AN ASSOCIATION MODEL FOR RESPONSE AND TRAINING VARIABLES IN PAIRED-ASSOCIATE LEARNING

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This paper reports an attempt to apply a simple association learning model to the analysis of the influence of response and training variables on paired-associate learning. The issues under investigation are old ones but have not been resolved satisfactorily in the past. With the aid of the elementary learning model, the problems are posed clearly and, the data willing, adequately resolved by the use of a few simple and intuitively compelling assumptions about learning.

The first problem that led to this investigation concerns the relationship between the number of response alternatives (N) and error rate in paired-associate learning. Experimental results (Noble, 1955; Riley, 1952) are in agreement in showing that the number of errors subjects make before reaching some criterion of learning is greater the larger the number of response alternatives. There is little agreement, however, as to the interpretation of this fact.

Two possible factors could be involved. First, the effectiveness of a reinforced trial in increasing performance (i.e., the learning rate constant) may be influenced by N; and second, N may influence the probability of being correct by sheer guessing on items that are yet unlearned. It is a reasonably safe assumption that N has the second effect on chance guessing. Previous data are unclear on whether N also influences the first factor, the effectiveness of a reinforced trial. This is not an easy question to answer since it is difficult to separate the effects of these two factors in the data. Guessing occurs only on unlearned items but there is no way to tell by direct observation just how many items have been learned and how many have been guessed correctly on any given trial. Moreover, there appears to be little hope that more refined or ingenious experimental procedures will enable us to unconfound these two factors so that we may crucially test the hypothesis that N affects the learning rate.

This is the type of situation in which a theoretical model of learning can make a strategic contribution. Indeed, without the aid of some formal model of the learning process, the question of the effect of N can neither be posed clearly nor answered clearly. With the aid of a theory one can make suitable allowance for the guessing factor and thus make an assessment of the contribution of learning and guessing at every stage of the experiment.

The model to be presented has been formally treated in more detail in a previous paper (Bower, 1961). Here the theory will be presented informally and only those implications relevant to the present discussion will be introduced. For a more formal statement of the axioms and theorems, the reader may consult the prior report. The basic notion of the model is the assumption that each stimulus item and its correct response become associated on an all-or-none basis. Considering a single item, it can be in either of

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1 This investigation was supported in part by a grant, M-3849, from the National Institute of Health, United States Public Health Service.
two states at the beginning of each trial: conditioned to its correct response or not conditioned. If the item is conditioned at the beginning of a trial, then the correct response occurs. If the item is not conditioned, then the probability of the correct response depends somewhat upon the experimental procedure. In experiments by the writer, the subjects were told the $N$ responses (Integers 1, 2, . . . $N$) available to them and were told to respond on every trial regardless of whether they knew the correct answer. If the $N$ numbers occur equally often as the to-be-learned responses to the items, then the probability that the subject will guess correctly on an unlearned item is $1/N$ on the simplest assumptions; $^2$ correspondingly, his probability of guessing incorrectly is $1 - 1/N$. The following discussion of the model is oriented specifically towards such an experimental procedure.

With the theory formulated in this way, one cannot uniquely specify the subject's state of conditioning on a given item from the knowledge that he made a correct response, since this correct response may have come about by guessing on an unlearned item. Thus, the theory does not imply that the first correct response will be followed by correct responses on all subsequent trials. However, if the subject makes an error, then we can make a determinate inference that the item was not conditioned at the beginning of the trial.

We have introduced the notions of the two states of conditioning and the probability of the correct response given the item's state of conditioning at the beginning of each trial. We assume that each item in initially unconditioned and that the effect of successive reinforced trials is to provide repeated opportunities for the item to become conditioned. The single parameter of the theory is the learning rate constant ($c$) which represents the probability that an unconditioned item becomes conditioned as the result of a single reinforced trial. The probability that a reinforced trial fails to condition the correct response to an unlearned item is $1 - c$. The probability that an item is still not conditioned after $n$ reinforced trials is $(1 - c)^n$. If $c$ is larger than zero, then this probability approaches zero as $n$ becomes large; that is, given a large number of reinforced trials, it is certain that the item will become conditioned on some one of these reinforced trials. When the item becomes conditioned, the probability of a correct response jumps from $1/N$ up to 1.

From the considerations above we

$^2$ The arguments to be developed apply only to those conditions in which the response alternatives are immediately available to the subject and he is permitted time to give a relevant response on each trial before the terminal event (e.g., reinforcement) occurs. This is the procedure most familiar to those who run animal subjects in choice situations. A more frequent procedure in human learning experiments has been to give the subject a fixed time interval in which he may respond, and the terminal event (e.g., correct response in paired associates, next item in a serial list) follows after that time interval regardless of whether the subject makes a relevant response. To remark on the sociology of experimenters, there seems to be an empirical correlation between the use of experimenter vs. subject-controlled exposure time and whether the relevant responses per se (as well as the S-R associations) must be learned in the experimental situation: when the responses must be learned, experimenters usually control exposure time; when the response alternatives are immediately available, either via instruction or construction, the subject is allowed to control exposure time. Clearly, however, there is no necessary entailment between these procedural aspects, and one may expect the empty cells of the $2 \times 2$ contingency table to be filled by future experimentation.
may obtain an expression for $q_n$, the probability that the subject responds incorrectly to a given item on Trial $n$ of the experiment. To obtain $q_n$ we note that the probability that the item has failed to be conditioned during the preceding $n - 1$ trials is $(1 - c)^{n-1}$; and if the item is not conditioned then the probability of guessing incorrectly is $1 - 1/N$. Hence, the probability of an error on Trial $n$ is given by the product of these two factors:

$$q_n = (1 - 1/N) (1 - c)^{n-1} \quad [1]$$

Using this elementary association model, the first question of this investigation can be restated in a clear manner: Is $c$ independent of $N$? If $c$ is independent of $N$, then, given an estimate of $c$ obtained from one group, we should be able to predict in advance the performance of other groups trained with differing numbers of response alternatives by simply adjusting the $N$ factor in Equation 1. To the extent that these free predictions are accurate, we would have good evidence for the assertion that $N$ affects guessing probabilities but has no influence on learning rate per se.

