Political Science 452: Text as Data

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Where We've Been, Where We're Going

- Class 1: Finding Text Data
- Class 2: Representing Texts Quantitatively
- Class 3: Dictionary Methods for Classification
- Class 4: Comparing Language Across Groups
- Class 5: Texts in Space
- Class 6: Clustering
- Class 7: Topic models
- Class 8: Supervised methods for classification
- Class 9: Ensemble methods for classification
- Class 10: Scaling Speech

Texts and Geometry

Term Document Matrix

Docs	Word1	Word2	...	Word M
Doc1	1	0	...	0
Doc2	0	3	...	1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
DocN	0	0	...	4

Texts and Geometry

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  - Kernel Trick: richer comparisons of documents (Spirling Paper)
  - Basis for clustering, supervised learning

Texts in Space

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$$\begin{aligned}
\textbf{Doc1} \cdot \textbf{Doc2} &= (1, 1, 3, \ldots, 5)^{'} (2, 0, 0, \ldots, 1) \\
&= 1 \times 2 + 1 \times 0 + 3 \times 0 + \ldots + 5 \times 1
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&= 1 \times 2 + 1 \times 0 + 3 \times 0 + \ldots + 5 \times 1 \\
&= 7
\end{aligned}
$$

Length of document:

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\begin{aligned}
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Cosine of the angle between documents:

$$
\begin{aligned}
\cos\theta &\equiv \left(\frac{\textbf{Doc1}}{||\textbf{Doc1}||}\right) \cdot \left(\frac{\textbf{Doc2}}{||\textbf{Doc2}||}\right) \\
&= \frac{7}{6 \times 2.24} \\
&= 0.52
\end{aligned}
$$

Measuring Similarity

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- Minimum: documents have no words in common (orthogonal)
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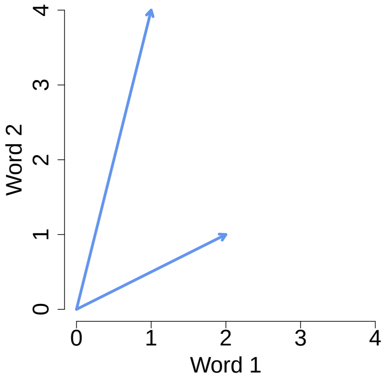
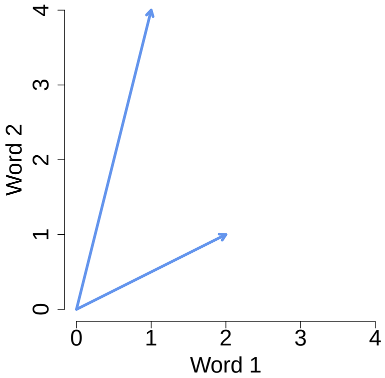
- ? $s(a, b) = s(b, a)$.

Measuring Similarity



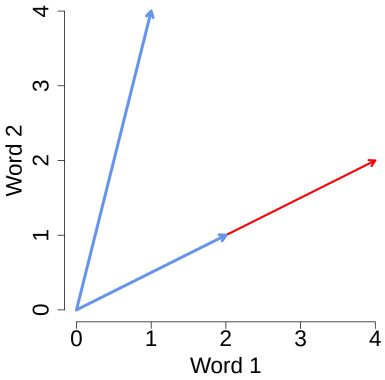
Measure 1: Inner product

Measuring Similarity

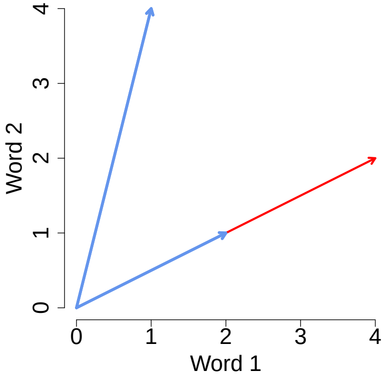


Measure 1: Inner product

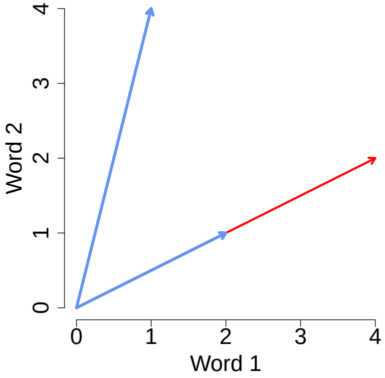
$$(2,1)' \cdot (1,4) \quad = \quad 6$$



Word 2
0
1
2
3
4
0
1
2
3
4
Word 1



Problem(?): length dependent



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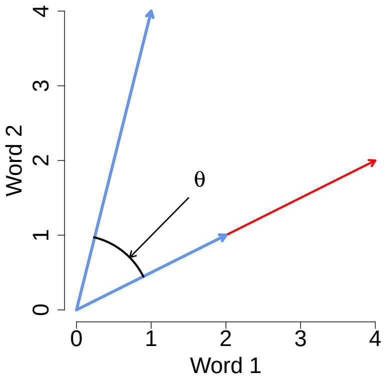
$$(4,2)^{'}(1,4) \quad = \quad 12$$



