9. Simple Rules for Monetary Policy

John B. Taylor, May 10, 2013

Woodford, *AER* (2001) overview paper

Purpose is to "consider to what extent this prescription

$$(1) \qquad i_t = 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - \bar{y}_t)$$

resembles the sort of policy that economic theory would recommend"

But first, let's review how this "sort of policy" was originally derived from what "economic theory would recommend."

Dynamic
Stochastic
Forward-looking
Lucas critique
Time inconsistency
Sticky prices

RULE

November 1992

$$r = p + .5y + .5(p - 2) + 2 \qquad (1)$$

$$r = 1.5p + .5y + 1$$

where

- r is the federal funds rate,
- p is the rate of inflation over the previous four quarters
- y is the percent deviation of real GDP from a target.

That is,

- y = 100(Y - Y*)/Y* where
- Y is real GDP, and
- Y* is trend real GDP (equals 2.2 percent per year from 1984.1 through 1992.3).

So where did it come from?

- Searching for optimal rules
  - By simulating alternative rules in dynamic stochastic monetary models with rational expectations & sticky prices
  - Finding rules which performed well: small Var(y) & Var(p)
  - Example: Model comparison at Brookings
    - 9 multi-country models (7 with RE) including Taylor multi-country model
- Finding that good policy rules had certain properties

  r rather than M, no forex, react to y <u>and</u> p, (coef on p) >1

  -- Zero bound on interest rate: switch to M-growth

  Then rounded off the coefficients for practical work
- Then showing policy was close in 1987-92
  - but it was not an estimated rule
  - it was meant to be normative, not positive

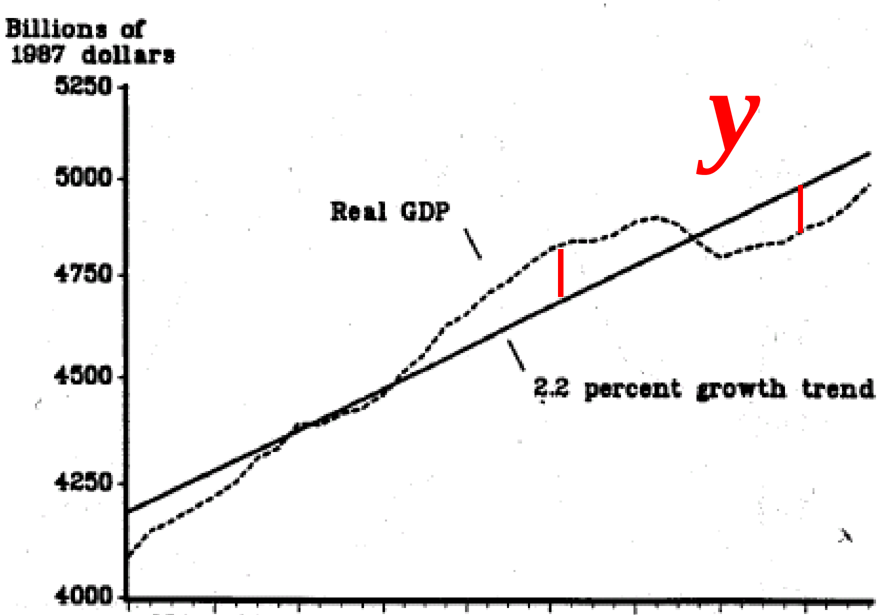


Figure 2. Real GDP and 2.2 percent growth trend.

