RESPONSE STRENGTHS AND CHOICE PROBABILITY: A CONSIDERATION OF TWO COMBINATION RULES

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The area of preferential choice behavior has long been a focus of much interest and activity among experimental psychologists. To people of my persuasion, simple choice experiments are to be thought of as placing into competition two or more instrumental responses, each of which may vary independently in its strength. To handle these situations, the theorist must propose some solution to the following question: Given two competing responses with known strengths, how are we to use this knowledge to predict choice between the two; that is, what is the composition, or combination, rule by which choice probability is related to the strengths of the separate responses?

Within this area of reference, there are today two major theories. One theory is due to Thurstone [11], and it assumes that the probability of choosing response A over B is an increasing function of the difference in strengths of the two responses. The second major theory, which has been proposed by Luce [7], assumes that the choice of A over B is determined by the ratio of their strengths; thus, if A and B have equal strengths, then A is chosen half the time; if A is three times stronger than B, then A will be chosen three-quarters of the time, and so on.

The natural testing grounds for both of these theories has been the experimental arrangement we call 'paired comparisons'. An experiment on paired comparisons has a simple structure. A number of commodities are presented to the subject a pair at a time, and choice probabilities are obtained for all possible pairs. The result of this experiment is a matrix of pairwise choice probabilities, and the test of a theory is how well it can reproduce this matrix after the response strengths have been estimated. A number of experiments like this have been carried out comparing the two theories. The upshot of these theoretical considerations has been essentially this: if the data are based on large sample sizes so that the estimates of pairwise preference are reasonably reliable, then both theories fit the data about equally well and it is difficult to discriminate between them. The two sets of predictions generally differ by less than one percentage point so that it would take literally thousands of observations on each pairwise choice

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probability to gain sufficient reliability to accept one theory and reject the other.

Since for practical purposes the two theories do not differ in their account of such results, we must look elsewhere to find reasons for favoring one or the other. Lacking any further constraints, the choice between the two alternatives would seem to be determined by considerations of ease, convenience, and parsimony of assumptions. On these terms alone, the ratio rule would be favored, since (*a*) it is easier to work with mathematically, and (*b*) Luce has shown that it follows from a single assumption about choice behavior, whereas the Thurstone theory requires a number of assumptions, some of which appear unrealistic for finite sets of choice alternatives.[1] Moreover, if a subject's choices among a set of alternatives satisfy Luce's axiom, then we can assign numerals (strengths, utilities) to these alternatives which reflect his choices and which are unique except for multiplication by a positive constant (i.e., ratio measurement is guaranteed).

An alternative line of approach to selection between the Luce and Thurstone rules would be to seek further constraints which a suitable composition rule must satisfy. Such extra constraints could arise from two different sources. One source might be highly confirmed behavioral laws relating response strength to experimental variables. Given such laws for single responses, then the various combination rules may be differentiated by their predictions about certain choice experiments. To cite one example, if we had a well-confirmed law that drive and incentive motivation interact multiplicatively in determining response strength, as Hull [6] has supposed, then the Thurstone rule would imply that choice between alternative rewards would be an increasing function of drive level, whereas the Luce rule would predict no effect of drive level on choice probability in this situation. Other examples of this sort could be cited, but in each case the validity of the test presumes the prior existence of valid theoretical laws about the strengths of single instrumental responses. However, there is some doubt even among behavior theorists whether such well-confirmed theoretical laws can be provided in the near future.

The second source of extra constraints could arise from a more detailed analysis and representation of the component behaviors of a subject during the act of choosing, and it is to this task that we now turn. In the remainder of this paper, we present a detailed representation (model) of what goes on during a single act of choice by an individual, we discuss some direct empirical support for this model, and finally we show that Luce's ratio rule falls out directly as a theorem from this representation of the behavior of a subject at a choice point.

The model [2] began as a rather modest attempt to describe the behavior of a rat at the choice point of a *T*-maze, so let us turn our attention to this

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[1]This brief sentence suffices here for complex issues, but can be unpacked by the interested reader by referring to Luce [7], Adams and Messick [1], and Marschak [8].

furry rodent as he putters about in the maze. The maze has right and left pathways leading off from the choice point; to be concrete, suppose the left alley is painted white and the right one is painted black. As any seasoned rat-watcher can tell you, the animal's relevant behaviors at the choice point can be categorized, more or less exhaustively, into two classes; orienting to, or looking at, or noticing the alternative choice stimuli, and finally approaching one of the stimuli he is looking at. The animal may spend a good deal of time looking back and forth between the two pathways before he finally approaches one. For reasons indigenous to experimental psychologists, this looking back and forth at the choice point has been labeled vicarious trial-and-error behavior, or VTE for short.

