function of the infinite horizon model. We assume balanced growth $\gamma_{kT} = \gamma_{cT} = \gamma_A$, and feed the resulting two equations to a Dynare perturbation. Assuming $c_T = C_T(k_T, z_T)$ and $c_{T+1} = C_T(k_{T+1}, z_{T+1})$, we obtain the usual stationary solution to

$$u'(c_T) = \beta E_T \left[u'(c_{T+1}\gamma_A)(1 - \delta + z_{T+1} f_k(k_{T+1}\gamma_A, A_T\gamma_A)) \right],$$
$$c_T = (1 - \delta) k_T + z_T f(k_T, A_T) - k_{T+1}\gamma_A.$$

Unless $\gamma_A = 1$, the model does not have a balanced growth and our approximation does not coincide with the infinite horizon solution at T. But the turnpike theorem implies that the specific terminal condition assumed at T does not affect significantly the solution up to τ provided that $\tau \leq T$. There are ways of constructing more accurate terminal conditions at additional costs.[8]

Step II: Finding a path of decision functions. In Step II, we start from the constructed terminal condition for T and proceed backward to compute the path of the decision functions for $t = T-1, T-2, ..., 0$ by iterating backward on

$$u'(c_t) = \beta E_t \left[u'(C_{t+1}(k_{t+1}, z_{t+1}))(1 - \delta + z_{t+1} f_k(k_{t+1}, A_{t+1})) \right], \tag{38}$$
$$k_{t+1} = (1 - \delta) k_t + z_t f(k_t, A_t) - c_t. \tag{39}$$

In particular, for period $T - 1$, given $c_T = C_T(k_T, z_T)$, Dynare produces the decision function for $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$, in period $T - 2$, given $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$ we find $c_{T-2} = C_{T-2}(k_{T-2}, z_{T-2})$ and so on until the entire solution path is constructed.

Perturbation solutions we construct are obtained around a deterministic growth path. We consider five alternative methods for constructing such a path. We either assume some exogenous growth rates or precompute the growth rates endogenously by shutting down uncertainty in the model. Also, our methods differ in a way the policy functions are specified. In particular, for each deterministic growth-path specification, we have two versions of the algorithm: one in which a next-period policy function takes into account the volatility of uncertainty σ, and the other in which it does not setting $\sigma = 0$. Why might we want to handle the volatility differently? Perturbation policy functions of second and higher orders of approximation are not passing in general through a deterministic steady state of the model. Even in the balanced growth model, if true policy functions for period $t + 1$ are combined with the model's equations written for period t, the deterministic steady state would not be a solution of the deterministic version of the combined system of equations. This feature can be overcome by recognizing explicitly that $C_{t+1}(\cdot)$ depends on σ and by setting σ to zero when computing the deterministic steady state.

Methods 1 and 2. Methods 1 and 2 find local approximations of today's consumption policy function $C_t(k_t, z_t)$ in period t from equations (38) and (39) given the next-period function $C_{t+1}(k_{t+1}, z_{t+1})$. The difference between the two methods lies only in the point around which the local approximation is taken and it is related to our implementation in Dynare.

In period t, Method 1 finds local approximation around a point $(k_t^*, 1)$ that solves the following system of two equations for c_t^* and k_t^*:

$$u'(c_t^*) = \beta u'(C_{t+1}(k_t^*, 1)) \left[1 - \delta + f_k(k_t^*, A_{t+1}) \right], \tag{40}$$
$$k_t^* = (1 - \delta) k_t^* + f(k_t^*, A_t) - c_t^*. \tag{41}$$

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[8] MMTT (2020) offer an alternative way of constructing a terminal condition. Namely, they assume that the solution is stationary in periods T, $T + 1$ and $T + 2$ provided that it is adjusted to growth. This gives 4 equations (Euler equation and constraint) for T and $T + 1$, which can be solved with respect to steady state k_T^*, c_T^* and growth rates γ_{kT} and γ_{cT}.

Here today's and tomorrow's capital are the same and equal to k_t^* because we assume that the growth rate of capital is one.

