Abstracts

Speaker: Noga Alon, Princeton U. and Tel Aviv U.

Title: Benny and List Coloring

Abstract: The list chromatic number of a graph G is the minimum k so that for every assignment of a list of k colors to any vertex of G there is a proper vertex coloring assigning to each vertex a color from its list. This notion was introduced by Vizing and by Erdos, Rubin and Taylor in the late 70s and its study combines combinatorial, probabilistic and algebraic techniques. I will describe some of the contribution of Benny Sudakov to the subject and discuss several old and several new problems and results in the area.

Speaker: Michael Krivelevich, Tel Aviv University

Title: A hitting time result for Hamiltonicity in subgraphs of dense graphs

Abstract: Hamiltonicity questions have long been one of Benny's favorite research subjects, the passion I was happy to share, in particular by working him in this direction extensively. This talk is too about Hamiltonicity.

Let G be a graph on n vertices with minimum degree at least $\left(\frac{1}{2}+\epsilon\right)n$, for some fixed $\epsilon > 0$. Then G is Hamiltonian due to Dirac's theorem.

Consider now the random subgraph process on G: let σ be a random permutation of the edges of G, start with the empty graph on $V(G)$, and add edges of G one by one according to σ, thus getting a nested sequence (G_i) of subgraphs of G. We show that for a fixed positive integer k, with high probability with respect to a random permutation σ exactly at the first moment i the minimum degree of G_i becomes $2k$, the subgraph G_i contains k edge-disjoint Hamilton cycles. This extends the result of Tony Johansson who proved it for $k = 1$ (i.e. for the appearance of a Hamilton cycle), and answers a question by Alan Frieze.

A joint work with Yahav Alon.

Speaker: David Conlon, Caltech and University of Oxford

Title: Sidorenko's conjecture for blow-ups

Abstract: A celebrated conjecture of Sidorenko and Erdős–Simonovits states that, for all bipartite graphs H, quasirandom graphs contain asymptotically the minimum number of copies of H taken over all graphs with the same order and edge density. This conjecture has attracted considerable interest over the last decade and is now known to hold for a broad range of bipartite graphs, with the overall trend saying that a graph satisfies the conjecture if it can be built from simple building blocks such as trees in a certain recursive fashion.

Our contribution here, which goes beyond this paradigm, is to show that the conjecture holds for any bipartite graph H with bipartition $A \cup B$ where the number of vertices in B of degree k satisfies a certain divisibility condition for each k. As a corollary, we have that for every bipartite graph H with bipartition $A \cup B$, there is a positive integer p such that the blow-up H_A^p formed by taking p vertex-disjoint copies of H and gluing all copies of A along corresponding vertices satisfies the conjecture. Another way of viewing this latter result is that for every bipartite H there is a positive integer p such that an L^p-version of Sidorenko's conjecture holds for H.

Joint work with Joonkyung Lee.

Speaker: Matthew Kwan, Stanford University

Title: Ramsey Graphs

Abstract: An n-vertex graph is called C-Ramsey if it has no clique or independent set of size $C \log n$. As one of the first applications of the probabilistic method, Erdos proved that 2-Ramsey graphs exist, but our understanding of Ramsey graphs is still extremely limited. For example, there are still no known explicit constructions of C-Ramsey graphs, for any constant C.

Benny and his coauthors have had a large impact in an ongoing line of research towards understanding the structure of Ramsey graphs and in particular showing that they must satisfy certain "richness" properties characteristic of random graphs. I'll talk about some of Benny's earlier work on this subject, some of my recent joint work with Benny, and some new directions joint with Lisa Sauermann.

Speaker: David Gamarnik, MIT

Title: The Hidden Clique Problem. 20 years later

Abstract: In their seminal paper of 1998 Alon, Krivelevich and Sudakov have established that a clique imbedded into a random graph can be recovered using a polynomial time (spectral) method when its size is at least the square root of the size of the graph. Repeated failed attempts to break this square root barrier raised the possibility that the problem is algorithmically hard below the square root regime, but this unfortunately still remains only a conjecture. The question gained prominence not only in the field of random graphs, but also in the fields of theoretical computer science and the high-dimensional statistics, earning the status of the "mother" of many possibly average-case hard inference problems.

As an attempt to explain the conjectured hardness below the square root threshold, and inspired by insights from the field of spin glasses, we study the solution space geometry of the hidden clique problem, both below and above the conjecturally hard threshold. Specifically, we consider the densest subgraph problem as a function of an intersection size with the planted clique. Based on the first moment approximation, we prove that the model exhibits a certain monotonicity phase transition around the conjectured hardness threshold, and for certain parameter choices exhibits the Overlap Gap Property (OGP): every sufficiently dense subgraph either contains a substantial fraction of the hidden clique or has almost no intersection with it. The evidence of OGP thus obtained is then rigorously verified using the second moment method, albeit only in the case when the hidden clique size is a small power of the graph size.

Joint work with Ilias Zadik (MIT/NYU).