In abstract terms, we may think of our rat at the choice point as being at any time in one of several orienting states: looking straight ahead, looking to the right at the black arm of the maze, or looking to the left at the white arm. When he is oriented toward, say, the black alley, he either reorients or he approaches it, and thus the trial terminates. The original idea of this model was to consider the subject to be performing a random walk over these orienting states before he finally approaches one of the stimuli. In the language of random walks, there would be two absorbing states here, approaching the black or approaching the white, since either of these events terminates the trial. A diagram of the process is shown at the top of Fig. 1. This diagram is supposed to be a skeleton of the animal's behavior at the choice point; indeed, I've laid out the skeleton so that it still resembles a *T*-maze.

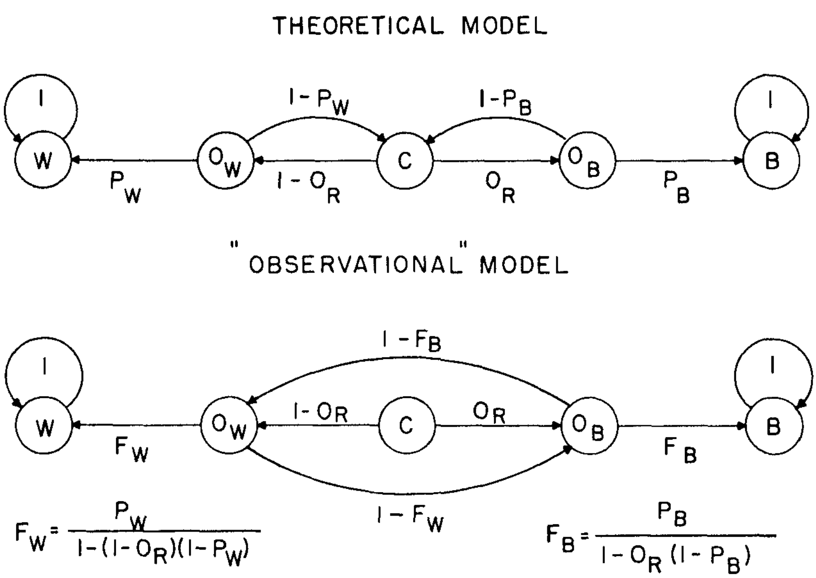

FIG. 1 State diagrams of the random walk process. The probabilities of the various transitions among states are indicated beside the appropriate transition arrow. The observational model at the bottom is derived from the theoretical model at the top of the figure. (See text for explanation.)

The animal begins the trial in the center state, C, oriented straight ahead. With probability o_R he orients to the right, where he sees the black alley; he is then in state O_B. When oriented to the black alley, he may approach it with probability p_B, but with probability $1 - p_B$ he reorients back to the center and the random walk continues. It continues, in fact, until the subject approaches the black or white alley, and thus terminates the trial. 'Choosing the black alley' is to be identified with absorption of the process in state B after a random walk over the transient states. According to this conception, then, the subject's eventual choice is regarded to be the outcome of the interaction of a number of molecular behaviors that go on at the choice point. If we knew the transition probabilities between these states, then in principle we would know everything about the subject's behavior at the choice point. In particular, we can calculate the probability that the subject ends the trial by choosing the black alley rather than the white one. Of more immediate interest here is the fact that the model predicts the distribution of the number of transitions between the various orienting states before absorption occurs. It is the empirical accuracy of these predictions which I propose to offer as independent support for this model of choice behavior.

Suppose we concentrate on the number of times the subject switches his orientation between the black and white stimuli; this would be the number of VTE's on a given trial. One can easily derive the expected distribution of this random variable and if we knew the values of the underlying transition probabilities we could compare our predictions with the data. However, one practical problem with this strategy is that we cannot hope to estimate directly the underlying approach probabilities from watching the rat. The practical difficulty is that we probably would be unable to detect all transitions of a 'loop' form, say, from O_B to C and back to O_B, and these would be necessary to obtain accurate estimates of the approach probabilities. The only behaviors we can hope to record reliably are the complete shifts in orientation from one side of the maze to the other side. These movements are gross enough to be seen and recorded without serious error.

