Peer Pressure

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Abstract

We present a model where agents care about their neighbors' actions and can pressure them to take certain actions. Exerting pressure is costly for the exerting agent and it can impact the pressured agents by either lowering the cost of taking the action (which we call "positive pressure") or else by raising the cost of not taking the action (which we call "negative pressure"). We show that when actions are strategic complements, agents with lower costs for taking an action pressure agents with higher costs, and that positive pressure can improve societal welfare. More generally, we detail who gains and who loses from peer pressure, and identify some circumstances under which pressure results in fully (Pareto) optimal outcomes as well as circumstances where it does not. We also point out differences between positive and negative pressure.

Keywords: Peer pressure, peer effects, complementarities, externalities, public goods, strategic complements, strategic substitutes

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*Toni passed away in November of 2007, when we were in the midst of this project. His friendship, energy, and talents are sorely missed.

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1 Introduction

The catalog of settings where peers influence our decisions is large and varied, including our choices to engage in criminal behavior, smoke, perform charitable acts, follow styles and trends, educate ourselves, select a certain profession, adopt a new technology or buy a given product.[1] In understanding how peer influence operates it is useful to distinguish between two different ways in which peers affect decisions: passively and actively. Most of the literature has focused on the passive case, where an agent's behavior is influenced by others' behaviors, but not necessarily because of explicit lobbying by others. This includes learning effects, where information about the benefits of taking an action are communicated from one agent to another, as well as externalities where an agent's relative payoffs from different actions are affected by the behavior of others. Beyond such passive peer effects, there are also active peer effects where an agent takes a deliberate action at a cost to him or herself in order to influence other agents' choices of action. This includes behavior such as helping to subsidize another agent's action, actively lobbying another agent or even bullying or daring another agent in order to influence his or her behavior. Such active peer influence, although quite prevalent, is less studied from both an empirical and theoretical perspective than passive peer effects.[2] Since such "peer pressure" is important in determining the behavior and welfare of a given individual and, through its external effects, of a whole society, it is important to model and understand it. In this paper, we provide a simple model of peer pressure and investigate its properties.

A central question that we investigate here is whether or not peer pressure can lead to higher welfare relative to settings without any peer pressure. The idea is that in settings with externalities, the ability for one agent to pressure a second agent could get the second agent to internalize the impact that his or her actions have on the first agent. For instance, if one agent would like to have a second one join him or her in attending some event, then offering to pay for some of the second agent's expenses could lead to a welfare improving outcome. Thus, even though the term "pressure" embodies some negative connotations, the presence of externalities suggests that the ability of agents to influence each other's decisions could be welfare improving.[3]

[1] For surveys on various aspects of peer effects see the Handbook of Social Economics (forthcoming).

[2] There are, of course, exceptions and some of the empirical studies cited below measure active forms of peer pressure. For example, Brown, Clasen, and Eicher (1986, p. 523) measure peer pressure through surveys that identify pressure as "when people your own age encourage or urge you to do something or to keep from doing something else, no matter if you personally want to or not."

[3] Some of the empirical work on peer pressure among teens finds some aspects of welfare improving in that there is more perceived pressure for peer involvement than for misconduct

To investigate this question about the properties and potential welfare improvements of peer pressure, we build a model of active peer pressure where agents, at a cost, may change other agents' costs or benefits from various actions. We distinguish between two types of peer pressure: positive and negative. In both cases, one agent chooses an amount of pressure to exert on another agent and incurs a cost for such pressure. In the case of positive pressure, the pressure exerted by one agent reduces the cost that other agents face for taking a particular action, thus encouraging other agents to take the action. Essentially, this is like subsidizing other agents' activity. In the case of negative pressure, the pressure exerted by one agent increases other agents' costs of not taking an action. That is, one way to encourage an agent to take an action is to make it more costly for him or her not to take the action. There is empirical evidence that both sorts of peer pressures are observed (e.g., see Brown (1982), Brown, Clasen, and Eicher (1986), and Santor, Messervey, and Kusumakar (2000)), and so it is important to understand both types of peer pressure. While one might be tempted to conclude that positive peer pressure will be beneficial and negative peer pressure will be harmful, the picture is more nuanced. In particular, conclusions of whether or not peer pressure is welfare enhancing or inhibiting depends on the setting and the type of pressure. We find that positive peer pressure will generally lead to Pareto improvements relative to an absence of any pressure, but can fall short of leading to fully Pareto efficient outcomes. Negative peer pressure can lead to Pareto efficient outcomes, but differentially impacts agents, benefiting some and hurting others. Moreover, there are not necessarily clear Pareto rankings between positive and negative pressure.

To fix ideas, consider an example of smoking. Agents can either smoke or not. There are externalities and so generally agents prefer to have other agents not smoke. There are two ways in which agents can promote this outcome: they can either make it costly for other agents to smoke by harassing them if they do, advertizing the negative affects of smoking, fining individuals for smoking in public places and so forth; or else they can help agents not to smoke by subsidizing programs to help others quit smoking and generally rewarding nonsmoking behavior.[4] In the case of negative pressure where smoking is penalized, agents who quit smoking due to pressure end up with a lower expected utility than in the world where they are not pressured, while the pressuring agents are weakly better off (as otherwise they could choose not to exert the pressure).[5] Under

(e.g., see Brendt (1979) and Brown, Clasen, and Eicher (1986)).

[4]The analysis in the paper concentrates on the case of positive externalities. However, this application can be remapped into one with positive externalities by thinking of smoking as a default action and noting the improved welfare of others if an agent stops smoking.

[5]Clearly, this assumes rational agents who are choosing to quit or not to quit. The analysis would change if one presumes that agents are not rational in their decision making and do not

positive pressure all agents would be (weakly) better off and so there would be a Pareto improvement due to the pressure. Thus, we see a different pattern between positive and negative peer pressure. This does not necessarily imply that positive pressure is always an unambiguously better instrument, as it may be that there are different impacts on the pressuree per unit of money or utility spent by the pressurer when one compares positive versus negative peer pressure. That is, the technology for harassing, policing, fining, etc., might be more cost effective than the technology for subsidization of good behavior. This example is just meant to be suggestive, but provides an idea of the type of questions that the model provides insight into.

The most closely related predecessor to our analysis is a paper by Kandel and Lazear (1992), who studied peer pressure in agency problems. Kandel and Lazear point out the advantages of peer pressure in improving performance of groups of agents who can observe each other's actions, and they show that various psychological pressures such as guilt and shame, as well as other sorts of monitoring, can lead partnerships to have higher productivities than other sorts of organizational structures. We provide a model of peer pressure in a wider setting, model pressure differently, and investigate its equilibrium and welfare properties, as well as distinguishing between positive and negative pressure, and so our results do not overlap with those of Kandel and Lazear (1992).

Our paper is also related to a broader class of common agency models where some players can try to influence others' actions by offering them payments to take certain actions. This includes analyses by Prat and Rustichini (2004) who document the efficiency of transfers in set of common agency problems; as well as the role of transfers in more general game theoretic settings as analyzed by Jackson and Wilkie (2005). Although those papers are related in that some agents can make payments to influence other agents' actions, the models and analyses are quite different to that developed here, both in structure and intended application. Most importantly, the ways in which transfers can be made, distinctions between positive and negative pressure, as well as how the conclusions depend on complementarities, are special to our paper. For example, in Jackson and Wilkie (2005) an agent can make transfers that are fully contingent on all actions, and not just another agent's action. The interpretation of such contingencies is quite different than pressuring another agent and allows agents to do things like commit not to play certain actions by promising to pay large amounts if they take those actions. Thus, one cannot apply the Jackson and Wilkie results to conclude things about peer pressure. In addition, our focus here is primarily on games of strategic complements, and the more structured setting will allow for more direct

properly account for their own utility.

conclusions.

2 A Model of Peer Pressure

2.1 Actions and Base Payoffs

A community of $n \geq 2$ agents each choose whether or not to undertake a given action.

Each agent i chooses an action $x_i \in \{0, 1\}$. Agents' payoffs depend on other agents' actions. The base payoff (ignoring peer pressure) to agent i is

$$v_i(x_i, x_{-i}) - c_i x_i$$

where c_i is a cost parameter that is specific to the agent and the function v_i captures other aspects of the payoffs of taking action x_i.