Problem(?): length dependent

$$(4,2)^{'}(1,4) = 12$$
$$a \cdot b = ||a|| \times ||b|| \times \cos\theta$$

Cosine Similarity

$\cos\theta$: removes document length from similarity measure

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$$\cos\theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right)$$

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\begin{aligned}
\cos\theta &= \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right) \\
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(0.89, 0.45)^{'}(0.24, 0.97) &= 0.65
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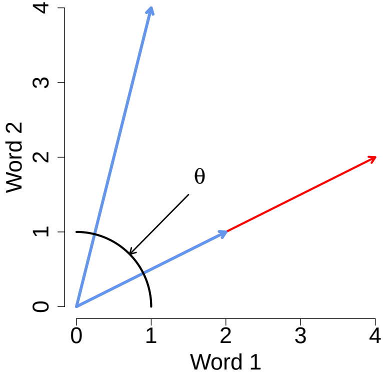

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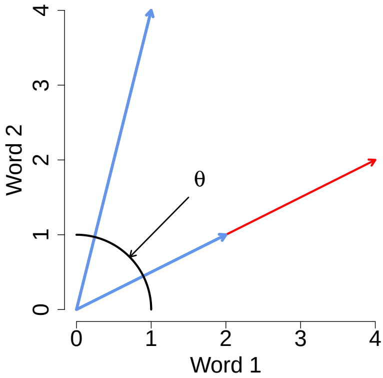

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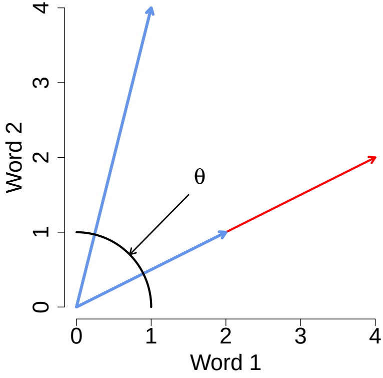

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$\cos\theta \rightarrow$ Inverse distance on Hypersphere

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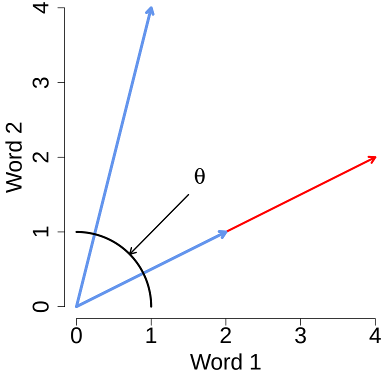

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von Mises Fisher distribution : distribution on sphere surface

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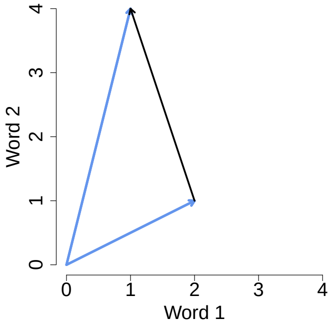

Measure distance or dissimilarity between documents

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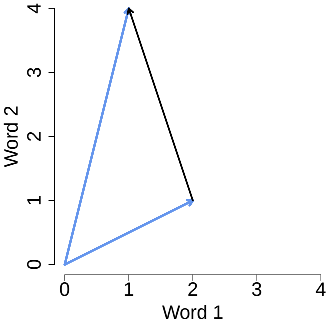

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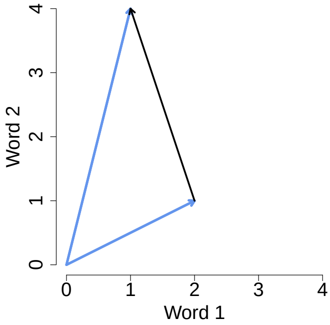

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$$
\begin{aligned}
||\mathbf{a} - \mathbf{b}|| &= \sqrt{(a_1 - b_1)^2 + (a_2 + b_2)^2 + \ldots + (a_M - b_M)^2} \\
||(1,4) - (2,1)|| &= \sqrt{(1-2)^2 + (4-1)^2} \\
&= \sqrt{10}
\end{aligned}
$$