For puposes of VTE predictions, then, it is of practical advantage to replace temporarily the underlying model with what I have called the 'observational' model in the lower part of Fig. 1. The observational model is derived strictly from the one above it, except that here we are considering the probability of an eventual shift between the two orienting states. For example, $1 - F_B$ represents the probability of eventually switching over to the white alley after the subject starts out by orienting to the black alley. The formulas at the bottom of Fig. 1 give the expressions for F_B and F_W calculated from the underlying model.

From this representation, the distribution of the number of VTE's is easily derived as a function of F_B and F_W; since these latter quantities can be estimated from the data, we are in a position to predict the distribution of VTE's.

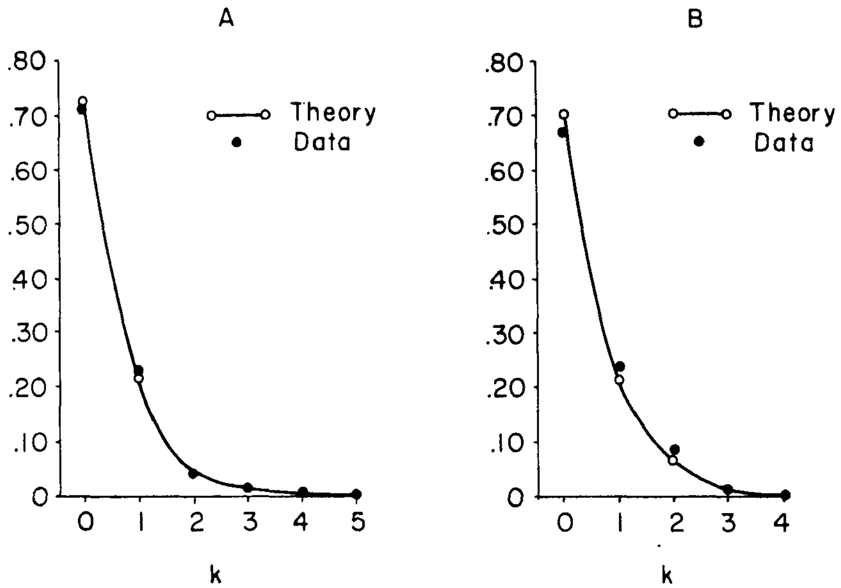

FIG. 2 Observed and predicted distributions on the number of VTE's in two experiments with rats in a T-maze.

In Fig. 2 are shown some predicted and obtained distributions from two groups of 15 rats learning T-maze problems. These data were collected during the middle course of learning and the basic probabilities were estimated from the data. It is clear from Fig. 2 that the model accurately predicts the VTE distributions obtained in these experiments. We have conducted several other experiments in which the predicted VTE distributions fit as well as or better than in these two studies. Since the model has performed well on a number of such occasions, there is little doubt that some kind of random-walk process is the appropriate model for choice point behavior.

As a bonus for the model's accounting for VTE's, it also gives a reasonably good account of choice times. In several experiments with rats we have obtained correlations in the high nineties between average choice time and VTE's, suggesting that decision time measures are almost redundant when VTE's are recorded. Another nice feature here is that the model delivers the correct prediction that choices of the more probable response are generally quicker than choices of the less probable response.

The previous remarks apply to observations of choice behavior taken over a single experimental trial during which we may assume that the basic transition probabilities remain constant. We have a static model, telling us what will be observed when the response tendencies are such and so. The model can be applied, of course, to those dynamic situations in which the response tendencies are changing from trial to trial as the result of experimental manipulations. An obvious example of a dynamic situation is a learning experiment in which the subject is rewarded or not depending upon which alternative he chooses. The model makes a number of unique predictions about the results of some learning experiments. Space limitations do not permit a full discussion here of many of these cases, but we shall briefly discuss two cases to illustrate the type of predictions derivable from the model.

The first condition to be discussed is that in which the subject is learning to choose, say, the black alley as it is shifted from side to side in the maze; this condition is contrasted with that in which the black-white cues are in fixed spatial positions (right or left of the choice point). The second case to be discussed is that in which, following initial learning, the reward significance of the two brightness cues is reversed.