We restrict attention to situations where v_i is nondecreasing in x_{-i} for each x_i. This embodies situations with positive externalities, which captures many important applications. [6]

In addition to externalities, incentives for agents to pressure each other arise from the interaction of their payoffs. If one agent's action has no impact on any other agent's payoff, then that agent would not be pressured. Thus, in order to understand peer pressure, we need to keep track of how one agent's action impacts another agent's payoff.

In particular, we focus on the case where $v_i(1, x_{-i}) - v_i(0, x_{-i})$ is nondecreasing in x_{-i} for all i. This is the well-known situation of strategic complements.

We comment on the extension to the case strategic substitutes in the conclusion.

In showing some of the results below, it will be useful to define a class of games where the payoff to agents taking action 0 is independent of x_{-i}.

A game is a *participation game* if $v(0, x_{-i})$ is independent of x_{-i}.

In a participation game, if an agent takes action 1, like attending a social event, then he or she cares about how many other agents take that action; but in contrast, if the agent takes action 0, such as staying home, then he or she does not care about how many other agents take action 1 or 0.

Another useful definition is that of benefit symmetry. Payoffs exhibit *benefit symmetry* if all agents have the same payoff function $v_i = v$ which depends on x_{-i} only via $\sum_{j \neq i} x_j$, so that $v_i(x_i, x_{-i}) = v(x_i, \sum_{j \neq i} x_{-i})$.

[6] One cannot simply relabel actions to accommodate negative externalities, as for instance, strategic incentives might still exhibit complementarities while externalities are negative. Thus, such cases need to be investigated independently, and are left for future research.

Under benefit symmetry, the benefits from actions are similar across agents, and the heterogeneity enters only in terms of the cost of the action for different agents.

2.1.1 Examples

The following examples provide a glimpse of some applications covered by the model.

EXAMPLE **1** *Symmetric Games and Thresholds*

Caring about other players symmetrically implies that

$$v_i(1, x_{-i}) - c_i \geq v_i(0, x_{-i}) \text{ if and only if } \sum_{j \neq i} x_j \geq t_i(c_i)$$

where $t_i(c_i)$ is a threshold. In particular, if more than $t_i(c_i)$ other players choose action 1, then it is best for player i to choose 1, and if fewer than $t_i(c_i)$ other players choose action 1 then it is better for player i to choose action 0. A special case is where $v_i(1, x_{-i}) = a_i \left(\sum_{j \neq i} x_j\right)$ and $v_i(0, x_{-i}) = 0$; and then the threshold is $t_i = c_i/a_i$.

EXAMPLE **2** *A Two-Person Coordination Game*

A special case is such that there are two agents ($n = 2$), and then payoffs are represented as

		Agent 2′s Action: 1	0
Agent 1′s Action	1	$a_1 - c_1, a_2 - c_2$	$-c_1, 0$
	0	$0, -c_2$	$0, 0$

This is a standard coordination game whenever $a_i > c_i$.

EXAMPLE **3** *Graphical Games.*

In the case of a graphical game[7] there is a network such that each agent only cares about the choices of his or her direct neighbors in the network. That is, for each agent i there is a set of other agents $N_i \subset N \setminus \{i\}$ who are i's neighbors in the network, such that $v_i(x_i, x_{-i})$ depends only on x_i and x_{N_i}.

[7] Graphical games were first defined by Kearns, Littman, and Singh (2001). For more discussion of such games see Jackson (2008).

2.2 Positive and Negative Peer Pressure

In addition to choosing an action, agents can choose to "pressure" other agents, which is modeled as follows. Agent i exerts an amount of pressure $p_{ij} \in [0, M]$ on agent j. Exerting pressure is a costly activity for agent i who incurs a cost p_{ij}.

The effect of pressure by an agent i on agent j is that it changes the incentives for j to take a higher action. In terms of strategic considerations, it is irrelevant as to whether we think of pressure as making it less costly to take a higher action or more costly to take a lower action. However, when we do welfare calculations this is an important distinction. Thus, we distinguish between positive and negative peer pressure.

Under *positive peer pressure*, the pressure from other agents lowers the cost for a given agent of taking action 1. Under positive peer pressure the overall payoff to an agent is

$$v_i(x_i, x_{-i}) - \left(c_i - \sum_j p_{ji} \right) x_i - \sum_j p_{ij}. \tag{1}$$

An example of positive peer pressure is where one agent wants to attend a concert and would prefer to be accompanied by a second agent who prefers not to attend the concert. The first agent offers to pay for the second agent's ticket in order to encourage the second agent to attend the concert. More generally, the applications covered by this sort of peer pressure include any offers by agents to subsidize the activities of others, including, for instance, countries offering aid to other countries to help influence their actions.[8]

Under *negative peer pressure* the pressure from other agents raises the cost for a given agent taking action 0. Under negative peer pressure the overall payoff to an agent is written as

$$v_i(x_i, x_{-i}) - c_i x_i - \left(\sum_j p_{ji} \right) (1 - x_i) - \sum_j p_{ij}. \tag{2}$$

Such negative peer pressure appears in a variety of collective action problems. A classic example is of "burning bridges," with stories of Roman generals burning their boats upon an invasion so as to eliminate the option of retreat (which can be viewed as making the action of retreat prohibitively expensive) and thus to encourage the soldiers to fight to the death. Burning boats is clearly a costly act for the generals and it does not lower the cost of fighting for the soldiers, but instead raises the cost of not fighting: encouraging the soldiers to fight when

[8] For an example where charities expend resources to influence agents' utility from donating to the charity, see Weinberg (2006).

they might otherwise have fled. A well-known example of the destruction of ships is by Cortés in 1519 in Mexico. While the full story is more nuanced than the legend (e.g., see Reynolds (1959)), the accounts indicate that Cortés disabled his ships with the intent of affecting his soldiers' behavior. Often such examples are discussed as evidence of the value of commitment. The more general approach here entails not committing to a given action by removing other options (often by a player him or herself and sometimes with "motivational" reasons in mind), but on having some players try to influence others' behaviors by making some actions costly and thus making them relatively unattractive and other actions more attractive. More generally, other examples include various sorts of bullying and threats that sometimes are exhibited in various social interactions.[9,10]

Under either positive or negative peer pressure the difference between taking action 1 and action 0 is

$$d_i(x_{-i}, p) = v_i(1, x_{-i}) - v_i(0, x_{-i}) - c_i + \sum_j p_{ji}. \tag{3}$$

It is clear that under either form of peer pressure, increased pressure from other agents makes action 1 more attractive to a given agent.[11]

Finally, note that the cost of pressuring here is incurred regardless of whether it has an impact. That is, under positive pressure an agent pays to subsidize another agent's action 1 regardless of whether that other agent takes that action. If we allowed an agent to save the cost of positive peer pressure if another agent did not end up taking action 1, then that would not substantively affect the actions supported in equilibrium or the payoffs.[12] In terms of negative peer pressure, allowing for payment only in the event that the action is taken can change the conclusions. First, part of the inefficiency associated with negative peer pressure is that an agent incurs a cost (like the Cortés example above, where

[9] For an interesting example where firms try to pressure undesired workers to quit by making their work unpleasant, see Wasmer (2008).

[10] Note that we allow agents to pressure themselves. While one might want to rule this out in some settings, there are others where it makes sense. For instance, burning boats or bridges affects the general's ability to leave and not just the soldiers. Generally, "pressure" encompasses a broad array of activities that might be costly up front and change incentives later on, including forms of costly self-commitment.

[11] Note that although we allow agents to pressure others to take action 1 or not to take action 0, we do not consider the reverse. For instance, if 0 and 1 represent two different technologies, and some agents have naturally strong preferences for technology 1 and others for technology 0, then it is conceivable that these strongly predisposed agents would compete in pressuring the remaining agents to try to get them to adopt a given technology and not the other. Such competition is outside of the scope of this paper, but such an extension is worth considering.

[12] There are some new equilibria that arise where an agent promises to subsidize actions that he or she knows will never be taken, but that does not have any consequence for the conclusions of the paper.

ships were lost). If Cortés could somehow have committed to destroying the ships only in the case where some soldiers tried to use them for escape, that would have saved the cost of losing the ships. Beyond this, allowing for threats that never have to be paid for can change the nature of the game:, an agent could commit to making some action outrageously costly for another agent, knowing that this cost will never have to be paid. Having to pay the cost up front puts a sort of credibility check on the pressuring agent, making sure that the cost incurred is something that the pressuring agent is willing to pay in order to achieve a given outcome.

