1. Prove that for all integers n ≥ 1 we have
   \[ \sum_{k=n}^{2n-1} (2k+1) = 3n^2 \]

2. Let x, y ∈ R. Prove (using the axioms only) the following statement:
   \( (\forall \varepsilon > 0 : x < y + \varepsilon) \Rightarrow x \leq y \)

3. Let x, y ∈ R. Prove that
   \[ ||x| - |y|| \leq |x - y| \]

4. Let S be a bounded nonempty set and let a, b ∈ R with a > 0. Prove that
   \[ \sup (aS + b) = a \cdot \sup(S) + b \]
   \[ \sup \{ ax + b : x \in S \} \]

5. Let \( (x_n) \) be a recursive sequence.
   \[ x_0 = 1, \quad x_1 = 1 \]
   \[ x_n = x_{n-1} \cdot x_{n-2} + 1 \]
   a. Prove that \( (x_n) \) does not converge.
   b. Are there initial values \( x_0 = a \) and \( x_1 = b \) that would make the sequence converge?

6. Let \( (s_n) \) be a sequence,
   \[ A = \sup \{ s_n : n \geq 0 \} \]
   \[ B = \limsup (s_n) \]
   a. What is the relationship between A and B?
      i. A = B always
      ii. A ≤ B always
      iii. A ≥ B always
      iv. None of these
   b. If you choose ii - iv, give an example where A ≠ B.

7. Let \( L = \lim (1 + \frac{1}{n})^n \) (this exists). Show that \( L \geq 2 \).