Resource Allocation in Cooperative Communications

Ankit Singh Rawat, Kartik Venkat

Department of Electrical Engineering
Indian Institute of Technology
Kanpur, India-208016
ankitsr, kartikv@iitk.ac.in

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Objectives of communication:
- High Data Rate.
- Low bit Error Rate (reliability).

Channel impairments (e.g. shadowing, multi-path fading etc) make it hard to communicate in wireless.

Diversities in wireless channel exploited to counter these.

Multiple input multiple output (MIMO) ⇒ Spatial Diversity.
Cooperative Communications

- Cooperative Communications $\Rightarrow$ alternative way to achieve the spatial diversity
- Wireless devices ‘overhear and transmit’

![Diagram of cooperative communications network with nodes s, r_1, r_2, r_3, r_4, d, s_1, s_2, and d, showing cooperative relays h_1d, h_2d, h_3d, h_4d, and h_{s1d}, h_{s2d}, h_{s12} with two cooperative sources s_1 and s_2.]
Challenges

- Which Relay Functionality to use at cooperating nodes?
- Sharing of network resources (potential relays and their power)
  - not many researchers have addressed this issue in a multi-user network
- MAC layer issues in cooperative communications
- Distributed nature of the cooperative network makes synchronization and perfect CSI acquisition difficult
How do we allocate the relays in a multi user network?

- **Centralized Scheme** A central unit, with complete knowledge of participating users - channel gain, resource constrains performs node assignment

- **Decentralized Scheme** No central authority governing allocation. A localized scheme to distribute power, resources

What performance metric do we want to optimize in the network?
Figure: A Typical Wireless Ad-Hoc Network
System Model

Figure: source-relay-destination

$h_{sr}, h_{rd}, h_{sd}$: Channel Gains

$\gamma_{sr}, \gamma_{rd}, \gamma_{sd}$: SNR
Relay Functionality

- **Amplify and Forward**

  \[
  \gamma_{AF} = \gamma_{sd} + \frac{\gamma_{sr} \gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}}
  \]

- **Decode and Forward**

  \[
  \gamma_{DF} = \gamma_{sd} + \min(\gamma_{sr}, \gamma_{rd})
  \]

Here \(\gamma_{AF}\) and \(\gamma_{DF}\) denote the Effective SNR.
Optimal Relay Assignment

Objective: To maximize Sum Throughput

We are given $N_s$ source-destination pairs and $N_r$ relays. We are given the throughput $C_{i\phi}$ for the direct channel between the source and destination, as well as the throughput $C_{ij}$ between the $i^{th}$ source-destination communicating through the $j^{th}$ relay channel, for all $j$ and $i$.

Assign each source-destination pair to at most one unique relay.
**Mathematical Formulation**

**Determine** \( \vec{\alpha} = \{ \alpha_1, \alpha_2 ... \alpha_{N_s} \} \), \( i = 1, 2 .... N_s \),
where \( \alpha_j \in \{ \phi_1, \phi_2, ..., \phi_{N_s}, 1, 2 .... N_r \} \) such that

\[
\vec{\alpha} = \max_{\alpha}( \sum_{i=1}^{N_s} C_i \alpha_i )
\]

**Constraints**

\( \alpha_i \neq \alpha_j \) when \( i \neq j : i, j \in \{1, 2, ....N_s\} \) \hspace{1cm} (1)

\( \alpha_i \neq \phi_j \) when \( j \neq i \) \hspace{1cm} (2)

In other words, there exists no other node-relay assignment which can have greater sum throughput.
Centralized Scheme

Central Unit has knowledge to compute the throughput of each possible communication link in the network

- $N_s$ source destination pairs
- $N_r$ potential relays

How can we perform the sum throughput maximizing assignment?
One possible Solution

- Compute Objective Function for all possible assignments
- Select the assignment which maximizes required metric (in this case - sum throughput)

Very inefficient solution as there are \( O\left(\frac{(N_s+N_r)!}{N_s!}\right) \) possible assignments

Running time exponential in input size!
We can use graph theory to develop an efficient solution.

Above problem can be modeled as a Weighted Bipartite Graph Matching problem.
Adding Nodes

Source-Destination pairs as NODES
Potential Relay’s

Figure: Adding Vertices to Graph G
Adding Edges and Weights

\[ G = (V, E) \]

\[ \forall e(i,j) \in E, \text{ define } w : E \rightarrow \mathbb{R}, \text{ as } w_{ij} = S - C_{ij}, \]

\[ S = \max(C_{ij}) \]
Definitions and Objective

- **Bipartite Graph** A bipartite graph $G$ can be partitioned into two disjoint vertex sets $V = X \cup Y$, such that every edge $e \in E$ has one end in $X$ and the other in $Y$.