2.3 The Timing of Actions and Equilibrium

Actions take place in two stages.

- In the first stage, agents simultaneously choose how much to pressure other agents. So, agent i chooses $p_i = (p_{i1} \ldots, p_{in}) \in [0, M]^n$. The pressuring activity is publicly observed, so all agents see the pressures exerted by all agents.
- In the second stage agents simultaneously choose actions x_i.

The following notation will be useful. Let $\sigma_i(p)$, where $\sigma_i : [0, M]^{n^2} \rightarrow [0, 1]$, denote the probability with which player i plays action 1 in a subgame following a pressure vector p. Let $u_i(\sigma, p)$ denote the expected utility of player i in a subgame following a pressure vector p when agents are following strategies σ. Let $\phi_i \in \Delta([0, M]^n)$ denote a mixed strategy for agent i in the first (pressure) stage. Let $U_i(\phi, \sigma)$ denote the expected payoff to player i when the vector of strategies $\phi = (\phi_1, \ldots, \phi_n)$ are played in the first stage, and $\sigma = (\sigma_1, \ldots, \sigma_n)$ describes play in the second stage as a function of the realized peer pressure.

We examine subgame perfect equilibrium of this game and the following refinements.

2.4 Pareto Perfect Equilibria and Maximal Equilibria

In some cases there exist multiple equilibria of the subgame and of the overall game. In particular, the multiplicity of equilibria in the subgames following the pressuring stage can end up producing some strange behaviors in the first stage. Just as an example, consider a coordination game where the second stage is used as a sort of blackmail to induce unnecessary positive peer pressure in the first stage. That is, if one agent provides substantial positive peer pressure to another then they coordinate on a good equilibrium in the second stage, while otherwise they coordinate on a bad equilibrium in the second stage. This is illustrated in the following example.

Example 4 *Multiplicity of Equilibria, Blackmail, and Equilibrium Refinements*

Consider a coordination game with two agents, such that $v_i(1,1) = 10$ and $v_i(\cdot,\cdot) = 0$ otherwise. Let $c_1 = 2 = c_2$. These payoffs are represented as follows.

		Agent 2′s Action	
		1	0
Agent 1′s	1	$8,8$	$-2,0$
Action	0	$0,-2$	$0,0$

Without any peer pressure there are two pure strategy Nash equilibria of this game, one where both agents choose action 1 and the other where both agents choose action 0. Here, the equilibrium where both players play action 1 is both payoff and risk dominant.

Now, let us consider the game where this is augmented with the possibility of positive peer pressure. The following is a subgame perfect equilibrium. If $p_{12} \geq 1$, then in the second stage the equilibrium (1,1) is played, while otherwise (0,0) is played (unless $p_{21} > 2$ in which case (1,1) is the unique equilibrium in the second stage). In the first stage $p_{12} = 1$ and $p_{21} = 0$.

In this equilibrium, agent 1 is "coerced" into helping to subsidize agent 2's action, even though the action is already part of an equilibrium. The coercion is via a threat of playing a bad equilibrium in the second stage if agent 1 does not follow the prescribed strategy. While there may be some interest in such equilibria, and they are not entirely implausible, this introduces a large multiplicity of equilibria in the peer pressure games that complicate the analysis. Most importantly, the reasoning behind such equilibria are quite different from the basic incentives of agents to try to help sustain play that would not otherwise be possible. In order to isolate the basic incentives of agents to use peer pressure, we consider a refinement of equilibrium that picks out a single equilibrium in the above game. It is that of Pareto perfection, due to Bernheim, Peleg and Whinston (1987). It requires that agents not play equilibria that are Pareto dominated in any second stage subgame, and then that the overall equilibrium is not Pareto dominated by another equilibrium that satisfies the same requirement. It is defined more formally below.

In this game, Pareto perfection requires that (1,1) be played in all subgames. Thus, there will be no peer pressure in the first stage and the unique Pareto perfect equilibrium of this game is no pressure in the first stage and then (1,1) played in all subgames in the second stage.

More formally, in our class of games, Pareto perfection (Bernheim, Peleg and Whinston (1987)) is defined as follows.

Let PE^0 be the subgame perfect equilibria of the overall game such that the continuation strategies $\sigma(\cdot)$ are (Borel) measurable functions of p. Let PE^1 denote a subset of PE^0 defined as follows. An equilibrium $(\phi, \sigma) \in PE^0$ is in PE^1 if there does not exist a p and another equilibrium $(\phi', \sigma') \in PE^0$ such that $u_i(\sigma', p) \geq u_i(\sigma, p)$ for all i with strict inequality for some i.

A *Pareto perfect equilibrium* is an equilibrium $(\phi, \sigma) \in PE^1$ such that there is no other equilibrium $(\phi', \sigma') \in PE^1$ such that $U_i(\phi', \sigma') \geq U_i(\phi, \sigma)$ for all i with strict inequality for some i.

There is a close relationship between Pareto efficiency and a maximality condition in the second stage. In particular, for any given level of p chosen in the first stage, there exists a pure strategy "maximal equilibrium" $\overline{x}(p)$, such that if $\sigma_i(p) > 0$ for any equilibrium in the subgame, then $\overline{x}(p) = 1$. Thus, a maximal equilibrium is one such that the actions of all of the agents in some second stage subgame are as high as in any equilibrium in that subgame. Given the nonnegative externalities, a maximal equilibrium is Pareto undominated by any other equilibrium (within the second stage).

A *maximal equilibrium* in the overall game is a subgame perfect equilibrium where the maximal equilibrium is played in every second-stage subgame, on and off the equilibrium path. The following lemma points out a relationship between Pareto perfection and maximal equilibria.

LEMMA **1** *If v_i is increasing in x_{-i} for each i, then all Pareto perfect equilibria are maximal equilibria.*

The proof of Lemma 1 is straightforward and thus omitted. Note that the converse is not true, so that there are maximal equilibria that are not Pareto perfect equilibria. For example suppose that it is a dominant strategy for two players to take action 1, but that the third faces a cost of action 1 higher than the benefit, even when the other two take action 1. It could be that the necessary pressure to induce the third agent to take action 1 is substantial enough that it requires both of the first two agents to contribute and that there are multiple equilibria of the peer pressure phase, some of which are Pareto dominated by others.

The condition that v_i be increasing rules out situations where agents are indifferent as to others actions in which case nonmaximal equilibria could be Pareto perfect.

2.5 Equilibrium Existence

Before proceeding to the analysis of peer pressure, we state a result concerning existence of equilibria to ensure that the analysis that follows is not vacuous.

There are no off-the-shelf existence theorems that apply in this setting, even to establish existence of subgame perfect equilibrium. The game is a discontinuous one, as when pressure reaches certain thresholds it suddenly becomes attractive for agents to take actions. Even though equilibria in the second stage are upper semicontinuous as a function of first-period actions, the fact that there are multiple agents taking simultaneous actions prevents this correspondence from being convex and hence this precludes using standard fixed point theorems or even results such as that of Simon and Zame (1990) (for some background on this see Harris, Reny and Robson (1995)). While there are some theorems on existence of subgame perfect equilibria in discontinuous extensive form games, they are specialized and none cover the class of games we examine here. Most importantly, such theorems either involve convexity or lower semicontinuity conditions, neither of which are satisfied here. Moreover, we wish to establish existence of Pareto perfect equilibrium and not just subgame perfect equilibrium. To that end, we prove existence directly, and the techniques used here should be useful more generally when the variations on lower semicontinuity conditions used in the literature fail.

PROPOSITION **1** *There exists a Pareto perfect equilibrium of the overall game (of either positive or negative peer pressure), and there exists such an equilibrium that is a maximal equilibrium and is in pure strategies in the second stage.*

The proof begins by showing that there exists a pure strategy Nash equilibrium in the second stage, regardless of the pressure vector in the first stage (e.g., the maximal equilibrium). In fact, by standard results (e.g., Milgrom and Shannon (1994) and Vives (1990)) the pure strategy equilibria form a complete lattice, and so there are maximal and minimal equilibria. The proof then shows that upper semicontinuity and Reny's (1999) payoff security condition hold when agents anticipate that the maximal equilibrium will be played in the second stage.

