Writing in the Major

The Writing in the Major (WIM) project is a short essay on a mathematical topic. All students enrolled in Math 110 must complete a WIM essay. The paper must be typeset in LaTeX and should be roughly 4–7 pages long.

Your work will be evaluated both in terms of mathematical content/correctness and clarity of exposition. The goal is for you to explore a topic that interests you in some depth and show that you can explain it in a way that is not only clear and engaging, but mathematically rigorous as well. Your work should generally be understandable to other students in this course, but because of the diverse backgrounds of students in 110, some variation in the technicality of the topics you select will of course be expected.

We expect that students observe the conventions for proper mathematical writing. The project is an essay: it should be written in sentences and paragraphs, with due attention paid to spelling, grammar, and writing style. It should make proper use of math environments for math typesetting, and use LaTeX symbols properly.

Math papers are written in the inclusive first-person plural (“We will prove Theorem 2.5 in three steps”), to collectively refer to both the author and the reader. Use of passive voice is acceptable too but it can be cumbersome (“Theorem 2.5 will be proved in three steps”).

There are many resources available on good mathematical writing practices, with advice on (for example) how to incorporate equations and other symbolic math statements into your writing, and which shorthand is or is not considered appropriate in a formal math paper. Some links to get you started are posted under Mathematical Writing on the course website. Additionally, the Hume Center has a number of resources available to help students with their writing assignments.

Choosing a topic

A list of suggested topics (with scant descriptions/blurbs) is available on the course website. If you would like to write on a topic that is not suggested on that list, please clear it with me first (I will probably say yes)!

You may choose any topic you like (relevant to Math 110), but be aware that some topics are more technically demanding than others, especially with respect to the abstract algebra required to fully understand them. Even so, you should pick something which challenges you more than a little, so that you can learn new things!

Structure of your paper

Your project should include:

- **A title.** A good title should be specific enough to capture the content and tone of the paper. It should contain keywords (so your paper is easily found by computer-search), and, ideally, will catch the reader’s attention. Include your name and the current date with the title.
• **An abstract.** The abstract is a brief paragraph that summarizes the main results in your paper. It should appear right after the title.

• **An introduction.** The introduction should engage the reader, motivate the content of the paper, and summarize the main results. (At the research level, the introductions to most math papers get far wider readership than the rest of the paper does so put substantial thought and work into our introductions). Ideally the introduction will include some or all of:
  
  – *Motivation.* Explain the importance of the subject of the paper, its applications, and why it has wider interest to the mathematical community.
  
  – *Background.* Explain the historical context for the main topic of the paper.
  
  – *Main results.* An informal description of the main mathematical results of the paper. Sometimes the main theorems are even in the introduction and the proofs come later, in the main body of the paper.
  
  – *Outline.* A brief summary of the paper, outlining its structure, its flow, where main results are stated, and where those results are proved.

• **Sections.** Organize the paper into numbered and titled sections, starting with the introduction. LaTeX also allows for numbered subsections (and even subsubsections—though you will probably not need these for a paper of this length).

• **Theorem, definition, and proof environments.** State your definitions using the LaTeX definition environment, and state results using the lemma, proposition, theorem, and corollary environments. Write proofs in the proof environment. These formatting conventions help make the paper more readable, clarify its organization, and also make it easier to refer forwards or backwards to key definitions and results.

• **Self-contained theorem statements.** Each theorem statement should be self-contained, mathematically correct and complete. This has the advantage that a reader can use your paper as a quick reference—looking directly at the statements of the theorems without having to read or re-read the whole section. It is okay to use terminology in the theorems that the reader might not know without reading the whole paper, but any variable or notation used in stating the theorem should be defined in the statement of the theorem, even if it were defined earlier in the paper.

Any hypotheses should be stated explicitly, even if the same assumptions were in place in the paragraph before. (Eg., “Assume and are relatively prime.”) The same holds for the statement of lemmas, propositions, and corollaries.

As a shortcut, if many of the same assumptions/notation are used in several propositions, you can say “Theorem Y. Assume the hypotheses [and/or notation] of Theorem X.”

• **User-friendliness.** It can take considerable thought to communicate mathematical concepts effectively. Remember that the target audience is another Math 110 student. Some strategies to consider:

  – Aim to make statements simple and direct.
– Explicitly spell out details or implications that might not be obvious to someone learning the material for the first time.

– Use the LaTeX’s label and ref commands to let you refer to sections, definitions, theorems, and other results by number. (“Combining this result with Theorem 2.2 in Section 2, we conclude that . . .”)

– Break down difficult arguments into smaller steps. Modularize the arguments by making preliminary steps into lemmas or propositions. If there is a particular mathematical fact that you invoke repeatedly, make it into a numbered proposition for easy reference.

– Put thought into the notation you use. This can be as simple as choosing conventional variables—for example, it is easy for an experienced reader to remember that \( n \) denotes an integer and \( p \) a prime integer. When the arguments become more involved—with large numbers of variables, complicated functions, complicated interdependencies, and multiple indices—then decisions about how to set up notation can go far to either aid or hinder the reader’s understanding of the argument.

– Use concrete examples to illustrate the main theorems or procedures. For example, if you are describing a primality test, then show how the test would apply to an actual integer, say, \( n = 13697 \). Or, if a proof involves a subtle argument, consider also showing how the argument applies in a concrete situation.

– It is of course not considered mathematically rigorous to prove a result or describe an algorithm only in the case of a particular example, but including a particular example as well as the general argument can be an excellent pedagogical technique to help the reader grasp the concept.

– If your topic includes some very technical theorems, you may cite them from other sources without re-proving them in whole yourself.

– It is not necessary to repeat material that was covered in Math 110 unless you consider the concepts to be central to your exposition.

– Similarly, if there are certain special cases where the arguments are easier, it can be enlightening for the reader to include these (even if they are, strictly speaking, mathematically redundant). For example, “We first give the proof in the special case where the integer \( n \) is a positive prime number \( p \). . .”

– Pictures or diagrams can be powerful tools to help the reader visualize a concept or procedure. You can include pictures files (such as pdf, png, and jpg) in LaTeX documents using the graphicx package.

– Balance mathematical precision and readability. Mathematicians place utmost value on accuracy and precision, but this can make statements hard for the reader to decipher, especially when they are steeped in dense notation and technical hypotheses. Make hard-to-parse results more accessible by following the theorem with an informal rephrasing of the result, a statement of the result in an easy special case, a main corollary of the result, an analogy to a more familiar result, or anything else that may give the reader perspective on ‘how to think about’ the statement.

– Bibliography. List any books, papers, or other sources you consulted while writing your paper. Moreover, give specific references for any facts you cite or mathematical results
that you state without proving. There is a LaTeX tool called bibtex designed to manage bibliographical information, but you may also list references by hand if this is easier. You can use whichever citation style you choose.

Your references list is not included in the 4–7 page suggested length.

**Deadlines**

- Your topic must be decided on by May 14th, submitted as part of Problem Set 5.
- Your first draft will be due June 1st.
- Your final draft will be due June 11th.

**Grading criteria**

A separate document concerning how your projects will be graded will be uploaded to the course website. You will receive a preliminary WIM grade on your first draft along with comments from the reader (Naomi and/or myself), and applying that feedback to your final draft to our satisfaction will overwrite that grade with a higher one.