Measures of Dissimilarity

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Minkowski (p) metric

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d_p(\mathbf{a}, \mathbf{b}) &= \left(\sum_{i=1}^{M} (a_i - b_i)^p \right)^{1/p} \\
d_p((1,4),(2,1)) &= ((1-2)^p + (4-1)^p)^{1/p}
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- $\cos \theta = 1 \iff \mathbf{a} = \mathbf{b}$

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- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

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\mathrm{n}_j &= \text{No. documents in which word } j \text{ occurs} \\
\mathrm{idf}_j &= \log \frac{N}{n_j}
\end{aligned}
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Ex. If all statements about OBL contain `Bin Laden` than this contributes nothing to similarity/dissimilarity measures

Inverse document frequency:

$$\begin{aligned} \mathrm{n}_j &= \text{No. documents in which word } j \text{ occurs} \\ \mathrm{idf}_j &= \log \frac{N}{n_j} \\ \mathbf{idf} &= (\mathrm{idf}_1, \mathrm{idf}_2, \ldots, \mathrm{idf}_M) \end{aligned}$$

Weighting Words: TF-IDF Weighting

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- Other functional forms are fine, embed assumptions about penalization of common use

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\mathbf{a}_{\text{idf}} \cdot \mathbf{b}_{\text{idf}} &= (\mathbf{a} \times \mathbf{idf})^{'}(\mathbf{b} \times \mathbf{idf}) \\
&= (\text{idf}_1^2 \times a_1 \times b_1) + (\text{idf}_2^2 \times a_2 \times b_2) + \ldots + (\text{idf}_M^2 \times a_M \times b_M)
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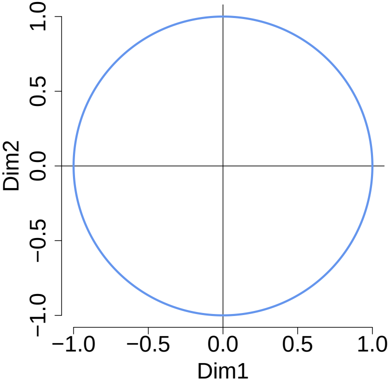
$\leadsto$ You can use $\boldsymbol{\Sigma}$ to modify similarity measures

Some Intuition: The Unit Circle



$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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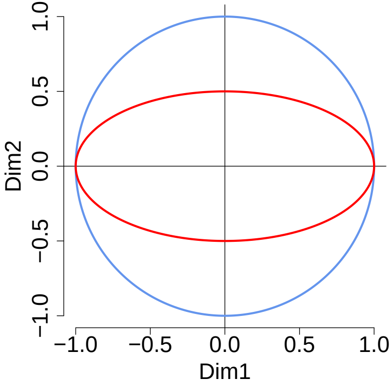

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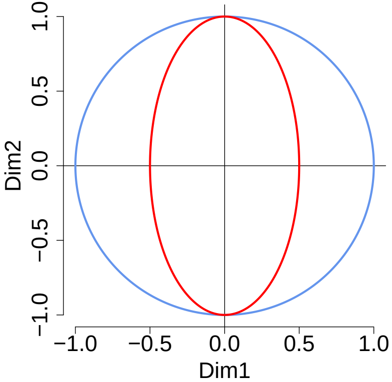

$$\mathbf{\Sigma} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$$

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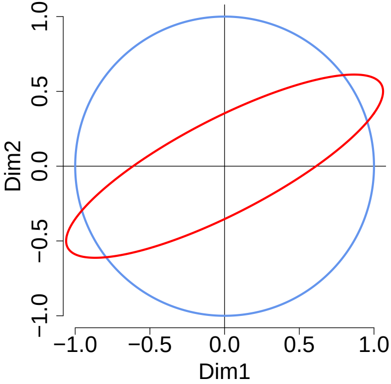

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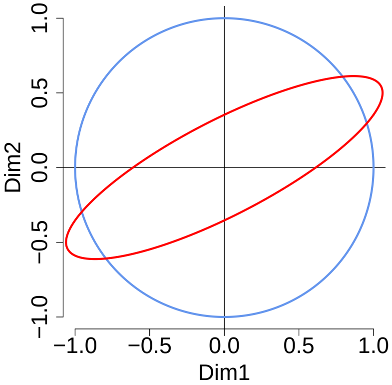

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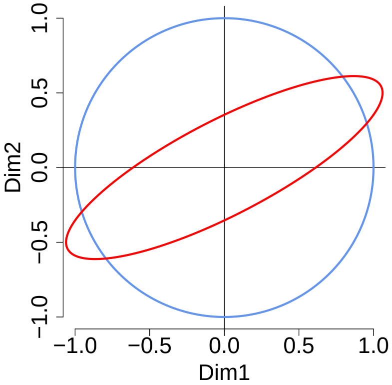

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Lower Triangle contains unique information $N(N-1)/2$

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Learning low-dimensional structure of **D**.