The second question prompting this investigation concerns the relation between several different training conditions and performance in paired-associate learning. The two major conditions of training studied to date are the correction and noncorrection conditions. The more frequently used condition is the correction procedure in which subject is informed of the correct response on every trial regardless of whether he responds correctly. In noncorrection training, the subject is told right or wrong depending on whether his response is correct or incorrect. In the following, we will use the word “reinforcement” in the general sense intended by Estes (1960), viz., as operations exerting certain general quantitative effects upon response probabilities. In paired-associate experiments, the reinforcing operation is that of informing subjects of the correct response or in some way ensuring that this response is the last one to occur in the presence of a stimulus item. With this interpretation, then, on every trial under a correction procedure the correct response is reinforced. The noncorrection training procedure differs in that the correct response is reinforced only on those trials when it occurs; since all incorrect responses are followed by the experimenter simply saying “Wrong,” no explicit reinforcement operation is involved. We will refer to such trials as nonreinforced trials. Within the model, an account of the noncorrection procedure takes the form of specific assumptions about the formation of associations following reinforced and nonreinforced trials. Considerations of parsimony lead us to assume that the effect on conditioning of a reinforced trial is the same whether the reinforcement occurs in the context of a correction or noncorrection procedure, viz., the probability that an unlearned item becomes conditioned to its reinforced response is $c$, the same constant as in the case of the correction procedure. A new assumption that is required concerns the effect of a nonreinforced trial, when the experimenter says “Wrong” following an error. The assumption that has been made elsewhere (Thorndike, 1932) and that is made here also, is that a nonreinforced trial in this situation results in no net change in the probability of a correct response. This assumption is not offered as a universal interpretation of nonreinforcement; however, within the specific situation to which the model will be applied (i.e., the alternative responses are equiprobable initially and a num-
number of different S-R events occur between successive presentations of any given stimulus) there is some basis for thinking that the no change assumption may be approximately correct.

From these assumptions about the noncorrection procedure, a difference equation can be derived expressing the change in the average probability of an error from Trial $n$ to Trial $n+1$:

$$q_{n+1} = q_n\left[q_n\right] + (1 - q_n)\left[(1 - c)q_n\right] \quad [2]$$

The first term on the right-hand side of Equation 2 exhibits the assumption of no change (i.e., $q_{n+1} = q_n$) when the subject makes an error on Trial $n$, with probability $q_n$; the second term shows that if a success occurs (with probability $1 - q_n$), then with probability $c$ the item becomes conditioned (if it is not already) and with probability $1 - c$ the reinforcement was ineffective and no change occurs on that particular trial. An explicit solution of Equation 2 is not available. However, Miller and McGill (1952; see also Bush & Mosteller, 1955, p. 181), have derived the following recurrence relation for Equation 2 (the constants have been changed appropriately):

$$q_n = (1 - 1/N) [1 - (1 - c)^{n-1}]q_{n-1} + (1 - 1/N)(1 - c)^{n-1} \quad [3]$$

Equation 3 can be used to compute successive values of the average error probability once $N$ and $c$ are known. In general, the predicted learning curve is S shaped, a feature which is consistent with the expectation of a low rate of improvement early in training when the subject is trying to discover the correct response. The point of inflection of the S curve will be positively related to $N$. These features seem in qualitative agreement with noncorrection results reported by Noble (1955), although his experimental procedure was more complex than the one under present consideration.

At this point in the analysis, Experiment I was carried out with four groups of subjects learning the same list of 10 paired associates. The variables were the number of response alternatives (the first three or first eight integers) and correction vs. noncorrection training procedure. For brevity, the four groups will be referred to by the symbols 3-C, 8-C, 3-NC and 8-NC, where 3 or 8 represent the number of responses, and C and NC designate correction and noncorrection training, respectively. A more extensive discussion of the procedure and results of Experiment I will be deferred till later; it will be sufficient here to note one critical result of Experiment I since it was this fact that led to Experiment II. The fact was that strong evidence was obtained to support the assumption of no change in success probability following a nonreinforced trial in the noncorrection training groups; that is, with either three or eight response alternatives, saying “Wrong” followed a subject’s incorrect response had no effect on his probability of success on the next trial.

This no change result supports our earlier assumption about nonreinforced trials, but how are we to understand it? On rational grounds, one might expect that subjects would tend to eliminate a response which was followed by “Wrong.” For example, if subjects in the 3-NC group tended to eliminate their first erroneous response, then the probability of the correct response on the next trial should be around one-half or at least greater than one-third. However, the results of Experiment I clearly showed that this increase did not occur, and
we seek some explanation for why it did not occur.

An explanation may be found perhaps by attending to the responses evoked in the subject by saying to him "Wrong" after he has responded, e.g., "three." The reason for this inquiry is that, holding to a strict contiguity interpretation for the formation of association, it is these terminal reactions to the stimulus which have an opportunity to become conditioned. Often these implicit reactions to "Wrong" are primarily emotional; in this regard, it may be reported that subjects frequently volunteered the information, "I know I've been getting that item wrong, but I can't remember what number I said last time." Clearly, if the subjects' implicit reactions are primarily emotional, then we may expect no change in the recorded response probabilities on the next trial. However, if subjects implicitly react to "three is wrong" with the response "one or two is correct," then the contiguity interpretation would imply that the probability of response "three" would decrease and the probability of responses "one" and/or "two" would correspondingly increase.

According to this interpretation, the noncorrection procedures in Experiment I did not insure responses of the form, e.g., "one or two is correct" after the experimenter said "Wrong" to "three." For one of the groups in Experiment II, conditions were arranged to insure the occurrence of responses in this form following errors. There were three response alternatives and subjects were instructed that one and only one number was correct for each nonsense syllable. If, for example, the subject responded with "three," instead of saying "Wrong," the experimenter said "one or two is correct." It should be noted that, in a formal sense, the subject gains no more information about the correct response with this procedure than when the experimenter says "Wrong," as in the noncorrection procedure. Comparisons of the results of this procedure with the standard noncorrection procedure are given below.