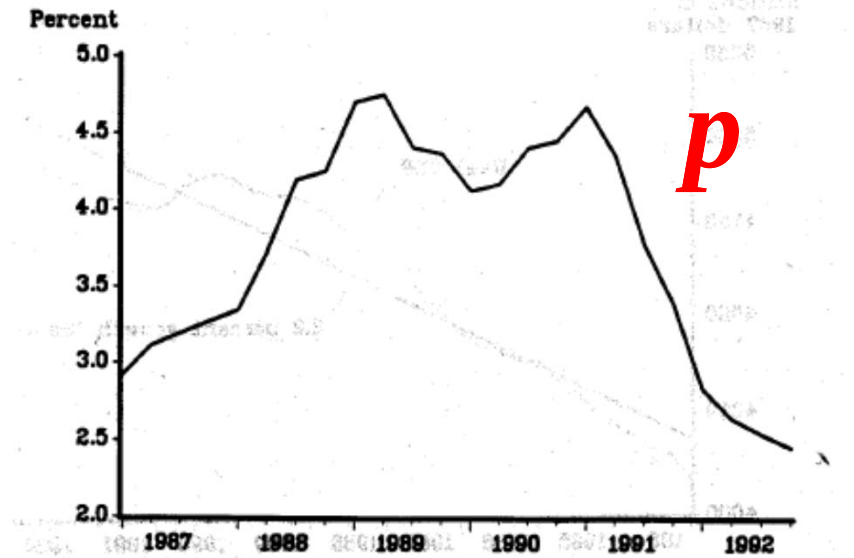
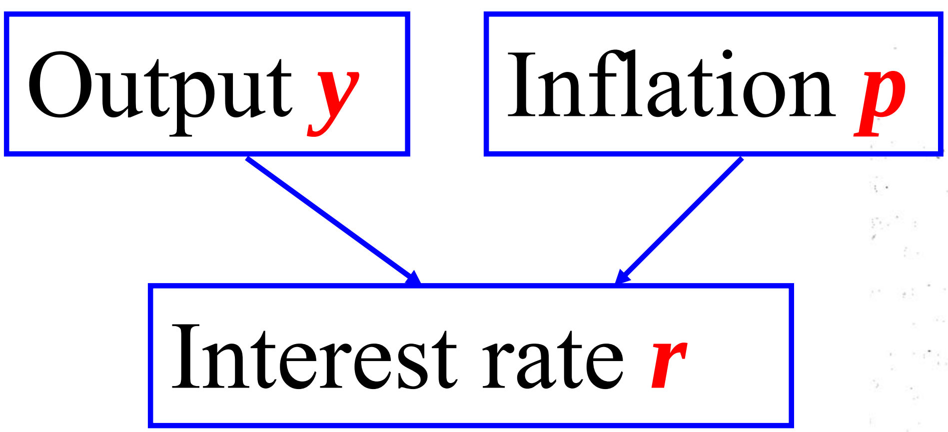


Figure 3. Inflation during previous 4 quarters (GDP deflator).



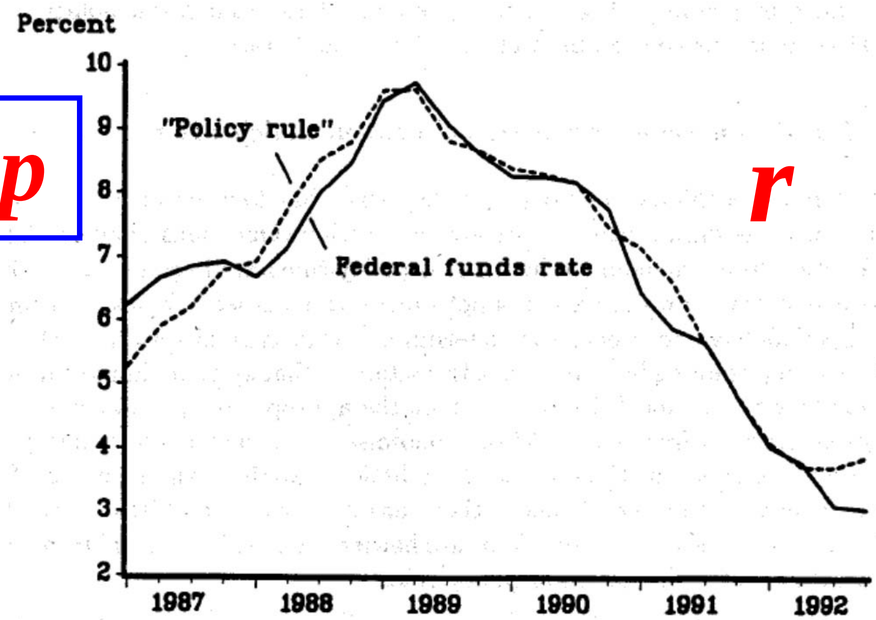


Figure 1. Federal funds rate and example policy rule.

From Taylor (1992)

Woodford (2001): What can we learn about the rule from this model?

$$i_t = i_t^* + \phi_\pi(\pi_t - \overline{\pi}) + \phi_y(y_t - y_t^n)$$
$$\pi_t = \kappa(y_t - y_t^n) + \beta E_t \pi_{t+1}$$
$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + g_t$$
where $\kappa, \sigma, \phi_y, \phi_\pi$ are positive and $0 < \beta < 1$

$$\left[\begin{array}{l} \sum_{t=0}^{\infty} \beta^t u(C_t) \\ A_{t+1} = (1+\rho_t)A_t + Y_t - C_t \\ \dfrac{u_c(C_t)}{u_c(C_{t+1})} = \beta(1+\rho_t) = \beta(1+i_t)/(1+\pi_{t+1}) \\ \text{take log linear approximation} \end{array}\right.$$