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Key question in Manifold learning (low-dimensional representation of high dimensional data):

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Key question in Manifold learning (low-dimensional representation of high dimensional data):

What are the set of points in $\Re^J$ that "best" approximate points in $\Re^M$?

Classic Multidimensional Scaling Algorithms

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`cmdscale` command in `R`

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Why?

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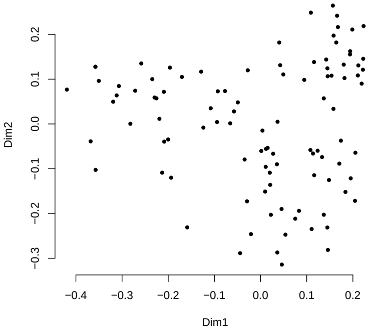
- Many equivalent ways to place documents at same relative positions

Visualizing Documents from Frank Lautenberg

Cosine dissimilarity, Classic MDS



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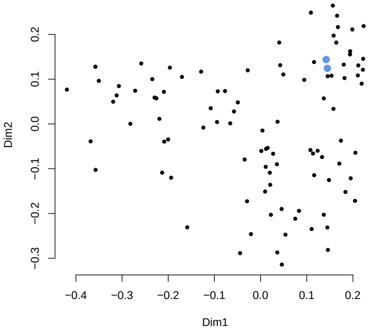

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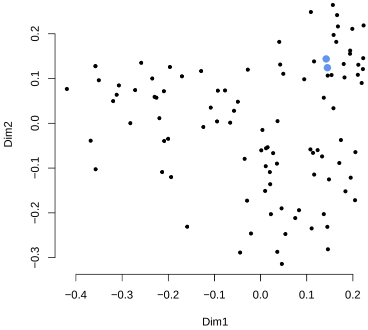

"The intolerance and discrimination we have seen from the Bush administration against gay and lesbian Americans is astounding, and anything but compassionate,"

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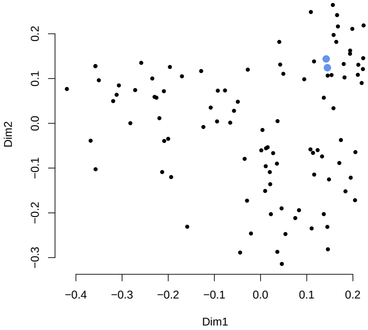

"Such a narrow-minded statement from the U.S. Secretary of Education is unacceptable...For Secretary Paige to say that the upbringing of one class of children offers superior morality compared to other children is offensive and hurtful to people of all other persuasions in America."

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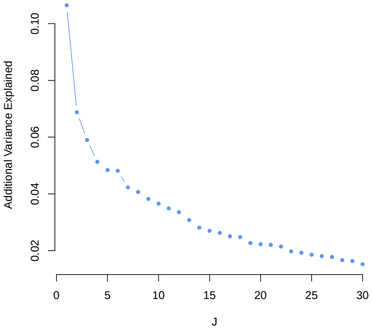

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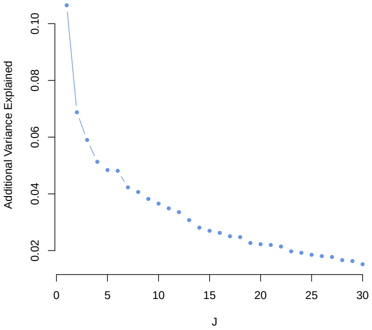

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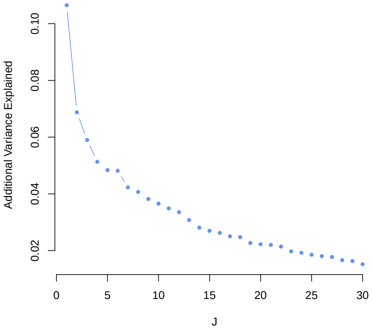

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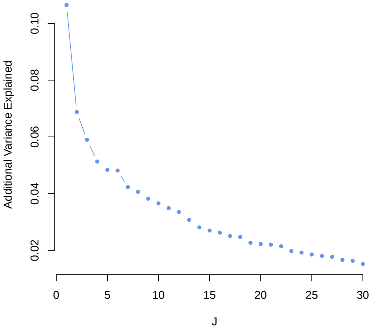