The second condition run in Experiment II was aimed at a slightly different question. The question was whether there is some more basic way of specifying reinforcement contingencies in this situation rather than merely listing them, correction and noncorrection. The formulation proposed here is that this basic variable is the degree to which the experimenter specifies the correct response following an error by the subject; to describe it in another way, the variable is the size of the subset of alternatives within which the one correct alternative is said to lie. The correction and noncorrection procedures occupy the extreme poles of this dimension; in the correction procedure, the experimenter uniquely specifies the correct response; in the noncorrection procedure, when the experimenter says "Wrong," he implicitly specifies "one of the other $N - 1$ alternatives is correct." To illustrate the construction of intermediate values of this training dimension, suppose there were eight response alternatives and that the subject responds incorrectly with "one" to a given item; then the experimenter might say "three or six is correct," or "six or eight or two or five is correct," and so on. For the second group in Experiment II, with eight responses, the experimenter said two numbers following errors. The subjects were instructed that one of the numbers was correct and that the other number was a distractor, but there were no cues.

3 However, see the related discussion above concerning the subject's reactions to "Wrong."
as to which number was the correct one. Of course, if the subject gave the correct number, the experimenter indicated this to him.

It is clear that such a continuum of training conditions can be constructed. It is also clear that variations in this training variable should produce graded variations in performance intermediate between the two extremes produced by the correction and non-correction procedures. To describe some of the factors involved in this prediction, consider the reasons for expecting the partial correction group listed above (call it the 8-P group) to make fewer errors than the 8-NC group from Experiment I.

First, a subject in the 8-P condition is expected to make fewer errors before his first correct response because there is at least some likelihood that the correct association will be formed following an error. Specifically, we assume that before the first correct response the probability that the item becomes conditioned following an error is \(\frac{c}{2}\), where \(c\) is the same learning constant as before. In general, if the subject is told \(k\) alternatives, one of which is correct, then we assume that the probability of the correct association being formed is \(\frac{c}{k}\) on each trial before the first success. This formulation is equivalent to assuming that the subject selects at random one of the \(k\) possibly correct responses to rehearse on a given trial, with probability \(\frac{1}{k}\) he selects the correct response, and with probability \(c\) rehearsal of this correct response results in conditioning.

The preceding analysis applies on trials before the first correct response occurs. After the subject has been once informed of the correct response, his probability of recognizing it among the two numbers the experimenter says (following a subsequent error, if any) will be greater than one-half. If the subject recognizes the correct number and then rehearses it, the effect is much the same as a reinforced trial. In general, we may let \(r\) represent this recognition probability following the first correct response; because of implicit rehearsal, \(r\) is also the probability of a reinforced trial on incorrect trials following the first correct response. Specifically, we assume for the present case that \(r\) is unity; that is, following the first correct response, we assume that subject can recognize with Probability 1 the correct number among the two numbers that the experimenter says following an error. If the subject recognizes the correct response and rehearses it, then with probability \(c\) the conditioned association is formed.

To summarize the discussion for the partial correction groups, we have assumed that prior to the occurrence of the first correct response the probability of conditioning is \(\frac{c}{2}\) on each trial; on the trial of the first correct response and those trials following the first correct response and those trials following the probability of conditioning is \(c\). In a later section we suggest a number of ways to test the details of these assumptions.

In the experiments to be described there were six independent groups; if the model and theoretical assumptions are correct, then the data from the six groups can be reproduced after estimating the constant \(c\) from one of the groups selected arbitrarily. One can easily recognize the advantages of casting our theoretical assumptions in explicit form within such a model. Not only can we clearly pose the questions of how \(N\) and its interactions with training conditions affect error rate, but we have also developed a conceptual framework within which it is possible to get an answer to these questions. The basic learning parameter,
$c$, plays a central role in the theory and is not just a curve-fitting constant; for a homogeneous population of subjects with the same learning materials, the underlying theory constrains $c$ to be the same for all conditions. If indeed such parameter invariance obtains (i.e., if $c$ can be transposed from one condition to the others), then a strong appeal can be made for the simplicity and power of the underlying theory within which $c$ derives its meaning.

**The Experiments**

The subjects, 88 Stanford undergraduates, were required to learn a list of 10 paired associates to a criterion of two successive correct runs through the entire list. The stimulus items were nonsense syllables of 0–5% association value chosen for low intralist similarity. The responses were the first three or first eight integers, paired randomly with the stimuli for each subject. With three response alternatives, two of the three responses were used for three stimuli each and the third response, selected randomly for each subject, was paired with the remaining four stimulus items. With eight response alternatives, all eight numbers were first paired with eight randomly selected syllables for each subject, and then the remaining two syllables were assigned to different but randomly selected numbers for each subject.

The subjects, run individually, were instructed to learn a list of 10 nonsense syllables and their associated number responses, the response alternatives being either one, two, or three (or one, two, . . . , seven, or eight), and that the syllables and numbers had been paired off randomly. They were asked to respond within 2 or 3 seconds after the stimulus card was shown, and to guess a number if they didn’t know the answer. The result of this practice is that the exposure time cannot be specified exactly; however, it was noted that the subjects followed instructions and responded within 2 or 3 seconds in the large majority of cases.

For all three training conditions, if the subject gave the correct number to a card, the experimenter repeated it, e.g., “three is correct.” The three conditions differed on trials when the subject responded incorrectly: in the correction conditions, the experimenter said the correct number; in the non-correction conditions, the experimenter said “Wrong” following errors; and in the partial correction condition, the experimenter said two numbers following errors, e.g., “one or three is correct.” The subjects in the partial conditions were informed that only one of the numbers the experimenter said would be correct and the other number was a distractor (chosen at random) and that half the time the correct number would be said first and half the time it would be said second. The 10 stimulus cards were shuffled for 15 seconds between runs through the deck. The entire list was repeated until the subject went through two consecutive cycles without any errors.