Consider first the learning of a brightness discrimination in which approaches to the black alley are rewarded while approaches to the white alley are not rewarded. The result of these contingencies will be an increase over successive trials in the subject's tendency to approach the rewarded black alley and a decrease in his tendency to approach the non-rewarded white alley. If the rewarded black alley is shifted from right to left at random in the maze, then over successive trials the subject's orientation probability, o_R, will not vary appreciably from one-half because he has not been consistently reinforced for looking right or left at the choice point. However, if the rewarded black alley is always on the right side of the maze ('position' learning), then orientations to the right are consistently reinforced and o_R would be expected to increase to unity under such conditions. It is this latter fact which differentiates position learning from stimulus discrimination learning.

In Fig. 3 are shown some hypothetical curves illustrating the qualitative differences to be expected for these two cases. The solid curves are for position learning and the dashed curves for stimulus discrimination learning conditions. For both conditions we have plotted the probability of the correct response (upper curves, to be referred to the right ordinate) and the average or expected number of VTE's (lower curves, to be referred to the left ordinate). For this illustration, it has been assumed, as in previous work [2], that reward and nonreward increase and decrease, respectively, the approach probabilities by linear transformations.[2]

The first thing to notice in Fig. 3 is that the probability of the correct response is expected to increase faster in conditions of position learning, and this is a well-established qualitative result in animal learning [10]. The expected difference here is due to the fact that, in position learning, the subject not only is learning which stimulus to approach and which to avoid, but is also learning which way to orient at the choice point to expose himself to the rewarded stimulus. Corresponding to the expected changes in the orienting and approach probabilities, the average number of VTE's for

[2]Fortunately, the qualitative results given in Figs. 3 and 4 do not depend to any great extent upon the form or details of the conditioning laws we use in transforming the basic probabilities following the reward-nonreward outcomes.

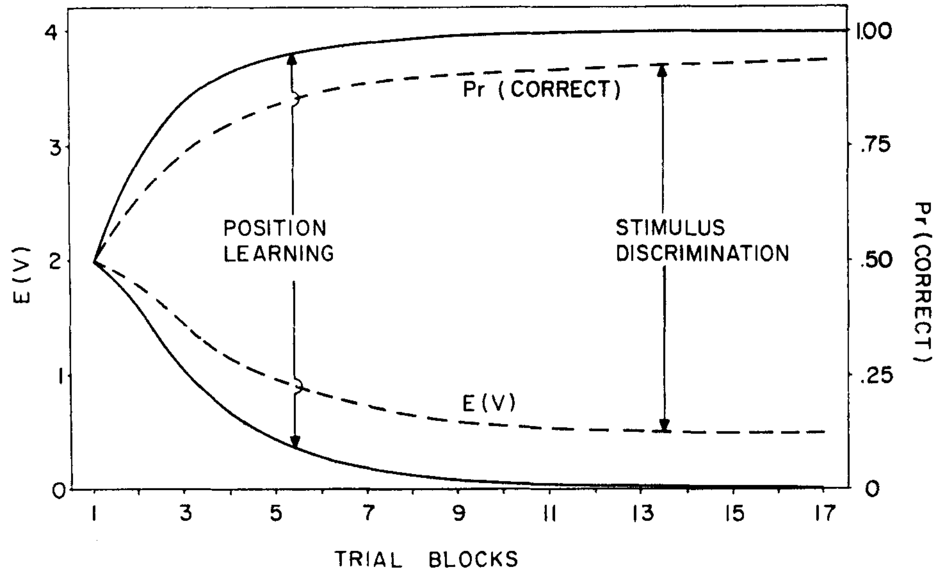

FIG. 3 Hypothetical curves comparing VTE (left ordinate) and choice probability (right ordinate) for conditions of position learning (solid curves) and stimulus discrimination learning (dashed curves). Successive blocks of training trial are plotted on the abscissa.

position learners is expected to drop rapidly to zero. By contrast, the initial orientation at the choice point of the subjects in the stimulus discrimination conditions will continue to be random, so that at asymptote their initial orientation on half the trials will be to the nonrewarded white alley, and one VTE will occur on such trials. Hence, the asymptote of E(V) is .50 for these conditions, and this result is well-established empirically.

