

Interference Mitigation Through Limited Transmitter Cooperation

I-Hsiang Wang, *Student Member, IEEE*, and David N. C. Tse, *Fellow, IEEE*

Abstract—Interference limits performance in wireless networks and cooperation among receivers or transmitters can help mitigate interference by forming distributed MIMO systems. Earlier work [1] shows how limited receiver cooperation helps mitigate interference. The scenario with transmitter cooperation, however, is more difficult to tackle. In this paper we study the two-user Gaussian interference channel with conferencing transmitters to make progress towards this direction. We characterize the capacity region to within 6.5 bits/s/Hz, regardless of channel parameters. Based on the bounded-gap-to-optimality result, we show that there is an interesting reciprocity between the scenario with conferencing transmitters and the scenario with conferencing receivers and their capacity regions are within a bounded gap to each other. Hence, in the interference-limited regime, the behavior of the benefit brought by transmitter cooperation is the same as that by receiver cooperation.

Index Terms—Capacity to within a bounded gap, distributed MIMO system, interference management, transmitter cooperation.

I. INTRODUCTION

IN modern wireless communication systems, interference has become the major factor that limits performance. Interference arises whenever multiple transmitter-receiver pairs are present and each receiver is only interested in retrieving information from its own transmitter. Due to the broadcast and superposition nature of wireless channels, one user's information-carrying signal causes interference to other users. The *interference channel* is the simplest information theoretic model for studying this issue, where each transmitter (receiver) is assumed to be isolated from other transmitters (receivers). In various practical scenarios, however, they are not isolated and *cooperation* among transmitters or receivers can be induced. For example, in downlink cellular systems, base stations are connected via infrastructure backhaul networks.

In our previous work [1], we have studied the two-user Gaussian interference channel with conferencing receivers to

understand how limited receiver cooperation helps mitigate interference. We proposed good coding strategies, proved tight outer bounds and characterized the capacity region to within 2 bits/s/Hz. Based upon the bounded-gap-to-optimality result, we identify two regions regarding the gain from receiver cooperation: linear and saturation regions. In the linear region, receiver cooperation is *efficient*, in the sense that the growth of each user's over the air data rate is roughly linear with respect to the capacity of receiver-cooperative links. The gain in this region is the *degrees-of-freedom* gain that distributed MIMO systems provide. In the saturation region, receiver cooperation is *inefficient* in the sense that the growth of user data rate becomes saturated as one increases the rate in receiver-cooperative links. The gain is the *power* gain which is bounded, independent of the channel strength. Furthermore, until saturation the degree-of-freedom gain is either *one cooperation bit buys one over-the-air bit* or *two cooperation bits buy one over-the-air bit*.

In this paper, we study its reciprocal problem, the two-user Gaussian interference channel with conferencing transmitters, to investigate how limited transmitter cooperation helps mitigate interference. A natural cooperative strategy between transmitters is that, prior to each block of transmission, two transmitters hold a conference to tell each other part of their messages. Hence the messages are classified into two kinds: 1) *cooperative* messages, which are those known to both transmitters due to the conference; and 2) *noncooperative* ones, which are those unknown to the other transmitter since the cooperative link capacities are finite. On the other hand, messages can also be classified based on their target receivers: 1) *common* messages, which are those aimed at both receivers; and 2) *private* ones, which are those aimed at their own receiver. Hence in total there are four kinds of messages for each user and seven codes for the whole system.¹ Now the question is, how do we encode these messages?

Generally speaking, Gaussian interference channels with transmitter cooperation are more difficult to tackle than Gaussian interference channels with receiver cooperation. Take the following extreme case. When transmitters can cooperate in an unlimited fashion, the scenario reduces to the MIMO Gaussian broadcast channel. When receivers can cooperate in an unlimited fashion, the scenario reduces to MIMO Gaussian multiple access channel. The capacity region of the latter is fully characterized in the 1970's [2], [3], while that of the former has not been solved until recently [4]. This is due to difficulties both in achievability and outer bounds.

¹There is only one cooperative common code carrying both cooperative common messages.

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The authors are with the Wireless Foundations, Department of EECS, University of California at Berkeley, Berkeley, CA 94720 USA (e-mail: ihsiang@eecs.berkeley.edu; dtse@eecs.berkeley.edu).

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Similar phenomenon arises between Gaussian interference channels with conferencing transmitters and Gaussian interference channels with conferencing receivers. Compared with the scenario with conferencing receivers [1] where each user just has two kinds of messages (common and private), in the scenario with conferencing transmitters not only does the message structure in the strategy become more complicated due to the collaboration among transmitters, but it is also more difficult to prove the outer bounds since the transmitters are potentially correlated. In order to overcome the difficulties, we first study an auxiliary problem in the *linear deterministic* setting [5], [6]. We first characterize the capacity region of the linear deterministic interference channel with conferencing transmitters and then make use of the intuition there to design good coding strategies and to prove outer bounds in the Gaussian scenario. Eventually the proposed strategy in the Gaussian setting is a simple superposition of a pair of *noncooperative* common and private codewords and a pair of *cooperative* common and private codewords. For the noncooperative part, Han-Kobayashi scheme [7] is employed and the common-private split is such that the private interference is at or below the noise level at the unintended receiver [8]. For the cooperative part, we use a simple linear beamforming strategy for encoding the private messages, superimposed upon the common codewords. By choosing the power split and beamforming vectors cleverly, the strategy achieves the capacity region universally to within 6.5 bits, regardless of channel parameters. The 6.5-bit gap is the worst-case gap which can be loose in some regimes and it is vanishingly small at high SNR when compared to the capacity.

With the bounded-gap-to-optimality result, we observe an interesting uplink-downlink reciprocity between the scenario with conferencing receivers and the scenario with conferencing transmitters: for the original and reciprocal channels, the capacity regions are within a bounded gap to each other. Hence the fundamental gain from transmitter cooperation at high SNR is the same as that from receiver cooperation [1].

A. Related Works

Conferencing among transmitters is first studied by Willems [9] in the context of *multiple access channels*, where the capacity region is characterized. The capacity of the Gaussian MAC with conferencing transmitters, however, has not been characterized explicitly in a computable form until recently by Bross *et al.* [10], where the authors show that the optimization on auxiliary random variables can be reduced to finding the optimal Gaussian input distribution. On the other hand, the extension to the compound MAC has been done by Maric *et al.* [11].

Works on Gaussian *interference channel* with transmitter cooperation can be roughly divided into two categories. One set of works investigate cooperation in interference channels with a setup where the cooperative links share the same band as the links in the interference channel. Høst-Madsen [12] proposes cooperative strategies based on decode-forward, compress-forward and dirty paper coding and derives the achievable rates. The recent work by Prabhakaran *et al.* [13] characterizes the sum capacity of Gaussian interference channels with reciprocal in-band transmitter cooperation to within a bounded gap. The

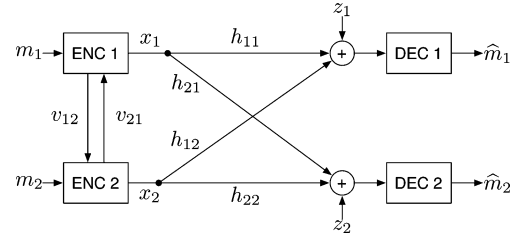


Fig. 1. Channel model.

other set of works focus on conferencing transmitters, that is, cooperative links are orthogonal to each other as well as the links in the interference channel. Some works are dedicated to achievable rates. Cao *et al.* [14] derive an achievable rate region based on superposition coding and dirty paper coding. Some works consider special cases of the channel. One such special case attracting particularly broad interest is the *cognitive interference channel*, where one of the transmitters (the cognitive user) is assumed to have full knowledge about the other's transmission (the primary user). It is equivalent to the case where transmitter cooperation is unidirectional and unlimited. As for the cognitive interference channel, Maric *et al.* [11] characterize the capacity region in the strong interference regime. Wu *et al.* [15] and Jovičić *et al.* [16] independently characterize the capacity region when the interference at the primary receiver is weak. Very recently, Rini *et al.* [17] characterize the capacity region to within a bounded gap, regardless of channel parameters. On the other hand, works on the case with limited cooperative capacities are not rich in the literature. Bagheri *et al.* [18] investigate symmetric Gaussian interference channel with unidirectional limited transmitter cooperation and characterize the sum capacity to within a bounded gap.

Our main contribution in this paper is characterizing the capacity *region* of two-user Gaussian interference channel with conferencing transmitters to within a bounded gap for *arbitrary* channel strength and cooperative link capacities. The rest of the paper is organized as follows. After we formulate the problem in Section II, we investigate the auxiliary linear deterministic channel in Section III. Then we carry the intuitions and techniques to solve the original problem in Section IV and characterize the capacity region to within a bounded gap. In Section V we discuss the interesting uplink-downlink reciprocity.

II. PROBLEM FORMULATION

A. Channel Model

The Gaussian interference channel with conferencing transmitters is depicted in Fig. 1.

The links among transmitters and receivers are modeled as the *normalized* Gaussian interference channel:

$$y_1 = h_{11}x_1 + h_{12}x_2 + z_1, \quad y_2 = h_{21}x_1 + h_{22}x_2 + z_2$$

where the additive noise processes $\{z_i[k]\}$, ($i = 1, 2$), are independent $\mathcal{CN}(0, 1)$, i.i.d. over time. In this paper, we use $[\cdot]$ to denote time indices. Transmitter i intends to convey message m_i

to receiver i by encoding it into a block codeword $\{x_i[k]\}_{k=1}^N$, with transmit power constraints

$$\frac{1}{N} \sum_{k=1}^N |x_i[k]|^2 \leq 1, \quad i = 1, 2$$

for arbitrary block length N . Note that outcome of the encoder depends on both messages. Messages m_1, m_2 are independent. Define channel parameters

$$\text{SNR}_i := |h_{ii}|^2, \quad \text{INR}_i := |h_{ij}|^2, \quad i, j = 1, 2, \quad i \neq j.$$

The cooperative links between transmitters are noiseless with finite capacity C_{ij}^B from transmitter i to j . Encoding must satisfy causality constraints: for any time index $k = 1, 2, \dots, N$, $v_{ij}[k]$ is only a function of $\{m_i, v_{ji}[1], \dots, v_{ji}[k-1]\}$.

B. Notations

We summarize below the notations used in the rest of this paper.

- For a real number a , $(a)^+ := \max(a, 0)$ denotes its positive part.
- For a real number a , $\lfloor a \rfloor$ denotes the closest integer that is not greater than a .
- For sets $A, B \subseteq \mathbb{R}^k$ in k -dimensional space, $A \oplus B := \{a+b : a \in A, b \in B\}$ denotes the direct sum of A and B .
- With a little abuse of notations, for $x, y \in \mathbb{F}_q$, $x \oplus y$ denotes the modulo- q sum of x and y .
- Unless specified, all the logarithms $\log(\cdot)$ are of base 2.

III. LINEAR DETERMINISTIC INTERFERENCE CHANNEL WITH CONFERENCING TRANSMITTERS

As discussed in Section I, we shall first study an auxiliary problem, *linear deterministic* interference channel with conferencing transmitters, to overcome the complications both in achievability and outer bounds.

The corresponding linear deterministic channel (LDC) is parametrized by nonnegative integers $n_{11}, n_{21}, n_{22}, n_{12}, k_{12}$, and k_{21} , where

$$n_{ij} := (\lfloor \log |h_{ij}|^2 \rfloor)^+, \quad i, j \in \{1, 2\}$$

correspond to the channel gains in logarithmic-two scale and

$$k_{12} := \lfloor C_{12}^B \rfloor, \quad k_{21} := \lfloor C_{21}^B \rfloor$$

correspond to the cooperative link capacities. An illustration is depicted in Fig. 2(a) along with an example in Fig. 2(b). Each circle or diamond represents a bit. The bit emitting from a single circle at transmitters will broadcast noiselessly through the edges to the circles at receivers. Multiple incoming bits at a circle are summed up using modulo-two addition and produce a single received bit. The diamonds represent the bits exchanged between transmitters. In Fig. 2(b), Tx1 can send one bit to Tx2

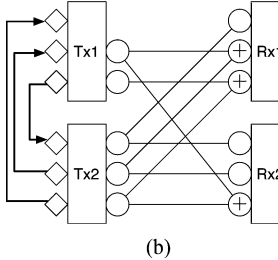
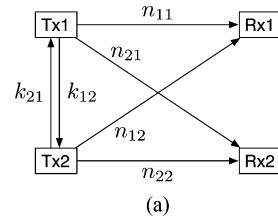


Fig. 2. Linear deterministic interference channel with conferencing transmitters. (a) Channel model. (b) Example channel.

and Tx2 can send two bits to Tx1. For more details about this model, we point the readers to [5], [6], [19].

The following theorem characterizes the capacity region of this channel.

Theorem 3.1: Nonnegative (R_1, R_2) is achievable if and only if it satisfies the constraints listed at the bottom of the next page.

A. Motivating Examples

Before going into technical details of proving the achievability and outer bounds, we first give several examples to motivate the scheme as well as the outer bounds. In the discussions below, bit $a_i \in \mathbb{F}_2$ denotes an information bit for user 1 and similarly $b_i \in \mathbb{F}_2$ denotes an information bit for user 2. The index i denotes the i -th level from the most significant bit (MSB) at the user’s transmitter. If i becomes larger than the total number of levels available at the user’s transmitter, the corresponding bit has to be relayed to the final destination via the other transmitter, as we will see in the sequel.

Achievability: The first example channel is depicted in Fig. 2(b), where $n_{11} = 2, n_{12} = 3, n_{21} = 1, n_{22} = 3, k_{12} = 1, k_{21} = 2$. We shall use this example to argue intuitively the need of *cooperative common* messages, shed some light on how cooperative messages should be encoded and illustrate the twofold usage of transmitter cooperation—nulling out interferences and relaying additional information.

To achieve the rate point $(R_1, R_2) = (2, 3)$, one simple strategy is depicted in Fig. 3(a). In this coding scheme, we identify the message structure in Table I. Note that transmitter 2 sends b_2 and b_3 to transmitter 1 so that it can carry out proper precoding to null out interference b_2 and b_3 at receiver 1. Similarly transmitter 1 sends a_1 to transmitter 2 so that it can null out interference a_1 at receiver 2.

On the other hand, to achieve the rate point $(R_1, R_2) = (3, 1)$, one simple strategy is depicted in Fig. 3(b). In this coding

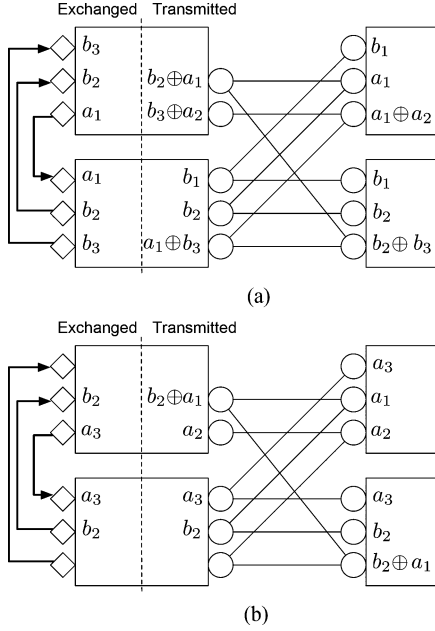


Fig. 3. Coding strategies for example channel in Fig. 2. (a) Achieving $R_1 = 2$, $R_2 = 3$. (b) Achieving $R_1 = 3$, $R_2 = 1$.

TABLE I
MESSAGE STRUCTURE IN FIG. 3(A)

Cooperative common	Cooperative private	Noncooperative common	Noncooperative private
None	a_1 and (b_2, b_3)	b_1	a_2

scheme, we identify the message structure in Table II. Note that to support a third bit a_3 for user 1, it has to occupy the topmost circle level at transmitter 2 and both receivers, since the direct link from transmitter 1 to receiver 1 has only two levels. Hence, receiver 2 inevitably will decode bit a_3 , which is then classified as cooperative common. From this example we see that *cooperative common* messages are needed and their signal should occupy the levels that appear at both receivers cleanly. For the

TABLE II
MESSAGE STRUCTURE IN FIG. 3(B)

Cooperative common	Cooperative private	Noncooperative common	Noncooperative private
a_3	b_2	a_1	a_2

cooperative private parts, the example suggests that one should design precoding cleverly such that interference is nulled out at the unintended receiver. Based upon these intuitions, we propose an explicit scheme in Section III-E.

In the above example, we can see that the usage of the cooperative links is twofold: 1) null out interference, as in Fig. 3(a); and 2) relay additional bits, as the link from transmitter 1 to 2 in Fig. 3(b). This observation is also useful in motivating outer bounds.

Fundamental Tradeoff on $2R_1 + R_2$ and $R_1 + 2R_2$: For outer bounds, the main difference from the interference channel *without* cooperation [19], [20] is that there are two different types of bounds on $2R_1 + R_2$ (and $R_1 + 2R_2$ correspondingly). Below we demonstrate the two different types of fundamental tradeoff on $2R_1 + R_2$ through two examples.