To understand Method 2, recall that the consumption decision function obtained by perturbation depends on the uncertainty parameter σ and is given by $C_{t+1}(.,.;\sigma)$ in period $t+1$; see (32) for a general representation.[9] In Method 2, we perturb around a point that is computed taking $C_{t+1}(.,.;\sigma)$ without the effect of uncertainty, $\sigma = 0$; this approach is similar to finding a deterministic steady state first (as $\sigma = 0$). In other words, the approximation is conducted around a point $\left(k_t^\dagger, 1\right)$ that solves the following system of two equations for $c_t^\dagger$ and $k_t^\dagger$

$$u'(c_t^\dagger) = \beta u'(C_{t+1}\left(k_t^\dagger, 1; 0\right))\left[1 - \delta + f_k\left(k_t^\dagger, A_{t+1}\right)\right], \tag{42}$$

$$k_t^\dagger = (1 - \delta) k_t^\dagger + f\left(k_t^\dagger, A_t\right) - c_t^\dagger. \tag{43}$$

Evidently, the first-order perturbation solutions obtained by Method 1 and 2 are identical, as such solutions do not depend on uncertainty.[10]

Methods 3 and 4. These two methods explicitly account for time-varying growth rates $\{\gamma_{kt}\}_{t=1}^T$ (recall that for both Methods 1 and 2 we assume that growth rates are equal to unity). Similarly to the latter methods, our Methods 3 and 4 differ in points around which we find Taylor's expansions and parallel to Methods 1 and 2, respectively. To construct a path of growth rates $\{\gamma_{kt}\}_{t=1}^T$, both Methods 3 and 4 solve a deterministic version of the model. Namely, we shut down uncertainty by assuming $z_t = 1$ for all t, set $\tilde{c}_{T+1}$ and $\tilde{k}_{T+1}$ equal to the steady state of the stationary model in the terminal period, and solve the following system of equations:[11]

$$u'(\tilde{c}_t) = \beta u'(\tilde{c}_{t+1})\left(1 - \delta + f\left(\tilde{k}_{t+1}, A_{t+1}\right)\right),$$

$$\tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + f\left(\tilde{k}_t, A_t\right) - \tilde{c}_t.$$

Given the solution $\left\{\tilde{k}_{t+1}\right\}_{t=1}^T$, we compute the growth rates as $\gamma_{kt} = \tilde{k}_{t+1}/\tilde{k}_t$. Both Methods 3 and 4 take $\{\gamma_{kt}\}_{t=1}^T$ as given.

In period t, Method 3 perturbs the solution around a point $(k_t^*, 1)$ that solves for k_t^* and c_t^* the following system of two equations:

$$u'(c_t^*) = \beta u'(C_{t+1}(\gamma_{kt}k_t^*, 1))(1 - \delta + f(\gamma_{kt}k_t^*, A_{t+1})), \tag{44}$$

$$\gamma_{kt}k_t^* = (1 - \delta) k_t^* + f(k_t^*, A_t) - c_t^*. \tag{45}$$

Note that a variable k_{t+1}^* is replaced by $\gamma_{kt}k_t^*$ meaning that we take into account growth when computing the point of approximation. In turn, Method 4 finds a perturbation solution around a point $\left(k_t^\dagger, 1; 0\right)$ and finds $c_t^\dagger$ and $k_t^\dagger$ by solving

$$u'(c_t^\dagger) = \beta u'(C_{t+1}\left(\gamma_{kt}k_t^\dagger, 1; 0\right))1 - \delta + f\left(\gamma_{kt}k_t^\dagger, A_{t+1}\right), \tag{46}$$

$$\gamma_{kt}k_t^\dagger = (1 - \delta) k_t^\dagger + f\left(k_t^\dagger, A_t\right) - c_t^\dagger. \tag{47}$$

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[9] Note that the dependence of $C_{t+1}(k_t^*, 1)$ on σ is implicit in Method 1, i.e., we mean $C_{t+1}(k_t^*, 1; \sigma)$ there.

[10] Note, however, that higher-order approximations will differ between the two methods not only because the intercepts associated with uncertainty are distinct (equal to $C_{\sigma\sigma,t+1}(k_t^*, 1)\sigma^2$ and $C_{\sigma\sigma,t+1}\left(k_t^\dagger, 1; 0\right)\sigma^2$ for Method 1 and Method 2, respectively) but also because the points around we approximate differ.

[11] To implement this step in Dynare, we just solve a system of equations backward in terms of variables $\left\{\tilde{c}_t, \tilde{k}_t\right\}$.

Method 5.   Method 5 is close to Method 3, but the path for growth rates is computed iteratively. We begin by exogenously fixing the path $\left\{\gamma_{k,t}\right\}_{t=1}^{T}$ and obtaining the policy functions for a stochastic version of the model; this is similar to Method 3. As a next step, we simulate the model with the realized values of shocks which are set to zero, we compute the growth rates of capital over this simulated path, and we obtain the policy functions for a stochastic version of the model. We can repeat this step as many times as necessary. We do not offer any counterpart of Method 5 (i.e., Method 6) that corrects for volatility as it is the case of the methods above because the stochastic growth path is computed in a stochastic version of the model, in which the growth path is obtained endogenously.

