Writing your final reports

As we approach the end of SURIM, it is time to write the final report for your project. These reports are meant to provide complete and self-contained expository accounts of what you learned, discovered, and accomplished this summer.

Below I outline my expectations and provide some tips for writing the first draft of your final report. Your first draft is due to me on Tuesday, August 21st, by 5 PM. Please send them to me by email, attaching both the PDF output and the TeX/image source files. I will be reading everyone’s first drafts closely during the 9th week of the program and providing (probably a good deal of) comments on your work. The final drafts will be due on the last day of the program (August 31st).

Effective technical communication is an essential part of research, and writing your report will give you important experience in this area. In addition to what I’ve written below, more advice on how to write/communicate mathematics can be found in “Writing a research paper in mathematics” by Ashley Reiter, and in “How to write mathematics,” by Paul Halmos. Both of these are linked from the SURIM 2018 website.

Length

There is no length requirement or limit, but I am expecting somewhere between 10 and 15 pages from each project (this expectation does not include any pages that are just data/tables/appendices or references). It should be long enough to give the reader a complete picture of your research experience this summer, and should allow them to continue your work should they wish. If your draft is too short or too long, I will certainly let you know.

Abstract

Your reports should begin with an abstract that briefly describes your project in a few (perfectly crafted) sentences. Writing abstracts is a bit of an art, and I’ll do my best to help you create something succinct and beautiful for your final draft. These will appear in the SURIM archives (along with your reports), so ideally, they should give a compelling summary of your project.

Introduction

The introduction should give a detailed summary of your project and place it in the wider context of ongoing research. It should not contain any proofs or any significant technical development.

- Emphasize the motivation for the problem(s) you investigated this summer: Where did this problem come from? Why do you care about this problem? Why would the mathematical community at large care about this problem? What aspects did you explore that may be new/unexplored?
- Give an overview of the history of the problem and previous work, with citations.
- Once the problem has been introduced, state the main theorem(s you have proven (or used centrally).
• If your main results require a lot of technical language that you haven’t set up yet, that’s fine, simply point the reader to the section with the relevant definitions, or give informal definitions in the surrounding prose (before or after). For example: “By an elliptic curve with CM, we mean an elliptic curve whose endomorphism ring is strictly larger than \( \mathbb{Z} \).” Another option: You can always give an imprecise version of the theorem in your introduction, directing the reader to the more precise version that you state later. For example, you could write something like “under certain conditions” without specifying those conditions precisely, if, to someone reading your introduction, they would be overly complicated or abstruse.

• The proofs of your main results should appear in the main body of your report, not here.

• The end of your introduction should include an outline of your report. Be sure, for example, to say which theorems are proven in which sections.

**Background / Basic Theory**

This section should be a somewhat formal development of the material that is prerequisite for understanding the results in your report.

You should start with the very basics and work up to “standard results” that you would expect anyone familiar with the area to know already, but you should focus mainly on the parts of the theory you will actually need to use when proving your results. The idea is that the report should be as self-contained as possible, within reason. (However, don’t rewrite a whole textbook on your topic!)

Example: if your paper is on elliptic curves, this is the section where you would define “an elliptic curve.” You might be tempted to define “abelian variety” as well, but you should do so only if there is a compelling need (you need deep results about abelian varieties, or if it somehow makes your exposition about elliptic curves simpler).

It is totally acceptable to punt annoying prerequisite results to the references, but you should explicitly state any not-commonly-known result that you use in a crucial way.

**Main body of report**

Split this up into sections however you think makes sense. I’ll provide suggestions if I think you need more or fewer sections, or if they need to be permuted. Here are some ideas for sections you might want to include within the main body:

• Previous work in more detail (including important results in your area, and proofs of them if they are important or contain ideas you will use later).

• Technical development (if your main proof makes use of several lemmas, sometimes it is a good idea to put them all in one section together).

• Proof of main result(s).

• Corollaries, discussion, and future directions.
Here are some tips for this part of the report:

• Always begin a new section with a few sentences that outline the purpose of the section and how it relates to others.

• If you have references that you could point to that cover some or all of the material in the section mention it at the beginning of the section.

• If you have running assumptions, like “throughout this section, $E$ will denote an elliptic curve with CM by the imaginary quadratic ring $\mathcal{O}$,” then put these assumptions at the beginning. It might be good to remind the reader of these assumptions on occasion, particularly when forgetting about them could cause confusion, and you should almost always restate all hypotheses in the statements of theorems.

• Don’t forget to pepper your report with plenty of well-chosen examples. While proofs are “how mathematics is done,” examples are usually how mathematics is learned.

Resources (Data, code, etc)

Be sure to also include any data or code produced during the course of your project. If the code or data are compact enough, it is alright to include them in the report. If your files are Quite Large, we will figure out a way to host them (but still mention that these resources are available).

Acknowledgements

Be sure to thank anyone who has helped you in the course of this project.

References

• Everything you write should be attributed either to a reference or, if it is an original result, proven in a self-contained way in your report.

• Every reference should be referenced at some point in your report. The reference could be as simple/off-hand as “for a more complete exposition on the theory of elliptic curves, see [Silverman] or [Koblitz].”