The first type of tradeoff does not involve the information that flows in the cooperative links. Consider the example channel with $n_{11} = n_{22} = 5$, $n_{12} = n_{21} = 3$, $k_{12} = k_{21} = 1$. We first consider the case *without* cooperation. Two corner points of the capacity region are $(R_1, R_2) = (4, 2)$ and $(5, 0)$ and the optimal strategies are depicted in Fig. 4(a) and (b), respectively. To enhance user 1's rate from 4 to 5 bits, the bit a_3 has to be turned on and causes collisions at the third level at receiver 1 and the fifth level at receiver 2. Transmitter 2 then has to turn off bit b_1 to avoid destroying bit a_3 and b_5 cannot be decoded since it is corrupted by a_3 . Now consider the case *with* cooperation. Two corner points of the capacity region are $(R_1, R_2) = (4, 4)$ and $(5, 2)$ and the optimal strategies are depicted in Fig. 5(a) and (b), respectively. Note that to enhance user 1's rate from 4 to 5 bits, again the bit a_3 has to be turned on and again causes collisions at the same places as in the case without cooperation. Note

$$R_1 \leq \min \left\{ \max(n_{11}, n_{12}), n_{11} + k_{12} \right\}$$

$$R_2 \leq \min \left\{ \max(n_{22}, n_{21}), n_{22} + k_{21} \right\}$$

$$R_1 + R_2 \leq (n_{11} - n_{21})^+ + \max(n_{22}, n_{21}) + k_{12} \quad (1)$$

$$R_1 + R_2 \leq (n_{22} - n_{12})^+ + \max(n_{11}, n_{12}) + k_{21} \quad (2)$$

$$R_1 + R_2 \leq \max \left\{ n_{12}, (n_{11} - n_{21})^+ \right\} + \max \left\{ n_{21}, (n_{22} - n_{12})^+ \right\} + k_{12} + k_{21} \quad (3)$$

$$R_1 + R_2 \leq \begin{cases} \max \{ n_{11} + n_{22}, n_{12} + n_{21} \}, & \text{if } n_{11} + n_{22} \neq n_{12} + n_{21} \\ \max \{ n_{11}, n_{12}, n_{21}, n_{22} \}, & \text{if } n_{11} + n_{22} = n_{12} + n_{21} \end{cases} \quad (4)$$

$$2R_1 + R_2 \leq \max(n_{11}, n_{12}) + \max \left\{ n_{21}, (n_{22} - n_{12})^+ \right\} + (n_{11} - n_{21})^+ + k_{12} + k_{21} \quad (5)$$

$$R_1 + 2R_2 \leq \max(n_{22}, n_{21}) + \max \left\{ n_{12}, (n_{11} - n_{21})^+ \right\} + (n_{22} - n_{12})^+ + k_{21} + k_{12}$$

$$2R_1 + R_2 \leq n_{21} + \max \left\{ n_{11} + (n_{22} - n_{21})^+, n_{12} \right\} + (n_{11} - n_{21})^+ + k_{12}$$

$$R_1 + 2R_2 \leq n_{12} + \max \left\{ n_{22} + (n_{11} - n_{12})^+, n_{21} \right\} + (n_{22} - n_{12})^+ + k_{21} \quad (6)$$

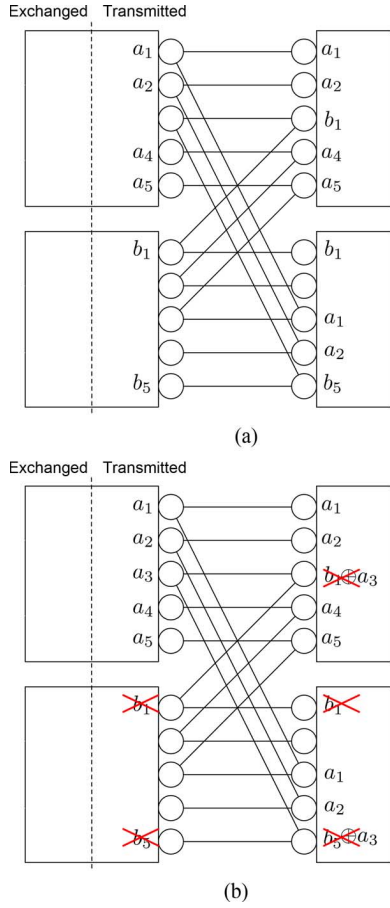


Fig. 4. Example channel without transmitter cooperation: Tradeoff from $(R_1, R_2) = (4, 2)$ to $(5, 0)$. (a) Achieving $R_1 = 4, R_2 = 2$. (b) Achieving $R_1 = 5, R_2 = 0$.

that the bits exchanged in the cooperative links remain the same and hence the information that flows in the cooperative links is not involved in this tradeoff. Furthermore, the tradeoff is qualitatively the same as that in the case *without* cooperation. Later we will see that this type of outer bound on $2R_1 + R_2$ can be generalized from the $2R_1 + R_2$ bound in deterministic interference channel *without* cooperation [20] and the proof technique is quite similar.

The second type of tradeoff is a new phenomenon in interference channel with cooperation and involves the information that flows in the cooperative links. Consider the example channel in Fig. 2(b). The two rate points $(R_1, R_2) = (2, 3)$ and $(R_1, R_2) = (3, 1)$ are on the boundary of the capacity region. To enhance user 1's rate from 2 to 3 bits, since the number of levels from transmitter 1 to receiver 1 is only 2, the third bit a_3 has to be relayed from transmitter 2 to receiver 1. Hence, the topmost level at transmitter 2 has to be occupied by information exclusively for user 1, that is, a_3 and at receiver 2 the topmost level is no longer available for user 2. On the other hand, since the cooperative link from transmitter 1 to transmitter 2 is now occupied by a_3 , the opportunity of nulling out the interference at the third level at receiver 2 is eliminated. As a consequence, the only available level for user 2 at receiver 2 is the second level and user 2 has to back off its rate from 3 to 1. Note that the key difference from the first type of tradeoff is that, at the rate point $(R_1, R_2) = (2, 3)$ the cooperative link from transmitter 1 to 2 is

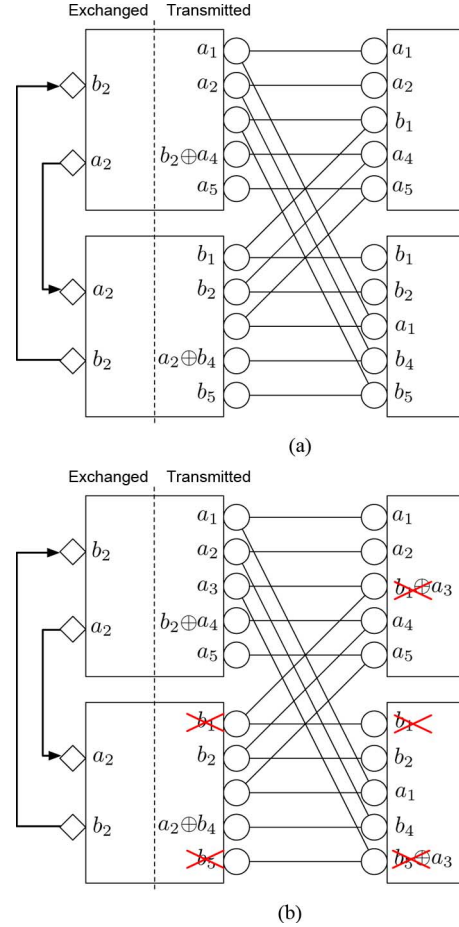


Fig. 5. Example channel with transmitter cooperation: Tradeoff from $(R_1, R_2) = (4, 4)$ to $(5, 2)$. (a) Achieving $R_1 = 4, R_2 = 4$. (b) Achieving $R_1 = 5, R_2 = 2$.

used for *nulling out interference*, while at $(R_1, R_2) = (3, 1)$ it is used for *relaying additional bits*. Hence, the information that flows in the cooperative links *is* involved in this tradeoff and the tradeoff is qualitatively different from that in the case *without* cooperation. As we will show later, to prove this type of outer bound on $2R_1 + R_2$, we need to develop a new technique for giving side information to the receivers.

B. Outer Bounds

To prove the converse part of Theorem 3.1, instead of giving full details of the proof,² here we describe the techniques used in the proof. These techniques will be reused for proving outer bounds in the Gaussian problem.

Bounds on R_1 and R_2 : These bounds are straightforward cut-set bounds.

Bounds on $R_1 + R_2$: Bound (4) is a standard cut-set bound. Its value is the rank of the *system transfer matrix*, with both transmit signals as input and both receive signals as output assuming full cooperation. It is quite straightforward to see that $n_{11} + n_{22} \neq n_{12} + n_{21}$ if and only if the system matrix is full rank and the value of its rank is the right-hand side (RHS) of (4).

Bound (1) is obtained by providing side information (m_2, v_{12}^N) to receiver 1 so that receiver 1 is not interfered by transmitter 2 at all. This leads to the part

²We will provide full details when we deal with the Gaussian problem.

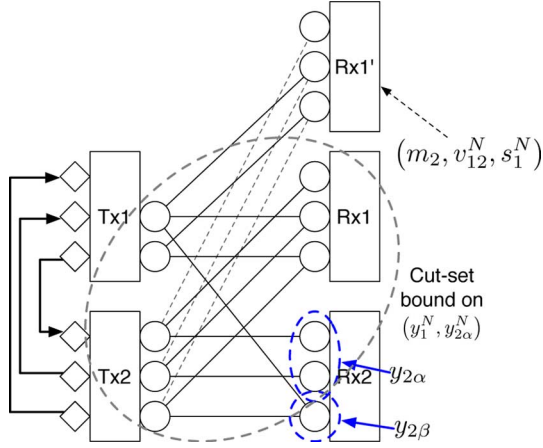


Fig. 6. Side information structure for outer bound (6).

$(n_{11} - n_{21})^+ + \max(n_{22}, n_{21})$, which is identical to the Z-channel bound in interference channel without cooperation. Giving the side information enhances the sum rate by at most $H(v_{12}^N | m_2) \leq Nk_{12}$ bits. Similar arguments works for bound (2).

Bound (3) is obtained by providing side information $(v_{12}^N, v_{21}^N, s_1^N)$ and $(v_{12}^N, v_{21}^N, s_2^N)$ to receiver 1 and 2, respectively, where s_1^N denotes the interference caused by transmitter 1 at receiver 2 (and vice versa for s_2^N). Giving side information (v_{12}^N, v_{21}^N) to both receivers enhances the sum rate by at most $H(v_{12}^N, v_{21}^N) \leq N(k_{12} + k_{21})$ bits. Finally, we are able to prove the bound by making use of the Markov relations observed first in [9] for the MAC with conferencing transmitters, which states that given the conferencing signals, the transmit signals and messages at two transmitters are independent: $(m_i, x_i^N) - (v_{12}^N, v_{21}^N) - (m_j, x_j^N)$, for $(i, j) = (1, 2)$ or $(2, 1)$.

Bounds on $2R_1 + R_2$ and $R_1 + 2R_2$: By symmetry we shall focus on the bounds on $2R_1 + R_2$.

For linear deterministic interference channel *without* cooperation, the outer bound on $2R_1 + R_2$ is proved by first creating a copy of receiver 1 and then giving proper side information to these three receivers [20]. The side information structure is the following: give side information (x_2^N, s_1^N) to one of the two receiver 1's and side information s_2^N to receiver 2.

As discussed in the previous section, there are two types of tradeoff on $2R_1 + R_2$. They correspond to bound (5) and bound (6), respectively. Bound (5) can be obtained via a similar technique as that in [20]. The side information structure is the following: give side information $(m_2, s_1^N, v_{12}^N, v_{21}^N)$ to one of the two receiver 1's and $(s_2^N, v_{12}^N, v_{21}^N)$ to receiver 2. The role of (m_2, v_{12}^N) is the same as x_2^N and the additional side information (v_{12}^N, v_{21}^N) is to make the transmitters conditionally independent. We then make use of the above Markov property to complete the proof.

Bound (6), which corresponds to the second type of tradeoff discussed earlier, is obtained by splitting receiver 2's signal into two parts: $y_2^N = (y_{2\alpha}^N, y_{2\beta}^N)$, where $y_{2\alpha}^N$ is the part of transmitter 2's signal that is not corrupted by s_1^N , the interference from transmitter 1. Then we apply a cut-set bound argument on one of the two receiver 1's and $y_{2\alpha}^N$ and provide side information (m_2, v_{12}^N, s_1^N) to the other receiver 1. Fig. 6 provides an illustra-

tion. Since this kind of side information structure has not been reported in literature, we detail the proof below:

Proof: If (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
 N(2R_1 + R_2 - \epsilon_N) &\leq 2I(m_1; y_1^N) + I(m_2; y_2^N) \\
 &\leq I(m_1; y_1^N, s_1^N | m_2, v_{12}^N) + I(m_1; v_{12}^N | m_2) + I(m_1; y_1^N) \\
 &\quad + I(m_2; y_{2\alpha}^N, y_{2\beta}^N) \\
 &= H(x_1^N, s_1^N | m_2, v_{12}^N) + I(m_1; y_1^N) + I(m_2; y_{2\alpha}^N) \\
 &\quad + I(m_2; y_{2\beta}^N | y_{2\alpha}^N) + I(m_1; v_{12}^N | m_2) \\
 &\stackrel{(a)}{\leq} H(x_1^N, s_1^N | m_2, v_{12}^N) + I(m_1, m_2; y_1^N, y_{2\alpha}^N) \\
 &\quad + I(m_2, v_{12}^N; y_{2\beta}^N | y_{2\alpha}^N) + H(v_{12}^N | m_2) \\
 &\stackrel{(b)}{\leq} H(x_1^N, s_1^N | m_2, v_{12}^N) + H(y_1^N, y_{2\alpha}^N) \\
 &\quad + H(y_{2\beta}^N) - H(y_{2\beta}^N | y_{2\alpha}^N, m_2, v_{12}^N) + H(v_{12}^N) \\
 &\stackrel{(c)}{=} H(x_1^N, s_1^N | m_2, v_{12}^N) + H(y_1^N, y_{2\alpha}^N) \\
 &\quad + H(y_{2\beta}^N) - H(s_1^N | m_2, v_{12}^N) + H(v_{12}^N) \\
 &= H(x_1^N | s_1^N, m_2, v_{12}^N) + H(y_1^N, y_{2\alpha}^N) + H(y_{2\beta}^N) + H(v_{12}^N) \\
 &\leq N \left\{ (n_{11} - n_{21})^+ + \max \left\{ n_{11} + (n_{22} - n_{21})^+, n_{12} \right\} \right\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to a simple fact that $I(m_1; y_1^N) + I(m_2; y_{2\alpha}^N) \leq I(m_1, m_2; y_1^N, y_{2\alpha}^N)$. (b) is due to that conditioning reduces entropy. (c) holds since $y_{2\alpha}^N$ is a function of (m_2, v_{12}^N) . ■

Let us revisit the example in Fig. 2(b) and demonstrate that bound (6) is active. Plugging the channel parameters into Theorem 3.1, we see that *without* bound (6), the region is

$$R_1 \leq 3, R_2 \leq 3, R_1 + R_2 \leq 5$$

and the rate point $(3, 1)$ is not on its boundary. In this example, $y_{2\alpha}$ spans the topmost two levels at receiver 2. Hence, $H(y_1^N, y_{2\alpha}^N) \leq 4N$ and $2R_1 + R_2 \leq 1 + 4 + 1 + 1 = 7$ which is active in the capacity region

$$R_1 \leq 3, R_2 \leq 3, R_1 + R_2 \leq 5, 2R_1 + R_2 \leq 7.$$

C. Achievability via Linear Reciprocity

Unlike the linear deterministic interference channel with conferencing receivers, it is not straightforward to directly show that linear strategies achieves the capacity in the case with conferencing transmitters. We can overcome this by using *linear reciprocity* of linear deterministic networks [21] and prove the achievability part of Theorem 3.1. We sketch the idea of the proof as follows.

First it is not hard to show that linear strategies are optimal for the reciprocal channel, that is, the linear deterministic interference channel with conferencing receivers. In such linear strategies, each user modulates its information bits (message) onto the transmit signal vector via a linear transformation. Each receiver, serving as a relay, linearly transforms its received signal and sends it to the other receiver through the finite-capacity link. Since the channel is linear and deterministic, the exchanged signals between receivers are again linear transformations of the

transmit information bits. Finally, each receiver solves all its received linear equations of the transmit information bits (one set from the other receiver and the other from the transmitters) and recovers its desired message. Note that the decoding process is again a linear transformation. By choosing these linear transformations (encoding, relaying and decoding) properly, the scheme achieves the capacity.

Next by linear reciprocity, we immediately show that the capacity region of the reciprocal channel (the linear deterministic interference channel with conferencing receivers) is an achievable region of the original channel. The strategy is again linear. Each transmitter sends a linear transformation of its information bits to the other transmitter through the finite-capacity link. Then it sends out a linear transformation of the received bits from the other transmitter and its own information bits to the receivers. Finally, each receiver solves the linear equations it receives to recover its desired message. It remains to show that this region coincides with that given in Theorem 3.1, which is a straightforward calculation.

Note that in such linear strategies, there is no need to split the messages at the transmitters and the decoding process at the receivers can be viewed as *treating interference as noise*. This is first observed in Lecture Notes 6 in [22] for linear deterministic interference channels without cooperation. This implies that the complicated message structure described in Section I is not necessary for linear deterministic interference channel with conferencing receivers or transmitters.

To this end, there are two paths towards constructing good coding strategies in the Gaussian scenario. The first approach is deriving structured lattice strategies based on the capacity-achieving linear strategies of the corresponding linear deterministic channels. This approach, however, requires an explicit description of linear transformations in the capacity-achieving linear strategies for the LDC. The second approach is deriving Gaussian random coding strategies, which is the conventional approach for additive white Gaussian noise networks. In this paper, we will take the second approach. For this purpose, however, the proof of achievability via linear reciprocity does not give much insight. Below we give an alternative proof of achievability, which provides guidelines for designing good Gaussian random coding strategies in the Gaussian interference channel with conferencing transmitters.

D. Alternative Proof of Achievability

To get a better handle to deal with the design of good Gaussian random coding schemes, we propose a general coding strategy that applies both the linear deterministic channel (LDC) and the Gaussian channel. The strategy is based on Marton's coding scheme for general broadcast channels [23] and superposition coding. It is described as follows: (Notations: subscript o stands for *cooperative common*, subscript h stands for *cooperative private*, subscript c stands for *noncooperative common* and subscript p stands for *noncooperative private*).

- 1) First, generate the cooperative common vector codeword $\underline{x}_o^N(m_{1o}, m_{2o})$ according to $p(\underline{x}_o^N) = \prod_{k=1}^N p(\underline{x}_o[k])$. Denote $m_o := (m_{1o}, m_{2o})$.
- 2) Second, for each cooperative common m_o , generate the cooperative vector codeword $\underline{x}_{oh}^N(m_{1h}, m_{2h}, m_o)$

based on Marton's coding scheme according to conditional distribution $p(\underline{x}_{oh}^N, u_1^N, u_2^N | \underline{x}_o^N(m_o)) = \prod_{k=1}^N p(\underline{x}_{oh}[k], u_1[k], u_2[k] | \underline{x}_o(m_o)[k])$, where the auxiliary codewords are $u_1^N(\tilde{m}_{1h}, m_o)$ and $u_2^N(\tilde{m}_{2h}, m_o)$.

- 3) Third, at transmitter i , for $i = 1, 2$, generate the noncooperative common codeword $x_{ic}^N(m_{ic})$ according to distribution $p(x_{ic}^N) = \prod_{k=1}^N p(x_{ic}[k])$.
- 4) Fourth, at transmitter i , for each m_{ic} generate the noncooperative codeword $x_{icp}^N(m_{ip}, m_{ic})$ according to $p(x_{icp}^N | x_{ic}^N(m_{ic})) = \prod_{k=1}^N p(x_{icp}[k] | x_{ic}(m_{ic})[k])$.
- 5) Finally, superimpose these two codewords to form the transmit codewords:

$$\begin{aligned} x_1^N(m_o, m_{1h}, m_{2h}, m_{1c}, m_{1p}) &= \underline{x}_{oh}^N[1] + x_{1cp}^N \\ x_2^N(m_o, m_{1h}, m_{2h}, m_{2c}, m_{2p}) &= \underline{x}_{oh}^N[2] + x_{2cp}^N. \end{aligned}$$

Remark 3.2: Note that in Step 4), say at transmitter 1, we can use Gelfand-Pinsker coding (dirty paper coding) to generate the noncooperative private codeword so that it can be protected against known interference at transmitter 1, which is caused by the cooperative private auxiliary codeword of the other user, that is, u_2 . Throughout the paper, however, we will choose (u_1, u_2) cleverly such that the effect of u_2 is zero-forced *exactly* in LDC and *approximately* in the Gaussian setting and hence Gelfand-Pinsker coding does not provide significant improvement.