3 Peer Pressure

With existence of equilibrium established, we now examine the structure of equilibrium.

3.1 Who Pressures Whom

Let us start by noting some properties of equilibrium, and in particular, which agents exert pressure and which are pressured.

The following example illustrates some of the insights into the structure of pressure in equilibrium.

Example 5 *Peer Pressure and Contagion Effects.*

Consider a game where $n = 3$ and $v_i(x) = x_i m$ where $m = \sum_i x_i$ is the number of agents choosing action 1. Let $c_1 = 0$, $c_2 = 2.5$, and $c_3 = 3.5$.

In this game if an agent takes action 1 then his or her payoff is simply the total number of people taking action 1 less the cost of taking action 1, while if an agent takes action 0 then the agent's payoff is 0. In the absence of any peer pressure there is a unique equilibrium $x = (1, 0, 0)$, where only agent 1 chooses action 1. It is dominant for agent 1 to take action 1 and for agent 3 to take action 0, and thus the unique best response for player 2 is to choose action 0. With peer pressure, there is a maximal equilibrium which is Pareto perfect where $p_{13} = .5$ and $x = (1, 1, 1)$. In fact, there is a whole set of pure strategy maximal (and Pareto perfect) equilibria where $p_{13} + p_{23} = .5$. Here, even though without peer pressure agent 2 will not take action 1, agent 2 is not pressured in equilibrium, as by pressuring agent 3 agent 1 realizes that this will be enough to lead agent 2 to also take action 1. Thus, by pressuring a higher cost agent, middle cost agents can be led to take higher actions due to the complementarities. This means that generally it will not be the marginal non-participating agents (from the no-pressure game) who will be pressured, but sometimes infra-marginal players.

This example also points out the role of maximal equilibria and Pareto perfection. With peer pressure there are other subgame perfect equilibria that are not Pareto perfect and not maximal. For example there is an equilibrium where agent 1 pressures both agents 2 and 3 so that $p_{12} = p_{13} = .5$. This is supported with the off-equilibrium-path behavior of minimal equilibria. In that case, if agent 1 pressures agent 2 by less than .5 and agent 3 by less than 1.5, then it is an equilibrium for neither agent 2 or 3 to take action 1. This is not Pareto perfect, as in any subgame where $p_{13} \geq .5$ the equilibrium continuation is Pareto dominated by the maximal equilibrium.

Proposition **2** *Consider a pure strategy maximal and/or Pareto perfect equilibrium.*

(I) If an agent i is pressured (so that $\sum_j p_{ji} > 0$), then in the second stage agent i takes action 1 and is indifferent between action 1 and 0.

(II) If the game is a participation game, then an agent i who exerts pressure chooses action 1 (so $p_{ij} > 0$ for some j implies $x_i(p) = 1$) and an agent j who is pressured does not exert any pressure (so $p_{ij} > 0$ for some j implies $\sum_i p_{ji} = 0$).

(III) If the game is a participation game with positive peer pressure and benefit symmetry, and the equilibrium is both Pareto perfect and maximal, then

there exist cost thresholds $c^1 \leq c^2$ such that agents can be partitioned as follows. Agents with costs above c^2 take action 0 and agents with cost below c^2 take action 1. All agents with costs in (c^1, c^2) are pressured and take action 1, and all agents exerting pressure have costs no higher than c^1.[13] *Moreover, if any agents take action 1 on the equilibrium path, then there exist some agents with costs no higher than c^1.*

Proposition 2 shows that we can nicely order the agents into those with low costs who are not pressured and some of whom exert pressure, those with a middle range of costs who are pressured, and those with very high costs who are too costly to pressure.

The ideas behind the proposition are as follows. First, if an agent is pressured, then the agent must be indifferent. If the agent were pressured to the point of strictly preferring an action, then some other agent could save pressuring cost by slightly reducing the pressure exerted. Second, in participation games, only agents choosing action 1 would ever exert pressure because only they are affected by others' actions. Moreover, it must be unpressred agents who do the pressuring. To see that, note that given our first conclusion that pressured agents are indifferent between choosing an action or not, their ending utility is the same if they choose action 1 or 0 in the second stage. If they were exerting pressure, they could simply not exert the pressure and choose 0 in the second stage and end up getting the same second-stage utility but save on the cost of pressure. Thus, all the pressuring is done by agents who are choosing action 1 and who are not pressured themselves. With symmetry, we can then order the agents as in (III), as pressuring agents (those with costs below c^1 who choose action 1 without any pressure) should then choose to pressure the "cheapest" agents (those between c^1 and c^2) by Pareto perfection.[14]

The proposition does not hold for non-maximal equilibria, as there are examples where equilibrium selection in the second stage depends on the first stage actions, such as Example 4. The equilibrium noted in that example fails to satisfy (I), and slight variations on the equilibrium constructed there violate (II).

To see the role of the participation game requirement in parts (II) and (III) of Proposition 2, consider the following example which shows that in non-

[13] It is possible that some agents with costs exactly equal to c^2 are pressured while others are not. This happens only when the return to pressuring an agent is exactly equal to the costs for those exerting the pressure, and generically in the specification of costs this would not occur.

[14] This is the point at which positive peer pressure is essential. It could be that agents need to coordinate to pressure some agent to take action 1. Which agent they coordinate upon leaves the pressured agent indifferent under positive pressure, and thus the cheapest one is the best for the pressuring agents. However, under negative pressure a change in the pressured agents is not Pareto comparable when those agents are negatively affected and thus is not clearly ordered by Pareto perfection.

participation games it is possible to have agents pressuring each other at the same time.

EXAMPLE **6** *Mutual Peer Pressure.*

Consider a game where $n = 2$; $v_i(\cdot, 0) = 0$, $v_i(0, 1) = 1$, $v_i(1, 1) = 1.5$ for both i where the first entry in v_i is own action and the second entry is the other agent's action, and $c_1 = c_2 = .6$.

Here, each agent benefits from the other agent taking the action. In the absence of peer pressure, the game is essentially a prisoner's dilemma and it is a dominant strategy for each agent to take action 0. With peer pressure, the (unique) pure strategy maximal equilibrium outcome is for each agent to pressure the other at a level of .1 and to take action 1.

3.2 The Efficiency of Equilibria

We now examine the question of the efficiency properties of peer pressure.

PROPOSITION **3** *(i) Under positive peer pressure, a pure strategy maximal and/or Pareto perfect equilibrium leads to weakly higher payoffs for all agents than any equilibrium in the game without peer pressure.*

(ii) Under negative peer pressure, there are examples of pure strategy maximal and/or Pareto perfect equilibria where some agents are better off and others are worse off than without peer pressure, and such that the sum of utilities of agents are either higher or lower than in games without peer pressure. There are examples where a pure strategy maximal equilibrium[15] of the game with negative peer pressure is Pareto dominated by the maximal equilibrium in the game without peer pressure.

(iii) In a pure strategy maximal and/or Pareto perfect equilibrium of a participation game with negative peer pressure, relative to the maximum equilibrium without pressure:

- *pressured agents are worse off,*
- *agents taking action 0 are indifferent*
- *agents who are not pressured and take action 1 are weakly better off.*

So, we find that positive peer pressure offers improvements relative to the world without pressure while negative peer pressure will generally be Pareto

[15]This equilibrium is the limit of Pareto perfect equilibria, but is itself not Pareto perfect. Thus, even under Pareto perfection one can come arbitrarily close to having an equilibrium that is Pareto dominated by an equilibrium of the game without any peer pressure.

noncomparable to the world without pressure and can in some extreme cases even be worse (if the pressuring agents are just indifferent to pressuring). The intuition behind Proposition 3 is as follows. First, (i) follows from the fact that pressured agents are weakly better off (due to positive externalities) from positive peer pressure and those exerting pressure are better off as otherwise they would choose not to exert the pressure. In contrast, under negative peer pressure the pressurees are worse off than under no pressure since they would naturally have strictly chosen action 0 and only choose 1 because other agents have made their other option worse. The pressurers are still (weakly) better off, but this leads to a more ambiguous comparison. The following example illustrates some of the points of the Proposition.