Numerical results.   In this section, we present the results of our numerical analysis. We assume the standard utility and production functions:

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \quad f\left(k, A\right) = A^{1-\alpha}k^{\alpha}.$$

For all the experiments, we fix the parameters $\{\alpha, \beta, \delta, \rho\}$ at the following values:

$$\alpha = 0.36, \quad \beta = 0.99, \quad \delta = 0.025, \quad \rho = 0.95.$$

We vary the values of the remaining parameters $\{\gamma, \sigma_{\varepsilon}, \gamma_A, T\}$; in the benchmark case, we set them to the following values:

$$\gamma = 5, \quad \sigma_{\varepsilon} = 0.03, \quad \gamma_A = 1.01, \quad T = 200.$$

We simulate the model's solution for different values of the terminal date T. For all simulations, we use the same initial condition (k_0, z_0) and the same sequence of productivity shocks $\{z_t\}_{t=1}^{\mathcal{T}}$.

To see whether our perturbation-based method computes accurate solutions, we obtain an (almost) exact solution by exploiting the property of balanced growth. For this purpose, we first introduce labor-augmenting technical change into the model, then derive the first-order conditions, and finally, detrend them. The resulting stationary model is solved by a very accurate standard projection method with Smolyak grid, third-order polynomial approximation, and 10-node Gauss-Hermite quadrature (the maximum residuals in the model's equations are of order 10^{-9} in $\log_{10}$ units). We compare the simulated series generated by such a projection method with those of our perturbation method on a fixed sequence of shocks of length $\mathcal{T}$.

In Table 2, we report absolute unit-free mean and maximum differences between our approximate and balanced growth ("exact") solutions (in $\log_{10}$ units) on a simulated path $[0, \mathcal{T}]$ with $\mathcal{T} \in \{50, 100, 150, 175, 200\}$. We consider both, first- and second-order approximations.

As is evident from the table, the first-order perturbation solutions are significantly less accurate than the second-order solutions; the difference between the two can reach two orders of magnitude. However, in terms of running times (both solution and simulation), the two solutions are roughly comparable. It is not faster to obtain a first- than second-order solution because each perturbation step takes just few seconds and the largest share of time is spent on finding different decision rules for each period. For second-order approximations, the most basic method, Method 1, yields very accurate solutions: the mean difference from the exact solution is at most 1 percent across the considered simulation lengths, while the maximum difference reaches 1.5 percent. The ranking of the methods in terms of accuracy varies with time horizon $\mathcal{T}$. For example, for $\mathcal{T} = 200$, Method 1 is the least accurate method, followed by Method 2, and then by Methods 4 and 3 (we look at the maximum errors). However, the ranking between Methods 2 and 4 reverses when the other Ts in the table are considered. Methods 3 and 5 are about the same in terms of accuracy and they are the most accurate.

Figure 10 plots our first- and second-order solutions for capital of the nonstationary model (produced by Method 5), as well as the exact solution of the balanced growth model (produced by the standard projection method); the left panel displays the growing solutions, while the right panel contains the detrended

	Method 1		Method 2		Method 3		Method 4		Method 5	

First-order solution

Errors, in $\log_{10}$ units

Horizon	Mean	Max		Mean	Max		Mean	Max
[0, 50]	-1.41	-1.13		-1.50	-1.22		-1.53	-1.23
[0, 100]	-1.24	-0.97		-1.33	-1.11		-1.35	-1.12
[0, 150]	-1.14	-0.77		-1.25	-1.08		-1.27	-1.08
[0, 175]	-1.07	-0.58		-1.22	-0.94		-1.23	-0.97
[0, 200]	-1.04	-0.58		-1.19	-0.94		-1.20	-0.97

Running time, in seconds

	Method 1	Method 2	Method 3	Method 4	Method 5
Solution	161.57	157.60	294.28	288.03	317.38
Simulation	0.0387	0.0275	0.0346	0.0293	0.0271

Second-order solution

Errors, in $\log_{10}$ units

Horizon	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
[0, 50]	-2.28	-2.03	-2.83	-2.53	-3.48	-2.95	-2.79	-2.24	-3.51	-2.92
[0, 100]	-2.12	-1.90	-2.77	-2.53	-3.30	-2.93	-2.46	-2.05	-3.30	-2.91
[0, 150]	-2.05	-1.80	-2.75	-2.53	-3.26	-2.88	-2.33	-2.03	-3.26	-2.88
[0, 175]	-2.00	-1.71	-2.71	-2.15	-3.14	-2.23	-2.38	-2.03	-3.14	-2.23
[0, 200]	-2.04	-1.71	-2.61	-1.79	-3.07	-2.23	-2.43	-2.03	-3.09	-2.23