According to the theory, the exact path of the VTE curve during learning will depend considerably upon the initial strengths of the approach tendencies to the alternative choice stimuli. For the cases shown in Fig. 3 it was assumed that the initial values of p_B and p_W were low and equal. A completely different course of learning and VTE is expected for those conditions in which the approach tendency to the incorrect stimulus is initially higher than that to the correct stimulus. Such conditions might be expected to obtain in the stimulus discrimination problem, for example, if the reward values of the black and white alleys are reversed after the subject has learned previously to approach one and avoid the other. In Fig. 4 are shown the expected results for reversal learning under two conditions: in one condition, depicted by the dashed lines, the subject simply receives no reward for incorrect choices (and is not permitted to correct his errors); in the other condition, shown in solid curves, the subject is punished, say, by an electric shock every time he makes an error. In terms of conditioning assumptions, the punishment is conceived simply as reducing more rapidly the subject's tendency to approach the incorrect stimulus.

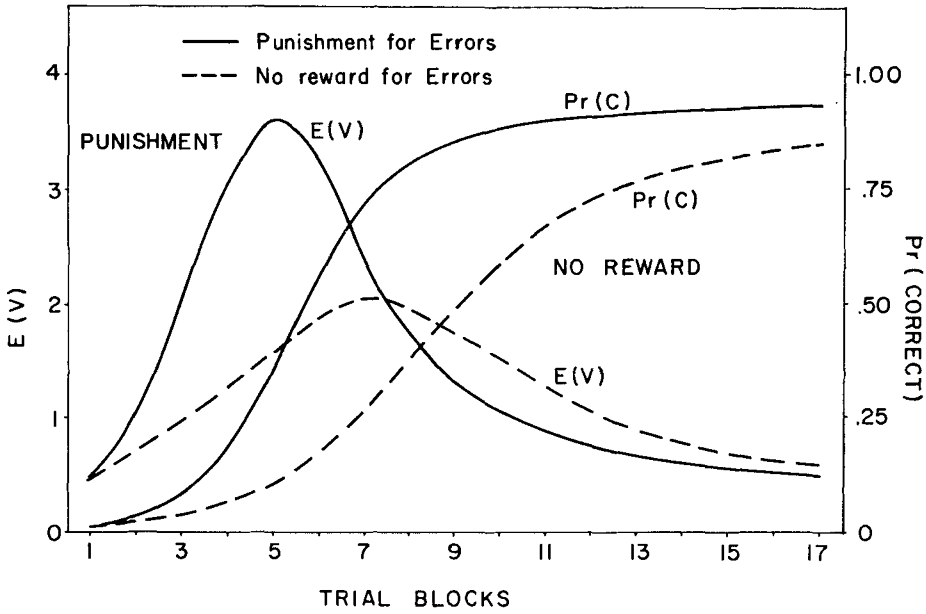

FIG 4. Hypothetical curves showing VTE and choice probability during reversal of a previously learned stimulus discrimination. Two conditions are compared: no reward for errors (dashed curves); and no reward plus punishment for errors (solid curves).

There are several features of Fig. 4 which may be noted. First, the curve for the probability of the correct response during reversal learning is expected to be *S*-shaped rather than negatively accelerated as it was during initial learning when the two responses begin at nearly equal strengths. Secondly, the course of VTE during reversal learning differs from that in initial learning; the E(V) rises to a maximum as the strong incorrect response tendency is reduced, and then falls off as the correct response is strengthened. Finally, the effect of punishment for incorrect responses, as compared with just nonreward for errors, is to increase the absolute amount of VTE behavior and to lead to faster learning of the new correct response.

The qualitative aspects of these last predictions seem to be in general accord with the scant amount of VTE data that have been collected during reversal learning. An experiment by Davenport [4] has demonstrated in rats the hump-shaped VTE curve during reversal learning. Punishment for errors during reversal learning is well-known to increase the rate of learning; VTE during punished reversal learning has not been compared with that using nonreward only, but relevant comparisons during original learning [9] have shown that punishment results in increased levels of VTE behavior.

The previous discussion has provided what I consider to be independent empirical support for this model of choice point behavior. Let us now return to the earlier discussion of different combination rules for determining choice probability given the respective strengths of the competing responses.

One interesting feature of the choice point model is that it yields as a special case Luce's ratio rule for determining choice probabilities. Suppose we label the alternative choice stimuli as S_1 and S_2, rather than talking about black and white alleys. The probability of absorption at S_1 when a choice is required between S_1 and S_2 may be derived to be

$$P\{1|1,\ 2\} = \frac{o_1 p_1}{o_1 p_1 + (1-o_1) p_2}. \tag{1}$$

In this expression, o_1 represents the initial probability of orienting to S_1. For most of the situations we shall discuss, in particular for paired comparisons, we may reasonably assume that o_1 is one-half; that is, we may assume that the subject orients at random to the alternative stimuli. With this condition, then, the probability of choosing alternative 1 over 2 turns out to be a simple function of the ratio of the two approach tendencies.