Example **7** *Contrasting Positive and Negative Peer Pressure.*

Let $n = 2$ and both agents have the same $v_i = v$ where $v(1, 0) = 1$, $v(1, 1) = 2$, and $v(0, \cdot) = 0$. Thus, this is a participation game of strategic complements.

Let $c_1 = 0$ and $c_2 > 0$.

If $c_2 < 2$ then (1,1) is the unique equilibrium outcome with or without pressure.

If $c_2 > 2$, then (1,0) is the unique equilibrium without pressure.

If $c_2 > 3$, then (1,0) remains the unique equilibrium with or without pressure, as affecting agent 2's choice would require $p_{12} > 1$, and it cannot be in agent 1's interest to pressure 2, and it is clearly never in 2's interest to pressure 1.

Consider $3 > c_2 > 2$. In all equilibria[16] of the peer pressure game $p_{12} = c_2 - 2$ and the actions are $(1, 1)$. The resulting payoffs are $(4 - c_2, 0)$ in the positive peer pressure game and $(4 - c_2, 2 - c_2)$ in the negative peer pressure game.

In the positive peer pressure game this leads to a Pareto improvement relative to not having peer pressure, while agent 1 is better off and agent 2 is worse off in the negative peer pressure game. The total utility in the negative peer pressure game is higher with pressure if $c < 2.5$ and lower if $c > 2.5$.

If $c_2 = 3$ then there is a maximal equilibrium of either peer pressure game where $p_{12} = 1$ and (1,1) is played. In the case of positive peer pressure this leads to payoffs of (1,0), while in the negative peer pressure game it leads to payoffs of (1,-1). In the first case, agents are indifferent between the games with or without pressure, while in the second case the payoffs are Pareto dominated by those in the game without peer pressure.

While the above results suggest that positive peer pressure can offer Pareto improvements, one might also conjecture that it leads to full Pareto efficiency,

[16]The equilibria is effectively unique, except for some possible off the equilibrium path specifications for situations where players are indifferent between 1 and 0 and can mix at different levels.

at least within the constraints of the transfers than can be made in the peer pressure game. The intuition is that pressure allows agents to help other agents internalize the externalities that they impose on others. However, this conjecture turns out to be false, as shown in the following examples.

Example **8** *Constrained Pareto Inefficiency with Positive Pressure*

Let $n = 4$ and $c_1 = 0$, $c_2 = 10$ and $c_3 = c_4 = 13.8$ and consider a participation game with positive pressure and symmetric benefits such that the symmetric benefit function v satisfies $v(1, 0) = 1$, $v(1, 1) = 2$, $v(1, 2) = 10$, $v(1, 3) = 11$, and $v(0, \cdot) = 0$.

Here there is no maximal or Pareto perfect equilibrium where all four agents choose action 1 in the second stage with probability one. To see this, first note that we can restrict attention to pure strategy equilibria. To see this, note that if there exists an equilibrium where agents mix in the first stage and all four agents choose action 1 in the second stage, then it must be that almost all pressure vectors of any agent have the same total pressure (or else some would be more expensive than others but would still lead to the same actions). Thus, there would also exist a pure strategy equilibrium where the agents each play a pressure vector in the support of their strategies. So, without loss of generality, consider a pure strategy equilibrium. Next, note that in any equilibrium where all four agents choose action 1 in the second stage it must be that $p_{13} + p_{23} \geq 2.8$ and $p_{14} + p_{24} \geq 2.8$. Note also that agent 2 can guarantee him or herself a nonnegative payoff by not pressuring and choosing 0 in the second stage, and so the total pressure that agent 2 will exert on agents 3 and 4 is at most 1 in total, and so $p_{23} + p_{24} \leq 1$. Thus, in order to have all four agents choose action 1 in the second stage, agent 1 must exert a pressure of at least 4.6 in total on agents 3 and 4, and so $p_{13} + p_{14} \geq 4.6$, and so either $p_{13} \geq 2.3$ or $p_{14} \geq 2.3$. Suppose, without loss of generality, that $p_{14} \geq 2.3$. Let agent 1 deviate and offer $p'_{14} = 0$, and $p'_{13} = 3.8 - p_{23}$. Agent 1's payoff is then $10 - (3.8 - p_{23})$ in any Pareto perfect equilibrium or maximal equilibrium (as such equilibria will have the first three agents choose action 1), while before it was at most $11 - (2.8 - p_{23}) - 2.3$, which is strictly lower.

Thus, there is no maximal or Pareto perfect equilibrium where all four agents choose 1.[17] Note that the total payoffs where all four agents choose 1 is strictly higher than that of any other configuration of agents participating, and so any

[17] The maximality/Pareto perfection is important. If agents anticipate that all agents will take action 0 if agent 1 deviates from subsidizing both agents 3 and 4, then that can sustain agent 1's subsidization; but that requires expectation of a nonmaximal and Pareto inefficient equilibrium (relative to other equilibria) in the second stage.

maximal equilibrium is (strictly) Pareto dominated by some non-equilibrium configuration of pressure and having all four agents participate in the second stage. Therefore, all maximal equilibria and all Pareto perfect equilibria in this example are *constrained Pareto inefficient* in that for any maximal equilibrium there exists some (non-equilibrium) prescription of strategies that leads to higher payoffs for all players.

The above example just shows that it is possible to have all maximal and or Pareto perfect equilibria be constrained inefficient. The following example is one where all equilibria are constrained inefficient.

EXAMPLE **9** *Constrained Pareto Inefficiency*

Let $n = 4$ and $c_1 = 0 = c_2 = 0$, while $c_3 = c_4 = 4.5$ and consider a game with positive pressure.

Let $v_i(0, \cdot) = 0$, $v_i(1, 0) = v_i(1, 1) = 1$, and $v_i(1, 2) = v_i(1, 3) = 3$, for all i with the only exception being that $v_2(1, 2) = 1$.

Note that the unique constrained Pareto efficient outcome is to have all four agents participate. It is clear that both 1 and 2 participate in any Pareto efficient allocation. Next, note that then it must also be that 3 should participate as that increases 1's utility by 2 and only has a next cost of 1.5 for 3 and so 1 could subsidize the action. Similarly, then it must be that 4 also participates, since that benefits agent 2 by 2 and only has a net cost of 1.5 to agent 4.

Let us argue that there is no equilibrium (of any kind) where both 3 and 4 participate with probability one. Note that agents 1 and 2 will take action 1 in any equilibrium. Next, note that it cannot be that p_{13} and p_{14} are both positive (with positive probability), as if that were the case, by dropping one of them to 0 and increasing the other by some small ε there is a unique equilibrium in the subgame (with probability one) that is for three agents to take action 1 and so agent 1 is strictly better off. This implies that in order to have all agents take action 1, either $p_{23} \geq 1.5$ or $p_{24} \geq 1.5$ (with probability one). Without loss of generality, suppose that $p_{24} \geq 1.5$. In that case, if agent 1 is exerting positive pressure, then agent 1 is better off setting $p_{13} = 0$ and $p_{14} = \varepsilon$ which still results in a unique equilibrium in the subgame where agents 1, 2, and 4 take action 1, and agent 1 saves in payment while not losing any utility from the complementarities. So agent 1 is not exerting any peer pressure. But then agent 2 will exert a maximum total pressure of 2 (2's maximum potential benefit from pressuring agents 3 and 4), which is less than is needed to get both agents 3 and 4 to choose action 1.

Let us close this section with an observation about the case of two agents. Note that the above examples involved more than two agents. With just two agents, we can deduce that Pareto efficient outcomes will be sustained with positive peer pressure (and not always with negative peer pressure), as the above examples hinge on issues of how the costs of pressuring should be split. A total payoff maximizing pair of actions can be sustained as an equilibrium with just two agents with positive pressure, since an agent simply pays the other agent to take an action if the positive externality induced exceeds the cost.

4 Extensions and Discussion

Our analysis of peer pressure has provided basic existence results, insights into who pressures whom, and results showing that while peer pressure can lead to Pareto improvements in some cases, it does not always lead to full efficiency. Moreover, there are clear differences between positive and negative peer pressure. This model suggests further analysis of the yet largely unmodeled phenomenon of peer pressure. With this in mind, we close the paper with a discussion of some extensions of the model.