The numbers of subjects in each group were 14 in the 3-C, 8-C, and 8-NC groups; 15 in the 3-P and 8-P groups; and 16 in the 3-NC group. Assuming that each item and each subject may be characterized by the same value of $c$, the data consist of 140, 150, or 160 sequences of correct and incorrect responses for the various experimental conditions. Before the model could be evaluated, the learning rate constant, $c$, had to be estimated. The data from the 3-C group were selected for this purpose;

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*This experiment was carried out with the assistance of Takao Umemoto.*
the least squares estimate of \( c = .218 \) was obtained by fitting the learning curve (i.e., percentage correct vs. trials) for this group.

The first predictions concern the average total errors per item before achieving the learning criterion. The predictions for the correction and non-correction groups were obtained by summing Equations 1 and 3, respectively, over trials. The predictions for the partial groups were obtained by calculating the average errors before the first success (using \( c/2 \) as the rate constant) and the average errors following the first success (using \( c \) as the rate constant) and then adding these two numbers. The observed means \((M)\) and standard errors of the means \((\sigma_M)\) are shown in Table 1 along with the predicted means \((P)\).

It is clear from Table 1 that the theory performs adequately in predicting average total errors using the single estimate of \( c \). In all six cases the predicted value falls within one standard error of the observed mean. The differential effect of \( N \) upon error rate is amplified as one proceeds from the correction through the partial to the non-correction training procedures.

A second indication of the goodness of fit of the theory is shown graphically in Figures 1 and 2 which present observed and predicted values for cumulative errors as a function of trials. The smooth curves connect the predicted values while the unconnected points are the empirical values. The predicted values for the correction and non-correction conditions were obtained by cumulating successive values of error probabilities calculated from Equations 1 and 3, respectively. The theoretical curve for the partial correction conditions is not expressible in a simple equation but successive values can be obtained. The first several values will be derived here to illustrate the procedure. Define \( Q_n \) as the probability that the item is not yet conditioned by the beginning of Trial \( n \). The probability of an error on Trial \( n \) would then be \((1 - 1/N)Q_n \). The probability of an error on Trial \( n \) would then be \( (1 - 1/N)Q_n \). The first several values of \( Q_n \) for the partial correction conditions are:

\[
\begin{align*}
Q_1 & = 1 \\
Q_2 & = \frac{1}{N}(1 - c) + \left(1 - \frac{1}{N}\right) \times \left(1 - \frac{c}{2}\right) \\
Q_3 & = \frac{1}{N}(1 - c)^2 + \left(1 - \frac{1}{N}\right) \times \left(1 - \frac{c}{2}\right) \times \frac{1}{N}(1 - c) \\
& \quad + \left(1 - \frac{1}{N}\right)^2 \left(1 - \frac{c}{2}\right)^2 \\
\end{align*}
\]

\[\text{TABLE 1} \]

<table>
<thead>
<tr>
<th>Training condition</th>
<th>Response alternatives</th>
<th>3</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction</td>
<td>( M = 2.94 )</td>
<td>( P = 3.03 )</td>
<td>( P = 4.00 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_M = .19 )</td>
<td>( \sigma_M = .25 )</td>
<td></td>
</tr>
<tr>
<td>Partial</td>
<td>( M = 3.74 )</td>
<td>( P = 3.88 )</td>
<td>( P = 5.80 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_M = .21 )</td>
<td>( \sigma_M = .35 )</td>
<td></td>
</tr>
<tr>
<td>Noncorrection</td>
<td>( M = 5.65 )</td>
<td>( P = 5.59 )</td>
<td>( P = 9.54 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_M = .34 )</td>
<td>( \sigma_M = .61 )</td>
<td></td>
</tr>
</tbody>
</table>

by assumption

\[
\begin{align*}
Q_1 & = 1 \\
Q_2 & = \frac{1}{N}(1 - c) + \left(1 - \frac{1}{N}\right) \times \left(1 - \frac{c}{2}\right) \\
Q_3 & = \frac{1}{N}(1 - c)^2 + \left(1 - \frac{1}{N}\right) \times \left(1 - \frac{c}{2}\right) \times \frac{1}{N}(1 - c) \\
& \quad + \left(1 - \frac{1}{N}\right)^2 \left(1 - \frac{c}{2}\right)^2 \\
\end{align*}
\]

The standard errors for the 3-C and 8-C groups can be predicted by the theory and turn out to be somewhat larger than the observed values (predicted values were .24 and .30 for 3-C and 8-C groups, respectively). Deriving comparable variance predictions for the partial and non-correction conditions presents complex mathematical problems which have not been solved to date.
FIG. 1. Average cumulative errors per item plotted as a function of trials for subjects having three response alternatives. (The smooth curves represent predicted values; the dots near a curve represent the corresponding observed values.)

FIG. 2. Average cumulative errors per item plotted against trials for subjects having eight response alternatives. (The smooth curves represent predicted values; the dots near a curve are the corresponding observed values.)
The value of $Q_2$ is the sum of two probabilities: (a) that a correct guess occurs on Trial 1 but conditioning failed to occur with probability $1 - c$, and (b) that subject guessed incorrectly on Trial 1 and fails to condition the correct response with probability $1 - c/2$. The value of $Q_3$ is the sum of the joint probabilities of three events: (a) correct on Trial 1, fails to condition on Trials 1 and 2; (b) error on Trial 1, success on Trial 2, fails to condition after either trial; (c) errors on both trials, not conditioned on either trial. The general rule for obtaining $Q_n$ is that prior to the first correct response the conditioning probability is $c/2$ and that on the trial of the first correct response and thereafter the conditioning probability is $c$. By multiplying these $Q_n$ values by $1 - 1/N$ and summing them, the predicted curves for the partial correction conditions are obtained. As Figures 1 and 2 show, for five of the six groups the fit of predicted to observed values is satisfactory; the correspondence to the 8-NC points would not be impressive were it not for the fact that the same $c$ constant was used in generating all the curves under appropriate boundary conditions.

