Math Camp

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Interpreting Causal Effects

Rubin Causal Model:

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- $Y_i(1)$: response under treatment

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$Y_i(1)$ - $Y_i(0)$
ATE $= E[Y(1) - Y(0)]$

Observational Studies and Causal Inference

Experimental studies:

Observational Studies and Causal Inference

Experimental studies:

- Treatment under control of analyst
- Random assignment, estimate

$$\widehat{ATE} = E[Y(1)|T = 1] - E[Y(0)|T = 0]$$

Because $E[Y(1)] = E[Y(1)|T = 1] = E[Y(1)|T = 0]$ and $E[Y(0)] = E[Y(0)|T = 0] = E[Y(0)|T = 1]$

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- Many, many, potential strategies for limiting bias
- Regression is just one method

The Problem of Selection

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Experiments:

- Control assignment, learn about counterfactual values

Observational studies:

- Assignment not controlled
- Problem: how do we learn about counterfactuals in the face of selection?

Manski Bound

What can we learn in the face of selection?

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- Interval that contains true ATE with probability 1
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- Requires no additional assumptions

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Note that:
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So, we can form bounds

$$\begin{aligned} E[Y(1)|T=1]\pi - (1-\pi)E[Y(0)|T=0] - \pi \\ < ATE \end{aligned}$$

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$$[\text{Average(Treat)}\pi - (1-\pi)\text{Average(Control)} - \pi, \quad \text{Average(Treat)}\pi - (1-\pi)\text{Average(Control)} + 1 - \pi]$$

and has length 1.

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- $\pi = 0.06$

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- $\pi = 0.06$
- $E[Y(1)|T=1] = 18/21 = 0.857$
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Bound is then:

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$$[-0.476, 0.523]$$

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- Further assumptions can narrow bounds
- Law of Decreasing Credibility: the credibility of inference decreases with the strength of the assumptions maintained

Causal Inference Via Stratification

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Proposition

Suppose there exists S such that 1) and 2) hold. Then, we can obtain unbiased estimates for

1) $E[ATE|S=1] \equiv$ *ATE, Given* $S=1$
2) $E[ATE|S=0] \equiv$ *ATE, Given* $S=0$
3) *ATE*

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   - Average(Treatment) - Average(Control) , for all units with $S = 1$

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   - Calculate Average(Treatment) - Average(Control) within each strata, $E[ATE|S]$

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3) Average Treatment Effect
   - Calculate Average(Treatment) - Average(Control) within each strata, $E[ATE|S]$
   - $ATE = \sum_{s=0}^{1} E[ATE|S] \times Pr(S)$

Proof

$$E[Y(1)|S = 1, D = 1] - E[Y(0)|S = 0, D = 0]$$

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&\quad +\Pr(D=0|S=1)E[Y(0)|S=1,D=0]\} \\
&= E[ATE|S=1]
\end{aligned}$$

Causal Inference via Stratification: Example

We are interested in the causal effect of incumbency on reelection.

- $T = 1$, Incumbent
- $T = 0$, Challenger

$Y_i(T)$ result of election.
Incumbency obviously not assigned at random.
But suppose we have a dichotomous measure of candidate quality

- $S = 1$, High quality
- $S = 0$, Low quality

And that incumbency is as good as random, given S.
We're interested in obtaining

$$ATE = E[Y(1) - Y(0)]$$

What if we don't condition on candidate quality?

Joint Distribution of Treat and Strata

	$T = 1$	$T = 0$	$P(S)$
$S = 1$	0.4	0.2	0.6
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Confound effect of T with differences across S

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Exact Matching, Basic idea:

- Identify all characteristics (covariates) that affect both outcome and treatment assignment (hint, more than 1!)
- For all treated units, identify control unit with same characteristics
- Exact match: units in the same strata

What do we do?