Running time, in seconds

	Method 1	Method 2	Method 3	Method 4	Method 5
Solution	167.99	167.18	308.87	296.46	337.60
Simulation	0.0256	0.0249	0.0326	0.0312	0.0348

Notes: Mean and Max are, respectively, the average and maximum of absolute difference between the P-EFP and exact solutions (in $\log_{10}$ units) on a stochastic simulation of 200 observations.

Table 2: Difference of a simulated solution path from the balanced growth path in $\log_{10}$ units
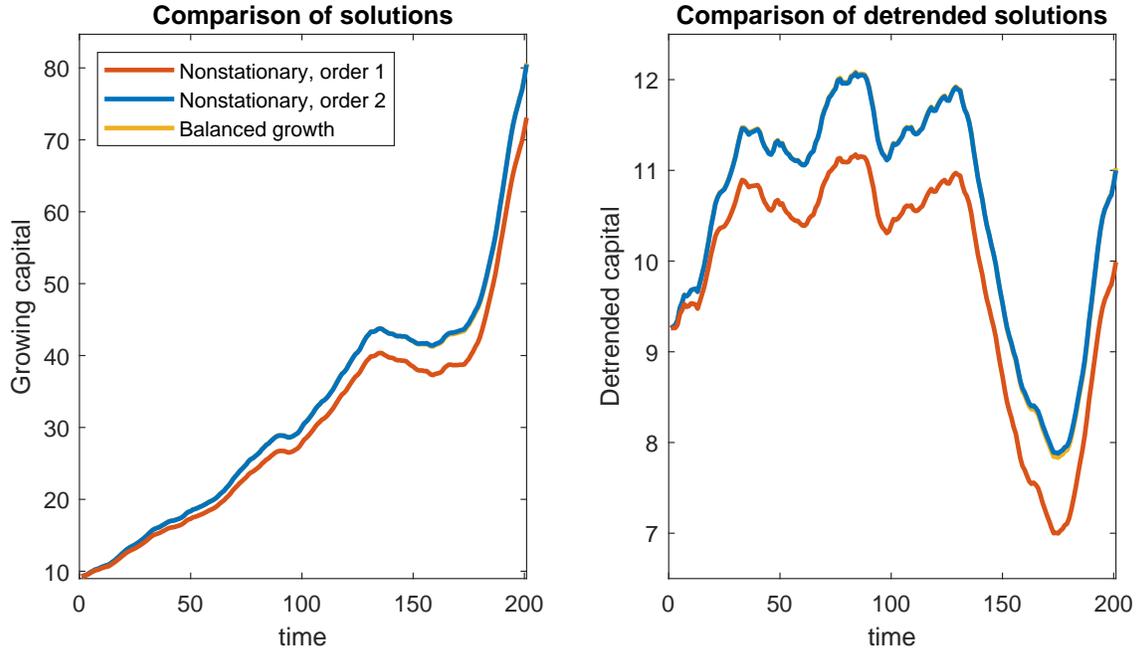
Figure 10: Comparison of the nonstationary P-EFP solutions computed by Method 5 and the balanced growth solution

solutions. One striking feature of our solutions is that its second-order approximation is virtually identical to the exact solution (blue and yellow lines coincide). In turn, the first-order solution is a visible upward shift of the other two solutions, and therefore, can imply substantial inaccuracy.

An important question is: How does our perturbation solutions compare to the existing methods that can solve nonstationary models?

In Table 3, we make a comparison of our perturbation method to three other methods, an extended path method of Fair and Taylor (1983) method, a naive method and a global EFP method of Maliar et al. (2020). Fair and Taylor's (1983) method solves for a path of variables and not functions (as our method does). A naive method finds a different solution for each period t under the assumption that the t-period level of technology prevails in each subsequent period. For each of the methods, we use $T = 200$ in the solution procedure, and we simulate the model for $\mathcal{T} \in \{50, 100, 150, 175, 200\}$.