Equations 2 and 3 for the noncorrection conditions were derived with the assumption that no change resulted from nonreinforced trials. To the extent that these equations adequately describe performance in the noncorrection conditions, the assumption of no change is supported. However, additional data bearing upon this assumption may be obtained by analysis of the early trials before learning has occurred. Beginning with the first trial, consider the sequence of probabilities of a correct response following 0, 1, 2, ... consecutive errors. As soon as a subject responds correctly to a given item, that item is dropped from later computations. If the randomization of responses to stimuli has been effective, then the first value of this series should be close to $1/N$, the probability of guessing the correct number by chance. Moreover, if the assumption of no change is correct, then the probabilities of a success following 1, 2, 3, ... consecutive errors should remain around $1/N$ for the noncorrection groups, deviating from this value only by random sampling fluctuations. The results of this analysis are shown in Table 2 for the noncorrection groups. It should be realized that going down the column these estimates become progressively less reliable because the number of cases involved (beginning with 160 or 140) decreases. These series of estimates for both three and eight response alternatives reveal no significant trends away from the first value. Hence, the results provide additional support for the assumption of no change following nonreinforced trials in the noncorrection conditions.

An analysis related to that above can be given for the correction and partial correction groups. Because the success rate was higher for these groups, there was insufficient data to obtain reliable estimates of single-trial probabilities such as shown in Table 2 for the noncorrection groups. How-
TABLE 3
PREDICTED AND OBSERVED CONDITIONAL PROBABILITIES OF SUCCESS FOLLOWING AN ERROR ON THE PRECEDING TRIAL

<table>
<thead>
<tr>
<th>Training condition</th>
<th>Response alternatives</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Observe</td>
<td>Predict</td>
<td>Observe</td>
</tr>
<tr>
<td>Correction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before first success</td>
<td>.48</td>
<td>.48</td>
<td>.35</td>
</tr>
<tr>
<td>After first success</td>
<td>.35</td>
<td>.35</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>.46</td>
<td>.48</td>
<td>.38</td>
</tr>
</tbody>
</table>

However, this difficulty may be circumvented by considering over all trials the probability of a success given an error on the preceding trial. For the correction group, this probability should be a constant, \( c + (1 - c)p_0 \), where \( p_0 \) is the probability of guessing correctly. For the partial correction conditions, the probability of a success following an error should be \( c/2 + (1 - c/2)p_0 \) before the first success and should be \( c + (1 - c)p_0 \) after the first success. Such predictions are very sensitive to small changes in \( p_0 \).

Our efforts to randomly assign responses to stimuli were directed towards making \( p_0 \) be close to \( 1/N \) in value. However, with even a moderate-sized sample, small deviations of the actual probability of correct guessing inevitably occur. Hence, a fairer test of the model (rather than a test of the randomization procedure) is provided by predictions based on empirical estimates of \( p_0 \). Using the first-trial estimates of \( p_0 \) (which on the whole were close to \( 1/N \)), the predicted probabilities of success following an error were obtained and are compared with the observed values in Table 3. The correspondence between predicted and observed values is quite satisfactory; we may note that the rather complex assumptions about conditioning in the partial correction conditions receive substantial support from these data.

Another way to test the assumptions of the model involves predicting the number of errors before the first success. Since for the partial and non-correction groups the stochastic process is not complicated prior to their first success, it is possible to predict both the mean and variance of this statistic for these groups. Define \( F \) to be the number of errors before the first success, and \( p_o \) to be the probability of a correct guess. Then it can be shown (Bower, 1961) that the mean and variance of \( F \) for the non-correction groups are:

\[
M = \frac{1 - p_0}{p_0}, \quad V = \frac{1 - p_0}{p_0^2} \tag{5}
\]

Similarly, for the correction groups, the mean and variance of \( F \) are:

\[
M = \frac{1 - p_0}{p_0 + c(1 - p_0)}, \quad V = M + M^2(1 - 2c) \tag{6}
\]

The expressions for the partial correction groups are the same as in Equation 6 except with \( c \) replaced by \( c/2 \).

The predicted values of the mean and standard error of \( F \) for the six groups are shown in Table 4 along with the observed means and standard errors. For the noncorrection groups, \( p_o \) was estimated as the average of the values in Table 2 since the theory says those values represent random fluctuations around \( p_0 \).

Inspection of Table 4 shows that in
the six cases the predicted mean $F_t$ is within a standard error of the observed mean, and the predictions of the standard errors are also close to the observed standard errors. In general, the average $F_t$ increases with $N$ and the differential effect of $N$ is amplified under the partial and non-correction training conditions.

It should be noted that at an empirical level $N$ has its primary influence upon errors before the first success, and has practically no influence upon errors following the first success. The reader may convince himself of this fact by subtracting the mean $F_t \pm \sigma_F$ values in Table 4 from the mean total error values in Table 1. At first glance, this observation seems to invalidate the model by showing that $N$ only influences the discovery stage but not the fixation stage of learning after the first correct response has occurred. In fact, however, theoretical predictions of average errors following the first success correspond closely to the observed values; of course, this must be so since the theory predicts $F_t$ and total errors reasonably well. The reason we expect errors following the first success to be relatively constant over $N$ is that, according to the theory, a sampling bias is involved in comparing the three and eight alternative conditions at this point. Errors following the first success will occur only for items whose first correct response came about by guessing, and for these guessed correct items we expect more errors with eight than with three response alternatives. Let $g_N$ represent the probability that with $N$ alternatives the first correct response comes about by guessing and let $e_N$ represent the average number of subsequent errors for an item whose first correct response occurred by guessing. Then the expression for the average number of errors following the first correct response (call it $w_N$) is:

$$w_N = g_N e_N + (1 - g_N) \cdot 0 = g_N e_N \quad [7]$$

That is, a proportion $g_N$ of first correct responses come about by guessing and we expect an average of $e_N$ more errors for these items; a proportion $1 - g_N$ of the first correct responses come about by conditioning on the preceding trial and for these items we expect zero subsequent errors. According to the theory, $e_s$ will be larger than $e_a$; however, $g_s$ will be smaller than $g_a$; that is, given a first correct response, it is more likely to have occurred by guessing when $N$ is three than when $N$ is eight. Therefore, the fact that the products $g_s e_s$ and $g_a e_a$ are approximately equal is neither surprising nor a refutation of the theory.