4.1 Strategic Substitutes

Although, games of strategic substitutes differ from games of strategic complements, the basic sorts of conclusions that are reached with complements extend also to substitutes. One can construct analogs of Examples 8 and 9 where even constrained Pareto efficiency is not reached. There are also settings where positive peer pressure is improving, and although negative peer pressure can lead to Pareto efficient outcomes in some particular cases, it is not always sure to do so. To illustrate this, we specialize to a canonical class of games of strategic substitutes: best-shot public goods games, which are defined as follows.

Agent i either contributes to the public good ($x_i = 1$) or not ($x_i = 0$) and then gets a payoff of

$$\begin{array}{ll} 1 - c_i & \text{if } x_i = 1, \\ 1 & \text{if } x_i = 0 \text{ and } x_j = 1 \text{ for some } j \neq i, \\ 0 & \text{if } x_j = 0 \text{ for all } j. \end{array}$$

Thus, an agent benefits if any player contributes to the public good, but does not see any additional benefits beyond having one person contribute. For example, this might be gathering information that is then freely communicated to other agents, or performing a task that just needs to be done once, or buying a product that can be shared freely among the agents. Taking the action 1 is costly and a player would prefer that some other agent take the action.[18]

[18]See Hirshleifer (1983) for background on best-shot public goods.

Existence is proven by construction of equilibria, which is part of the proof of the following proposition which discusses the efficiency results.

Proposition 4 *Consider a best-shot public goods game. Under either positive or negative peer pressure there exist Pareto perfect pure strategy equilibria that result in a total utility maximizing action choice (where a single minimum cost agent takes action 1 if* $n > \min_i c_i$*, and no agent takes action 1 if* $n < \min_i c_i$*). Under positive peer pressure, there exist such equilibria that are unconstrained Pareto efficient, while under negative peer pressure there are cost configurations (e.g.,* $2 > \min_i c_i > 1$*) where no equilibrium is constrained Pareto efficient.*[19]

The proof is relatively straightforward: if the lowest cost agent has a cost above 1 and no higher than n, then other agents share in pressuring a lowest cost agent to take action 1. Otherwise no pressure is exerted. On the equilibrium path, a total utility maximizing profile of action is chosen. The off-the-equilibrium path play is specified in the proof. The inefficiency of negative peer pressure stems from the fact that the negative pressure is costly for all involved and needs to be exerted in some cases to induce efficient actions in the second stage.

Note that this contrasts with the case of strategic complements. With strategic complements it was low cost agents who pressured high cost agents, while in the case of strategic substitutes it can be high cost agents who are pressuring low cost agents to take an action. The clean split of agents into categories that we saw in Proposition 2 no longer holds, as without an analog of participation games (which are not so natural in the case of public goods), the externalities of one agent taking action 1 affects all agents and so it could be a quite complicated set of pressures that are exerted, although with some symmetry in payoffs efficiency involves lowest cost agents taking action 1.

4.2 Endogenous Groups, Matching, and Networks

In settings where agents can choose with whom they associate, there are some interesting differences between positive and negative peer pressure.

There are two important drivers of endogenous groups with peer pressure. First, pressured agents benefit from positive pressure while they can suffer from negative pressure. Thus, to avoid being pressured, agents might leave a group when facing negative peer pressure, but not when facing positive peer pressure.

[19] Unconstrained Pareto efficiency refers to a utility profile that is not Pareto dominated by any (possibly nonequilibrium) utility profile where actions can be dictated and arbitrary (balanced) transfers across agents are possible, whereas constrained Pareto efficiency refers to a utility profile that is not Pareto dominated by what is feasible within the (possibly nonequilibrium) constraints of the given peer pressure game.

Second, exerting pressure is an expensive activity, regardless of the type of pressure. Thus, if agents face some limit on the number of others with whom they can interact, then agents will prefer to be matched with agents who do not need to be pressured, all else held equal. This leads to an assortative matching.

To get a feeling for these two main effects, consider a participation game of strategic complements. We augment the game of peer pressure with a pre-game stage where agents choose with whom they associate. Agents' payoffs are only affected by the actions of the other agents in their own group. We consider two variations on this to illustrate the two effects discussed above.

First, consider a society where agents partition themselves into groups. Let Π denote a partition of the set of agents $\{1, \ldots, n\}$. An agent i who is in a group $S \subset \{1, \ldots, n\}$ who takes action x_i has a base payoff (not including any peer pressure or cost of action) of $v_i(x^S, S)$ where x^S is the vector of actions taken by the agents in S. Correspondingly, fixing any given group S, the previous definitions of the peer pressure game extend, and so write $U_i(\phi^S, \sigma^S, S)$ to denote i's expected payoff when in a group S playing strategies (ϕ^S, σ^S) in the peer pressure game. There are a wide variety of solutions that one might consider for such an endogenous-group game. An obvious one is the following core-based concept.

An *endogenous-group equilibrium* for a society is a partition Π of $\{1, \ldots, n\}$ and a specification of a Pareto perfect equilibrium (ϕ^S, σ^S) for the peer pressure game for each group of agents $S \in \Pi$ such that there does not exist any group of agents $S' \subset \{1, \ldots, n\}$ and a Pareto perfect equilibrium $(\phi^{S'}, \sigma^{S'})$ in the peer pressure game for S' such that

$$U_i(\phi^{S'}, \sigma^{S'}, S') > U_i(\phi^{S_i}, \sigma^{S_i}, S_i),$$

for each $i \in S'$ where $S_i \in \Pi$ is the element of the partition containing agent i.

In this variable group setting, a participation game is such that such that $v_i(x^S, S) = 0$ whenever $x_i^S = 0$, regardless of S and the actions of other agents in S.

The following claim is straightforward and its proof is omitted.

CLAIM **1** *Consider a participation game of strategic complements. In an endogenous group equilibrium in a game with negative peer pressure, there is no peer pressure exerted on the equilibrium path. In contrast, there are examples of endogenous group equilibria in games with positive peer pressure where there is peer pressure exerted on the equilibrium path.*

The fact that there is never any negative peer pressure exerted follows easily from the fact that in a participation game of strategic complements and negative peer pressure pressured agents are worse off than if they were alone and took action 0 (e.g., see Proposition 3, (iii)). In fact this implies that any agents who would be pressured within some group will segment themselves away. Thus, in any case where negative peer pressure would occur among the grand coalition, an endogenous group equilibrium must involve multiple groups. In contrast, in the absence of congestion effects, under positive peer pressure it is possible for the grand coalition to form.

It is clear that the above claim abstracts away from a series of other things which might impact decisions of whom to associate with, but does capture some aspects: a non drug user might prefer not to have drug users as roommates in order to avoid being pressured (in a negative way) to use drugs.

In order to see the impact of the expense of exerting pressure, on group formation, we specialize the setting a bit more. In particular, one needs some congestion effects or limits to the numbers of others that a given agent wants to interact with. Natural settings of this type are marriage and roommate settings, where agents wish to be matched with at most one other agent.

In particular, consider a setting where $v_i(x^S, S) > v_i(x^{S'}, S')$ for any x^S and $x^{S'}$ when $|S'| > 2$ and $|S| \leq 2$. In such a setting, agents prefer any group with two or fewer agents to one with more than two. Let us call such a setting a *roommate* setting.

The endogenous group equilibrium definition extends directly to such a setting and generalizes the usual core-based stable matching definition.[20]

CLAIM **2** *Consider a participation game of strategic complements in a roommate setting, and suppose that payoffs exhibit benefit symmetry, $v_i(x^S, S)$ is increasing in x^S_{-i} when $x_i = 1$, and no agents have cost exactly equal to $c^* = v(1, 1, S) - v(0, 1, S)$ when $|S| = 2$ (where the first entry is the agent's own action). In an endogenous group equilibrium, all agents are matched into pairs or singletons and there is pressure exerted in at most one pair. The matching is assortative in that pairs are such that either both agents take action 1 or both agents take action 0, except for at most one pair.*

Again, the proof is straightforward and just sketched here. There cannot exist two different agents who have costs below c^* and who are each matched to agents who have costs above c^*. They would benefit from deviating and matching with each other as then they would each get $v(1, 1) - c_i$, while in the original matching their utility is lower than this (either having to pressure their partner, or having their partner choose 0 and possibly taking action 0 themselves). Thus, all pairs except for at most one are such that either both agents have costs below

[20]See Jackson and Watts (2005) for more discussion of endogenous matchings with the play of games.

c^* and choose action 1 in any Pareto perfect equilibrium and do not exert any pressure, or both agents have costs above c^* and take action 0 and do not exert any pressure in a Pareto perfect equilibrium.