As is seen from the table, among the three alternative methods, the ranking of the methods is always the same: the naive method is the least accurate and the global EFP is the most accurate, with Fair and Taylor's (1983) method being in between. The latter reaches a notorious accuracy of 0.0001 percent for $\mathcal{T} = 50$; the residuals increase to 3.5% for $\mathcal{T} = 200$. The main finding in the table is that for $\mathcal{T} = 175$ our second-order method is almost as accurate as third-degree solution obtained with the global EFP method, and for $\mathcal{T} = 200$, the second-order solution overpasses the third-degree global EFP solution by a half order of magnitude. Moreover, our perturbation solution is not only more accurate for longer $\mathcal{T}$ but also much faster. This is because of perturbation used as a basis of the method.

Appendix B

In this section, we present sensitivity experiments.

A gradual increase in inflation target implemented with probability. In Figures 11 and 12, we present the supplementary experiments for Section 3.2. Namely, we consider two experiments that are parallel to the one in Figure 3, where a gradual increase in the inflation target happens with probability

Table 3: Comparison of the P-EFP to the other methods

	Fair-Taylor (1983) method	Naive method	Global EFP	P-EFP	
Type of approximation	path	path	3rd order	1st order	2nd order

Maximum errors, in $\log_{10}$ units

[0, 50]	-1.29	-1.04	-6.35	-1.27	-2.24
[0, 100]	-1.18	-0.92	-4.76	-1.11	-2.05
[0, 150]	-1.14	-0.89	-3.22	-1.07	-2.03
[0, 175]	-1.14	-0.89	-2.47	-0.94	-2.03
[0, 200]	-1.14	-0.89	-1.51	-0.94	-2.03

Running time, in seconds

Solution	1.2(+4)	28.9	199.4	317.4	337.6
Simulation	-	2.6	0.0244	0.0271	0.0348
Total	1.2(+4)	31.5	199.4	317.4	337.6

Note: Maximum errors are the maximum of the absolute difference between the given and exact solutions (in $\log_{10}$ units) on a stochastic simulation of $\mathcal{T}$ observations.

of 50 percent. In Figures 11 and 12, such a gradual change occurs with probabilities 75 and 25 percent, respectively. As is seen from the figures, a larger probability of implementing a higher inflation target leads to slightly larger expansionary effects on output, consumption, investment, and commodity exports. Although the qualitative patterns are the same, the anticipation effects (changes up to the fifth period when the actual change takes place) are visibly larger with 75 percent probability than with 25 percent probability.