For decoding, receiver 1 looks for a unique message tuple $(m_o, \tilde{m}_{1h}, m_{1c}, m_{1p})$ such that

$$\left(y_1^N, \underline{x}_o^N(m_o), u_1^N(\tilde{m}_{1h}, m_o), x_{1c}^N(m_{1c}), x_{1cp}^N(m_{1p}, m_{1c}), x_{2c}^N(\tilde{m}_{2c}) \right)$$

is jointly typical, for some \hat{m}_{2c} . Receiver 2 uses the same decoding rule with index 1 and 2 exchanged.

Based on the above strategy, we have the following coding theorem.

Theorem 3.3 (Achievable Rates): A nonnegative rate tuple $(R_{1o}, R_{1h}, R_{1c}, R_{1p}, R_{2o}, R_{2h}, R_{2c}, R_{2p})$ is achievable if it satisfies the following for some nonnegative $(\tilde{R}_{1h}, \tilde{R}_{2h})$: (denote $R_o := R_{1o} + R_{2o}$)

Constraints at Receiver 1:

$$\begin{aligned} R_{1p} &\leq I(x_{1cp}; y_1 | x_{1c}, x_{2c}, u_1, \underline{x}_o) \\ \tilde{R}_{1h} &\leq I(u_1; y_1 | x_{1cp}, x_{1c}, x_{2c}, \underline{x}_o) \\ \tilde{R}_{1h} + R_{1p} &\leq I(u_1, x_{1cp}; y_1 | x_{1c}, x_{2c}, \underline{x}_o) \\ R_{2c} + R_{1p} &\leq I(x_{2c}, x_{1cp}; y_1 | x_{1c}, u_1, \underline{x}_o) \\ R_{1c} + R_{1p} &\leq I(x_{1c}, x_{1cp}; y_1 | x_{2c}, u_1, \underline{x}_o) \\ R_{2c} + \tilde{R}_{1h} &\leq I(x_{2c}, u_1; y_1 | x_{1cp}, x_{1c}, \underline{x}_o) \\ R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq I(x_{2c}, u_1, x_{1cp}; y_1 | x_{1c}, \underline{x}_o) \\ R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq I(x_{1c}, x_{1cp}, u_1; y_1 | x_{2c}, \underline{x}_o) \\ R_{1c} + R_{2c} + R_{1p} &\leq I(x_{1c}, x_{1cp}, x_{2c}; y_1 | u_1, \underline{x}_o) \\ R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq I(x_{1c}, x_{1cp}, x_{2c}, u_1; y_1 | \underline{x}_o) \\ R_o + \tilde{R}_{1h} &\leq I(\underline{x}_o, u_1; y_1 | x_{1cp}, x_{1c}, x_{2c}) \\ R_o + \tilde{R}_{1h} + R_{1p} &\leq I(\underline{x}_o, u_1, x_{1cp}; y_1 | x_{1c}, x_{2c}) \\ R_o + R_{2c} + \tilde{R}_{1h} &\leq I(\underline{x}_o, x_{2c}, u_1; y_1 | x_{1cp}, x_{1c}) \\ R_o + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq I(\underline{x}_o, x_{2c}, u_1, x_{1cp}; y_1 | x_{1c}) \\ R_o + R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq I(\underline{x}_o, x_{1c}, x_{1cp}, u_1; y_1 | x_{2c}) \\ R_o + R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq I(\underline{x}_o, x_{1c}, x_{1cp}, x_{2c}, u_1; y_1). \end{aligned}$$

Constraints at Receiver 2: Above with index 1 and 2 exchanged.

Constraints at Transmitters:

$$\begin{aligned} R_{1h} &\leq \tilde{R}_{1h} \\ R_{2h} &\leq \tilde{R}_{2h} \\ R_{1o} + R_{1h} &\leq C_{12}^B \\ R_{2o} + R_{2h} &\leq C_{21}^B \\ \tilde{R}_{1h} + \tilde{R}_{2h} - R_{1h} - R_{2h} &\geq I(u_1; u_2 | \underline{x}_o) \end{aligned}$$

for some $(u_1, u_2) - \underline{x}_{oh} - (y_1, y_2)$, $x_1 = x_{oh}[1] + x_{1cp}$, $x_2 = x_{oh}[2] + x_{2cp}$ and

$$\begin{aligned} &p(\underline{x}_{oh}, \underline{x}_o, u_1, u_2, x_{1c}, x_{1cp}, x_{2c}, x_{2cp}) \\ &= p(\underline{x}_o) p(\underline{x}_{oh}, u_1, u_2 | \underline{x}_o) p(x_{1c}) \\ &\quad \cdot p(x_{1cp} | x_{1c}) p(x_{2c}) p(x_{2cp} | x_{2c}). \end{aligned}$$

Proof: The proof is quite straightforward. It involves standard error probability analysis of superposition coding and Marton's coding scheme and hence is omitted here. Note we have in total 5 independent messages to be decoded at each receiver and hence in general there should be $2^5 - 1 = 31$ inequalities. However, say at receiver 1, decoding m_{2c} incorrectly is not accounted as an error. Furthermore due to the superposition coding of \tilde{m}_{1h} upon \underline{x}_o^N and the superposition coding of m_{1p} upon x_{1c}^N , we remove the inequality on R_{2c} and the $2^3 + 2^3 - 2 = 14$ inequalities involving R_o but not \tilde{R}_{1h} or involving R_{1c} but not R_{1p} . Hence in total we have $31 - 1 - 14 = 16$ inequalities at each receiver. ■

Below we show that with proper choices of distribution $p(\underline{x}_{oh}, \underline{x}_o, u_1, u_2, x_{1c}, x_{1cp}, x_{2c}, x_{2cp})$, the above coding strategy can achieve the capacity region of LDC. We shall distinguish into two cases: (1) system transfer matrix is full-rank and (2) system transfer matrix is not full-rank.

System Matrix is Full-Rank: $n_{11} + n_{22} \neq n_{12} + n_{21}$: In this case, for the cooperative part, we shall set \underline{x}_o to be running over all transmit levels and choose $(u_1, u_2) | \underline{x}_o \stackrel{d}{=} (y_{1h}, y_{2h})$ occupying the following numbers of least significant bits (LSB) at receiver 1 and 2, respectively

$$g_1 = \max \left\{ n_{11} - (n_{21} - n_{22})^+, n_{12} - (n_{22} - n_{21})^+ \right\} \quad (7)$$

$$g_2 = \max \left\{ n_{22} - (n_{12} - n_{11})^+, n_{21} - (n_{11} - n_{12})^+ \right\}. \quad (8)$$

Then we choose \underline{x}_h occupying the levels at transmitters so that it results in (y_{1h}, y_{2h}) at receivers. The cooperative codeword is generated according to the distribution of $\underline{x}_{oh} \stackrel{d}{=} \underline{x}_o + \underline{x}_h$. The addition here is bit-wise modulo-two. We observe the following:

Claim 3.4: Under i.i.d. Bernoulli half inputs, mutual information $I(u_1; u_2 | \underline{x}_o) = 0$ with the above choice if $n_{11} + n_{22} \neq n_{12} + n_{21}$ and hence u_1 and u_2 are independent conditioned on \underline{x}_o .

Remark 3.5 (Comments on Claim 3.4): Intuitively speaking, choosing $p(u_1, u_2, \underline{x}_o)$ as above has an effect that the interference caused by the other user's cooperative private signal is nulled out at the target receiver. The reason is that, the re-

ceived signal contributed by the cooperative private signals at a receiver, say, receiver 1, is $y_{1h} = u_1$ and the conditional independence of u_1 and u_2 given cooperative common signal \underline{x}_o implies that within y_{1h} there is no dependency on u_2 and hence, the interference caused by u_2 is nulled out.

Proof: For the case $\{n_{11} \geq n_{12}, n_{22} \geq n_{21}\}$, (g_1, g_2) becomes

$$\begin{aligned} g_1 &= \max \left\{ n_{11} - (n_{21} - n_{22})^+, n_{12} - (n_{22} - n_{21})^+ \right\} \\ &= n_{11} \\ g_2 &= \max \left\{ n_{22} - (n_{12} - n_{11})^+, n_{21} - (n_{11} - n_{12})^+ \right\} \\ &= n_{22} \end{aligned}$$

and the rank of the full system transfer matrix is $n_{11} + n_{22} = g_1 + g_2$. Hence, under i.i.d. Bernoulli half inputs

$$\begin{aligned} I(u_1; u_2 | \underline{x}_o) &= H(u_1 | \underline{x}_o) + H(u_2 | \underline{x}_o) - H(u_1, u_2 | \underline{x}_o) \\ &= n_{11} + n_{22} - (n_{11} + n_{22}) = 0. \end{aligned}$$

Similar argument works for the case $n_{11} \leq n_{12}, n_{22} \leq n_{21}$.

For the case $\{n_{11} \leq n_{12}, n_{22} \geq n_{21}\}$, (g_1, g_2) becomes

$$\begin{aligned} g_1 &= \max(n_{11}, n_{12} + n_{21} - n_{22}) \\ g_2 &= \max(n_{11} + n_{22} - n_{12}, n_{21}) \end{aligned}$$

and the rank of the transfer matrix is $g_1 + g_2$ again, since the subsystem (lies in the original system with (y_{1h}, y_{2h}) as output and the corresponding levels at transmitters as input) has channel parameters

$$n'_{11} = n_{11}, n'_{21} = n_{21}, n'_{12} = g_1, n'_{22} = g_2.$$

If $n_{11} + n_{22} > n_{12} + n_{21}$, then $g_1 = n_{11}$ and $g_2 = n_{11} + n_{22} - n_{12}$ and hence $n'_{11} = n'_{12}, n'_{22} > n'_{21}$.

Similarly, if $n_{11} + n_{22} < n_{12} + n_{21}$, then $g_1 = n_{12} + n_{21} - n_{22}$ and $g_2 = n_{21}$ and hence $n'_{11} < n'_{12}, n'_{22} = n'_{21}$.

Similar argument works for the case $n_{11} \geq n_{12}, n_{22} \leq n_{21}$.

Therefore, under i.i.d. Bernoulli half inputs, u_1 and u_2 are independent conditioned on \underline{x}_o from the analysis of the previous cases. ■

For the noncooperative part, we set $x_{1cp} \stackrel{d}{=} x_{1c} + x_{1p}$ such that x_{1c} and x_{1p} are independent. x_{1c} is allowed to occupy all levels at transmitter 1, while x_{1p} is allowed to occupy only the $(n_{11} - n_{21})^+$ LSB's. The addition here is bit-wise modulo-two. The same design is applied to user 2. As commented in Remark 3.5, with the above choice of y_{1h} and y_{2h} in (7) and (8), there are no redundancy between y_{1h} and y_{2h} and hence no interference from u_2 at receiver 1. Similar situation happens at receiver 2. Now take all inputs to be i.i.d. Bernoulli half across levels, we obtain a set of achievable rates from Theorem 3.3. After Fourier-Motzkin elimination, we show that the achievable region coincides with the region given in Theorem 3.1.

Lemma 3.6: The above strategy achieves the region given in Theorem 3.1 when $n_{11} + n_{22} \neq n_{12} + n_{21}$.

Proof: The details are left in Appendix A. ■

System Matrix is not Full-Rank: $n_{11} + n_{22} = n_{12} + n_{21}$: In this case, for the cooperative part, we shall again set \underline{x}_o to

be running over all transmit levels. The difference lies in the *cooperative private* part. Here we also choose $(u_1, u_2)|_{\underline{x}_o} \stackrel{d}{=} (y_{1h}, y_{2h})$, but occupying the following numbers of LSB's at receiver 1 and 2, respectively

$$g_1 = (n_{11} - n_{21})^+, g_2 = (n_{22} - n_{12})^+.$$

Claim 3.7: Under i.i.d. Bernoulli half inputs, mutual information $I(u_1; u_2 | \underline{x}_o) = 0$ with the above choice if $n_{11} + n_{22} = n_{12} + n_{21}$.

Proof: Since y_{ih} only occupies levels that appear at receiver i but do not appear at the other receiver, for $i = 1, 2$, hence they are conditionally independent given \underline{x}_o under Bernoulli half i.i.d. inputs. ■

For the noncooperative part, we use the same scheme as the previous case. Now take all inputs to be i.i.d. Bernoulli half across levels, we obtain achievable rates from Theorem 3.3. After Fourier-Motzkin elimination, we have the following lemma.

Lemma 3.8: The above strategy achieves the region given in Theorem 3.1 when $n_{11} + n_{22} = n_{12} + n_{21}$.

Proof: The details are left in Appendix A. ■

We conclude this section with the following remark.

Remark 3.9 (Implications on the Gaussian Problem): The two numbers g_1 and g_2 provide clues in determining the power allocated to the cooperative private codewords and the design of beamforming vectors in the Gaussian scenario. Take user 1 as an example. When $n_{11} + n_{22} \neq n_{12} + n_{21}$,

$$g_1 = \max \left\{ n_{11} - (n_{21} - n_{22})^+, n_{12} - (n_{22} - n_{21})^+ \right\} \\ = \max \{ n_{11} + n_{22}, n_{12} + n_{21} \} - \max \{ n_{22}, n_{21} \},$$

which corresponds to $\frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{SNR}_2 + \text{INR}_2}$. On the other hand, when $n_{11} + n_{22} = n_{12} + n_{21}$,

$$g_1 = (n_{11} - n_{21})^+ = (n_{12} - n_{22})^+$$

which corresponds to $\frac{\text{SNR}_1}{\text{INR}_2} = \frac{\text{INR}_1}{\text{SNR}_2} = \frac{\text{SNR}_1 + \text{INR}_1}{\text{SNR}_2 + \text{INR}_2}$. This implies that the power of u_1 conditioned on \underline{x}_o should be proportional to

$$\frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_1 + \text{INR}_1}{1 + \text{SNR}_2 + \text{INR}_2} \\ = \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + |h_{11}|^2 + |h_{22}|^2}{1 + \text{SNR}_2 + \text{INR}_2},$$

and that the beamforming vector should be a combination of zero-forcing and matched-filter vectors.

IV. GAUSSIAN INTERFERENCE CHANNEL WITH CONFERENCEING TRANSMITTERS

With the full understanding in the linear deterministic interference channel with conferencing transmitters, now we have enough clues to crack the original Gaussian problem. As for the outer bounds, we shall mimic the genie-aided techniques and the structure of side informations in Section III-B to develop the proofs. As for the achievability, we shall mimic the choice of auxiliary random variables and level allocation in Section III-D to construct good schemes in the Gaussian scenario. Moreover, the achievable rate regions obtained prior to Fourier-Motzkin elimination can be made equivalent symbolically and hence the proof of achieving approximate capacity in the Gaussian channel follows closely to the proof of achieving exact capacity in the linear deterministic channel. Although the Gaussian interference channel with conferencing transmitters and its corresponding linear deterministic channel are strongly related in coding strategies, proof of achievability and outer bounds, unlike the two-user Gaussian interference channel [19], their capacity regions are *not* within a bounded gap. Similar situations happen in MIMO channel, Gaussian relay networks [6] and the Gaussian interference channel with conferencing receivers [1], where explicit/implicit MIMO structures lie in the channel model.

Our main result is summarized in the following lemma and theorem.

Lemma 4.1 (Outer Bounds): If (R_1, R_2) is achievable, it satisfies the constraints listed as shown in (9)–(18) at the bottom of the next page.

Theorem 4.2 (Bounded Gap to Capacity): Outer bounds in Lemma 4.1 is within $\log 90 \approx 6.5$ bits per user to the capacity region.

A. Outer Bounds

Details of the proof of Lemma 4.1 are left in Appendix D. It follows closely to the techniques we develop in the proofs of the LDC outer bounds. The only twist is how to mimic the proof of bound (6), which is a new type of outer bound that does not appear in the case without cooperation. It corresponds to bound (17) here. Recall that in the proof there, we split receiver 2's signal into two parts: $y_2^N = (y_{2\alpha}^N, y_{2\beta}^N)$, where $y_{2\alpha}^N$ is the part of transmitter 2's signal that is not corrupted by s_1^N , the interference from transmitter 1. Such split is not possible in the Gaussian channel due to additive noise and carry-over in real addition. As shown in Appendix D, we will overcome this by providing the following side information to receiver 2:

$$\tilde{y}_2^N := h_{22}x_2^N + \tilde{z}_2^N$$

where $\tilde{z}_2 \sim \mathcal{CN}(0, 1 + \text{INR}_2)$, i.i.d. over time and is independent of everything else. This mimics the signal $y_{2\alpha}^N$ in LDC and helps us prove the $2R_1 + R_2$ outer bound.

B. Coding Strategy and Achievable Rates

We shall employ the coding strategy proposed in Section III-D. The analysis in the linear deterministic setting suggests that, for the cooperative private messages, in the Gaussian setting one may choose its bearing auxiliary random variables u_1 and u_2 to be conditionally independent given \underline{x}_o . This implies that a simple linear beamforming strategy is sufficient. On the other hand, the interference should be zero-forced approximately. Based on this observation, we implement the following strategy.

For the cooperative common signal, recall that in the LDC we allow \underline{x}_o to run over all transmit levels. To mimic it, in the Gaussian setting we choose \underline{x}_o to be Gaussian with zero mean and a covariance matrix which has diagonal entries (values of transmit power) that are comparable with the total transmit power. For simplicity, we choose the covariance matrix to be diagonal:

$$K_{\underline{x}_o} = \text{diag}(Q_{1o}, Q_{2o}), \quad Q_{io} = \frac{1}{4}, \quad i = 1, 2.$$

Here the value 1/4 is just a heuristic choice such that the transmit power constraints will be satisfied.

For the cooperative private signal, from the discussion in Remark 3.9, we shall make it a superposition of *zero-forcing* vectors

$$\underline{v}_{1z} = \begin{bmatrix} h_{22} \\ -h_{21} \end{bmatrix}, \quad \underline{v}_{2z} = \begin{bmatrix} -h_{12} \\ h_{11} \end{bmatrix}$$

and *matched-filter* vectors

$$\underline{v}_{1m} = \begin{bmatrix} h_{11}^* \\ h_{12}^* \end{bmatrix}, \quad \underline{v}_{2m} = \begin{bmatrix} h_{21}^* \\ h_{22}^* \end{bmatrix}.$$

For the auxiliary random variables u_1 and u_2 , we make them distributed as identical copies of user 1 and user 2's desired cooperative signals received at receiver 1 and 2, respectively. For example, u_1 would be the sum of the transmit cooperative common signal and user 1's cooperative private signal projected onto the channel vector $[h_{11} \ h_{12}]$.

