1. Prove that every graph on \( n \) vertices contains a path with three edges, or its connected components are triangles or stars. Deduce that any graph on \( n \) vertices with more than \( n \) edges contains a path with three edges.

2. Prove that every graph on \( n \) vertices with minimum degree greater than \( n/2 \) contains a cycle of length \( n \).

3. Prove that every graph on \( n \) vertices contains an \( \epsilon \)-regular pair of subsets each of order at least \( \delta n \) with \( \delta = 2^{-\epsilon-O(1)} \).

4. A subset \( X \subset V(G) \) of vertices of a graph is \( \epsilon \)-regular if the pair \( (X, X) \) is \( \epsilon \)-regular. Prove that for each \( \epsilon > 0 \) there is \( \delta > 0 \) such that every graph on \( n \) vertices contains a subset of vertices on at least \( \delta n \) vertices which is \( \epsilon \)-regular.

**Note:** It is open as to whether or not we can take \( \delta \) to be single-exponential, i.e., \( \delta = 2^{-\epsilon-O(1)} \). The best known bound is double-exponential.

5. Prove that every 3-uniform hypergraph on \( n \) vertices in which no six vertices contains three edges has \( o(n^2) \) edges.

6. Prove that for each \( \epsilon > 0 \) and graph \( H \) there is \( \delta > 0 \) such that every graph on \( n \) vertices contains an induced copy of \( H \) or has a vertex subset of size at least \( \delta n \) with edge density at most \( \epsilon \) or at least \( 1 - \epsilon \).

**Note:** It is open as to whether or not we can take \( \delta \) to be polynomial, i.e., \( \delta = \epsilon^{c(H)} \) for some \( C(H) \) depending only on \( H \). A polynomial bound would imply the Erdős-Hajnal conjecture, that every induced \( H \)-free graph on \( n \) vertices contains a clique or independent set of order \( n^{c(H)} \) for some \( c(H) > 0 \).

7. Prove that for any \( \epsilon > 0 \) there is \( \delta > 0 \) such that if a graph on \( n \) vertices has at least \( \epsilon n^2 \) edges, then it contains a \( d \)-regular subgraph with \( d \geq \delta n \).