A negative supply shock and a switch to price-level targeting. In Figure 7, we focus on a switch to price-level targeting after a negative foreign demand shock. Here, we present a supplementary experiment for Section 3.5. namely, we consider a negative supply shock instead.
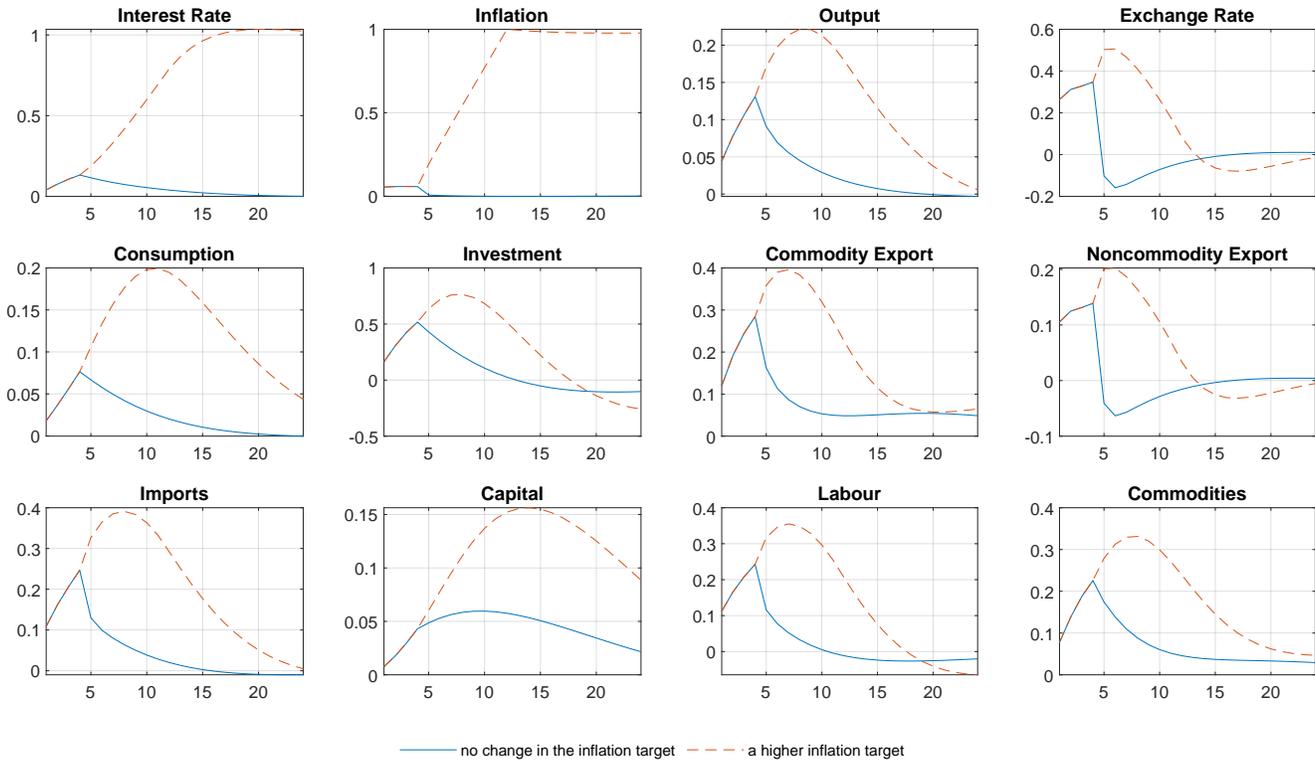
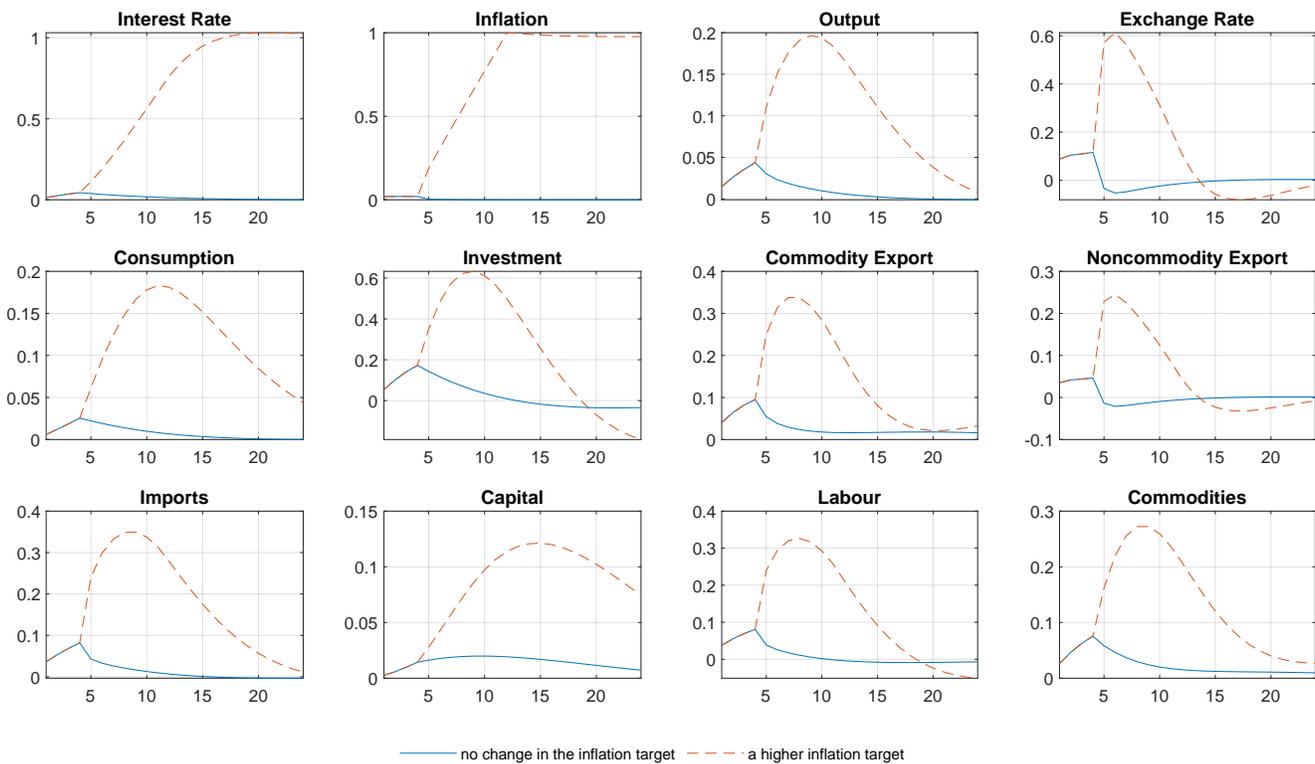Figure 11: A gradual increase in the inflation target (75% probability)