Hence we choose $(\underline{x}_{oh}, \underline{x}_o, u_1, u_2)$ be jointly Gaussian such that

$$\underline{x}_{oh} \stackrel{d}{=} \underline{x}_o + \underbrace{w_{1z}\underline{v}_{1z} + w_{2z}\underline{v}_{2z} + w_{1m}\underline{v}_{1m} + w_{2m}\underline{v}_{2m}}_{\underline{x}_h}$$

$$u_1 \stackrel{d}{=} [h_{11} \ h_{12}] (\underline{x}_o + \underline{v}_{1z}w_{1z} + \underline{v}_{1m}w_{1m})$$

$$u_2 \stackrel{d}{=} [h_{21} \ h_{22}] (\underline{x}_o + \underline{v}_{2z}w_{2z} + \underline{v}_{2m}w_{2m})$$

$$R_1 \leq \min \left\{ \log(1 + \text{SNR}_1) + C_{12}^B, \log\left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1}\right) \right\} \quad (9)$$

$$R_2 \leq \min \left\{ \log(1 + \text{SNR}_2) + C_{21}^B, \log\left(1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_2\text{INR}_2}\right) \right\} \quad (10)$$

$$R_1 + R_2 \leq \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + \log\left(1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_2\text{INR}_2}\right) + C_{12}^B \quad (11)$$

$$R_1 + R_2 \leq \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + \log\left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1}\right) + C_{21}^B \quad (12)$$

$$R_1 + R_2 \leq \log\left(1 + \frac{\text{SNR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1}}{1 + \text{INR}_2} + \text{INR}_1\right) + \log\left(1 + \frac{\text{SNR}_2 + 2\sqrt{\text{SNR}_2\text{INR}_2}}{1 + \text{INR}_1} + \text{INR}_2\right) + C_{12}^B + C_{21}^B \quad (13)$$

$$R_1 + R_2 \leq \log\left(1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_1\text{INR}_1} + 2\sqrt{\text{SNR}_2\text{INR}_2} + |h_{11}h_{22} - h_{12}h_{21}|^2\right) \quad (14)$$

$$2R_1 + R_2 \leq \left\{ \begin{array}{l} \log\left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1}\right) + \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) \\ + \log\left(1 + \frac{\text{SNR}_2 + 2\sqrt{\text{SNR}_2\text{INR}_2}}{1 + \text{INR}_1} + \text{INR}_2\right) + C_{12}^B + C_{21}^B \end{array} \right\} \quad (15)$$

$$R_1 + 2R_2 \leq \left\{ \begin{array}{l} \log\left(1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_2\text{INR}_2}\right) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) \\ + \log\left(1 + \frac{\text{SNR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1}}{1 + \text{INR}_2} + \text{INR}_1\right) + C_{21}^B + C_{12}^B \end{array} \right\} \quad (16)$$

$$2R_1 + R_2 \leq \left\{ \begin{array}{l} \log\left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + \text{SNR}_1\text{SNR}_2}{+ \text{INR}_1\text{INR}_2 + \text{SNR}_1\text{INR}_2 + 2(1 + \text{INR}_2)\sqrt{\text{SNR}_1\text{INR}_1}}\right) \\ + \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + 1 + C_{12}^B \end{array} \right\} \quad (17)$$

$$R_1 + 2R_2 \leq \left\{ \begin{array}{l} \log\left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + \text{SNR}_1\text{SNR}_2}{+ \text{INR}_1\text{INR}_2 + \text{SNR}_2\text{INR}_1 + 2(1 + \text{INR}_1)\sqrt{\text{SNR}_2\text{INR}_2}}\right) \\ + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + 1 + C_{21}^B \end{array} \right\} \quad (18)$$

where w_{1z}, w_{2z}, w_{1m} , and w_{2m} are independent Gaussians and independent of everything else, with variances $\theta_{1z}, \theta_{2z}, \theta_{1m}$, and θ_{2m} , respectively. Their values are chosen such that the total transmit power constraint will be met and the conditional variances of u_1 and u_2 conditioned on \underline{x}_o behave as we predicted in Remark 3.9. With this guideline, we choose

$$\begin{aligned}\theta_{1z} &= \frac{\frac{1}{4}}{(1 + \text{SNR}_2 + \text{INR}_2)} \\ \theta_{1m} &= \frac{\frac{1}{4}}{(\text{SNR}_1 + \text{INR}_1)(1 + \text{SNR}_2 + \text{INR}_2)} \\ \theta_{2z} &= \frac{\frac{1}{4}}{(1 + \text{SNR}_1 + \text{INR}_1)} \\ \theta_{2m} &= \frac{\frac{1}{4}}{(\text{SNR}_2 + \text{INR}_2)(1 + \text{SNR}_1 + \text{INR}_1)}.\end{aligned}$$

Again, the factor $1/4$ is just a heuristic choice such that the transmit power constraints will be satisfied.

For the noncooperative part, we set $x_{ic} \sim \mathcal{CN}(0, Q_{ic})$, where $Q_{ic} = 1/4 - Q_{ip}$, for $i = 1, 2$. $x_{icp} \stackrel{d}{=} x_{ic} + x_{ip}$, where $x_{ip} \sim \mathcal{CN}(0, Q_{ip})$ is independent of x_{ic} and $Q_{ip} = \min(1/4, 1/\text{INR}_j)$, for $(i, j) = (1, 2)$ or $(2, 1)$. The choice of Q_{ip} is such that the interference caused by the other user's *noncooperative* private signal is at or below the noise level at the receiver.

At this stage, we shall check that the total transmit power constraint is met with the above heuristic choices of factors. We only need to show that the power for \underline{x}_h at each transmitter is at most $1/2$, which is pretty straightforward.³

Note that the variances of u_1 and u_2 conditioned on \underline{x}_o are

$$\begin{aligned}K_{u_1|\underline{x}_o} &= \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_1 + \text{INR}_1}{4(1 + \text{SNR}_2 + \text{INR}_2)}, \\ K_{u_2|\underline{x}_o} &= \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_2 + \text{INR}_2}{4(1 + \text{SNR}_1 + \text{INR}_1)}\end{aligned}$$

matching our prediction in Remark 3.9.

With this encoding, the interference caused by the other user's *cooperative* private signal should be nulled out approximately, that is, its variance is at or below the noise level. To see this, the received signals are jointly distributed with $(\underline{x}_o, u_1, u_2, x_{1c}, x_{1cp}, x_{2c}, x_{2cp})$ such that

$$\begin{aligned}y_1 &\stackrel{d}{=} u_1 + \hat{z}_1 + h_{11}x_{1cp} + h_{12}x_{2cp} + z_1 \\ y_2 &\stackrel{d}{=} u_2 + \hat{z}_2 + h_{21}x_{1cp} + h_{22}x_{2cp} + z_2,\end{aligned}$$

where the interferences caused by undesired cooperative private signals are

$$\hat{z}_1 = (h_{11}h_{21}^* + h_{12}h_{22}^*)w_{2m}, \quad \hat{z}_2 = (h_{21}h_{11}^* + h_{22}h_{12}^*)w_{1m}$$

³We have $Q_{1h} = \frac{\text{SNR}_2 + \frac{\text{SNR}_1}{\text{SNR}_1 + \text{INR}_1}}{4(1 + \text{SNR}_2 + \text{INR}_2)} + \frac{\text{INR}_1 + \frac{\text{INR}_2}{\text{SNR}_2 + \text{INR}_1}}{4(1 + \text{SNR}_1 + \text{INR}_1)} \leq \frac{1}{2}$ and vice versa for Q_{2h} .

at receiver 1 and 2, respectively. Note that the variance of these terms are upper bounded by a constant, since

$$\begin{aligned}|h_{11}h_{21}^* + h_{12}h_{22}^*|^2 &= |h_{21}h_{11}^* + h_{22}h_{12}^*|^2 \\ &= (\text{SNR}_1 + \text{INR}_1)(\text{SNR}_2 + \text{INR}_2) \\ &\quad - |h_{11}h_{22} - h_{12}h_{21}|^2 \\ &\leq (\text{SNR}_1 + \text{INR}_1)(\text{SNR}_2 + \text{INR}_2).\end{aligned}$$

Hence

$$\begin{aligned}\sigma_1^2 &:= \text{Var}(\hat{z}_1) \\ &= \frac{|h_{11}h_{21}^* + h_{12}h_{22}^*|^2}{4(\text{SNR}_1 + \text{INR}_1)(1 + \text{SNR}_2 + \text{INR}_2)} \leq \frac{1}{4} \\ \sigma_2^2 &:= \text{Var}(\hat{z}_2) \\ &= \frac{|h_{21}h_{11}^* + h_{22}h_{12}^*|^2}{4(1 + \text{SNR}_1 + \text{INR}_1)(\text{SNR}_2 + \text{INR}_2)} \leq \frac{1}{4}\end{aligned}$$

and in effect the interference is nulled out approximately.

Remark 4.3: When the cooperative link capacities are sufficiently large and the channel becomes a two-user Gaussian MIMO broadcast channel with two transmit antennas and single receive antenna at each receiver, the proposed scheme in Theorem 3.3 is capacity-achieving. Dirty paper coding among cooperative private messages is needed to achieve the capacity of Gaussian MIMO broadcast channel *exactly* [4], that is, u_1 and u_2 is not independent conditioned on \underline{x}_o and \underline{x}_o is made zero. As shown in Appendix B and C, however, linear beamforming strategies along with superposition coding suffice to achieve the capacity *approximately*. We conjecture that dirty paper coding among cooperative private messages will lead to a better rate region and smaller gap to the outer bounds, while the procedure of computing the achievable region becomes complicated.

We have designed a coding strategy and its configuration which met the observation and intuition from the analysis of LDC and it turns out that it achieves the capacity to within a bounded gap. This completes the proof of Theorem 4.2. The proof is broken into two parts: (1) the computation of the achievable rate region and (2) the evaluation of the gap among inner and outer bounds. Details are left in Appendix B and C, respectively.

V. UPLINK-DOWNLINK RECIPROCITY

Recall that in Section III-C we have demonstrated the reciprocity between linear deterministic interference channel with conferencing receivers and the linear deterministic interference channel with conferencing transmitters. In this section, we show that a similar reciprocity holds in the Gaussian case.

For the channel described in Section II, we define its *reciprocal* channel as the Gaussian interference channel with conferencing receivers [1] with the 2-by-2 channel matrix

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^H = \begin{bmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{bmatrix}$$

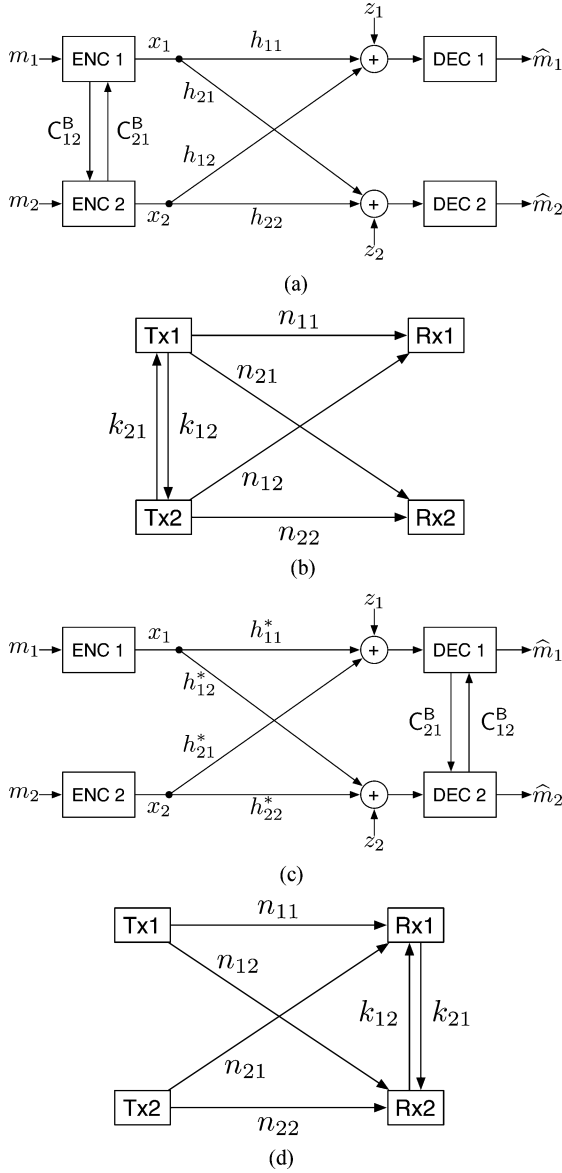


Fig. 7. Uplink-downlink reciprocity. (a) Original Gaussian downlink scenario. (b) Original LDC downlink scenario. (c) Reciprocal Gaussian uplink scenario. (d) Reciprocal LDC uplink scenario.

and cooperative link capacities C_{21}^B from receiver 1 to 2 and C_{12}^B from receiver 2 to 1. Note that for the reciprocal channel, the channel matrix is the Hermitian of the original one and the cooperative link capacities are swapped. Motivated by backhaul cooperation in cellular networks where cooperation is among base stations, we term the interference channel with conferencing receivers the *uplink* scenario and the interference channel with conferencing transmitters the *downlink* scenario. The original downlink and the reciprocal uplink scenarios are depicted in Fig. 7.

Theorem 5.1: The capacity regions of the original and the reciprocal channels are within a bounded gap, regardless of channel parameters.

Proof: Details are left in Appendix E. ■

TABLE III
NOTATIONS

p_1	t_1	m_1
$(n_{11} - n_{21})^+$	$\max(n_{12}, p_1)$	$\max(n_{11}, n_{12})$
l_1	s_1	
$\max(n_{11}, g_1)$	$\max(n_{12}, g_1)$	
p_2	t_2	m_2
$(n_{22} - n_{12})^+$	$\max(n_{21}, p_2)$	$\max(n_{22}, n_{21})$
l_2	s_2	
$\max(n_{22}, g_2)$	$\max(n_{21}, g_2)$	

The reciprocity implies immediately that the gain from transmitter cooperation shares the same characteristics as that from receiver cooperation, that is, the degree-of-freedom gain is either one bit or half a bit per cooperation bit until saturation and the power gain is bounded no matter how large the cooperative link capacities are after saturation.

Remark 5.2: As mentioned in Section III-C, there is an *exact* reciprocity between the linear deterministic downlink scenario and the uplink scenario. Not only are the capacity regions of the original and the reciprocal channel the same, but the capacity-achieving linear schemes are also reciprocal. On the other hand, for the Gaussian downlink scenario and the uplink scenario, combining the results in this paper and [1], it seems such reciprocity in the proposed strategies does not exist, since the message structures are different. Although the strategies proposed in this paper and [1] are not reciprocal, we conjecture that such reciprocity may be obtained via structured lattice strategies derived from capacity-achieving linear schemes of the corresponding linear deterministic channels. Such conversion has been applied successfully in [24] to construct lattice coding strategies for many-to-one and one-to-many Gaussian interference channels.

APPENDIX A

PROOF OF ACHIEVABILITY IN THEOREM 3.1

A) *Proof of Lemma 3.6:* Plugging in the configuration, we have the following achievable rates from Theorem 3.3: for some nonnegative $(\tilde{R}_{1h}, \tilde{R}_{2h})$, (notations are listed in Table III)

Constraints at Transmitters:

$$\begin{aligned}
 R_{1h} &\leq \tilde{R}_{1h} \\
 R_{2h} &\leq \tilde{R}_{2h} \\
 R_{1o} + R_{2o} &= R_o \\
 R_{1o} + R_{1h} &\leq k_{12} \\
 R_{2o} + R_{2h} &\leq k_{21} \\
 \tilde{R}_{1h} + \tilde{R}_{2h} - R_{1h} - R_{2h} &\geq 0
 \end{aligned}$$

Constraints at Receiver 1:

$$\begin{aligned}
 R_{1p} &\leq p_1 \\
 R_{2c} + R_{1p} &\leq t_1 \\
 R_{1c} + R_{1p} &\leq n_{11} \\
 R_{1c} + R_{2c} + R_{1p} &\leq m_1
 \end{aligned}$$

$$\begin{aligned}
 \tilde{R}_{1h} &\leq g_1 \\
 \tilde{R}_{1h} + R_{1p} &\leq g_1 \\
 R_{2c} + \tilde{R}_{1h} &\leq s_1 \\
 R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq s_1 \\
 R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq l_1 \\
 R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
 R_o + \tilde{R}_{1h} &\leq m_1 \\
 R_o + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
 R_o + R_{2c} + \tilde{R}_{1h} &\leq m_1 \\
 R_o + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
 R_o + R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
 R_o + R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1
 \end{aligned}$$

Constraints at Receiver 2: Above with index 1 and 2 exchanged.

After Fourier-Motzkin elimination, we have the following achievable rates and identify all redundant terms. The claims used below to show the redundancy are proved in the end of this section. We use symbol ‘‘Red’’ to denote ‘‘redundant.’’

(1) R_1 and R_2 :

$$\begin{aligned}
 R_1 &\leq n_{11} + k_{12} & R_2 &\leq n_{22} + k_{21} \\
 R_1 &\leq m_1 & R_2 &\leq m_2 \\
 R_1 &\leq p_1 + t_2 + k_{12} \text{ (Red)} & R_2 &\leq p_2 + t_1 + k_{21} \text{ (Red)} \\
 R_1 &\leq p_1 + s_2 + k_{12} \text{ (Red)} & R_2 &\leq p_2 + s_1 + k_{21} \text{ (Red)} \\
 R_1 &\leq g_1 + t_2 + k_{12} \text{ (Red)} & R_2 &\leq g_2 + t_1 + k_{21} \text{ (Red)} \\
 R_1 &\leq g_1 + s_2 + k_{12} \text{ (Red)} & R_2 &\leq g_2 + s_1 + k_{21} \text{ (Red)}.
 \end{aligned}$$

To show the redundancy, we need to prove the following claim

Claim A.1:

- $p_1 + t_2 \geq n_{11}$
- $p_2 + t_1 \geq n_{22}$
- $g_1 \geq p_1, g_2 \geq p_2$
- $s_1 \geq t_1, s_2 \geq t_2$

(1) $R_1 + R_2$:

$$\begin{aligned}
 R_1 + R_2 &\leq t_1 + t_2 + k_{12} + k_{21} \\
 R_1 + R_2 &\leq p_1 + m_2 + k_{12} \\
 R_1 + R_2 &\leq p_2 + m_1 + k_{21} \\
 R_1 + R_2 &\leq g_1 + m_2 \\
 R_1 + R_2 &\leq g_2 + m_1 \\
 R_1 + R_2 &\leq s_1 + t_2 + k_{12} + k_{21} \text{ (Red)} \\
 R_1 + R_2 &\leq s_2 + t_1 + k_{12} + k_{21} \text{ (Red)} \\
 R_1 + R_2 &\leq s_1 + s_2 + k_{12} + k_{21} \text{ (Red)}
 \end{aligned}$$

(2) $2R_1 + R_2$ and $R_1 + 2R_2$:

$$\begin{aligned}
 2R_1 + R_2 &\leq p_1 + m_1 + t_2 + k_{12} + k_{21} \\
 2R_1 + R_2 &\leq g_1 + m_1 + t_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + R_2 &\leq p_1 + m_1 + s_2 + k_{12} \\
 2R_1 + R_2 &\leq g_1 + m_1 + s_2 + k_{12} \text{ (Red)} \\
 2R_1 + R_2 &\leq p_1 + l_1 + m_2 + k_{12}
 \end{aligned}$$

$$\begin{aligned}
 R_1 + 2R_2 &\leq p_2 + m_2 + t_1 + k_{12} + k_{21} \\
 R_1 + 2R_2 &\leq g_2 + m_2 + t_1 + k_{12} + k_{21} \text{ (Red)} \\
 R_1 + 2R_2 &\leq p_2 + m_2 + s_1 + k_{21} \\
 R_1 + 2R_2 &\leq g_2 + m_2 + s_1 + k_{21} \text{ (Red)} \\
 R_1 + 2R_2 &\leq p_2 + l_2 + m_1 + k_{21}.
 \end{aligned}$$

(3) $2R_1 + 2R_2$:

$$\begin{aligned}
 2R_1 + 2R_2 &\leq p_1 + s_1 + t_2 + m_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq l_1 + t_1 + p_2 + m_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq p_1 + s_1 + s_2 + m_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq l_1 + t_1 + g_2 + m_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq t_1 + m_1 + p_2 + s_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq s_1 + m_1 + p_2 + s_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq p_1 + m_1 + l_2 + t_2 + k_{12} + k_{21} \text{ (Red)} \\
 2R_1 + 2R_2 &\leq g_1 + m_1 + l_2 + t_2 + k_{12} + k_{21} \text{ (Red)}.
 \end{aligned}$$

To show the redundancy, we need to prove the following claim.