Suppose we look at some further consequences of this model. Consider first the paired-comparison experiment with a number of commodities, the subjects indicating their choices for each possible pair. In this case, we may think of the approach probabilities as reflecting the subject's evaluation of the commodities. The first result to be discussed concerns the relationship among the pairwise choice probabilities for any three commodities, say i, j, and k. Suppose we take the ratio of the probability of choosing i over j to the probability of choosing j over i. The value of this ratio, which we may call w_{ij}, is

$$w_{ij} = \frac{P(i|i,\ j)}{P(j|i,\ j)} = \frac{p_i}{p_j}. \tag{2}$$

Using these w_{ij} numbers, we may prove a theorem which Estes [5] has called the product rule and which Luce [7] calls the triple condition. The theorem is

$$w_{ik} = w_{ij} \cdot w_{jk}. \tag{3}$$

PROOF:

$$\frac{p_i}{p_k} = \frac{p_i}{p_j} \cdot \frac{p_j}{p_k}.$$

The product rule may be regarded as a probabilistic form of transitivity: it says that if we know the probabilities that i is chosen over j, and j is chosen over k, then we can predict the probability that i is chosen over k.

Some experimental results on the product rule are shown in Table 1. At the top are the results of two rat studies done in T-mazes, the first one by P. T. Young [12] and the second one by the author, and at the bottom are some results of two studies with human subjects in a mock consumer survey, the first of these done by Estes [5] and the second one by the author. In the study by Young, rats' preferences for sugar, wheat, and casein were determined by pairing each substance with the other two in independent compar-

TABLE 1

EXPERIMENTAL TESTS OF THE PRODUCT RULE

	w_{ij}	w_{jk}	predicted w_{ik}	observed w_{ik}
Young (rats)	$W_{SW} = \frac{.55}{.45}$	$W_{WC} = \frac{.70}{.30}$	$W_{SC} = \frac{.74}{.26}$	$\frac{.738}{.262}$
Bower (rats)	$W_{62} = \frac{.992}{.008}$	$W_{24} = \frac{.040}{.960}$	$W_{64} = \frac{.835}{.165}$	$\frac{.833}{.167}$
Estes (humans)	$W_{RB} = \frac{.71}{.29}$	$W_{BY} = \frac{.73}{.27}$	$W_{RY} = \frac{.87}{.13}$	$\frac{.89}{.11}$
Bower (humans)	$W_{GB} = \frac{.65}{.35}$	$W_{BY} = \frac{.55}{.45}$	$W_{GY} = \frac{.70}{.30}$	$\frac{.71}{.29}$

isons. The rats chose sugar over wheat 55 per cent of the time, and wheat over casein 70 per cent of the time. By the product rule, we may predict that sugar will be chosen over casein 74 per cent of the trials, and this prediction compares favorably with the observed percentage of 73.8 shown in the brackets to the side. The study by the author compared rats' asymptotic probability of choosing different amounts of food reward, 6, 4, or 2 tiny food pellets. Three groups of rats were run, one group corresponding to each of the three possible pairs. The results predicted here by the product rule are again quite accurate: 83.5 predicted versus 83.3 observed as the per cent choice of 6 over 4 pellets. In the mock consumer studies with college students, the subjects were asked to indicate their preferences among different pairs of color-combinations for automobiles. For example, in the first study, by Estes, the specific color-combinations compared were red-white, blue-gray, and yellow-black. Comparing the predicted and observed values in the bottom two studies, we see that the product rule performs about as well with the human subjects as it did with the rats. These data strongly suggest that the product rule may be regarded as a low-level law about choice probabilities in this type of situation.

As a final illustration of this model, let us consider its implications for situations in which the subject is required to choose from among more than two alternatives at a time. As one example of this, we may extend the ordinary paired comparisons experiment by having the subjects also indicate their first choices from among all possible sets of three commodities, from among all possible sets of four commodities, and so on. Unfortunately, theoretical and experimental treatment of such multiple choice problems has been seriously neglected in the history of psychology, so the data in this area are quite meager. This neglect has been partly due to the fact that prior to the appearance of Luce's axiom the Thurstone theory was the only usable choice axiom known to psychologists, and that theory has never extended to the multiple choice situation. I might add that the problems in so extending Thurstone's theory seem a bit formidable at the present.