Now substitute for i_t in the third equation using the first equation:

$$y_t = E_t y_{t+1} - \sigma(i_t^* + \phi_\pi(\pi_t - \overline{\pi}) + \phi_y(y_t - y_t^n) - E_t \pi_{t+1}) + g_t$$
$$(1 - \sigma\phi_y)y_t - \sigma\phi_\pi \pi_t = E_t y_{t+1} + \sigma E_t \pi_{t+1} - \sigma i_t^* + \sigma\phi_\pi \overline{\pi} + \sigma\phi_y(y_t^n) + g_t$$

When combined with second equation we obtain a linear vector rational expectaions model in the form:

$$E_t \mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{e}_t$$

where $\mathbf{z_t} = \begin{pmatrix} \pi_t \\ y_t \end{pmatrix}$ and $\mathbf{e}_t = \begin{pmatrix} \kappa y_t^n / \beta \\ \sigma(i_t^* - \phi_\pi \overline{\pi} - (\phi_y - \kappa/\beta)y_t^n) - g_t) \end{pmatrix}$

$$\mathbf{A} = \begin{pmatrix} 1/\beta & -\kappa/\beta \\ \sigma(\phi_\pi - 1/\beta) & 1 + \sigma\phi_y + \sigma\kappa/\beta \end{pmatrix}$$

Solving the Model: Making sure there is a unique solution

Observe that there are no lags in this model, so the conditions for uniqueness will be different from the rational expectations model with one lag and one lead considered earlier.

- In this case there are two leads.
- In continuous time one would say that there are two " jump" variables.

In the earlier case we needed one unstable root for uniqueness.

- Now we need two unstable roots to pin down the solution.
- To see this, assume that e_t is serially uncorreleted with zero mean.

- Look for a solution using undetermined coefficients :

$$\mathbf{z_t} = \boldsymbol{\Gamma}_{\mathbf{0}}\mathbf{e}_t + \boldsymbol{\Gamma}_1\mathbf{e}_{t-1} + \boldsymbol{\Gamma}_2\mathbf{e}_{t-2} + \dots$$

Substituting in for $\mathrm{E_t}\mathbf{z}_{t+1}$ and $\mathbf{z}_t$ in the model

$$E_{\mathrm{t}}\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{e}_t$$

results in

$$\boldsymbol{\Gamma}_1\mathbf{e}_t + \boldsymbol{\Gamma}_2\mathbf{e}_{t-1} + \boldsymbol{\Gamma}_3\mathbf{e}_{t-2} + \mathbf{\dots} = \mathbf{A}(\boldsymbol{\Gamma}_0\mathbf{e}_t + \boldsymbol{\Gamma}_1\mathbf{e}_{t-1} + \boldsymbol{\Gamma}_2\mathbf{e}_{t-2} + \mathbf{\dots}) + \mathbf{e}_t$$

which implies that

$$\boldsymbol{\Gamma}_1 = \mathbf{A}\boldsymbol{\Gamma}_0 + \mathbf{I}$$

Each element is an equation:
4 equations in 8 unknowns

$$\boldsymbol{\Gamma}_{i+1} = \mathbf{A}\boldsymbol{\Gamma}_i, \;\; i = 1,2,\dots$$

Each of these brings 4 more equations,
but with 4 more unknowns

$$\mathbf{A} = \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^{-1}$$

$$\mathbf{H}^{-1}\boldsymbol{\Gamma}_{i+1} = \boldsymbol{\Lambda}\mathbf{H}^{-1}\boldsymbol{\Gamma}_i, \;\; i = 1,2,\dots$$