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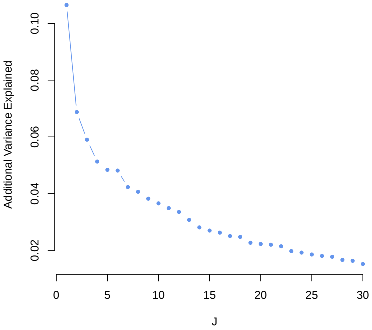

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`library(MASS)`

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sammon
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Sammon MDS

$$\text{Stress}_{\text{Sammon}}(\mathbf{x}) = \sum_{j=2}^{N} \sum_{i<j} \frac{(d(\mathbf{Doc}_j, \mathbf{Doc}_i) - d(\mathbf{x}_j, \mathbf{x}_i))^2}{d(\mathbf{Doc}_j, \mathbf{Doc}_i)}$$

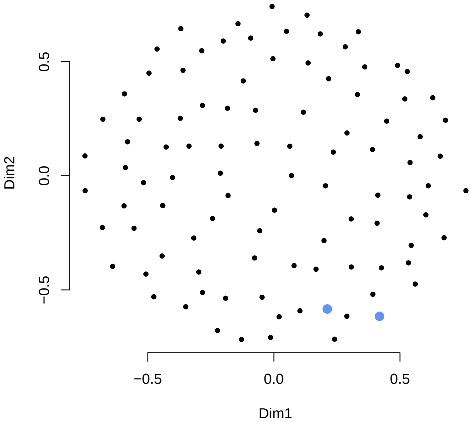
Algorithm "cares" more about small distances $\rightsquigarrow$ prioritizes approximations for small distances

`library(MASS)`

`sammon`

Pro tip: For all document $j \neq k$ $d(j,k) > 0$.

Comparing Sammon and Classic MDS



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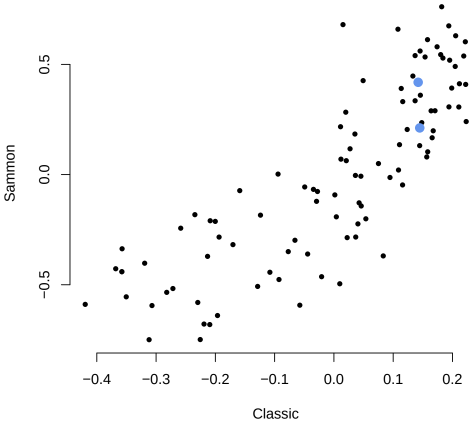

Spirling and Indian Treaties

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- Text representation and similarity calculation
- Projecting to low dimensional space

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How do we preserve word order and semantic language?
After stemming, stopping, bag of wording:

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- No Peace Between Us
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are identical.
Spirling uses complicated representation of texts to preserve word order $\rightsquigarrow$ quite useful

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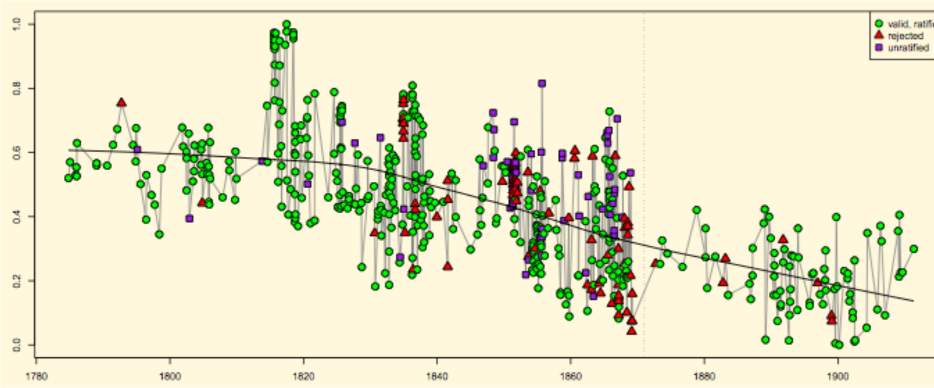
  - Justin gives all his money to Arthur
  - Discard word order: same sentence Kernel : different sentences.

Kernel Trick

Apply kernel methods to simultaneously represent texts, measure similarity

- Creates dissimilarity matrix
- We can use projection methods to scale documents
- Spirling (2011): essentially uses classic MDS on dissimilarity measure

Harshness of Indian Treaties → Credible US Threats



Where We've Been Where We're Going

Today:

- Distance
- Projection

Next weeks:

- Clustering
- Topic Models
- Supervised learning

All require understanding material this week