Claim A.2:

- $s_1 + t_2 \geq p_2 + m_1; s_2 + t_1 \geq p_1 + m_2$
- $l_1 + t_1 \geq p_1 + m_1; l_2 + t_2 \geq p_2 + m_2$

After removing the redundant terms, we have the following achievable region for LDC when $n_{11} + n_{22} \neq n_{12} + n_{21}$:

$$\begin{aligned}
 R_1 &\leq \min \{n_{11} + k_{12}, m_1\} \\
 R_2 &\leq \min \{n_{22} + k_{21}, m_2\} \\
 R_1 + R_2 &\leq \min \{g_1 + m_2, g_2 + m_1\} \\
 R_1 + R_2 &\leq t_1 + t_2 + k_{12} + k_{21} \\
 R_1 + R_2 &\leq \min \{p_1 + m_2 + k_{12}, p_2 + m_1 + k_{21}\} \\
 2R_1 + R_2 &\leq \min \{p_1 + l_1 + m_2 + k_{12}, p_1 + s_2 + m_1 + k_{12}\} \\
 2R_1 + R_2 &\leq p_1 + m_1 + t_2 + k_{12} + k_{21} \\
 R_1 + 2R_2 &\leq \min \{p_2 + l_2 + m_1 + k_{21}, p_2 + s_1 + m_2 + k_{21}\} \\
 R_1 + 2R_2 &\leq p_2 + m_2 + t_1 + k_{21} + k_{12}.
 \end{aligned}$$

To show that the above achievable region coincides with the rate region given in Theorem 3.1, the following facts are crucial.

Claim A.3:

- $g_1 + m_2 = g_2 + m_1 = \max(n_{11} + n_{22}, n_{12} + n_{21})$.
- $s_2 + m_1 = l_1 + m_2; s_1 + m_2 = l_2 + m_1$

With these facts, referring to Table III and checking with the outer bounds, we complete the proof.

B) Proof of Lemma 3.8: We have the following achievable rates: for some nonnegative $(\tilde{R}_{1h}, \tilde{R}_{2h})$,

Constraints at Transmitters:

$$\begin{aligned}
 R_{1h} &\leq \tilde{R}_{1h} \\
 R_{2h} &\leq \tilde{R}_{2h} \\
 R_{1o} + R_{2o} &= R_o \\
 R_{1o} + R_{1h} &\leq k_{12} \\
 R_{2o} + R_{2h} &\leq k_{21} \\
 \tilde{R}_{1h} + \tilde{R}_{2h} - R_{1h} - R_{2h} &\geq 0
 \end{aligned}$$

Constraints at Receiver 1:

$$\begin{aligned}
R_{1p} &\leq p_1 \\
R_{2c} + R_{1p} &\leq t_1 \\
R_{1c} + R_{1p} &\leq n_{11} \\
R_{1c} + R_{2c} + R_{1p} &\leq m_1 \\
\tilde{R}_{1h} &\leq p_1 \\
\tilde{R}_{1h} + R_{1p} &\leq p_1 \\
R_{2c} + \tilde{R}_{1h} &\leq t_1 \\
R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq t_1 \\
R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq n_{11} \\
R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
R_o + \tilde{R}_{1h} &\leq m_1 \\
R_o + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
R_o + R_{2c} + \tilde{R}_{1h} &\leq m_1 \\
R_o + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
R_o + R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq m_1 \\
R_o + R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq m_1
\end{aligned}$$

Constraints at Receiver 2: Above with index 1 and 2 exchanged.

After Fourier-Motzkin elimination and removing redundant terms based on facts derived in the previous analysis, we have the following achievable region:

$$\begin{aligned}
R_1 &\leq \min\{m_1, n_{11} + k_{12}\} \\
R_2 &\leq \min\{m_2, n_{22} + k_{21}\} \\
R_1 + R_2 &\leq \min\{p_1 + m_2, p_2 + m_1, t_1 + t_2 + k_{12} + k_{21}\} \\
2R_1 + R_2 &\leq p_1 + t_2 + m_1 + k_{12} \quad (\text{Red}) \\
R_1 + 2R_2 &\leq p_2 + t_1 + m_2 + k_{21} \quad (\text{Red})
\end{aligned}$$

which coincides with the outer bounds. To prove this, we need the following facts.

Claim A.4:

- $p_1 + m_2 = p_2 + m_1 = \max(n_{11}, n_{22}, n_{12}, n_{21})$
- $p_1 + t_2 = p_2 + n_{11}; p_2 + t_1 = p_1 + n_{22}$

With the first fact $p_1 + m_2 = p_2 + m_1 = \max(n_{11}, n_{22}, n_{12}, n_{21})$, we show that the sum rate inner bound coincides the outer bound. With the second fact $p_1 + t_2 = p_2 + n_{11}$, we show that the $2R_1 + R_2$ inner bound is redundant. Similarly the $R_1 + 2R_2$ inner bound is also redundant. This completes the proof.

C) Proof of the Claims:

Proof of Claim A.1:

- $p_1 + t_2 \geq n_{11}, p_2 + t_1 \geq n_{22}$

Proof:

$$\begin{aligned}
p_1 + t_2 &\geq (n_{11} - n_{21})^+ + n_{21} \geq n_{11} \\
p_2 + t_1 &\geq (n_{22} - n_{12})^+ + n_{12} \geq n_{22}.
\end{aligned}$$

- $g_1 \geq p_1, g_2 \geq p_2$

Proof:

$$\begin{aligned}
g_1 &= \max\{n_{11} - (n_{21} - n_{22})^+, n_{12} - (n_{22} - n_{21})^+\} \\
&\geq n_{11} - (n_{21} - n_{22})^+ \geq n_{11} - n_{21}.
\end{aligned}$$

On the other hand, $g_1 \geq 0$. Hence, $g_1 \geq (n_{11} - n_{21})^+ = p_1$. Similarly $g_2 \geq p_2$. ■

- $s_1 \geq t_1, s_2 \geq t_2$

Proof:

$$s_1 = \max(n_{12}, g_1) \geq \max(n_{12}, p_1) = t_1$$

since $g_1 \geq p_1$. Similarly $s_2 \geq t_2$. ■

Proof of Claim A.2:

- $s_1 + t_2 \geq p_2 + m_1; s_2 + t_1 \geq p_1 + m_2$

Proof: If $n_{21} \leq n_{22}$

$$\begin{aligned}
s_1 + t_2 &= \max\{n_{12}, n_{11} - (n_{21} - n_{22})^+\} + t_2 \\
&= m_1 + t_2 \geq m_1 + p_2.
\end{aligned}$$

If $n_{21} > n_{22}$ and $n_{22} \leq n_{12}$

$$\begin{aligned}
s_1 + t_2 &= \max\{n_{12}, n_{11} + n_{22} - n_{21}\} + n_{21} \\
&= \max\{n_{12} + n_{21}, n_{11} + n_{22}\} \\
&\geq 0 + \max(n_{11}, n_{12}) = p_2 + m_1.
\end{aligned}$$

If $n_{21} > n_{22}$ and $n_{22} > n_{12}$

$$\begin{aligned}
s_1 + t_2 &= \max\{n_{12}, n_{11} + n_{22} - n_{21}\} + n_{21} \\
&= \max\{n_{12} + n_{21}, n_{11} + n_{22}\} \\
&\geq \max\{n_{11} + n_{22} - n_{12}, n_{21}\} \\
&\geq \max\{n_{11} + n_{22} - n_{12}, n_{22}\} \\
&\geq n_{22} - n_{12} + \max(n_{11}, n_{12}) = p_2 + m_1.
\end{aligned}$$

Hence, $s_1 + t_2 \geq p_2 + m_1$. Similarly, $s_2 + t_1 \geq p_1 + m_2$. ■

- $l_1 + t_1 \geq p_1 + m_1; l_2 + t_2 \geq p_2 + m_2$

Proof: If $n_{21} \geq n_{22}$,

$$\begin{aligned}
l_1 + t_1 &= \max\{n_{11}, n_{12} - (n_{22} - n_{21})^+\} + t_1 \\
&= m_1 + t_1 \geq m_1 + p_1.
\end{aligned}$$

If $n_{21} < n_{22}$ and $n_{11} \geq n_{12}$

$$p_1 + m_1 = p_1 + n_{11} \leq t_1 + l_1.$$

If $n_{21} < n_{22}$ and $n_{11} < n_{12}$, then $m_1 = t_1 = n_{12}$, hence

$$p_1 + m_1 = p_1 + t_1 \leq n_{11} + t_1 \leq l_1 + t_1.$$

Hence, $l_1 + t_1 \geq p_1 + m_1$. Similarly, $l_2 + t_2 \geq p_2 + m_2$. ■

Proof of Claim A.3:

- $g_1 + m_2 = g_2 + m_1 = \max(n_{11} + n_{22}, n_{12} + n_{21})$.

Proof: Note that

$$\begin{aligned} \max(n_{22}, n_{21}) - (n_{21} - n_{22})^+ &= n_{22} \\ \max(n_{22}, n_{21}) - (n_{22} - n_{21})^+ &= n_{21}. \end{aligned}$$

Hence

$$\begin{aligned} g_1 + m_2 &= \max \left\{ n_{11} - (n_{21} - n_{22})^+, n_{12} - (n_{22} - n_{21})^+ \right\} \\ &\quad + \max(n_{22}, n_{21}) \\ &= \max \{ n_{11} + n_{22}, n_{12} + n_{21} \}. \end{aligned}$$

By symmetry, $g_2 + m_1 = \max(n_{11} + n_{22}, n_{12} + n_{21})$. ■

- $s_2 + m_1 = l_1 + m_2$; $s_1 + m_2 = l_2 + m_1$

Proof:

$$\begin{aligned} s_2 + m_1 &= \max \left\{ n_{21}, n_{22} - (n_{12} - n_{11})^+ \right\} \\ &\quad + \max(n_{11}, n_{12}) \\ &= \max \{ n_{21} + \max(n_{11}, n_{12}), n_{22} + n_{11} \} \\ &= \max \{ n_{21} + n_{11}, n_{21} + n_{12}, n_{22} + n_{11} \}; \\ &\quad l_1 + m_2 \\ &= \max \left\{ n_{11}, n_{12} - (n_{22} - n_{21})^+ \right\} \\ &\quad + \max(n_{22}, n_{21}) \\ &= \max \{ n_{11} + \max(n_{22}, n_{21}), n_{12} + n_{21} \} \\ &= \max \{ n_{11} + n_{22}, n_{11} + n_{21}, n_{12} + n_{21} \}. \end{aligned}$$

Hence $s_2 + m_1 = l_1 + m_2$. By symmetry $s_1 + m_2 = l_2 + m_1$. ■

Proof of Claim A.4:

- $p_1 + m_2 = p_2 + m_1 = \max(n_{11}, n_{22}, n_{12}, n_{21})$

Proof: If $n_{11} \geq n_{21} \geq n_{22}$, then $n_{11} \geq n_{12}$ (otherwise contradicts the assumption $n_{11} + n_{22} = n_{12} + n_{21}$) and

$$p_1 + m_2 = n_{11} - n_{21} + n_{21} = n_{11} = \max_{i,j \in \{1,2\}} \{n_{ij}\}.$$

If $n_{21} \geq n_{22}$ and $n_{21} \geq n_{11}$, then $n_{21} \geq n_{12}$ (contradiction otherwise) and

$$p_1 + m_2 = 0 + n_{21} = n_{21} = \max_{i,j \in \{1,2\}} \{n_{ij}\}.$$

If $n_{21} \leq n_{22}$ and $n_{21} \leq n_{11}$, then $n_{12} \geq n_{11}$ and $n_{12} \geq n_{22}$ (contradiction otherwise) and

$$p_1 + m_2 = n_{11} - n_{21} + n_{22} = n_{12} = \max_{i,j \in \{1,2\}} \{n_{ij}\}.$$

If $n_{11} \leq n_{21} \leq n_{22}$, then $n_{22} \geq n_{12}$ (contradiction otherwise) and

$$p_1 + m_2 = 0 + n_{22} = n_{22} = \max_{i,j \in \{1,2\}} \{n_{ij}\}.$$

Hence, $p_1 + m_2 = \max_{i,j \in \{1,2\}} \{n_{ij}\}$. Similarly, $p_2 + m_1 = \max_{i,j \in \{1,2\}} \{n_{ij}\}$. ■

- $p_1 + t_2 = p_2 + n_{11}$; $p_2 + t_1 = p_1 + n_{22}$

Proof: Note that $t_2 = \max \{ n_{21}, (n_{22} - n_{12})^+ \} = n_{21}$, since $n_{21} \geq 0$ and

$$n_{21} = n_{11} + n_{22} - n_{12} \geq n_{22} - n_{12}.$$

Hence

$$p_1 + t_2 = (n_{11} - n_{21})^+ + n_{21} = \max(n_{11}, n_{21}).$$

On the other hand

$$\begin{aligned} p_2 + n_{11} &= (n_{22} - n_{12})^+ + n_{11} = (n_{21} - n_{11})^+ + n_{11} \\ &= \max(n_{11}, n_{21}). \end{aligned}$$

Hence, $p_1 + t_2 = p_2 + n_{11}$. Similarly, $p_2 + t_1 = p_1 + n_{22}$. ■

APPENDIX B

PROOF OF THEOREM 4.2: ACHIEVABLE RATE REGION

Plug in Theorem 3.3 and evaluate, we obtain the following achievable rates.

Constraints at Transmitters:

$$\begin{aligned} R_{1h} &\leq \tilde{R}_{1h} \\ R_{2h} &\leq \tilde{R}_{2h} \\ R_{1o} + R_{2o} &= R_o \\ R_{1o} + R_{1h} &\leq C_{12}^B \\ R_{2o} + R_{2h} &\leq C_{21}^B \\ \tilde{R}_{1h} + \tilde{R}_{2h} - R_{1h} - R_{2h} &\geq 0 \end{aligned}$$

for some nonnegative $(\tilde{R}_{1h}, \tilde{R}_{2h})$.

Constraints at Receiver 1: See the equations at the bottom of the next page.

Constraints at Receiver 2: Above with index 1 and 2 exchanged.

Notice that $\text{SNR}_{ic} + \text{SNR}_{ip} = \text{SNR}_i/4$, $\text{INR}_{ic} + \text{INR}_{ip} = \text{INR}_i/4$ and $K_{u_i} \geq \text{SNR}_i/4 + \text{INR}_i/4$ for $i = 1, 2$. For simplicity, we consider the subset of the above region:

Constraints at Transmitters: The same as above.

Constraints at Receiver 1: Exactly the same expressions as those in Appendix A-A, but the notations are defined as

$$\begin{aligned} p_1 &:= \log \left(1 + \frac{\text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ g_1 &:= \log \left(1 + \frac{K_{u_1|z_o}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ t_1 &:= \log \left(1 + \frac{\text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ n_{11} &:= \log \left(1 + \frac{\frac{\text{SNR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ s_1 &:= \log \left(1 + \frac{K_{u_1|z_o} + \text{INR}_{1c}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ l_1 &:= \log \left(1 + \frac{K_{u_1|z_o} + \frac{\text{SNR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ m_1 &:= \log \left(1 + \frac{\frac{\text{SNR}_1}{4} + \text{INR}_{1c}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \end{aligned}$$

instead of those in Table III.

Constraints at Receiver 2: Above with index 1 and 2 exchanged.

Notice now the rate region is symbolically identical to that in LDC when the system matrix is full rank. Hence, after the Fourier-Motzkin procedure, we have achievable rates which are also symbolically identical to those in LDC. The only difference is that, “redundancy” is replaced by “approximate redundancy.” Proof of the claims to show the approximate redundancy will be given later in this section.

(1) R_1 and R_2 :

R_1 constraints:

$$\begin{aligned} R_1 &\leq n_{11} + C_{12}^B \\ R_1 &\leq m_1 \\ R_1 &\leq p_1 + t_2 + C_{12}^B \quad (\text{approx. Red}) \\ R_1 &\leq p_1 + s_2 + C_{12}^B \quad (\text{approx. Red}) \\ R_1 &\leq g_1 + t_2 + C_{12}^B \quad (\text{approx. Red}) \\ R_1 &\leq g_1 + s_2 + C_{12}^B \quad (\text{approx. Red}) \end{aligned}$$

R_2 constraints: Above with index 1 and 2 exchanged.

To show the approximate redundancy, we need to prove the following claim.

Claim B.1:

- $p_1 + t_2 \geq n_{11} - \log(9/4)$, $p_2 + t_1 \geq n_{22} - \log(9/4)$
- $p_1 + s_2 \geq n_{11} - \log(9/4)$, $p_2 + s_1 \geq n_{22} - \log(9/4)$
- $g_1 + t_2 \geq n_{11} - \log 9$, $g_2 + t_1 \geq n_{22} - \log 9$
- $g_1 + s_2 \geq n_{11} - \log 9$, $g_2 + s_1 \geq n_{22} - \log 9$

(1) $R_1 + R_2$:

$$\begin{aligned} R_1 + R_2 &\leq t_1 + t_2 + C_{12}^B + C_{21}^B \\ R_1 + R_2 &\leq p_1 + m_2 + C_{12}^B \\ R_1 + R_2 &\leq p_2 + m_1 + C_{21}^B \\ R_1 + R_2 &\leq g_1 + m_2 \\ R_1 + R_2 &\leq g_2 + m_1 \\ R_1 + R_2 &\leq s_1 + t_2 + C_{12}^B + C_{21}^B \quad (\text{approx. Red}) \\ R_1 + R_2 &\leq s_2 + t_1 + C_{12}^B + C_{21}^B \quad (\text{approx. Red}) \\ R_1 + R_2 &\leq s_1 + s_2 + C_{12}^B + C_{21}^B \quad (\text{approx. Red}). \end{aligned}$$

$$\begin{aligned} R_{1p} &\leq \log \left(1 + \frac{\text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ \tilde{R}_{1h} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{1c} + R_{1p} &\leq \log \left(1 + \frac{\text{SNR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o} + \text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_o + \tilde{R}_{1h} &\leq \log \left(1 + \frac{K_{u_1}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_o + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{2c} + R_{1p} &\leq \log \left(1 + \frac{\text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{2c} + \tilde{R}_{1h} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o} + \text{INR}_{1c}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o} + \text{SNR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{1c} + R_{2c} + R_{1p} &\leq \log \left(1 + \frac{\text{SNR}_{1c} + \text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1|\underline{x}_o} + \text{SNR}_{1c} + \text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_o + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1} + \text{SNR}_{1c} + \text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_o + R_{1c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1} + \text{SNR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ R_o + R_{1c} + R_{2c} + \tilde{R}_{1h} + R_{1p} &\leq \log \left(1 + \frac{K_{u_1} + \text{SNR}_{1c} + \text{INR}_{1c} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right). \end{aligned}$$

To show the approximate redundancy, we need to prove the following claim.