**TABLE 4**

<table>
<thead>
<tr>
<th>Training condition</th>
<th>Response alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td><strong>Correction</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td>$M = 1.33$</td>
<td>$M = 2.25$</td>
</tr>
<tr>
<td>$F = 1.39$</td>
<td>$F = 2.37$</td>
</tr>
<tr>
<td>$\sigma_M = .092$</td>
<td>$\sigma_M = .185$</td>
</tr>
<tr>
<td>$\sigma_F = .125$</td>
<td>$\sigma_F = .180$</td>
</tr>
<tr>
<td><strong>Partial</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td>$M = 2.12$</td>
<td>$M = 4.15$</td>
</tr>
<tr>
<td>$F = 2.16$</td>
<td>$F = 4.02$</td>
</tr>
<tr>
<td>$\sigma_M = .192$</td>
<td>$\sigma_M = .289$</td>
</tr>
<tr>
<td>$\sigma_F = .191$</td>
<td>$\sigma_F = .329$</td>
</tr>
<tr>
<td><strong>Noncorrection</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td>$M = 2.17$</td>
<td>$M = 6.20$</td>
</tr>
<tr>
<td>$F = 2.15$</td>
<td>$F = 6.38$</td>
</tr>
<tr>
<td>$\sigma_M = .201$</td>
<td>$\sigma_M = .511$</td>
</tr>
<tr>
<td>$\sigma_F = .206$</td>
<td>$\sigma_F = .558$</td>
</tr>
</tbody>
</table>

**DISCUSSION**

The overall results provide strong support for the validity of the model and the specific hypotheses about ex-
perimental variables. The data of six independent groups were adequately reproduced using a single estimate of the learning rate constant obtained from one of these groups. The fact that in 24 independent predictions the model came close to the data is sufficient justification for exploring further consequences of the theory.

The initial problem that led to this investigation was whether the number of response alternatives could be shown to affect learning rate on reinforced trials in addition to contributing to differential error probabilities due to guessing on unlearned items. The data supported the assumption that learning rate on reinforced trials was a constant independent of the number of response alternatives, and that the effect of $N$ upon error rate could be attributed to differential guessing probabilities on unlearned items. It should be noted that these conclusions are valid only for the experimental procedure used here in which the response alternatives are immediately available to the subject. For alternative procedures in which the subject is required to learn the responses (e.g., three or eight nonsense syllables) as well as the S-R associations, a number of complicating factors enter to obscure the picture and prevent resolution of the basic problem.

The second problem of this study was to account for the effect on performance of several training procedures. One may conceive of the correction, partial correction, and non-correction procedures as varying in the degree to which the correct response is specified following an incorrect response by the subject. This variable in turn determines the probability that the correct response is reinforced following incorrect responses. Thus, in the correction condition, the reinforcement probability is 1 on every trial; under partial correction, the reinforcement probability is $1/k$ before the first correct response and is essentially 1 afterwards due to a high recognition probability; under non-correction, the probability that the correct response is reinforced is essentially zero following errors. The data supported these hypotheses and gave additional support to the assumption that when the correct response was reinforced, the probability of conditioning was a constant, $c$, the same for all three conditions.

In addition to confirming these specific assumptions about $N$ and training conditions, the results lend support to the learning model in which these assumptions are embedded. In this one-element model, the probability of the correct response can have only two values, $1/N$ or 1. The stimulus element begins in the unconditioned state and each reinforced trial provides an opportunity for the element to become associated in all-or-none fashion with the correct response. When conditioning finally occurs, response probability jumps from $1/N$ to 1. Discontinuous learning theories have been contrasted frequently with "response strength" theories which assume that an associative factor (habit strength in Hull's theory, proportion of conditioned elements in Estes' linear model) increases in a cumulative manner with successive reinforcements. According to a strength theory, response probability make take on a large (possibly infinite) number of values ranging from $1/N$ up to 1. It is frequently difficult to distinguish this theory from the one-element model since they predict the same average learning curve over a group of subjects and items. Indeed, the present results on average errors could be fit about equally well
by the linear model. Since the discussion has been oriented towards the one-element model, perhaps it would be appropriate to record a few extra results which favor this model. For these purposes, let us consider two sequential statistics for which the two models deliver qualitatively different predictions.

The first statistic is the average number of errors (to perfect learning) following an error that occurs on Trial \( n \) of the experiment. The one-element model makes the rather counterintuitive prediction that this average number of subsequent errors is a constant independent of the trial number on which the leading error occurs. Thus, if we observe an error on Trial 10 we predict the same number of subsequent errors as if we had observed that error on Trial 1. The point of the matter is that when an error occurs on Trial \( n \) we know that the item was not conditioned before Trial \( n \), and we can set the clock back to Trial 1 as far as the model is concerned in predicting future errors on that item. In contrast, the associative strength approach (specifically, the linear model) predicts that the number of errors following an error on Trial \( n \) is a decreasing function of \( n \); that is, the greater the number of reinforced trials before a particular error, the higher the associative strength at that time, and hence the fewer the number of subsequent errors before perfect learning.

To obtain a sizable sample on which to test this critical point, data from 48 subjects in another 10-item paired-associate experiment \(^6\) (with \( N = 2 \)) were pooled with the 3-C and 8-C groups of the present experiment. The varying \( N \)'s will not affect the results on constancy vs. monotone decreasing aspects of errors following an error on Trial \( n \). Using this pool of 760 response sequences (10 items for each of 76 subjects), the distributions of the number of errors following an error occurring on Trial 1, on Trial 2, \ldots, on Trial 6 were obtained. The analysis was not carried beyond Trial 6 since the number of cases involved was decreasing so that estimates of means would be more unreliable. The estimates of the average number of errors following an error on Trial \( n \) are shown in Figure 3.