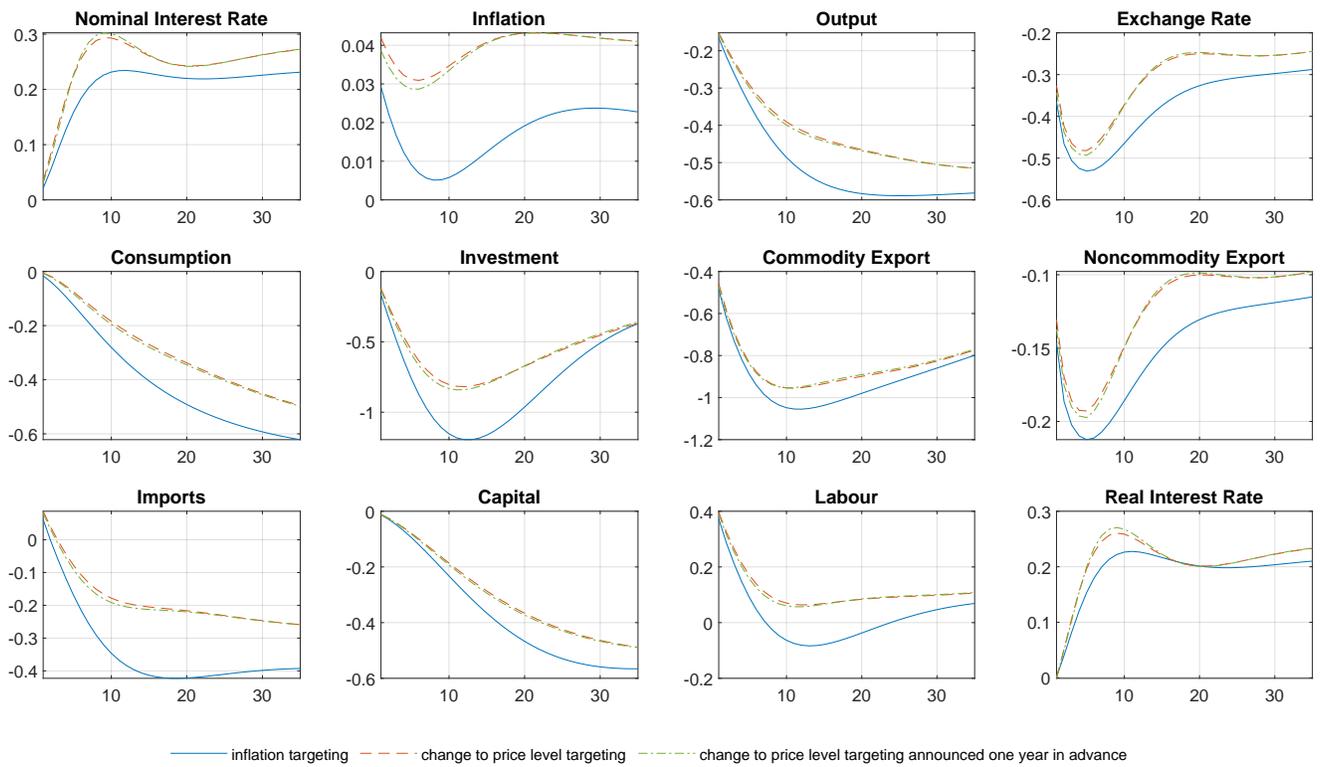Figure 12: A gradual increase in the inflation target (25% probability)

Figure 13: A negative supply shock and a switch to price-level targeting

Online appendix

For the reader's convenience, we provide a description of the calibration procedure, which is similar to LMM (2020).

5.1 Calibration

The model contains 61 parameters to be calibrated. Whenever possible, we use the same values of parameters in the scaled-down model as those in the full-scale model, and we choose the remaining parameters to reproduce a selected set of observations from the Canadian time series data. In particular, our calibration procedure targets the ratios of six nominal variables to nominal GDP $P_t^y Y_t$, namely, consumption $P_t C_t$, investment $P_t^i I_t$, noncommodity export $P_t^{nc} X_t^{nc}$, commodity export $P_t^{com} X_t^{com}$, import $P_t^m M_t$, total commodities $P_t^{com} COM_t$, and labor input $W_t L_t$. Furthermore, we calibrate the persistence of shocks so that the standard deviations of the selected bToTEM variables coincide with those of the corresponding ToTEM variables, namely, those of domestic nominal interest rate R_t, productivity A_t, foreign demand Z_t^f, foreign commodity price p_t^{comf}, and foreign interest rate r_t^f. The parameters choice is summarized in Tables 4 and 5 below.