Claim B.2: $s_1 \geq t_1 - \log 5, s_2 \geq t_2 - \log 5$

(1) $2R_1 + R_2$ and $R_1 + 2R_2$:

$$\begin{aligned} 2R_1 + R_2 &\leq p_1 + m_1 + t_2 + C_{12}^B + C_{21}^B \\ 2R_1 + R_2 &\leq g_1 + m_1 + t_2 + C_{12}^B + C_{21}^B \quad (\text{approx. Red}) \\ 2R_1 + R_2 &\leq p_1 + m_1 + s_2 + C_{12}^B \\ 2R_1 + R_2 &\leq g_1 + m_1 + s_2 + C_{12}^B \quad (\text{approx. Red}) \\ 2R_1 + R_2 &\leq p_1 + l_1 + m_2 + C_{12}^B \\ R_1 + 2R_2 &\leq p_2 + m_2 + t_1 + C_{12}^B + C_{21}^B \\ R_1 + 2R_2 &\leq g_2 + m_2 + t_1 + C_{12}^B + C_{21}^B \quad (\text{approx. Red}) \\ R_1 + 2R_2 &\leq p_2 + m_2 + s_1 + C_{21}^B \\ R_1 + 2R_2 &\leq g_2 + m_2 + s_1 + C_{21}^B \quad (\text{approx. Red}) \\ R_1 + 2R_2 &\leq p_2 + l_2 + m_1 + C_{21}^B. \end{aligned}$$

To prove the approximate redundancy, we need to show the following claim.

Claim B.3: $g_1 \geq p_1 - \log 5, g_2 \geq p_2 - \log 5$

(1) $2R_1 + 2R_2$:

$$\begin{aligned} 2R_1 + 2R_2 &\leq p_1 + s_1 + t_2 + m_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq l_1 + t_1 + p_2 + m_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq p_1 + s_1 + s_2 + m_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq l_1 + t_1 + g_2 + m_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq t_1 + m_1 + p_2 + s_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq s_1 + m_1 + p_2 + s_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq p_1 + m_1 + l_2 + t_2 + C_{12}^B + C_{21}^B \\ 2R_1 + 2R_2 &\leq g_1 + m_1 + l_2 + t_2 + C_{12}^B + C_{21}^B. \end{aligned}$$

All the above are approximately redundant.

To show the approximate redundancy, we need to prove the following claim.

Claim B.4:

- $s_1 + t_2 \geq p_2 + m_1 - \log 18, s_2 + t_1 \geq p_1 + m_2 - \log 18$
- $l_1 + t_1 \geq p_1 + m_1 - \log 12, l_2 + t_2 \geq p_2 + m_2 - \log 12$

We summarize in the lemma below an achievable rate region:

Lemma B.5: If (R_1, R_2) satisfies the following, it is achievable.

$$\begin{aligned} R_1 &\leq \min \{m_1, n_{11} + C_{12}^B - 2 \log 3\} \\ R_2 &\leq \min \{m_2, n_{22} + C_{21}^B - 2 \log 3\} \\ R_1 + R_2 &\leq t_1 + t_2 + C_{12}^B + C_{21}^B - 2 \log 5 \\ R_1 + R_2 &\leq \min \{p_1 + m_2 + C_{12}^B, p_2 + m_1 + C_{21}^B\} \\ &\quad - \frac{(\log 90)}{2} \\ R_1 + R_2 &\leq \min \{g_1 + m_2, g_2 + m_1\} \\ 2R_1 + R_2 &\leq p_1 + m_1 + t_2 + C_{12}^B + C_{21}^B - \log 5 \\ 2R_1 + R_2 &\leq \min \left\{ \begin{array}{l} p_1 + m_1 + s_2 + C_{12}^B - \log 5 \\ p_1 + l_1 + m_2 + C_{12}^B \end{array} \right\} \\ R_1 + 2R_2 &\leq p_2 + m_2 + t_1 + C_{12}^B + C_{21}^B - \log 5 \\ R_1 + 2R_2 &\leq \min \left\{ \begin{array}{l} p_2 + m_2 + s_1 + C_{21}^B - \log 5 \\ p_2 + l_2 + m_1 + C_{21}^B \end{array} \right\}. \end{aligned}$$

In Appendix C, we will show that the above achievable rate region is within a bounded gap from the outer bounds in Lemma 4.1. We close this section by the proof of the above mentioned claims.

Proof of the Claims: Prior to the proof of the above claims, we give a bunch of useful lemmas.

Lemma B.6:

$$\begin{aligned} &\log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+|h_{11}h_{22} - h_{12}h_{21}|^2} \right) \\ &\geq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + \log (1 + \text{SNR}_2 + \text{INR}_2). \end{aligned}$$

Proof: Consider the Gaussian interference channel without cooperation. We take independent Gaussian input signals. Note that

$$\begin{aligned} &\log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + \log (1 + \text{SNR}_2 + \text{INR}_2) \\ &= I(x_1; y_1, y_2 | x_2) + I(x_2; y_2) \\ &\leq I(x_1; y_1, y_2 | x_2) + I(x_2; y_2, y_1) \\ &= I(x_1, x_2; y_1, y_2) \\ &= \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+|h_{11}h_{22} - h_{12}h_{21}|^2} \right). \end{aligned}$$

■

Corollary B.7:

$$K_{u_1|\underline{x}_o} \geq \frac{\text{SNR}_1}{4(1 + \text{INR}_2)}, \quad K_{u_2|\underline{x}_o} \geq \frac{\text{SNR}_2}{4(1 + \text{INR}_1)}$$

Proof:

$$\begin{aligned} &1 + K_{u_1|\underline{x}_o} \\ &= \frac{3}{4} + \frac{\left\{ \begin{array}{l} 1 + \text{SNR}_2 + \text{INR}_2 \\ +|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_1 + \text{INR}_1 \end{array} \right\}}{4(1 + \text{SNR}_2 + \text{INR}_2)} \\ &\stackrel{(a)}{\geq} \frac{3}{4} + \frac{1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}}{4} = 1 + \frac{\text{SNR}_1}{4(1 + \text{INR}_2)} \end{aligned}$$

where (a) is due to Lemma B.6. Hence $K_{u_1|\underline{x}_o} \geq \frac{\text{SNR}_1}{4(1 + \text{INR}_2)}$. Similarly $K_{u_2|\underline{x}_o} \geq \frac{\text{SNR}_2}{4(1 + \text{INR}_1)}$. ■

Lemma B.8:

$$\begin{aligned} &2|h_{11}h_{22} - h_{12}h_{21}|^2 + 4\text{SNR}_1\text{SNR}_2 \\ &\geq \text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2, \\ &2|h_{11}h_{22} - h_{12}h_{21}|^2 + 4\text{INR}_1\text{INR}_2 \\ &\geq \text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2. \end{aligned}$$

Proof:

$$\begin{aligned} &|h_{11}h_{22} - h_{12}h_{21}|^2 \geq \text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2 \\ &\quad - 2\sqrt{\text{SNR}_1\text{SNR}_2\text{INR}_1\text{INR}_2} \\ &:= x + y - 2\sqrt{xy} \end{aligned}$$

where $x = \text{SNR}_1 \text{SNR}_2$ and $y = \text{INR}_1 \text{INR}_2$. Hence

$$\begin{aligned} & 2|h_{11}h_{22} - h_{12}h_{21}|^2 + 4\text{SNR}_1\text{SNR}_2 \\ & - (\text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2) \\ & \geq 2x + 2y - 4\sqrt{xy} + 3x - y = y(5u^2 - 4u + 1) \geq 0 \end{aligned}$$

where $u := \sqrt{x/y}$.

Similarly, we can prove that $2|h_{11}h_{22} - h_{12}h_{21}|^2 + 4\text{INR}_1\text{INR}_2 \geq \text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2$. ■

Proof of Claim B.1:

- $p_1 + t_2 \geq n_{11} - \log(9/4)$, $p_2 + t_1 \geq n_{22} - \log(9/4)$

Proof:

$$\begin{aligned} p_1 + t_2 &= \log\left(1 + \frac{\text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) + \log\left(1 + \frac{\text{INR}_{2c} + \text{SNR}_{2p}}{1 + \sigma_2^2 + \text{INR}_{2p}}\right) \\ &= \log\left(\frac{(1 + \sigma_1^2 + \text{SNR}_{1p} + \text{INR}_{1p})(1 + \sigma_2^2 + \frac{\text{INR}_{2c}}{4} + \text{SNR}_{2p})}{(1 + \sigma_1^2 + \text{INR}_{1p})(1 + \sigma_2^2 + \text{INR}_{2p})}\right) \\ &\geq \log\left(\frac{1 + \sigma_1^2 + \frac{\text{SNR}_{1c}}{4} + \text{INR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) - \log(1 + \sigma_2^2 + \text{INR}_{2p}) \\ &\geq n_{11} - \log\left(\frac{9}{4}\right). \end{aligned}$$

Similarly, $p_2 + t_1 \geq n_{22} - \log(9/4)$. ■

- $p_1 + s_2 \geq n_{11} - \log(9/4)$, $p_2 + s_1 \geq n_{22} - \log(9/4)$

Proof:

$$\begin{aligned} p_1 + s_2 &= \log\left(1 + \frac{\text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) + \log\left(1 + \frac{K_{u_2|\underline{x}_o} + \text{INR}_{2c}}{1 + \sigma_2^2 + \text{INR}_{2p}}\right) \\ &= \log\left(\frac{(1 + \sigma_1^2 + \text{SNR}_{1p} + \text{INR}_{1p})(1 + \sigma_2^2 + \frac{\text{INR}_{2c}}{4} + K_{u_2|\underline{x}_o})}{(1 + \sigma_1^2 + \text{INR}_{1p})(1 + \sigma_2^2 + \text{INR}_{2p})}\right) \\ &\geq n_{11} - \log\left(\frac{9}{4}\right). \end{aligned}$$

Similarly, $p_2 + s_1 \geq n_{22} - \log(9/4)$. ■

- $g_1 + t_2 \geq n_{11} - \log 9$, $g_2 + t_1 \geq n_{22} - \log 9$

Proof:

$$\begin{aligned} g_1 + t_2 &= \log\left(\frac{(1 + \sigma_1^2 + \text{INR}_{1p} + K_{u_1|\underline{x}_o})(1 + \sigma_2^2 + \frac{\text{INR}_{2c}}{4} + \text{SNR}_{2p})}{(1 + \sigma_1^2 + \text{INR}_{1p})(1 + \sigma_2^2 + \text{INR}_{2p})}\right) \\ &\geq \log\left(\frac{\sigma_1^2 + \text{INR}_{1p} + (1 + K_{u_1|\underline{x}_o})(1 + \frac{\text{INR}_{2c}}{4})}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\quad - \log(1 + \sigma_2^2 + \text{INR}_{2p}) \\ &\stackrel{(a)}{\geq} \log\left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_{1c}}{16}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) - \log(1 + \sigma_2^2 + \text{INR}_{2p}) \\ &\geq n_{11} - \log 9, \end{aligned}$$

where (a) is due to Corollary B.7. Similarly, $g_2 + t_1 \geq n_{22} - \log 9$. ■

- $g_1 + s_2 \geq n_{11} - \log 9$

Proof:

$$\begin{aligned} g_1 + s_2 &= \log\left(\frac{(1 + \sigma_1^2 + \text{INR}_{1p} + K_{u_1|\underline{x}_o})(1 + \sigma_2^2 + \frac{\text{INR}_{2c}}{4} + K_{u_2|\underline{x}_o})}{(1 + \sigma_1^2 + \text{INR}_{1p})(1 + \sigma_2^2 + \text{INR}_{2p})}\right) \\ &\geq \log\left(\frac{\sigma_1^2 + \text{INR}_{1p} + (1 + K_{u_1|\underline{x}_o})(1 + \frac{\text{INR}_{2c}}{4})}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\quad - \log(1 + \sigma_2^2 + \text{INR}_{2p}) \\ &\geq \log\left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_{1c}}{16}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) - \log(1 + \sigma_2^2 + \text{INR}_{2p}) \\ &\geq n_{11} - \log 9. \end{aligned}$$

Similarly, $g_2 + s_1 \geq n_{22} - \log 9$. ■

Remark B.9: If we want to follow the proofs in LDC closely, we can also prove the approximate redundancy by making use of the fact (to be proved later)

$$\begin{aligned} s_2 &\geq t_2 - \log 5, \quad s_1 \geq t_1 - \log 5, \\ g_1 &\geq p_1 - \log 5, \quad g_2 \geq p_2 - \log 5 \end{aligned}$$

which results in a looser upper bound on the gap to outer bounds.

Proof of Claim B.2:

Proof:

$$\begin{aligned} s_1 &= \log\left(1 + \frac{K_{u_1|\underline{x}_o} + \text{INR}_{1c}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &= \log\left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_{1c}}{4} + K_{u_1|\underline{x}_o}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\stackrel{(a)}{\geq} \log\left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_{1c}}{4} + \frac{\text{SNR}_{1c}}{4(1 + \text{INR}_{2c})}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\stackrel{(b)}{\geq} \log\left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_{1c}}{4} + \frac{\text{SNR}_{1p}}{5}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\geq t_1 - \log 5 \end{aligned}$$

where (a) is due to Corollary B.7 and (b) is due to the fact that $\frac{\text{SNR}_{1c}}{1 + \text{INR}_{2c}} \geq \frac{4\text{SNR}_{1p}}{5}$.

Similarly $s_2 \geq t_2 - \log 5$. ■

Proof of Claim B.3:

Proof:

$$\begin{aligned} g_1 &= \log\left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + K_{u_1|\underline{x}_o}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\stackrel{(a)}{\geq} \log\left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_{1c}}{4(1 + \text{INR}_{2c})}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\stackrel{(b)}{\geq} \log\left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_{1p}}{5}}{1 + \sigma_1^2 + \text{INR}_{1p}}\right) \\ &\geq p_1 - \log 5 \end{aligned}$$

where (a) is due to Corollary B.7 and (b) is due to the fact that $\frac{\text{SNR}_{1c}}{1 + \text{INR}_{2c}} \geq \frac{4\text{SNR}_{1p}}{5}$.

Similarly $g_2 \geq p_2 - \log 5$. ■

Proof of Claim B.4:

- $s_1 + t_2 \geq p_2 + m_1 - \log 18$, $s_2 + t_1 \geq p_1 + m_2 - \log 18$

Proof:

$$\begin{aligned} s_1 &= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_1}{4} + K_{u_1|\underline{x}_o}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ t_2 &= \log \left(\frac{1 + \sigma_2^2 + \frac{\text{INR}_2}{4} + \text{SNR}_{2p}}{1 + \sigma_2^2 + \text{INR}_{2p}} \right) \\ p_2 &= \log \left(\frac{1 + \sigma_2^2 + \text{INR}_{2p} + \text{SNR}_{2p}}{1 + \sigma_2^2 + \text{INR}_{2p}} \right) \\ m_1 &= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} + \frac{\text{INR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right). \end{aligned}$$

Hence, it suffice to compare $L = \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4} + K_{u_1|\underline{x}_o}\right) \left(1 + \sigma_2^2 + \frac{\text{INR}_2}{4} + \text{SNR}_{2p}\right)$ and $R = \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} + \frac{\text{INR}_1}{4}\right) \left(1 + \sigma_2^2 + \text{INR}_{2p} + \text{SNR}_{2p}\right)$.

Note that from Lemma B.8, if $\text{SNR}_2 \geq \text{INR}_2$,

$$\begin{aligned} &\frac{\text{INR}_1}{4} + K_{u_1|\underline{x}_o} \\ &= \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{INR}_1\text{INR}_2 + \text{SNR}_2\text{INR}_1 + \text{SNR}_1 + 2\text{INR}_1}{4(1 + \text{SNR}_2 + \text{INR}_2)} \\ &\geq \frac{\text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2 + 4\text{SNR}_2\text{INR}_1 + 4\text{SNR}_1 + 8\text{INR}_1}{16(1 + \text{SNR}_2 + \text{INR}_2)} \\ &\geq \frac{\text{SNR}_1 \max(\text{SNR}_2, 1) + 4\text{INR}_1 \max(\text{SNR}_2, 1)}{48 \max(\text{SNR}_2, 1)} \\ &\geq \frac{\text{SNR}_1 + \text{INR}_1}{48}. \end{aligned}$$

Also, $\text{INR}_2/4 \geq \text{INR}_{2p}$. Hence, $s_1 + t_2 \geq p_2 + m_1 - \log 12$.

If $\text{SNR}_2 < \text{INR}_2$,

$$\begin{aligned} R &= \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + \left(\frac{\text{SNR}_1}{4}\right) (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + \text{INR}_{2p} \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) + \left(\frac{\text{SNR}_1}{4}\right) \text{INR}_{2p} \\ &\leq 2 \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + \left(\frac{\text{SNR}_1}{4}\right) \left(\frac{5}{4} + \text{SNR}_{2p}\right) + \frac{\text{SNR}_1}{4} \\ &= 2 \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + \frac{9}{16}\text{SNR}_1 + \frac{\text{SNR}_1\text{SNR}_{2p}}{4} \end{aligned}$$

and

$$\begin{aligned} L &= \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + K_{u_1|\underline{x}_o} (1 + \sigma_2^2 + \text{SNR}_{2p}) \\ &\quad + \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4}\right) \left(\frac{\text{INR}_2}{4}\right) + K_{u_1|\underline{x}_o} \left(\frac{\text{INR}_2}{4}\right) \\ &\geq \frac{(1 + \sigma_1^2 + \frac{\text{INR}_1}{4})(1 + \sigma_2^2 + \text{SNR}_{2p})}{2} + \frac{\text{INR}_1}{8} \\ &\quad + \frac{K_{u_1|\underline{x}_o} \max(\text{INR}_2, 1)}{4} + \frac{\text{INR}_1\text{INR}_2}{16} + \frac{K_{u_1|\underline{x}_o}}{2} \end{aligned}$$

$$\begin{aligned} &\geq \frac{(1 + \sigma_1^2 + \frac{\text{INR}_1}{4})(1 + \sigma_2^2 + \text{SNR}_{2p})}{2} + \frac{\text{SNR}_1}{96} \\ &\quad + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + 2\text{INR}_1\text{INR}_2 + \text{SNR}_1 + \text{INR}_1}{48} \\ &\geq \frac{(1 + \sigma_1^2 + \frac{\text{INR}_1}{4})(1 + \sigma_2^2 + \text{SNR}_{2p})}{2} \\ &\quad + \frac{\text{SNR}_1\text{SNR}_2}{96} + \frac{\text{SNR}_1}{32}. \end{aligned}$$

Hence, $s_1 + t_2 \geq p_2 + m_1 - \log 18$.