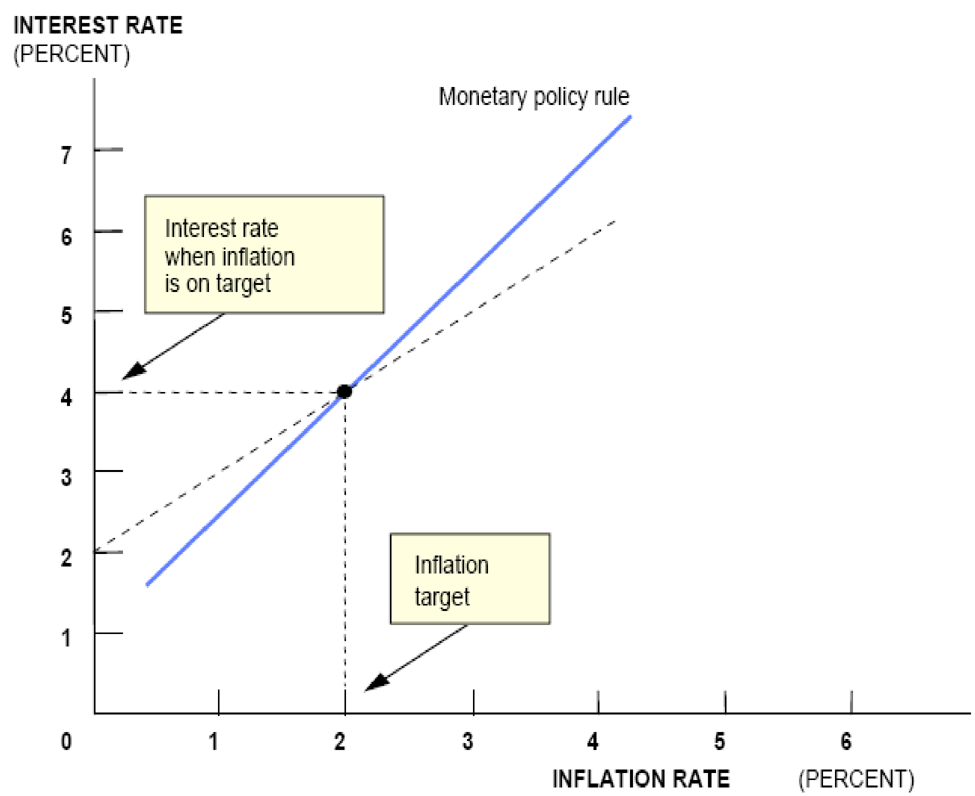
$$\Rightarrow \mathbf{H}^{-1}\boldsymbol{\Gamma}_1 = \mathbf{0}$$

However, if the roots of A are unstable
then there is only one value for
Γ that will not explode: $\mathbf{H^{-1}\,\Gamma_1 = 0}$

Lesson 1: An implication for the coefficients of optimal policy rule

The condition for a unique stationary solution is:

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1 \qquad (6)$$



Lesson 2: There may be inertia in the optimal rule

Observe that there is no lagged dependent variable in the Taylor rule
- The interest rate adjusts immediately, rather than gradually.
-Empirically, a lagged dependent variable (interest rate) appears in estimated policy rules
-- may be due to serial correlation of "errors" or to partial adjustment.

Such a partial adjustment might be optimal if policy makers could commit to it. Why?
- A rule which increases the interest rate later would increase expectations of such an interest rate increase in the future and begin to lower expected inflation and thus actual inflation without any adverse effects on output today.
-- This result can be illustrated in a very simple model…

Illustration of Gains From Inertia

The idea is to include terms in policy rules that exploit peoples' expectations. Simple example:

$y_t = \alpha(r_t + E_t r_{t+1}) + e_t$ (term structure - two periods - affects spending)

$r_t = ge_t + he_{t-1}$ (policy rule for interest rate)

$\min \operatorname{var}(y_t) + \lambda \operatorname{var}(r_t)$ (loss function)

In stochastic steady state:

$\operatorname{var}(y_t) = (\alpha(g+h)+1)^2 + \alpha^2 h^2$

$\operatorname{var}(r_t) = g^2 + h^2$

Optimal policy rule is thus:

$r_t = -(\alpha(\alpha^2+\lambda)e_t + \lambda\alpha e_{t-1})((\alpha^2+\lambda^2)+\alpha^2\lambda)^{-1}$

- Note that for $\lambda = 0$ we have $g = 1/\alpha$ and $h = 0$, but otherwise $h \neq 0$.

- Note that this reaction to e_{t-1} is optimal even though e_{t-1} is not in the basic model. Why?

By moving r_t in response to e_{t-1}, the policy signals that r_{t+1} it will react to $e_{t.}$ *Thus $E_t r_{t+1}$ moves with e_t*. This means that Er_{t+1} shares the work with r_t

But you have to keep with it.

Lesson 3: Rationale for the Loss function

$$(7) \qquad L_t = \pi_t^2 + \lambda(y_t - y_t^n - x^*)^2$$

Can this loss function be viewed as an approximation to maximizing the welfare of the representative individual in the economy?

- Woodford's answer is yes:

--- Target inflation rate is 0 because with staggered pricing, 0 minimizes price dispersion which reduces efficiency

--- The quadratic is an approximate cost of the dispersion.

- Similarly, the utility loss of movements of y away from the flexible price level y^n can be approximated by a quadratic.

--- But target is greater then y^n due to monopolistic competition assumption

--- And y^n is clearly not a linear trend, or any simple trend

--- It depends on technology changes, preference changes, etc.

--- Should not respond to all deviations