Most strata are empty, or only a few observations

- Bias-Variance tradeoff
  - Bias: assume same casual effect across strata
  - Variance: assume different causal effect across strata
- Modeling $E[Y|X_1, X_2, X_3, \ldots, X_K]$
  - Nonparametric (loess): different curse of dimensionality problem
  - High dimensional space is sparse, hard to borrow across

Solution: specify a model of how covariates relate to treatment

Linear Regression

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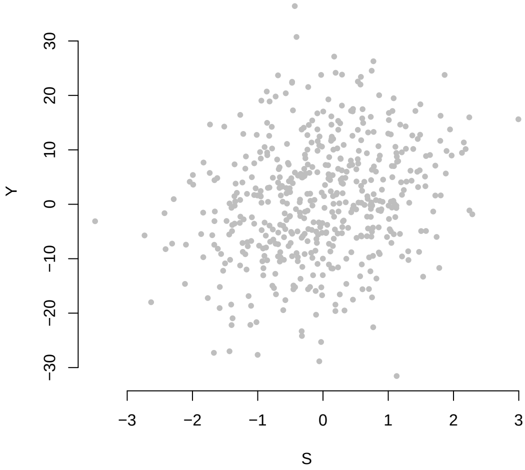
- Borrowing information across bins:
  - Assuming that means have a global and linear movement with S, β_1

Linear Regression



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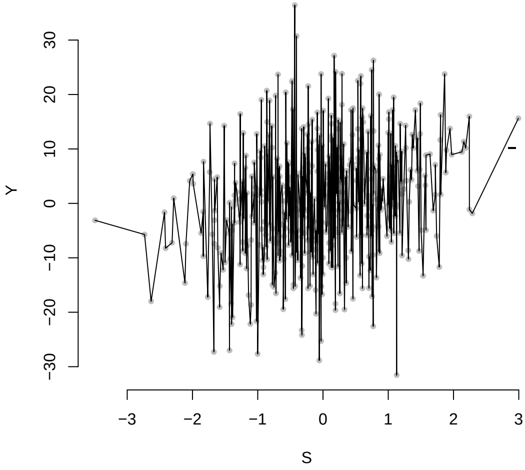

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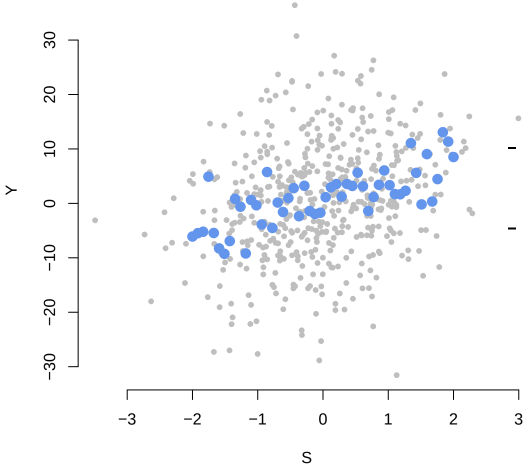

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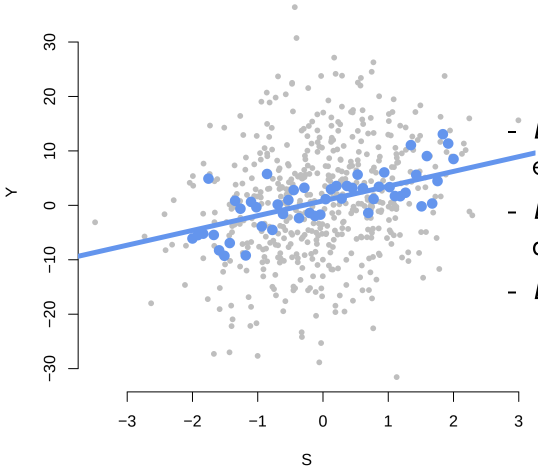

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For every random variable Y and S we can always write the random variable as,

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$$(\beta_0^*, \beta_1^*) = \texttt{argmin}_{\beta_0, \beta_1} \sum_{i=1}^{N} \epsilon_i^2 \quad (0.1)$$

For every random variable Y and S we can always write the random variable as,

$$\begin{aligned} Y_i &= \underbrace{Y_i - E[Y|S]}_{\epsilon_i} + E[Y|S] \\ Y_i &= E[Y|S] + \epsilon_i \\ Y_i &= \beta_0 + \beta_1 S_i + \epsilon_i \end{aligned}$$

