FAST LARGE-SCALE INVERSION FOR DEEP AQUIFER CHARACTERIZATION

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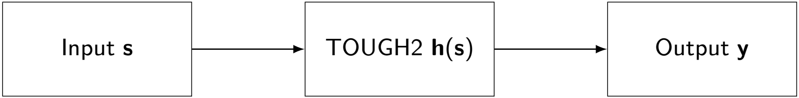
Introduction : Inverse Problem



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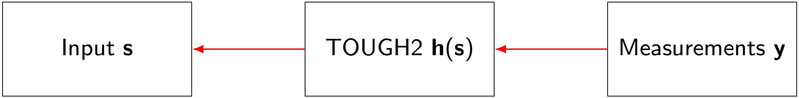

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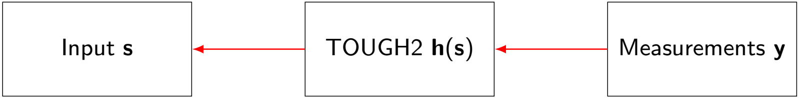

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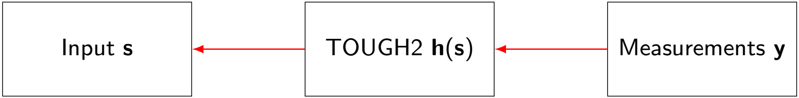

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- Therefore, **s** is characterized in a statistical framework
- requires $\mathcal{O}(min(n_{obs}, n_{unknowns}))$ TOUGH2 runs (or more)

Geostatistical Approach [Kitanidis, 1995]

Consider the measurement equation

$$\mathbf{y} = \mathbf{h}(\mathbf{s}) + \mathbf{v} \qquad \mathbf{v} \sim \mathcal{N}(0, \mathbf{\Gamma}_{\text{noise}})$$

where,

$$\begin{aligned}
\mathbf{y} &:= n_{obs} \text{ measurements, e.g., pressure or temperature} \\
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and the best estimate is given by

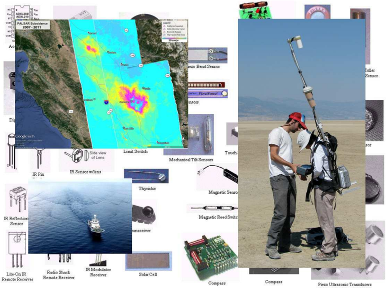
$$\hat{\mathbf{s}} = \mathbf{s}_{prior} + \mathbf{\Gamma}_{\text{prior}} \mathbf{J}^\top \xi$$

where $\mathbf{J}$ is Jacobian (sensitivity) matrix

$$\mathbf{J} = \frac{\partial \mathbf{h}}{\partial \mathbf{s}}_{\mathbf{s}=\hat{\mathbf{s}}}$$

Computational Challenges in Inverse Modeling

With advances in sensor technology and computational power (e.g.,$\min(n_{obs}, n_{unknowns}) \geq 10^6$), we need a scalable approach!



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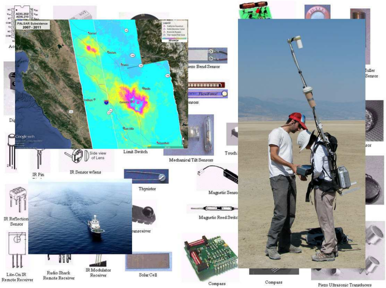

In geostatistical approach (as well as other approaches), computational costs from

- Jacobian-Covariance products (e.g., prior cross-covariance) $\mathbf{J}\boldsymbol{\Gamma}_{\text{prior}}$
- $\mathcal{O}(\min(n_{obs}, n_{unknowns}))$ forward model runs for $\mathbf{J}$
- $\mathcal{O}(n_{unknowns}^2)$ storage for $\boldsymbol{\Gamma}_{\text{prior}}$
- $\mathcal{O}(n_{unknowns}^2 n_{obs})$ multiplication for $\mathbf{J}\boldsymbol{\Gamma}_{\text{prior}}$

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For example, $n_{unknowns} = 6$ mil. and $n_{obs} = 100,000$, how can we compute

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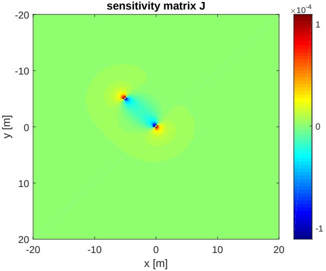

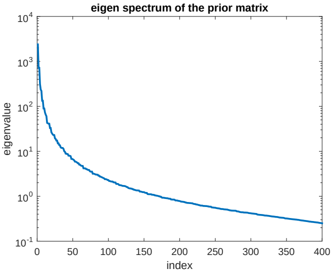

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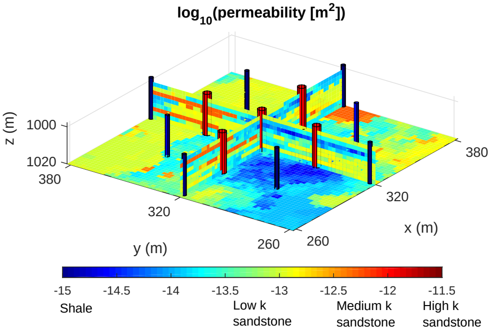
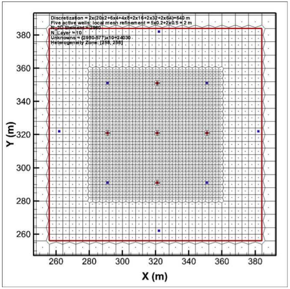
- previously applied to 3-D hydraulic tomography ($n_{unknown} \sim 3$ mil.), MRI-imaged tracer data inversion ($n_{obs} \sim 6$ mil.), arsenic-bearing mineral characterization (flow - transport - multi-species reaction) and so on

