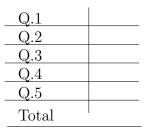
Mathematics Department Stanford University Math 61DM Practice Midterm Exam I

Unless otherwise indicated, you can use results covered in lecture and homework, provided they are clearly stated.

If necessary, continue solutions on backs of pages

Note: work sheets (the final two pages here) are provided for your convenience, but will not be graded



Name (Print Clearly):

I understand and accept the provisions of the honor code (Signed)

1 Let \Box denote the 1 × 1 square and Γ be the 2 × 2 square with one of the four 1 × 1 squares removed.

(a): Let f(n) be the number of ways to tile a $2 \times n$ rectangle by Γ . Determine and prove an explicit formula for f(n).

(b): Let g(n) be the number of ways to tile a $2 \times n$ rectangle by Γ and \Box .

(i) Determine a recursive formula for g(n).

(iii) Explain why there are constants a, b, c such that $g(n) = a\alpha^n + b\beta^n + c\gamma^n$ for all n, where α, β, γ are the solutions to $x^3 - x^2 - 4x - 2 = 0$.

2 Let

be a matrix with entried in \mathbb{R} . Using Gaussian elimination, find a basis for the null space N(A). What is the dimension of the column space C(A)? (Explain your answer.) Describe how you could find a basis for C(A). **3** Let $v = (a_1, a_2, a_3)^T$ and $w = (b_1, b_2, b_3)^T$ be two column vectors in \mathbb{R}^3 (written with respect to the standard basis), and define a function $T : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T(x) = (x \cdot w)v - (x \cdot v)w.$$

(Here $x \cdot w$ is the dot product of x with w.) Explain what it means for T to be a linear transformation, and show that it actually is one. Then calculate the matrix A associated to T with respect to the standard basis e_1, e_2, e_3 .

4 (a) What does the oddtown theorem say? Why is it true?

(b) Suppose a town with n citizens has m clubs C_1, \ldots, C_m of one type, and m clubs D_1, \ldots, D_m of another type such that

- for each i, C_i and D_i have an odd number of members in common, and
- if i > j, then clubs C_i and D_j have an even number of common members.

Prove that $m \leq n$.

5 Suppose that V is a k-dimensional vector space, and suppose that w_1, \ldots, w_ℓ are a set of linearly independent vectors in V. Give a detailed proof that $\ell \leq k$.

work-sheet 1/2

.

work-sheet 2/2

•