Parameter	Symbol	Value	Source
Rates			
– real interest rate	$\bar{r}$	1.0076	ToTEM
– discount factor	β	0.9925	ToTEM
– inflation target	$\bar{\pi}$	1.005	ToTEM
– nominal interest rate	$\bar{R}$	1.0126	ToTEM
– ELB on the nominal interest rate	R^{elb}	1.0076	fixed
Output production			
– CES elasticity of substitution	σ	0.5	ToTEM
– CES labor share parameter	δ_l	0.249	calibrated
– CES capital share parameter	δ_k	0.575	calibrated
– CES commodity share parameter	δ_{com}	0.0015	calibrated
– CES import share parameter	δ_m	0.0287	calibrated
– investment adjustment cost	χ_i	20	calibrated
– fixed depreciation rate	d_0	0.0054	ToTEM
– variable depreciation rate	$\bar{d}$	0.0261	ToTEM
– depreciation semielasticity	ρ	4.0931	calibrated
– real investment price	ι_i	1.2698	ToTEM
– real noncommodity export price	ι_x	1.143	ToTEM
– labor productivity	$\bar{A}$	100	normalization
Price setting parameters for consumption			
– probability of indexation	θ	0.75	ToTEM
– RT indexation to past inflation	γ	0.0576	ToTEM
– RT share	ω	0.4819	ToTEM
– elasticity of substitution of consumption goods	ε	11	ToTEM
– Leontieff technology parameter	s_m	0.6	ToTEM
Price setting parameters for imports			
– probability of indexation	θ^m	0.8635	ToTEM
– RT indexation to past inflation	γ^m	0.7358	ToTEM
– RT share	ω^m	0.3	ToTEM
– elasticity of substitution of imports	ε^m	4.4	
Price setting parameters for wages			
– probability of indexation	θ^w	0.5901	ToTEM

– RT indexation to past inflation	γ^w	0.1087	ToTEM
– RT share	ω^w	0.6896	ToTEM
– elasticity of substitution of labor service	ε^w	1.5	ToTEM
Household utility			
– consumption habit	ξ	0.9396	ToTEM
– consumption elasticity of substitution	μ	0.8775	ToTEM
– wage elasticity of labor supply	η	0.0704	ToTEM
Monetary policy			
– interest rate persistence parameter	ρ_r	0.83	ToTEM
– interest rate response to inflation gap	ρ_π	4.12	ToTEM
– interest rate response to output gap	ρ_y	0.4	ToTEM
Other			
– capital premium	κ^k	0.0674	calibrated
– exchange rate persistence parameter	$\varkappa$	0.1585	ToTEM
– foreign commodity price	$\bar{p}^{comf}$	1.6591	ToTEM
– foreign import price	$\bar{p}^{mf}$	1.294	ToTEM
– risk premium response to debt	ς	0.0083	calibrated
– export scale factor	γ^f	18.3113	calibrated
– foreign demand elasticity	ϕ	0.4	calibrated
– elasticity in commodity production	s_z	0.8	calibrated
– land	F	0.1559	calibrated
– share of other components of output	υ_z	0.7651	calibrated
– share of other components of GDP	υ_y	0.311	calibrated
– adjustment cost in commodity production	χ_{com}	16	calibrated
– persistence of potential GDP	φ_z	0.75	calibrated

Table 4: Calibrated parameters in endogenous model's equations

Parameter	Symbol	Value	Source
Shock persistence			
– persistence of interest rate shock	φ_r	0.25	ToTEM
– persistence of productivity shock	φ_a	0.9	fixed
– persistence of consumption demand shock	φ_c	0	fixed
– persistence of foreign output shock	φ_{zf}	0.9	fixed
– persistence of foreign commodity price shock	φ_{comf}	0.87	calibrated
– persistence of foreign interest rate shock	φ_{rf}	0.88	calibrated
Shock volatility			
– standard deviation of interest rate shock	σ_r	0.0006	calibrated
– standard deviation of productivity shock	σ_a	0.0067	calibrated
– standard deviation of consumption demand shock	σ_c	0.0001	fixed
– standard deviation of foreign output shock	σ_{zf}	0.0085	calibrated
– standard deviation of foreign commodity price shock	σ_{comf}	0.0796	calibrated
– standard deviation of foreign interest rate shock	σ_{rf}	0.0020	calibrated

Table 5: Calibrated parameters in exogenous model's equations