Beyond the group equilibrium and matching examples considered above, it would also be interesting to consider endogenous-network versions of graphical games with peer pressure.

4.3 Incomplete Information and Refinements

Much of the analysis here relies on a refinement of equilibrium to Pareto perfect and/or maximal equilibria. These make some sense in settings where agents can communicate or coordinate. If, in contrast, agents must choose actions without coordinating and with some uncertainty about others' payoffs and behaviors, then there are other considerations and refinements that might be more appropriate, such as Carlsson and van Damme's (1993) global games approach (and see Vives (2005) for more discussion about the general use in refining equilibria in games with strategic complementarities). Exactly what would happen with such an approach could depend on the type of uncertainty introduced (e.g., see Weinstein and Yildiz (2007) for results on how variations in the uncertainty can affect the equilibrium selection), and is an important question for future research.

5 Appendix

Proof of Proposition 1:

Following any p, let agents play the unique (pure strategy) maximal equilibrium. An algorithm for identifying such equilibria (e.g., see Jackson (2008)) is as follows. Start with all agents playing action 1, and denote this strategy profile $x^1(p)$. If for some agents, playing action 0 is a strict best response to other agents playing $x^1(p)$, then change those agents' actions to 0. It follows from the strategic complementarities that those agents will play action 0 in all equilibria. Let this profile of actions be $x^2(p)$. Iterate on this procedure. More formally, let $x^1(p) = (1, \ldots, 1)$ and for $k > 1$ let $x_i^k(p) = 1$ if $d_i(x_{-i}^{k-1}(p), p) \geq 0$ and $x_i^k(p) = 0$ otherwise. This will converge in at most n steps, and so let $\sigma^M(p)$ to be the mixed strategy profile that places probability one on the $x^n(p)$ found via this algorithm.[21]

[21]Note that there are only a finite set of points of discontinuity of $x^n(p)$ (as each player is faced only with a finite configuration of potential strategies of other players' pure strategies, and hence has at most a finite number of cost points where indifference occurs), and so this is a Borel measurable function.

Note that players' payoffs are each upper semicontinuous in p given that they anticipate play of the unique maximal equilibrium in the second stage. That is, $u_i(\sigma^M(p), p)$ is an upper semicontinuous function. This follows from the fact that players' payoffs are nondecreasing in other players' actions and that the maximal strategies are selected at any points of indifference (and hence at any points of discontinuity, and also that players are indifferent at points of discontinuity in their own action). This implies that $\sum_i U_i(\phi, \sigma^M)$ is upper semicontinous in ϕ (see Proposition 5.1 in Reny (1999)).

Fix the second period strategies σ^M, and consider the first period game where the anticipation is that σ^M describes the continuation play. We next show that there exists an equilibrium ϕ in the first stage, anticipating the σ^M continuation. If we can show that $U_i(\phi, \sigma^M)$ is payoff secure (in choices of mixed strategies ϕ), then by the upper semicontinuity and Proposition 5.1 and Corollary 5.2 in Reny (1999) it follows that there exists a Nash equilibrium ϕ for the first stage anticipating σ^M in the second stage.

To verify payoff security, consider any ϕ_{-i} and let p_i be a pure strategy best response to ϕ_{-i} (which exists given that pure strategies are a compact set and preferences are upper semicontinuous in p_i). Payoff security requires us to show that for any ε there exist p_i' such that $U_i(p_i', \phi_{-i}', \sigma^M) \geq U_i(p_i, \phi_{-i}, \sigma^M) - \varepsilon$ for ϕ_{-i}' in a small enough neighborhood of ϕ_{-i}. Let p_i^ε be defined by $p_{ij}^\varepsilon = p_{ij} + (\varepsilon/(2n))$. It then follows that for any k and p_{-i}, $p_{ik}^\varepsilon + \sum_j p_{jk} \geq p_{ik} + \sum_j p_{jk} + (\varepsilon/(2n))$. Thus, for any p_{-i}' within a small enough neighborhood of p_{-i}, it follows that $p_{ik}^\varepsilon + \sum_j p_{jk}' \geq p_{ik} + \sum_j p_{jk}$ and hence $x(p_i^\varepsilon, p_{-i}') \geq x(p)$. Given that i's utility is nondecreasing in x_{-i} and that any change in x_i is in terms of a best response, it follows that for ϕ_{-i}' in a small enough neighborhood of ϕ_{-i}, that $U_i(p^\varepsilon, \phi_{-i}', \sigma^M) \geq U_i(p, \phi_{-i}, \sigma^M) - \varepsilon$, and so payoff security is satisfied.[22]

The proof is concluded by noting that the set of vectors of utilities corresponding to equilibria that have Pareto efficient equilibria in all subgames (and are Borel measurable) is closed. Thus, we can find such a vector of utilities that is Pareto undominated among the set (which is nonempty given that the maximal equilibrium shown to exist above is in the set), and then find an associated equilibrium leading to that vector of utilities, resulting a Pareto perfect equilibrium. If such an equilibrium is not a maximal equilibrium it can be altered to be one by changing the equilibrium in any subgame to be a maximal equilibrium as the unique maximal equilibrium results in at least as high payoffs for all agents as any other equilibrium in the subgame, and so if the equilibrium is Pareto un-

[22]The transition from this holding for all p_{-i}'s and neighborhoods of them to mixed strategies is straightforward, and can be accomplished by partitioning the set of p_{-i}'s into small enough neighborhoods and then bounding the difference in payoff in each neighborhood and then summing.

dominated and not maximal, it must be result in the same payoffs for all agents as the maximal equilibrium. ▮

Proof of Proposition 2: Let us begin by proving (I). If some agent i is pressured in a maximal equilibrium, but is not indifferent, then some other agent j with $p_{ji} > 0$ could lower his or her pressure and still leave the maximal equilibrium in the second stage unchanged, which would be a contradiction of equilibrium. Next, let us show that the pressured agent must choose action 1. The same claim for a Pareto perfect equilibrium comes from noting that a Pareto perfect equilibrium has the same payoffs for all agents as a maximal equilibrium in all second-stage subgames. If the agent is indifferent, then in a maximal equilibrium the agent must take action 1. In the case of Pareto perfection, suppose that the agent did not take action 1. Then it must be that the agent's choice of action does not affect the pressuring agent's utility, or the equilibrium would be Pareto dominated. In that case, the pressuring agent could lower his or her utility and still end up with equivalent payoffs in a Pareto undominated equilibrium in the resulting subgame.

Next, we verify (II). In a participation game, if an agent i chooses action 0 in the second stage, then his or her payoff is unaffected by changes in the actions of the other agents and so the agent maximizes his or her payoff by setting $p_{ij} = 0$ for all j. Thus, a pressuring agent must choose action 1. To see that a pressured agent does not exert any pressure, suppose that i is pressured so that $\sum_j p_{ji} > 0$. Then by (I), the agent is indifferent between action 1 and 0 in the second stage and so the agent's payoff must be $v_i(0, x_{-i}(p)) - \sum_j p_{ij}$, which in a participation game is simply $v_i(0, \ldots, 0) - \sum_j p_{ij}$. If the agent were pressuring so that $\sum_j p_{ij} > 0$, then by deviating and setting $\sum_i p_{ij} = 0$ and choosing 0 in the second stage the agent's payoff would be $v_i(0, , \ldots, 0)$, and the agent would be better off.