Fortunately, extension of the choice-point model to the multiple choice case is a relatively easy matter. In fact, if we can assume that the subject orients at random to the alternative stimuli in the set presented to him, then the choice probabilities are a simple function of the ratios of the alternative approach tendencies. This result is shown in Equation 4.

$$(4)\quad P\{i|1, 2, \ldots, N\} = \frac{p_i}{p_1 + p_2 + \ldots + p_N} = \frac{1}{w_{1i} + w_{2i} + \ldots + w_{Ni}}.$$

When N stimuli are presented, the probability that the first choice is alternative i is given by p_i divided by the sum of the N approach tendencies. If we were to divide this expression by p_i then it is clear that it's just a function of the ratios of p_k to p_i. These ratios we have met before in the two choice problem where we called them w_{ki} and so forth. The implication of this simple result is that first choices from higher sets of alternatives should be predictable in a straightforward manner from pairwise choice probabilities, since all the w_{ki} can be estimated from pairwise choices. If the theory is correct, then it is enormously productive in terms of the number of predictions it can make. For example, with 6 alternative stimuli, there are 57 possible combinations or choice sets (pairs, triples, fours, etc.) which may be presented to the subject.[3] To describe the distribution of responses in all 57 choice sets would require 129 numbers. If the theory is right, then we can use 5 pairwise choice probabilities to estimate the ratios of the response tendencies, and then predict the other 124 probabilities. We would be getting roughly 25 predictions for each bit of data we use in estimating our parameters, and that is a fairly good operating characteristic for a theory.

One simple way by which part of this grandiose scheme may be tested is shown in Equation 5.

$$(5)\qquad w_{ij} = \frac{P(i|1, 2, \ldots, N)}{p(j|1, 2, \ldots, N)} = \frac{p_i}{p_j}.$$

Again let us take the ratio of the probability that the subject's first choice is alternative i when he chooses among N stimuli to the probability that his first choice is alternative j. The summation in the denominators of both probabilities cancels out and we are left simply with the ratio of p_i to p_j, which is the same as that given in Equation 2 for ratios of pairwise probabilities. The important point here is that this ratio is not affected by the number of alternatives that are added on to the choice set in addition to i and j. This constant ratio rule is often regarded as one version of Luce's choice axiom and we see that it also follows from the random walk model.

Empirical tests of the constant ratio rule are particularly scanty. There is one psychophysical experiment by Clarke [3] which gave generally confirmatory results. We attempted to test the constant ratio rule in a mock con-

[3]In general, for N alternatives, there will be $2^N - N - 1$ possible choice sets and $N-1$ free parameters (the w_{ij}'s) to be estimated from the data.

sumer survey, some of the data of which were reported in Table 1. There were 4 choice objects, in this case color-combinations for automobiles. With 4 objects you can construct 6 pairs, 4 triples, and the one set containing all 4 objects. Each of 170 subjects was required to indicate a first choice for each of these choice sets. From this data there are 6 independent probability ratios, and for each of these ratios we calculated the values for choice sets of size 2, 3, and 4. As one might have expected from sampling theory, ratios of probability estimates based on only 170 observations show considerable variability. However, to indicate the overall trend for these six series of probability ratios, I have taken the average ratios for pairs, for triples, and for the one quartet, and these values are shown in Table 2. Although one can't make a definite statistical statement about whether the slight discrepancies from constancy could have occurred by chance, I have accepted, for the moment at least, the conclusion that these ratios are constant, and that the observed deviations can be attributed to chance sampling error. Although this is not a particularly resounding conclusion, we can at least be assured that the data have not entirely defeated our expectations on the constant ratio rule.

TABLE 2

AVERAGE VALUES OF PROBABILITY RATIOS FOR CHOICE SETS OF SIZE 2, 3, AND 4

N	average w_{ij}
2	2.15
3	2.21
4	2.19

What the previous discussion has attempted to show is that there are extra constraints on the decision about suitable ways (rules) by which alternative response strengths may be combined to yield choice probabilities. The extra constraints arise from a more detailed analysis and representation of the (pre-decision) behaviors the subject engages in during the act of choice, and we have provided independent empirical support for this representation. The important result of interest here was that this model for choice behavior dictated a ratio rule for combining competing response strengths to determine choice probability. The result in this case is the same as that obtained by Luce through his axiomatic approach to choice behavior.

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