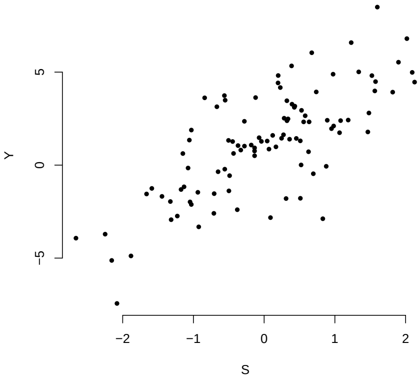
Where we have used our assumption about $E[Y|S] = \beta_0 + \beta_1 S_i$
We'll define our residuals to be,

$$\begin{aligned} \epsilon_i &= Y_i - (E[Y|X]) \\ &= Y_i - (\beta_0 + \beta_1 S_i) \end{aligned}$$

We are going to find the β_0^*, β_1^* that minimize the sum of squared residuals,

$$(\beta_0^*, \beta_1^*) = \texttt{argmin}_{\beta_0, \beta_1} \sum_{i=1}^{N} (Y_i - \beta_0 - \beta_1 S_i)^2$$

Graphically:



$$\beta_0^* = \overline{Y} - \beta_1^* \overline{S}$$
$$\beta_1^* = \frac{\sum_{i=1}^{N}(S_i - \overline{S})Y_i}{\sum_{i=1}^{n}(S_i - \overline{S})^2}$$
$$\beta_1^* = \frac{cov(S, Y)}{var(S)}$$

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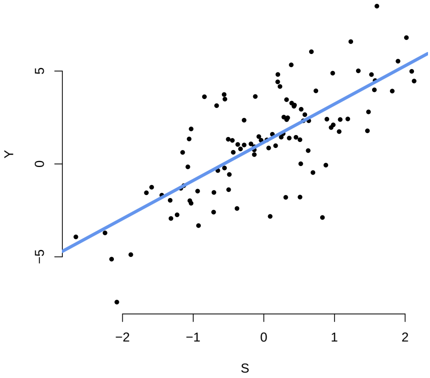

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You get one causal effect per regression:

- The whole point of the X's is just to replicate experimental conditions
- Not to estimate separate causal effects

TABLE 4. Panel Regression of County-level Unemployment and the Democratic Percent of the Two-party Vote for President, 1996–2008

	I Coefficient (standard error)[a]	II Coefficient (standard error)	III Coefficient (standard error)	IV Coefficient (standard error)
Unemployed (percent in county)	.176**		.203***	.198***
	(.086)		(.061)	(.058)
Change in unemployment from previous year		.113		.079
		(.105)		(.101)
State unemployment		−.067	−.229	−.243
		(.613)	(.687)	(.589)
Median household income ($1000s)	.030	.015	.030	.028
	(.036)	(.037)	(.037)	(.037)
Democratic vote in previous election	.906***	.914***	.905***	.906***
	(.018)	(.018)	(.084)	(.019)
Percent urban	.017	.017*	.017*	.017*
	(.009)	(.010)	(.010)	(.010)
Percent African American	.096***	.096***	.095***	.095***
	(.009)	(.008)	(.008)	(.008)
Percent without high school diploma	.085***	.097***	.083**	.085**
	(.030)	(.033)	(.035)	(.034)
Percent with four-year college degree or more	.130***	.129***	.132**	.132*
	(.047)	(.045)	(.053)	(.052)
Percent aged 18–30	.029	.013	.029	.026
	(.025)	(.027)	(.030)	(.029)
Percent 65 or older	−.013	−.029	−.013	−.016
	(.021)	(.023)	(.025)	(.024)
Constant	−8.29***	−6.67**	−7.30**	−7.14**
	(1.27)	(3.23)	(3.39)	(3.06)
N	12,444	12,444	12,444	12,444
N counties	3,111	3,111	3,111	3,111
N years (fixed)[b]	4	4	4	4
R-squared: overall (within years)	.84 (.92)	.84 (.91)	.84 (.92)	.84 (.92)

[a]Standard errors are bootstrapped with 250 replications; significance tests based on the normal distribution.
[b]State effects were fixed through inclusion of state-dummy variables not reported here.
*statistical significance at .10, two-tailed test;

See you in the spring!!