These data clearly favor the constancy prediction of the one-element model. Although the estimates fluctuate somewhat this can be attributed to sampling variability; even the largest difference (2.05 vs. 1.95) does not approach statistical significance \((t = 0.49)\). Also in Figure 3 is shown the rough order of magnitude of the numbers to be expected from the linear model. These predicted numbers are not exact because of different learning rates in the two experiments that were pooled. How-

\(^6\) These experiments were carried out with the assistance of Norman Karns and James Colloran.
ever, the average $c$ value was .25 and this value was used to obtain the values on the graph. Clearly the curve predicted by the linear model is quite discrepant from the data; accordingly, the one-element model appears to give a more adequate description of these data.

A second example of this constancy effect is shown in Figure 4 which presents similar data collected in an experiment on verbal discrimination learning. In this experiment 34 subjects learned 20 items, an item consisting of two different nonsense syllables printed on a card. The subject’s task was to read off each card that syllable arbitrarily designated as correct for that item; the subject repeatedly went through the items in random order until he achieved two consecutive errorless trials with the entire list. Since response learning per se is not involved in this situation, the one-element model was expected to apply. Since there were two response alternatives on each card, we make the natural assumption that the chance probability that subject selects the correct syllable is one-half. Detailed comparisons of predicted and observed statistics indicated excellent fit of the model; the results in Figure 4 are representative of the overall accuracy of the predictions for this set of data. An additional set of paired-associate learning data providing a comparison of the linear and one-element models has been reported in a previous paper (Bower, 1961); there again the one-element model was clearly superior to the linear model in predicting details of the data.

A second measure suggested by Estes (1960) for differentiating the one-element and linear models con-

![Figure 4](image)
cerns the probabilities of repetition and alternation of responses over a series of test trials following training. Consider a miniature experiment consisting of one or two reinforced trials followed by two test trials \((T_1\ and\ T_2)\) on which the subject is not informed whether his response is right or wrong. The comparison differentiating the two theories is the probability of correct on \(T_2\) given a correct and incorrect response, respectively, on \(T_1\). According to the one-element model, the probability of correct following incorrect on \(T_1\) should be around the chance guessing level \((1/N)\); the probability of correct following correct on \(T_1\) will be much higher, being somewhat less than one because we may expect some of the correct responses on \(T_1\) to have occurred by guessing. The predictions of these conditional probabilities by the linear model depends upon whether there are substantial subject and/or item differences in learning rate. If for the moment we neglect such differences, as we have in testing the one-element model, then the linear model implies that the conditional probabilities of success following either a success or failure on \(T_1\) will be equal.

Two experiments will be reported to this point. The first, performed with the assistance of Sharon Gadberry, consisted of 36 subjects learning two 10-item lists (nonsense syllables), the responses being the integers 1 to 10. Following two presentations of each S-R pair, the subjects received two test trials with the stimulus member of each pair. The pooled results of this experiment are shown in the first row of Table 5. Starting with 720 cases (36 subjects on two 10-item lists), the probability of correct on \(T_1\) was .586; of those correct, 89.6% were correct on \(T_2\); of those incorrect on \(T_1\), only 16.3% were correct on \(T_2\). These results support the one-element model. Two comments are required here: First, the value of .896 for correct following correct on \(T_1\) is about what would be expected if 10% of the correct responses on \(T_1\) had occurred by guessing; secondly, the value of .163 for correct following incorrect on \(T_2\) is a little higher than the chance level of .100; however, this is likely an artifact of assigning stimuli and responses in one-to-one correspondence so that when \(k\) items are learned the probability of guessing correctly on the unlearned items may be close to one in \(N-k\) instead of the a priori value of one in \(N\).

The difference between probabilities of success on \(T_2\) following a success and failure on \(T_1\) should be increased above those in the previous study if one requires the responses per se to be learned (e.g., as in word-word pairs). Under these circumstances, the chance level of guessing will be practically zero. Thus, if the subject fails to respond correctly on \(T_1\), the probability of being correct on \(T_2\) is essentially zero; by the same reasoning, if a correct response occurs on \(T_1\), it is most likely to have come about by condi-

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Number of cases</th>
<th>Percentage correct on (T_1)</th>
<th>Percentage correct after correct</th>
<th>Percentage correct after failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Nonsense syllables, 10 number responses</td>
<td>720</td>
<td>.586</td>
<td>.896</td>
<td>.163</td>
</tr>
<tr>
<td>II. Noun-noun pair</td>
<td>320</td>
<td>.900</td>
<td>.995</td>
<td>.030</td>
</tr>
</tbody>
</table>

TABLE 5

RESULTS OF TWO EXPERIMENTS ON THE PROBABILITY OF CORRECT ON \(T_2\) FOLLOWING CORRECT AND FAILURE ON \(T_1\)
tioning so that the probability of correct on T_2 for this item should be essentially unity if forgetting is negligible in this situation. Row two of Table 5 reports the results of an experiment performed under these conditions with the assistance of Judith Slagter. Thirty-two subjects received three presentations on each of 10 noun-noun pairs (e.g., moon-pin) followed by two test trials. The percentage correct on T_1 was substantially higher than in the first experiment with nonsense syllable-number pairs. However, the important points of interest are that the probability of correct on T_2 given a correct on T_1 is essentially unity, whereas the probability of correct following failure on T_1 is nearly zero.