- **Matching** A matching $M$ in $G$ is a subset of edges $M \subseteq E$ such that each vertex appears in at most one edge in $M$.
  - The *size* of a matching is $|M|$
  - The *cost* of a matching is $C_M = \sum_{e_{ij} \in M} w_{ij}$

**Aim:** To compute the *minimum cost* matching of size $N_s$
Solution to Minimum Cost Matching

- We shall **iteratively construct matchings** of larger size till we reach size $N_s$
- Given matching $M$, construct a **Residual Graph** $G_M$
- Find an **Augmenting Path** $P$ in $G_M$ to construct a matching $M'$ of size $|M| + 1$ and $C_{M'} = C_M + \text{cost}(P)$
Residual Graph

\[ M = \{(2, 1)\} \]
Augmented Matching

\[ M = \{(2, 1)\} \]

shortest path \( P \): \n\( \{(s, 1), (1, 2), (2, t)\} \)

\[ M' = \{(1, 2), (2, 1)\} \]
Graph Construction
Source nodes = N
Relay nodes = R

|M| = 0

|M| = N?

Yes

Construct Residual Graph

Find shortest path from s to t

Augment Matching with shortest path

|M| = |M| + 1

No

return Matching

END

BEGIN

Find shortest path from s to t

Augment Matching with shortest path

|M| = |M| + 1

Graph Construction
Source nodes = N
Relay nodes = R

|M| = 0

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# An Example

<table>
<thead>
<tr>
<th>Source-Destination</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>25</td>
<td>28</td>
<td>30</td>
<td>8</td>
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<tr>
<td>2</td>
<td>43</td>
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<td>60</td>
<td>39</td>
<td>37</td>
<td>23</td>
</tr>
</tbody>
</table>

Total Sum Throughput : 222
Running Time

Running Time of Algorithm

\[ \text{Running time (ms) [logscale]} \]

Number of source-destination pairs in network, \( N_s = N_r \)

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Centralized scheme for resource allocation assumes the existence of a central unit (CU).

CU solves the optimization problem of allocating the relays among the users maximizing certain objective function (sum throughput in this work).

Centralized information exchange and coordination, however does not scale well with the size of the network.

Solution is to look for a ‘decentralized (distributed)’ scheme.
Decentralized Scheme (contd..)

- Channel allocation matrix \((A_{N_s \times N_r})\) represents the present state of cooperative network:
  \[
  A_{ij} = \begin{cases} 
  1 & \text{if } j^{th} \text{ relay is allocated to } i^{th} \text{ s-d pair}, \\
  0 & \text{if } j^{th} \text{ relay is not allocated to } i^{th} \text{ s-d pair}.
  \end{cases}
  \tag{3}
  \]

  and \(\sum_{j=1}^{N_r} A_{ij}\) can take values in \(\{0, 1\}\).

- \(P_{ij}\) represents the power used by the \(j^{th}\) relay to forward the signal received from the \(i^{th}\) source.

- Capacity of the \(i^{th}\) user while using \(j^{th}\) relay, can be expressed as:
  \[
  C(i, j) = f(\gamma_{sd}, \gamma_{sr}, \text{SINR}_{rd}(i, j))
  \tag{4}
  \]

  where,

  \[
  \text{SINR}_{rd}(i, j) = \frac{P_{i,j}|h_{ij}|^2}{\sum_{n=1, n \neq i}^{N_s} A_{nj} P_{nj}|h_{ij}|^2 + N_o}
  \tag{5}
  \]
The problem of resource allocation can be modeled as an $N_s$ player game:

$$G : \left[ N_s, \{S_i\}_{i \in N_s}, \{U_i\}_{i \in N_s} \right]$$

where $N_s$ represents the set of $N_s$ sources, $S_i$ denotes the strategy set and $U_i$ the utility function of $i^{th}$ user.

- Strategy set of each user is identical:
  $$S_i = \{r_\phi, r_1, r_2, ..., r_3\}$$  \hspace{1cm} (6)

- We are concentrating on the following two possible utility functions:

  $$U_{1i}(A_{ij}, A_{-1}) = \begin{cases} C(i,j) - \beta P_{ij} & \text{if } A_{ij} = 1, \\ C(i, \phi) & \text{if } \sum_{j=1}^{N_r} A_{ij} = 0. \end{cases}$$  \hspace{1cm} (7)

  $$U_{2i}(A_{ij}, A_{-1}) = C(i,j) - (C_{-1} - \sum_{n=1, n \neq i}^{N} C_n)$$  \hspace{1cm} (8)
Plans for next semester

- Develop a generalized distributed scheme for relay allocation:
  - Analysis of the resource allocation in multi-user network modeled as a game.
  - Simulations to verify the findings in different settings.
- Outage optimal resource allocation scheme.
- Resource allocation with cooperative users paradigm (bandwidth sharing).
- Resource allocation with limited or outdated CSI.
Introduction
Relay Allocation
System Model
Centralized Scheme
Decentralized Scheme
Future Work
Thank you