In summary, $s_1 + t_2 \geq p_2 + m_1 - \log 18$ and similarly, $s_2 + t_1 \geq p_1 + m_2 - \log 18$. ■

• $l_1 + t_1 \geq p_1 + m_1 - \log 12, l_2 + t_2 \geq p_2 + m_2 - \log 12$

Proof:

$$\begin{aligned} l_1 &= \log \left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_1}{4} + K_{u_1|\underline{x}_o}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ t_1 &= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_1}{4} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ p_1 &= \log \left(\frac{1 + \sigma_1^2 + \text{INR}_{1p} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\ m_1 &= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} + \frac{\text{INR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right). \end{aligned}$$

Hence, it suffices to compare

$$\begin{aligned} L &= \left(1 + \sigma_1^2 + \text{INR}_{1p} + \frac{\text{SNR}_1}{4} + K_{u_1|\underline{x}_o}\right) \\ &\quad \cdot \left(1 + \sigma_1^2 + \frac{\text{INR}_1}{4} + \text{SNR}_{1p}\right) \end{aligned}$$

and

$$\begin{aligned} R &= \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} + \frac{\text{INR}_1}{4}\right) \\ &\quad \cdot (1 + \sigma_1^2 + \text{INR}_{1p} + \text{SNR}_{1p}). \end{aligned}$$

Note that from Lemma B.8, if $\text{SNR}_2 \leq \text{INR}_2$,

$$\begin{aligned} &\frac{\text{SNR}_1}{4} + K_{u_1|\underline{x}_o} \\ &= \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_1\text{SNR}_2 + \text{SNR}_1\text{INR}_2 + 2\text{SNR}_1 + \text{INR}_1}{4(1 + \text{SNR}_2 + \text{INR}_2)} \\ &\geq \frac{\text{SNR}_1\text{SNR}_2 + \text{INR}_1\text{INR}_2 + 4\text{SNR}_1\text{INR}_2 + 8\text{SNR}_1 + 4\text{INR}_1}{16(1 + \text{SNR}_2 + \text{INR}_2)} \\ &\geq \frac{\text{INR}_1 \max(\text{INR}_2, 1) + 4\text{SNR}_1 \max(\text{INR}_2, 1)}{48 \max(\text{INR}_2, 1)} \\ &\geq \frac{\text{INR}_1 + \text{SNR}_1}{48}. \end{aligned}$$

Also, $\text{INR}_1/4 \geq \text{INR}_{1p}$. Hence, $l_1 + t_1 \geq p_1 + m_1 - \log 12$.

If $\text{SNR}_2 > \text{INR}_2$,

$$\begin{aligned} R &= \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4}\right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\ &\quad + \left(\frac{\text{INR}_1}{4}\right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\ &\quad + \text{INR}_{1p} \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4}\right) + \left(\frac{\text{INR}_1}{4}\right) \text{INR}_{1p} \\ &\leq 2 \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4}\right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{\text{INR}_1}{4} \right) \left(\frac{5}{4} + \text{SNR}_{1p} \right) + \frac{\text{INR}_1}{4} \\
= & 2 \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
& + \frac{9}{16} \text{INR}_1 + \frac{\text{INR}_1 \text{SNR}_{1p}}{4},
\end{aligned}$$

and

$$\begin{aligned}
L = & \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
& + \left(\text{INR}_{1p} + K_{u_1|\underline{z}_o} \right) \left(\frac{\text{INR}_1}{4} \right) \\
& + \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) \left(\frac{\text{INR}_1}{4} \right) \\
& + \left(\text{INR}_{1p} + K_{u_1|\underline{z}_o} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
\geq & \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
& + \frac{\text{INR}_1}{4} \left(1 + K_{u_1|\underline{z}_o} + \frac{\text{SNR}_1}{4} \right) \\
\geq & \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
& + \frac{\text{INR}_1}{4} \left(1 + \frac{\text{SNR}_1}{48} \right) \\
\geq & \left(1 + \sigma_1^2 + \frac{\text{SNR}_1}{4} \right) (1 + \sigma_1^2 + \text{SNR}_{1p}) \\
& + \frac{\text{INR}_1}{4} \left(1 + \frac{\text{SNR}_{1p}}{12} \right).
\end{aligned}$$

Hence, $l_1 + t_1 \geq p_1 + m_1 - \log 12$.

In summary, $l_1 + t_1 \geq p_1 + m_1 - \log 12$ and similarly, $l_2 + t_2 \geq p_2 + m_2 - \log 12$. ■

APPENDIX C

PROOF OF THEOREM 4.2: CONSTANT GAP TO OUTER BOUNDS

(1) Bounds on R_1 :

- Consider the outer bound

$$R_1 \leq \log \left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1} \right)$$

and the inner bound

$$\begin{aligned}
R_1 \leq m_1 &= \log \left(1 + \frac{\text{SNR}_1 + \text{INR}_{1c}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\
&= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{SNR}_1 + \text{INR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right).
\end{aligned}$$

Note that

$$\begin{aligned}
& \log \left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1} \right) \\
& \leq \log (1 + \text{SNR}_1 + \text{INR}_1) + 1, \\
& \log \left(\frac{1 + \sigma_1^2 + \frac{\text{SNR}_1 + \text{INR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\
& \geq \log (1 + \text{SNR}_1 + \text{INR}_1) - \log 9.
\end{aligned}$$

Hence the gap is at most $\log 9 + 1 = 2 \log 3 + 1$.

- Consider the outer bound

$$R_1 \leq \log (1 + \text{SNR}_1) + C_{12}^B$$

and the inner bound

$$\begin{aligned}
R_1 &\leq n_{11} + C_{12}^B - 2 \log 3 \\
&= \log \left(1 + \frac{\frac{\text{SNR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) + C_{12}^B - 2 \log 3.
\end{aligned}$$

Note that

$$\log \left(1 + \frac{\frac{\text{SNR}_1}{4}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \geq \log (1 + \text{SNR}_1) - \log 9.$$

Hence, the gap is at most $2 \log 3 + \log 9 = 4 \log 3 \approx 6.34$

In summary, the gap is at most $4 \log 3 \approx 6.34$.

- (2) R_2 : similar to R_1 , the gap is at most $4 \log 3 \approx 6.34$.
(3) $R_1 + R_2$:

- Consider the outer bound (11) and the inner bound

$$R_1 + R_2 \leq p_1 + m_2 + C_{12}^B - \frac{(\log 90)}{2}.$$

Note that

$$\begin{aligned}
& \log \left(1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_2 \text{INR}_2} \right) \\
& \leq \log (1 + \text{SNR}_2 + \text{INR}_2) + 1, \\
m_2 &\geq \log (1 + \text{SNR}_2 + \text{INR}_2) - \log 9, \\
p_1 &= \log \left(1 + \frac{\text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\
&\geq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) - \log \left(\frac{9}{4} \right).
\end{aligned}$$

Hence the gap is at most $5 \log 3 - 1 + (\log 10)/2 \approx 8.586$.

- Consider the outer bound (13) and the inner bound

$$R_1 + R_2 \leq t_1 + t_2 + C_{12}^B + C_{21}^B - (2 \log 5).$$

Note that

$$\begin{aligned}
& \log \left(1 + \frac{\text{SNR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1}}{1 + \text{INR}_2} + \text{INR}_1 \right) \\
& \leq \log \left(1 + \frac{2\text{SNR}_1 + \text{INR}_1}{1 + \text{INR}_2} + \text{INR}_1 \right) \\
& \leq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} + \text{INR}_1 \right) + 1,
\end{aligned}$$

and

$$\begin{aligned}
t_1 &= \log \left(\frac{1 + \sigma_1^2 + \frac{\text{INR}_1}{4} + \text{SNR}_{1p}}{1 + \sigma_1^2 + \text{INR}_{1p}} \right) \\
&\geq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} + \text{INR}_1 \right) - \log 9.
\end{aligned}$$

Hence, the gap is at most $4 \log 3 + 2 + 2 \log 5 \approx 12.984$.

- Consider the outer bound (14) and the inner bound

$$R_1 + R_2 \leq \min \{g_1 + m_2, g_2 + m_1\}.$$

Note that

$$\begin{aligned} & \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + 2\sqrt{\text{SNR}_1 \text{INR}_1}}{+2\sqrt{\text{SNR}_2 \text{INR}_2} + |h_{11}h_{22} - h_{12}h_{21}|^2} \right) \\ & \leq \log (1 + 2\text{SNR}_1 + 2\text{INR}_1 + 2\text{SNR}_2 + 2\text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2) \\ & \leq \log (1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2) + 1, \end{aligned}$$

and

$$\begin{aligned} g_1 + m_2 & = \log \left(\frac{(1 + \sigma_1^2 + \text{INR}_{1p} + K_{u_1|z_o})(1 + \sigma_2^2 + \frac{\text{SNR}_2 + \text{INR}_2}{4})}{(1 + \sigma_1^2 + \text{INR}_{1p})(1 + \sigma_2^2 + \text{INR}_{2p})} \right) \\ & \geq \log \left(\left(1 + K_{u_1|z_o}\right) (1 + \text{SNR}_2 + \text{INR}_2) \right) - \log \left(\frac{81}{4} \right) \\ & = \log (1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2) - \log 81. \end{aligned}$$

Hence, the gap is at most $4 \log 3 + 1 \approx 7.34$.

In summary, the gap is at most $4 \log 3 + 2 + 2 \log 5 \approx 12.984$.

(4) $2R_1 + R_2$:

- Consider the outer bound (15) and the inner bound

$$2R_1 + R_2 \leq p_1 + m_1 + t_2 + C_{12}^B + C_{21}^B - (\log 5).$$

From previous arguments, one can directly see that the gap is at most

$$\begin{aligned} & \log \left(\frac{9}{4} \right) + (2 \log 3 + 1) + (2 \log 3 + 1) + \log 5 \\ & = 6 \log 3 + \log 5 \approx 11.832. \end{aligned}$$

- Consider the outer bound (17) and the inner bounds

$$2R_1 + R_2 \leq p_1 + m_1 + s_2 + C_{12}^B - (\log 5)$$

$$2R_1 + R_2 \leq p_1 + l_1 + m_2 + C_{12}^B.$$

Note that

$$\begin{aligned} & \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + \text{SNR}_1 \text{SNR}_2}{+ \text{INR}_1 \text{INR}_2 + \text{SNR}_1 \text{INR}_2 + 2(1 + \text{INR}_2)\sqrt{\text{SNR}_1 \text{INR}_1}} \right) \\ & \leq \log \left(\frac{1 + 2\text{SNR}_1 + 2\text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + \text{SNR}_1 \text{SNR}_2}{+ 2\text{INR}_1 \text{INR}_2 + 2\text{SNR}_1 \text{INR}_2} \right). \end{aligned}$$

For the inner bounds,

$$\begin{aligned} m_1 + s_2 & \geq \log \left((1 + \text{SNR}_1 + \text{INR}_1) \left(1 + \frac{\text{INR}_2}{4} + K_{u_2|z_o} \right) \right) \\ & \quad - (4 \log 3 - 2) \\ & = \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \frac{\text{SNR}_2}{4} + \frac{\text{INR}_2}{2}}{+ \frac{\text{SNR}_1 \text{INR}_2}{4} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{INR}_1 \text{INR}_2}{4}} \right) \\ & \quad - (4 \log 3 - 2) \end{aligned}$$

$$\begin{aligned} & \geq \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \frac{\text{SNR}_2}{4} + \frac{\text{INR}_2}{2}}{+ \frac{\text{SNR}_1 \text{INR}_2}{4} + \frac{\text{SNR}_1 \text{SNR}_2 + \text{INR}_1 \text{INR}_2}{16}} \right) \\ & \quad - (4 \log 3 - 2), \end{aligned}$$

and

$$\begin{aligned} l_1 + m_2 & \geq \log \left((1 + \text{SNR}_2 + \text{INR}_2) \left(1 + \frac{\text{SNR}_1}{4} + K_{u_1|z_o} \right) \right) \\ & \quad - (4 \log 3 - 2) \\ & = \log \left(\frac{1 + \text{SNR}_2 + \text{INR}_2 + \frac{\text{SNR}_1}{2} + \frac{\text{INR}_1}{4}}{+ \frac{\text{SNR}_1 \text{INR}_2}{4} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2 + \text{SNR}_1 \text{SNR}_2}{4}} \right) \\ & \quad - (4 \log 3 - 2) \\ & \geq \log \left(\frac{1 + \text{SNR}_2 + \text{INR}_2 + \frac{\text{SNR}_1}{2} + \frac{\text{INR}_1}{4}}{+ \frac{\text{SNR}_1 \text{INR}_2}{4} + \frac{\text{SNR}_1 \text{SNR}_2 + \text{INR}_1 \text{INR}_2}{16}} \right) \\ & \quad - (4 \log 3 - 2). \end{aligned}$$

Hence, the gap is at most $(4 \log 3 + 3) + \log(9/4) + \log 5 = 6 \log 3 + \log 5 + 1 \approx 12.832$.

In summary, the gap is at most $6 \log 3 + \log 5 + 1 \approx 12.832$.

(5) $R_1 + 2R_2$: similar to $2R_1 + R_2$, the gap is at most $6 \log 3 + \log 5 + 1 \approx 12.832$.

Combining the results, we characterize the capacity region to within a constant, which is

$$\begin{aligned} & \max \left\{ 4 \log 3, \frac{4 \log 3 + 2 + 2 \log 5}{2}, \frac{6 \log 3 + \log 5 + 1}{3} \right\} \\ & = 2 \log 3 + 1 + \log 5 = \log 90 \approx 6.5. \end{aligned}$$

APPENDIX D

PROOF OF LEMMA 4.1

We first state a useful fact [9]:

Fact D.1 (Conditional Independence Among Messages): The following Markov relations hold:

$$\begin{aligned} m_1 - (v_{12}^N, v_{21}^N) - m_2; \quad m_1 - (v_{12}^N, v_{21}^N) - x_2^N; \\ m_2 - (v_{12}^N, v_{21}^N) - x_1^N. \end{aligned}$$

The proof can be found in [9].

Below we start the proof of the outer bounds stated in Lemma 4.1.

Proof:

(1) R_1 bound (9): If R_1 is achievable, by Fano's inequality,

$$\begin{aligned} N(R_1 - \epsilon_N) & \leq I(m_1; y_1^N) \leq I(m_1; y_1^N | m_2) \\ & \leq I(m_1; y_1^N | m_2, v_{12}^N) + I(m_1; v_{12}^N | m_2) \\ & = h(y_1^N | m_2, v_{12}^N) - h(y_1^N | m_2, v_{12}^N, m_1) \\ & \quad + H(v_{12}^N | m_2) - H(v_{12}^N | m_2, m_1) \\ & \stackrel{(a)}{=} h(h_{11}x_1^N + z_1^N | m_2, v_{12}^N) \\ & \quad - h(z_1^N | m_2, v_{12}^N, m_1) + H(v_{12}^N | m_2) \end{aligned}$$

$$\begin{aligned}
&\stackrel{(b)}{\leq} h(h_{11}x_1^N + z_1^N) - h(z_1^N) + H(v_{12}^N) \\
&\leq N \log(1 + |h_{11}|^2) + NC_{12}^B \\
&= N \{\log(1 + \text{SNR}_1) + C_{12}^B\}
\end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to the fact that x_2^N is a function of (m_2, v_{12}^N) and (x_1^N, x_2^N, v_{12}^N) are all functions of (m_1, m_2) . (b) is due to conditioning reduces entropy and the fact that z_1^N is independent of everything else.

On the other hand, if R_1 is achievable

$$\begin{aligned}
N(R_1 - \epsilon_N) &\leq I(m_1; y_1^N | m_2) = h(y_1^N | m_2) - h(y_1^N | m_2, m_1) \\
&\leq h(y_1^N) - h(z_1^N | m_2, m_1) = h(y_1^N) - h(z_1^N) \\
&\leq \max_{|\rho| \leq 1} \{N \log(1 + |h_{11}|^2 + |h_{12}|^2 + 2\Re\{h_{11}h_{12}^*\rho\})\} \\
&= N \log(1 + |h_{11}|^2 + |h_{12}|^2 + 2|h_{11}||h_{12}|) \\
&= N \log(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1\text{INR}_1})
\end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$.

- (2) R_2 bound (10): They follow the same line as the R_1 bounds.
- (3) $R_1 + R_2$ bound (11) and (12): Let $s_1 := h_{21}x_1 + z_2$ and $s_2 := h_{12}x_2 + z_1$. If (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
N(R_1 + R_2 - \epsilon_N) &\leq I(m_1; y_1^N) + I(m_2; y_2^N) \\
&\leq I(m_1; y_1^N, s_1^N, v_{12}^N | m_2) + I(m_2; v_{12}^N; y_2^N) \\
&= I(m_1; y_1^N, s_1^N | v_{12}^N, m_2) + I(m_1; v_{12}^N | m_2) \\
&\quad + h(y_2^N) - h(y_2^N | m_2, v_{12}^N) \\
&= h(y_1^N, s_1^N | v_{12}^N, m_2) - h(z_1^N, z_2^N) + H(v_{12}^N | m_2) \\
&\quad - h(y_2^N) + h(s_1^N | m_2, v_{12}^N) \\
&= h(y_1^N | s_1^N, v_{12}^N, m_2) + h(y_2^N) - h(z_1^N, z_2^N) \\
&\quad + H(v_{12}^N | m_2) \\
&\leq h(h_{11}x_1^N + z_1^N | h_{21}x_1^N + z_2^N) + h(y_2^N) \\
&\quad - h(z_1^N, z_2^N) + H(v_{12}^N) \\
&\leq N \log\left(1 + \frac{|h_{11}|^2}{1 + |h_{21}|^2}\right) \\
&\quad + N \log(1 + |h_{21}|^2 + |h_{22}|^2 + 2|h_{21}||h_{22}|) + NC_{12}^B \\
&= N \{\text{RHS of (11)}\},
\end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$.

Similarly, we prove the outer bound (12).