One objection frequently raised to these foregoing comparisons is that if there are substantial subject and/or item differences in learning rates, then arguments based upon the pooled aggregate will tend to favor the one-element result. The point of the objection is this: if the linear model holds, and if differences in learning rates lead to a distribution over subjects of response probabilities, then considering the entire aggregate the conditional probability of a success following a success is expected to be higher than the conditional probability of success following a failure. This happens, according to the argument, because when we conditionalize upon the first success we are selecting predominately those protocols from the upper end of the distribution of response probabilities. However, granting that this argument is sound, it nevertheless helps not at all in rescuing the linear model from such data; it simply shifts the focus of the argument from conditional probabilities to joint probabilities of pairs of responses on T_1 and T_2. For convenience in the following, let \( p_{10} \) represent the joint probability of a success on T_1 and a failure on T_2, and let \( p_{01} \) be the joint probability of failure on T_1 and success on T_2. It is a simple matter to show that if the linear model holds, then \( p_{10} \) should equal \( p_{01} \) regardless of possible differences between subjects and/or items in learning rates or initial probabilities. Data from four studies previously published by Estes (1960, 1961) show that the required identity of \( p_{10} \) and \( p_{01} \) fails to appear in any of the four studies. For example, in one study (Estes, 1960, Figure 2, p. 215) the value of \( p_{10} \) was .132 while \( p_{01} \) was .046; in another study (Figure 6, p. 218) \( p_{10} \) was .098 while \( p_{01} \) was .003. A variety of ad hoc hypotheses about individual differences to supplement the linear model have been considered in a paper by Estes (1961); the outcome of those investigations was that no simple ad hoc supplements could bring the linear model into correspondence with these elementary data which were collected to test it directly.

The question about individual differences raised by the objection above can be answered by comparison with the variability expected from the model. Suppose this question is considered in the context of a more extended experiment such as that described for the 3-C group in the preceding pages. A summary measure of an individual's performance might be the total errors (T) he makes over the 10 items. If we assume within the model that all subjects and items are characterized by the same learning rate constant, \( c \), then a subject's T score represents the sum of 10 values sampled randomly and independently from a distribution which allegedly is the same for all subjects and items. The theoretical variance of these T
scores may be calculated using our single estimate of \( c \); this predicted variance may then be compared with the variance of the observed \( T \) scores by taking their ratio which will be distributed approximately as the \( F \) statistic. If the observed variance of \( T \) scores is much larger than that predicted, then we would tend to attribute the inflated variance to variations in \( c \) over subjects. This test has been applied to several sets of paired-associate learning data collected by the writer, including the data from the 3-C and 8-C groups of the present study for which the variance of \( T \) could be predicted. With the population of college students who have served as subjects in these various experiments, this test statistic has always yielded nonsignificant \( F \) values; that is, the variance of \( T \) between subjects does not differ significantly from what would be expected on the basis of random sampling from the stochastic process assumed by the model to be common to all subjects. A similar statement can be made about analogous tests for differences in item difficulty; this outcome was expected, of course, since precautions were taken in selecting stimuli which appeared to be equal in intralist similarity and association value. Alternative procedures for handling this question regarding subject and/or item differences in the RTT experiment were considered in a paper by Estes, Hopkins, and Crothers (1960).

In concluding this report, it might be appropriate to add a few general comments concerning the issues under discussion lest some misunderstandings arise. First, it is misleading to cast the issues under discussion at the level of all-or-none theories versus response-strength theories of learning; rather the point at issue is whether the appropriate model for the present experimental situation is a two-state or a multistate process, where “state” here refers to a particular value of response probability. The continuous linear model is the limiting case of the class of multistate models. The all-or-none assumption about conditioning with respect to the available stimuli is not a differentiating feature of these two classes of models. Since the early writings of Guthrie (1930) it has been clear that the all-or-none conditioning assumption would imply gradual and cumulative changes in response probabilities provided there is sufficient variability in the stimulus samples from trial to trial. Within the framework of statistical learning theory, the feature differentiating the two classes of models is how many independent stimulus components one must assume to represent adequately the course of learning in this or that experimental situation. The two-state model proposed here for elementary association learning assumes that each item may be represented by a single stimulus element within the model and that this element can be in either of two states. That the number of elements is the critical assumption is illustrated by the fact that the statistics in Figures 3 and 4 and in Table 5 would not discriminate qualitatively between the linear model and a two-element model (i.e., where each item is represented by two stimulus components, with a random one being sampled on each trial). For more extensive discussions of small element models the reader may refer to a paper by Estes (1959) or a book by Suppes and Atkinson (1960).

Secondly, it should be recognized that in this report we have demonstrated only that the one-element model adequately describes results
from elementary paired-associate learning experiments. The job of extending the range of applicability of a theory is an empirical project which must proceed piecemeal. In large part, the success in applying the model to a given learning situation will depend upon the simplicity of the situation and the degree of experimental control over stimulus variables. To cite a pertinent illustration: in training a rat to shuttle in response to a buzzer to avoid shock in a Mowrer-Miller shuttle box, conditioning is typically a gradual, "multi-stage" process (cf. Mowrer, 1960, p. 36); however, if experimental conditions are drastically simplified and precautions are taken to eliminate possible sources of interfering responses, then avoidance conditioning is a one-trial, "two-state" process (Maatsch, 1959). An analogous illustration for the eyelid conditioning situation has been reported by Voeks (1954).

In contrast to those situations in which a two-state behavioral process may possibly be achieved through experimental control, there is a large number of common learning situation for which such precise stimulus control probably can not be achieved, and it is unlikely that a simple two-state model could apply. These situations may be characterized generally as ones in which exposure to discriminative stimuli is subject-controlled; these situations range from those in which the critical cues are proprioceptive stimuli from the subject's current behavior, as in motor skill learning, to situations in which selected components of a complex stimulus array control behavior through the mediation of an overt or implicit observing response, as in complex concept learning. One example of this latter class would be that of a rat learning a black-white discrimination in a T maze; the stimuli effective at the moment of choice are multiple and vary from trial to trial depending upon the subject's vicarious trial and error behavior. For such situations more elaborate conceptual apparatus (cf. Audley, 1960; Bower, 1959; Spence, 1960) is required to analyze the interaction between observing behavior and instrumental, goal-directed behavior.

REFERENCES


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