(III) Consider a Pareto perfect and maximal equilibrium in a participation game of positive peer pressure such that all agents have the same v_i, denoted v which depends only on own action and the sum of actions of other players. Let p, x be the equilibrium path play, and let $m = \sum_i x_i$. We consider the case where $m \geq 1$ as otherwise simply set $c^2 = 0$ and $c^1 < 0$. First, note that if $c_i \leq v(1, m-1) - v(0, m-1)$ then $x_i = 1$ is a best response and played in the maximal equilibrium even if $\sum_j p_{ji} = 0$. It then follows from (I) that such an agent cannot be pressured (as if he or she were pressured then he or she would strictly prefer $x_i = 1$, which would contradict (I)). So, let $c^1 = v(1, m-1) - v(0, m-1)$. It follows that all agents with costs no higher than this level are not pressured and take action 1. It also follows from (II) that any agent who exerts pressure must have cost no higher than c^1 as agents with higher costs must either take action 0 or be pressured. Next, by the definitions of maximal equilibrium it follows that

$c_i > v(1,m) - v(0,m)$ for any i such that $x_i = 0$, and since this is a games of strategic complements $v(1,m)-v(0,m) \geq v(1,m-1)-v(0,m-1)$, and the latter is equal to c^1. If there are no pressured agents, let c^2 be the minimum cost among agents taking action 0, and the remaining conclusions of the proposition hold. So consider the case where there are some pressured agents. If there are no agents who take action 0, then let c^2 be a cost higher than any agent's cost and the remaining conclusions of the proposition hold. Otherwise, let c^2 be the minimum cost of any of the agents who take action 0. It follows from above that $c^2 > c^1$ and any agent with costs at or above c^2 who is not pressured takes action 0. It only remains to be shown that all agents who have costs at or above c^2 are not pressured, as then by the definition of c^2 it follows all agents with costs between c^1 and c^2 must be pressured (and by (I) take action 1). Suppose to the contrary that there exist agents j and k with $c_j < c_k$ such that agent k is pressured and takes action 1 while agent j is not pressured and takes action 0. Note that by our previous argument neither of these agents is then exerting any pressure as only agents with costs at or below c^1 exert pressure than both of these costs are above c^1. Consider a change in p to p' such that for each i: $p'_{ij} = p_{ik}c_j/c_k$ and $p'_{ik} = 0$ and $p'_{ih} = p_{ih}$ for any $h \notin \{j,k\}$. It follows that the maximal equilibrium involves the same sum of actions m (where j and k have swapped actions), that agents j and k are both indifferent between the resulting outcomes (each getting the value $v(0,0)$), and that all agents i exerting pressure are strictly better off as they exert less pressure. This is still an equilibrium, which then contradicts Pareto perfection.[23] ∎

Proof of Proposition 3: Let us begin by proving (i). Consider a maximal equilibrium of a game of strategic complements with positive peer pressure. Let x be the maximal equilibrium in the game without peer pressure. Note that $v_i(x) - c_i x_i$ is higher under x than under any other equilibrium without peer pressure, given the strategic complementarities and the fact that v_i is nondecreasing in x_{-i}. Consider any subgame in the game with peer pressure after pressure p' and let $x(p')$ denote the corresponding maximal equilibrium. Given that costs are now $c_i - \sum_j p'_{ji}$ for each i, and that $x(p')$ is a maximal equilibrium under p', it follows that $x(p') \geq x$.

Next note that by equilibrium

$$v_i(x(p)) - \left(c_i - \sum_j p_{ji}\right) x_i(p) - \sum_j p_{ij} \geq v_i(x(0,p_{-i})) - \left(c_i - \sum_j p_{ji}\right) x_i(0,p_{-i})$$

[23]In the absence of Pareto perfection, there are maximal equilibria where several agents coordinate on pressuring a single agent with a high cost, while they could pressure an agent with a lower cost instead, but due to the coordination issue no one of the pressuring agents has an improving deviation.

Next, given the strategic complementarities.

$$v_i(x(0,p_{-i})) - \left(c_i - \sum_j p_{ji}\right) x_i(0,p_{-i}) \geq v_i(x) - \left(c_i - \sum_j p_{ji}\right) x_i$$

Finally,

$$v_i(x) - \left(c_i - \sum_j p_{ji}\right) x_i \geq v_i(x) - c_i x_i$$

The last three inequalities provide the desired conclusion. The proof of the case of Pareto perfect equilibrium is an easy variation, noting that in any subgame a Pareto perfect equilibrium is either a maximal equilibrium or payoff equivalent (for all agents) to a maximum equilibrium.

(ii) is shown in Example 7.

To see (iii), first note that in a maximal and/or Pareto perfect equilibrium (see Proposition 2), a pressured agent must take action 1 and must be indifferent between taking action 1 and 0. Such an agent's payoff from taking action 0 in the negative pressure game is lower than it was in the game without pressure (given that it is a participation game so his or her payoff is independent of other players' actions, and he or she is pressured), and so the agent must have a lower payoff. An agent who is pressuring some other agent is weakly better off by an argument analogous to (i), as such an agent cannot be pressured and such an agent could always reduce pressure to 0 and still have at least as high a payoff as in the game without pressure. Similarly, an agent taking action 1 who does not exert any pressure sees a weak increase in x_{-i} due to the introduction of pressure and is thus weakly better off. Agents who take action 0 in a maximal and/or Pareto perfect equilibrium with negative pressure must not be pressured (again, see Proposition 2), and are thus indifferent between the equilibria of the two games as they must also take action 0 in a maximal equilibrium of the game without any pressure. ∎

Proof of Proposition 4: First, let us check that there is a Pareto perfect pure strategy equilibrium that results in a total utility maximizing action choice.

In a subgame following some p, define $x(p)$ as follows. Let i^* be the lowest indexed agent with cost equal to $\min_i c_i$.

(1) If there are some agents for whom $c_i - \sum_j p_{ji} < 0$ then have all such agents choose action 1 and all other agents choose action 0.

(2) If not case (1), and $c_{i^*} - \sum_j p_{ji^*} \leq 1$ then have i^* choose action 1 and all other agents choose action 0.

(3) If not case (1) or (2), and there are some agents for whom $c_i - \sum_j p_{ji} \leq 1$ then have the agent i with the minimum $c_i - \sum_j p_{ji}$ choose action 1 (and

if there are several such i's, then choose the one with the minimum i) and all other agents choose action 0.

(4) If not case (1), (2), or (3), then have all agents take action 0.

In the case where $n \leq \min_i c_i$, have no pressure in the first stage and have actions in the second stage as described by $x(p)$ above. It is clear that there are no beneficial deviations, as to change the second period actions requires pressure of at least $n - 1 \geq 1$.

In a case where $n > \min_i c_i > 1$, set $p_{ji^*} = c_{i^*} - 1/(n-1)$ for $j \neq i^*$, and otherwise set pressures to 0. In any subgame, follow the prescriptions of $x(p)$ as defined above. To see that this is an equilibrium, note that if some agent j deviates to lower p_{ji^*} (and not raise another p_{jk}), then all agents take action 0 in the following subgame and the resulting utility is at most 0 in the following subgame which is less than the equilibrium utility to j of $1-(c_{i^*}-1)/(n-1) > 0$. Also note that no agent j (including i^*) can gain by raising some p_{jk} for $k \neq i^*$ and possibly changing other pressures, as it either results in the same outcome with added cost, or else nobody taking 1, or else in agent k taking action 1 but with a pressure cost to j of more than 1 (as we need to be in case (2)). In any of these cases the resulting utility to j is either the same or negative.

Finally, consider a case where $\min_i c_i \leq 1$. Here have no pressure in the first stage follow $x(p)$ as described above in the second stage. In order to change the outcome, a pressure of greater than c_{i^*} needs to be exerted, and that leads to a worse outcome for any deviating agent.

Each of the above constructions involve Pareto undominated equilibria in the second stage (in all subgames). In the case of positive peer pressure, since each of these involve outcomes that maximize total utility, they cannot be Pareto dominated at the first stage, and hence are Pareto perfect.

In the case where there is negative peer pressure, any equilibrium that results in the total utility maximizing action must involve at least as much pressure on the agent taking the action, and so cannot Pareto dominate the equilibrium, and so the equilibrium is Pareto perfect.

To see the conclusions regarding constrained Pareto efficiency, simply note that any exerted pressure in the case of negative peer pressure is Pareto dominated by the same actions without any pressure, while any exerted pressure in the case of positive peer pressure (provided action 1 is taken by the pressured agent) transfers utility from one agent to another, and so the equilibria constructed above in the case of positive peer pressure are constrained Pareto efficient (and in fact unconstrained efficient). To see the conclusion that if $2 > \min_i c_i > 1$, then in the case of negative peer pressure all equilibria are necessarily constrained Pareto inefficient, note that an equilibrium must involve some pressure. If not, some agent would benefit by deviating and pressuring a minimum cost agent (which leads to a unique second stage equilibrium where that agent takes action 1). Thus, equilibrium involves some pressure, and the same actions without any pressure would offer a Pareto improvement. ∎

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