- (4) $R_1 + R_2$ bound (13): If (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
N(R_1 + R_2 - \epsilon_N) &\leq I(m_1; y_1^N) + I(m_2; y_2^N) \\
&\leq I(m_1; y_1^N | v_{12}^N, v_{21}^N) + I(m_2; y_2^N | v_{12}^N, v_{21}^N)
\end{aligned}$$

$$\begin{aligned}
&\quad + I(m_1; v_{12}^N, v_{21}^N) + I(m_2; v_{12}^N, v_{21}^N) \\
&\stackrel{(a)}{\leq} I(m_1; y_1^N | v_{12}^N, v_{21}^N) + I(m_2; y_2^N | v_{12}^N, v_{21}^N) \\
&\quad + I(m_1, m_2; v_{12}^N, v_{21}^N) \\
&\leq I(m_1; y_1^N, s_1^N | v_{12}^N, v_{21}^N) + I(m_2; y_2^N, s_2^N | v_{12}^N, v_{21}^N) \\
&\quad + I(m_1, m_2; v_{12}^N, v_{21}^N) \\
&= h(y_1^N, s_1^N | v_{12}^N, v_{21}^N) - h(y_1^N, s_1^N | v_{12}^N, v_{21}^N, m_1) \\
&\quad + h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - h(y_2^N, s_2^N | v_{12}^N, v_{21}^N, m_2) \\
&\quad + H(v_{12}^N, v_{21}^N) \\
&= h(y_1^N, s_1^N | v_{12}^N, v_{21}^N) - h(s_2^N, z_2^N | v_{12}^N, v_{21}^N, m_1) \\
&\quad + h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - h(s_1^N, z_1^N | v_{12}^N, v_{21}^N, m_2) \\
&\quad + H(v_{12}^N, v_{21}^N) \\
&\stackrel{(b)}{=} h(y_1^N, s_1^N | v_{12}^N, v_{21}^N) - h(s_2^N | v_{12}^N, v_{21}^N, m_1) - h(z_2^N) \\
&\quad + h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - h(s_1^N | v_{12}^N, v_{21}^N, m_2) - h(z_1^N) \\
&\quad + H(v_{12}^N, v_{21}^N) \\
&\stackrel{(c)}{=} h(y_1^N, s_1^N | v_{12}^N, v_{21}^N) - h(s_2^N | v_{12}^N, v_{21}^N) - h(z_2^N) \\
&\quad + h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - h(s_1^N | v_{12}^N, v_{21}^N) - h(z_1^N) \\
&\quad + H(v_{12}^N, v_{21}^N) \\
&= h(y_1^N | s_1^N, v_{12}^N, v_{21}^N) + h(y_2^N | s_2^N, v_{12}^N, v_{21}^N) \\
&\quad - h(z_1^N) - h(z_2^N) + H(v_{12}^N, v_{21}^N) \\
&\leq h(y_1^N | s_1^N) + h(y_2^N | s_2^N) - h(z_1^N) - h(z_2^N) + H(v_{12}^N) \\
&\quad + H(v_{21}^N) \\
&\leq N \{\text{RHS of (13)}\}
\end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to the fact that m_1 and m_2 are independent. (b) is due to the fact that z_1^N and z_2^N are independent to everything else, respectively. (c) is due to the following fact regarding the Markov relations:

- (5) $R_1 + R_2$ bound (14): If (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
N(R_1 + R_2 - \epsilon_N) &\leq I(m_1; y_1^N) + I(m_2; y_2^N) \\
&\leq I(m_1; y_1^N, y_2^N) + I(m_2; y_1^N, y_2^N) \\
&\leq I(m_1; y_1^N, y_2^N | m_2) + I(m_2; y_1^N, y_2^N) \\
&= I(m_1, m_2; y_1^N, y_2^N) \\
&= h(y_1^N, y_2^N) - h(z_1^N, z_2^N) \\
&\leq N \{\text{RHS of (14)}\}
\end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$.

- (6) $2R_1 + R_2$ bound (15) and $R_1 + 2R_2$ bound (16): If (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
N(2R_1 + R_2 - \epsilon_N) &\leq 2I(m_1; y_1^N) + I(m_2; y_2^N) \\
&\leq I(m_1; y_1^N) + I(m_1; y_1^N, s_1^N, v_{12}^N, v_{21}^N | m_2) \\
&\quad + I(m_2; y_2^N, s_2^N, v_{12}^N, v_{21}^N) \\
&\leq I(m_1, v_{12}^N, v_{21}^N; y_1^N) + I(m_1; y_1^N, s_1^N | v_{12}^N, v_{21}^N, m_2)
\end{aligned}$$

$$\begin{aligned}
 &+ I(m_2; y_2^N, s_2^N | v_{12}^N, v_{21}^N) + I(m_1; v_{12}^N, v_{21}^N | m_2) \\
 &+ I(m_2; v_{12}^N, v_{21}^N) \\
 = &h(y_1^N) - h(s_2^N | m_1, v_{12}^N, v_{21}^N) + I(m_1, m_2; v_{12}^N, v_{21}^N) \\
 &+ h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N, v_{21}^N) - h(z_1^N, z_2^N) \\
 &+ h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - h(s_1^N, z_1^N | m_2, v_{12}^N, v_{21}^N) \\
 \stackrel{(a)}{=} &h(y_1^N) - h(s_2^N | v_{12}^N, v_{21}^N) + H(v_{12}^N, v_{21}^N) \\
 &+ h(h_{11}x_1^N + z_1^N | s_1^N, m_2, v_{12}^N, v_{21}^N) \\
 &+ h(y_2^N, s_2^N | v_{12}^N, v_{21}^N) - 2h(z_1^N) - h(z_2^N) \\
 = &h(y_1^N) + h(h_{11}x_1^N + z_1^N | s_1^N, m_2, v_{12}^N, v_{21}^N) \\
 &+ H(v_{12}^N, v_{21}^N) + h(y_2^N | s_2^N, v_{12}^N, v_{21}^N) \\
 &- 2h(z_1^N) - h(z_2^N) \\
 \leq &N \{\text{RHS of (15)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to the Markovity in Fact D.1.

Similar arguments work for $R_1 + 2R_2$ bound (16).

- (7) $2R_1 + R_2$ bound (17) and $R_1 + 2R_2$ bound (18): Using the intuition from the study of linear deterministic channel, we give the following side information to receiver 2:

$$\tilde{y}_2^N := h_{22}x_2^N + \tilde{z}_2^N$$

where $\tilde{z}_2 \sim \mathcal{CN}(0, 1 + \text{INR}_2)$, i.i.d. over time and is independent of everything else.

Now, if (R_1, R_2) is achievable, by Fano's inequality

$$\begin{aligned}
 &N(2R_1 + R_2 - \epsilon_N) \\
 &\leq 2I(m_1; y_1^N) + I(m_2; y_2^N) \\
 &\leq I(m_1; y_1^N, s_1^N | m_2, v_{12}^N) + I(m_1; v_{12}^N | m_2) \\
 &\quad + I(m_1; y_1^N) + I(m_2; \tilde{y}_2^N, y_2^N) \\
 = &h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N) - h(z_1^N, z_2^N) \\
 &+ I(m_1; y_1^N) + I(m_2; \tilde{y}_2^N) + I(m_2; y_2^N | \tilde{y}_2^N) \\
 &+ I(m_1; v_{12}^N | m_2) \\
 \stackrel{(a)}{\leq} &h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N) - h(z_1^N, z_2^N) \\
 &+ I(m_1, m_2; y_1^N, \tilde{y}_2^N) + I(m_2, v_{12}^N; y_2^N | \tilde{y}_2^N) \\
 &+ I(m_1; v_{12}^N | m_2) \\
 = &h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N) \\
 &- h(z_1^N, z_2^N) + h(y_1^N, \tilde{y}_2^N) - h(z_1^N, \tilde{z}_2^N) \\
 &+ h(y_2^N | \tilde{y}_2^N) - h(y_2^N | \tilde{y}_2^N, m_2, v_{12}^N) \\
 &+ H(v_{12}^N | m_2) \\
 = &h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N) + h(y_1^N, \tilde{y}_2^N) \\
 &+ h(y_2^N | \tilde{y}_2^N) - h(s_1^N | \tilde{z}_2^N, m_2, v_{12}^N) + H(v_{12}^N | m_2) \\
 &- h(z_1^N, z_2^N) - h(z_1^N, \tilde{z}_2^N) \\
 \stackrel{(b)}{=} &h(h_{11}x_1^N + z_1^N, s_1^N | m_2, v_{12}^N) + h(y_1^N, \tilde{y}_2^N) \\
 &+ h(y_2^N | \tilde{y}_2^N) - h(s_1^N | m_2, v_{12}^N) + H(v_{12}^N | m_2)
 \end{aligned}$$

$$\begin{aligned}
 &- h(z_1^N, z_2^N) - h(z_1^N, \tilde{z}_2^N) \\
 = &h(h_{11}x_1^N + z_1^N | s_1^N, m_2, v_{12}^N) + h(y_1^N, \tilde{y}_2^N) \\
 &+ h(y_2^N | \tilde{y}_2^N) + H(v_{12}^N | m_2) \\
 &- h(z_1^N, z_2^N) - h(z_1^N, \tilde{z}_2^N) \\
 \leq &N \{\text{RHS of (17)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to a simple fact that $I(m_1; y_1^N) + I(m_2; \tilde{y}_2^N) \leq I(m_1, m_2; y_1^N, \tilde{y}_2^N)$ and that conditioning reduces entropy. (b) holds since \tilde{z}_2^N is independent of (m_2, v_{12}^N) and s_1^N .

Similar arguments work for $R_1 + 2R_2$ bound (18). ■

APPENDIX E PROOF OF THEOREM 5.1

In [1], we characterized the capacity region of the Gaussian interference channel with conferencing receivers to within 2 bits per user. Hence by Theorem 4.2, we only need to compare the outer bounds. Note that for the reciprocal channel, its channel parameters are

$$\begin{aligned}
 \text{SNR}'_1 &= \text{SNR}_1, \text{SNR}'_2 = \text{SNR}_2; \\
 \text{INR}'_1 &= \text{INR}_2, \text{INR}'_2 = \text{INR}_1; \\
 \text{C}_{12}^{\text{B}' } &= \text{C}_{21}^{\text{B}}, \text{C}_{21}^{\text{B}' } = \text{C}_{12}^{\text{B}}.
 \end{aligned}$$

Plugging these into the outer bounds in [1], we obtain the outer bounds for the reciprocal channel (see the equations at the bottom of the next page).

- (1) Bounds on R_1 and R_2 : Note that

$$\begin{aligned}
 &\log(1 + \text{SNR}_1 + \text{INR}_1) \\
 &\leq \log\left(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1}\right) \\
 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) + 1.
 \end{aligned}$$

Hence the gap is at most 1 bit.

- (2) Bounds on $R_1 + R_2$: Note that:

(a)

$$\begin{aligned}
 &\log\left(1 + \text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \\
 = &\left\{ \begin{aligned} &\log\left(\frac{(1 + \text{INR}_1)(1 + \text{INR}_2) + \text{SNR}_1}{1 + \text{INR}_1}\right) \\ &+ \log\left(\frac{(1 + \text{INR}_2)(1 + \text{INR}_1) + \text{SNR}_2}{1 + \text{INR}_2}\right) \end{aligned} \right\} \\
 = &\left\{ \begin{aligned} &\log\left(\frac{(1 + \text{INR}_1)(1 + \text{INR}_2) + \text{SNR}_1}{1 + \text{INR}_2}\right) \\ &+ \log\left(\frac{(1 + \text{INR}_2)(1 + \text{INR}_1) + \text{SNR}_2}{1 + \text{INR}_1}\right) \end{aligned} \right\} \\
 = &\log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) \\
 &+ \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) \\
 \leq &\left\{ \begin{aligned} &\log\left(1 + \frac{\text{SNR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1}}{1 + \text{INR}_2} + \text{INR}_1\right) \\ &+ \log\left(1 + \frac{\text{SNR}_2 + 2\sqrt{\text{SNR}_2 \text{INR}_2}}{1 + \text{INR}_1} + \text{INR}_2\right) \end{aligned} \right\}
 \end{aligned}$$

$$\leq \left\{ \begin{array}{l} \log \left(1 + \text{INR}_1 + \frac{2\text{SNR}_1 + \text{INR}_1}{1 + \text{INR}_2} \right) \\ + \log \left(1 + \text{INR}_2 + \frac{2\text{SNR}_2 + \text{INR}_2}{1 + \text{INR}_1} \right) \end{array} \right\}$$

$$\leq \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right)$$

$$+ \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) + 2$$

(b)

$$\log(1 + \text{SNR}_2 + \text{INR}_1) + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right)$$

$$= \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) + \log(1 + \text{SNR}_1 + \text{INR}_1)$$

$$\leq \left\{ \begin{array}{l} \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1}) \end{array} \right\}$$

$$\leq \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) + \log(1 + \text{SNR}_1 + \text{INR}_1) + 1$$

(c)

$$\log \left(\frac{1 + \text{SNR}_1 + \text{SNR}_2 + \text{INR}_1 + \text{INR}_2}{+|h_{11}h_{22} - h_{12}h_{21}|^2} \right)$$

$$\leq \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+2\sqrt{\text{SNR}_1 \text{INR}_1} + 2\sqrt{\text{SNR}_2 \text{INR}_2} + |h_{11}h_{22} - h_{12}h_{21}|^2} \right)$$

$$\leq \log \left(\frac{1 + 2\text{SNR}_1 + 2\text{INR}_1 + 2\text{SNR}_2 + 2\text{INR}_2}{+|h_{11}h_{22} - h_{12}h_{21}|^2} \right)$$

$$\leq \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+|h_{11}h_{22} - h_{12}h_{21}|^2} \right) + 1.$$

Hence the gap is at most 2 bits.

(3) Bounds on $2R_1 + R_2$ and $R_1 + 2R_2$:

Note that:

(a)

$$\left\{ \begin{array}{l} \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right) + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_2) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_1) + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(1 + \frac{\text{SNR}_2 + 2\sqrt{\text{SNR}_2 \text{INR}_2}}{1 + \text{INR}_1} + \text{INR}_2 \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_1 + 2\sqrt{\text{SNR}_1 \text{INR}_1}) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) + 1 + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_1) + 1 \end{array} \right\}$$

(b)

$$\left\{ \begin{array}{l} \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} + \text{INR}_1 + \text{SNR}_1 \right) \\ + \log \left(\frac{\text{INR}_2}{1 + \text{INR}_2} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}_2} \right) \\ + \log(1 + \text{SNR}_1 + \text{INR}_2) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + 2\text{INR}_2}{+ \text{SNR}_1 \text{INR}_2 + \text{INR}_1 \text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2} \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + 2\text{INR}_2}{+ \text{SNR}_1 \text{INR}_2 + \text{INR}_1 \text{INR}_2 + 2(\text{SNR}_1 \text{SNR}_2 + \text{INR}_1 \text{INR}_2)} \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+ \text{SNR}_1 \text{SNR}_2 + \text{INR}_1 \text{INR}_2 + \text{SNR}_1 \text{INR}_2 + 2(1 + \text{INR}_2)\sqrt{\text{SNR}_1 \text{INR}_1}} \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + 1 \end{array} \right\}$$

$$R_1 \leq \min \{ \log(1 + \text{SNR}_1) + C_{12}^B, \log(1 + \text{SNR}_1 + \text{INR}_1) \}$$

$$R_2 \leq \min \{ \log(1 + \text{SNR}_2) + C_{21}^B, \log(1 + \text{SNR}_2 + \text{INR}_2) \}$$

$$R_1 + R_2 \leq \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) + \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right) + C_{21}^B + C_{12}^B$$

$$R_1 + R_2 \leq \log(1 + \text{SNR}_2 + \text{INR}_1) + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) + C_{21}^B$$

$$R_1 + R_2 \leq \log(1 + \text{SNR}_1 + \text{INR}_2) + \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right) + C_{12}^B$$

$$R_1 + R_2 \leq \log(1 + \text{SNR}_1 + \text{SNR}_2 + \text{INR}_1 + \text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2)$$

$$2R_1 + R_2 \leq \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right) + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) + \log(1 + \text{SNR}_1 + \text{INR}_2) + C_{21}^B + C_{12}^B$$

$$R_1 + 2R_2 \leq \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) + \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right) + \log(1 + \text{SNR}_2 + \text{INR}_2) + C_{12}^B + C_{21}^B$$

$$2R_1 + R_2 \leq \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} + \text{INR}_1 + \text{SNR}_1 + \frac{\text{INR}_2}{1 + \text{INR}_2} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}_2} \right) + \log(1 + \text{SNR}_1 + \text{INR}_2) + C_{12}^B$$

$$R_1 + 2R_2 \leq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} + \text{INR}_2 + \text{SNR}_2 + \frac{\text{INR}_1}{1 + \text{INR}_1} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}_1} \right) + \log(1 + \text{SNR}_2 + \text{INR}_1) + C_{21}^B$$

$$\leq \left\{ \begin{array}{l} \log \left(\frac{1 + 2\text{SNR}_1 + 2\text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+ \text{SNR}_1 \text{SNR}_2 + 2\text{INR}_1 \text{INR}_2 + 2\text{SNR}_1 \text{INR}_2} \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + 1 \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(\frac{1 + 2\text{SNR}_1 + 2\text{INR}_1 + \text{SNR}_2 + \text{INR}_2}{+ 2\text{SNR}_1 \text{INR}_2 + 4|h_{11}h_{22} - h_{12}h_{21}|^2} \right) \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + 1 \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} \log \left(\frac{1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + 2\text{INR}_2}{+ \text{SNR}_1 \text{INR}_2 + \text{INR}_1 \text{INR}_2} \right) \\ + |h_{11}h_{22} - h_{12}h_{21}|^2 \\ + \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) + 4 \end{array} \right\}.$$

Hence the gap is at most 4 bits.

In summary, we have

$$\bar{\mathcal{C}}_{\text{Rx}} \subset \bar{\mathcal{C}}_{\text{Tx}} \subset \bar{\mathcal{C}}_{\text{Rx}} \oplus [0, \tau] \times [0, \tau] \quad (19)$$

where $\tau = 4/3$ bits.

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I-Hsiang Wang (S'10) received the B.Sc. degree in electrical engineering from National Taiwan University, Taiwan, in 2006. Since 2006, he has been with University of California at Berkeley and is currently working towards the Ph.D. degree in the Department of Electrical Engineering and Computer Sciences.

His research interests include network information theory, wireless networks, coding theory and network coding.

Mr. Wang received a 2-year Vodafone Graduate Fellowship in 2006.

David N. C. Tse (M'96–SM'07–F'09) received the B.A.Sc. degree in systems design engineering from the University of Waterloo, Waterloo, ON, Canada, in 1989, and the M.S. and Ph.D. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1991 and 1994, respectively.

From 1994 to 1995, he was a Postdoctoral Member of Technical Staff at AT&T Bell Laboratories. Since 1995, he has been with the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley, where he is currently a Professor.

Dr. Tse received a 1967 NSERC 4-year graduate fellowship from the government of Canada in 1989, a NSF CAREER award in 1998, the Best Paper Awards at the Infocom 1998 and Infocom 2001 conferences, the Erlang Prize in 2000 from the INFORMS Applied Probability Society, the IEEE Communications and Information Theory Society Joint Paper Award in 2001, the Information Theory Society Paper Award in 2003, and the 2009 Frederick Emmons Terman Award from the American Society for Engineering Education. He has given plenary talks at international conferences such as ICASSP in 2006, MobiCom in 2007, CISS in 2008, and ISIT in 2009. He was the Technical Program Cochair of the International Symposium on Information Theory in 2004 and was an Associate Editor of the IEEE TRANSACTIONS ON INFORMATION THEORY from 2001 to 2003. He is a coauthor, with P. Viswanath, of the text *Fundamentals of Wireless Communication*, which has been used in over 60 institutions around the world.