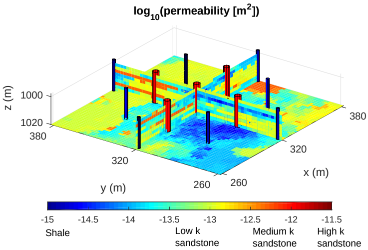
Application : aquifer characterization using CO_2 leakage monitoring network

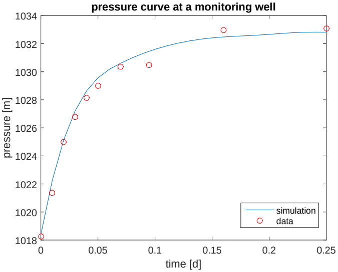




- 120 m x 120 m x 20 m permeability field estimation
- 640 m x 640 m x 20 m model domain
- 5 pumping & 8 monitoring wells
- "True" field from TPROGS [Carle and Fogg, 1996], $n_{unknowns} = 24{,}040$
- pressure and heat tracer data generated from TOUGH2-MP EOS1

Pressure Data Inversion





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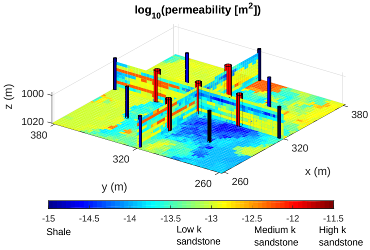

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- total inversion $\sim$ 300 TOUGH2 runs for each test
- On a 16-core workstation, it took less than 2 hours.

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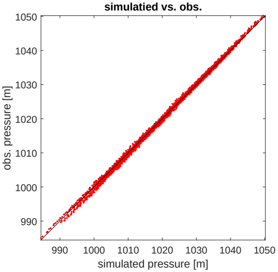

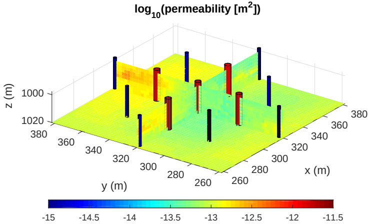

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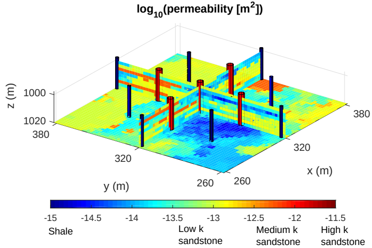

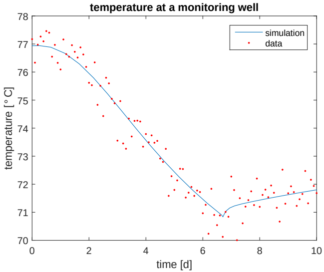

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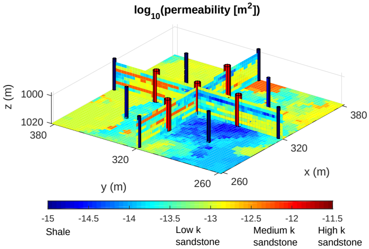

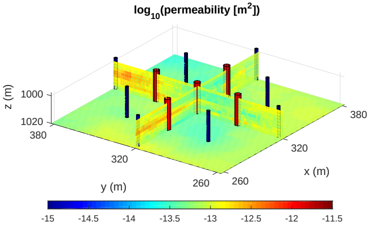

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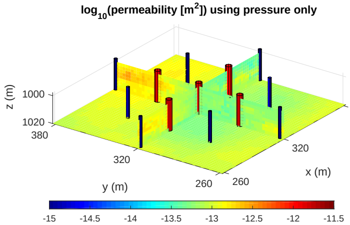

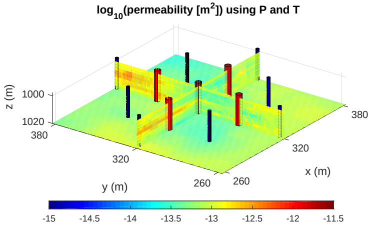

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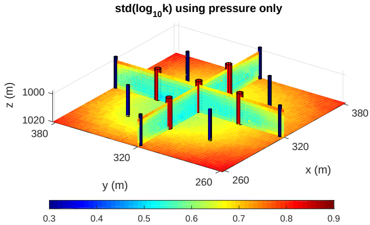

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- Estimation uncertainty is reduced with additional temperature data

Conclusion

Contributions

- use of TOUGH2-MP as a black box to perform joint inversion
- total $\sim \mathcal{O}(100)$ TOUGH2-MP executions for complex inverse problems
- Potential benefits using temperature tracer data to improve the inversion in CO2 leakage monitoring network

References

- Lee and Kitanidis, Large-Scale Hydraulic Tomography and Joint Inversion of Head and Tracer Data using the Principal Component Geostatistical Approach (PCGA), *WRR*, 2014
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