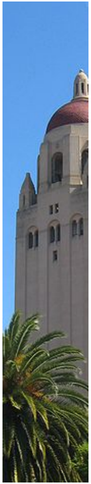

Discussion of Jones and Liu, “Growth with Capital-Embodied Technical Change”

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Background: Uzawa's Theorem

- Uzawa (1961) on neoclassical growth model:

$$Y_t = F(B_t K_t, A_t L_t)$$

  - Interior stable factor shares: BK and AL must grow at the same rate — balance
  - K accumulates endogenously $\Rightarrow K$ inherits AL trend
  - So B_t must stabilize for balanced growth (or Cobb-Douglas)

- Is a new computer a higher B or a higher A?

- But why would B ever be constant?

Uzawa (continued)

$$Y_t = F(B_t K_t, A_t L_t)$$

- Acemoglu (2003 JEEA)
  - A 2-dimensional Romer model: entrepreneurs can increase A or B
  - Surprise: they endogenously choose to stabilize B and only increase A
- However, extremely fragile!
  - Breaks if model is semi-endogenous growth instead of fully endogenous
  - Breaks if any asymmetry in the idea production functions of A versus B

Uzawa (continued)

- Grossman, Helpman, Oberfield, Sampson (2017 AER)

$$Y_t = F((1-s_t)^{\alpha} B_t K_t, A_t L_t/(1-s_t)^{\beta})$$

- Add a third factor "schooling" s_t.
- If it enters production in just the right way, you can get a BGP
  - $\dot{s}_t = \theta(1-s_t)$: schooling rises, but at a decreasing rate
  - $1-s_t$ falls at a constant exponential rate so $(1-s_t)^{\alpha} B_t$ constant $\Rightarrow$ satisfies Uzawa
- Aghion-Jones-Jones (2019) and Jones-Liu (2022) have closely-related math, but in a very different economic environment!

Background: Automation

- Acemoglu and Restrepo (2018, 2019, 2020, 2021, 2022)
  - Foundational work in this literature, building on Zeira (1998)
- Aghion, B. Jones, and C. Jones (2019) is the direct predecessor to the present paper
  - Many common ingredients
  - Let me show the similarities and then highlight the point of departure

AJJ Economic Environment

Final good $$Y_t = \left(\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}$$ where $\sigma < 1$

Tasks $$y_{it} = \begin{cases} K_{it} & \text{if automated} \quad i \in [0, \beta_t] \\ L_{it} & \text{if not automated} \quad i \in [\beta_t, 1] \end{cases}$$

Capital accumulation $$\dot{K}_t = I_t - \delta K_t$$

Resource constraint (K) $$\int_0^1 K_{it} di = K_t$$

Resource constraint (L) $$\int_0^1 L_{it} di = L$$

Resource constraint (Y) $$Y_t = C_t + I_t$$

Allocation $$I = \bar{s}_K Y$$

Automation and growth

- Combining equations

$$Y_t = \left[\beta_t \left(\frac{K_t}{\beta_t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\beta_t) \left(\frac{L}{1-\beta_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- How β interacts with K: two effects
  - β: what fraction of tasks have been automated
  - β: Dilution as $K/\beta \Rightarrow K$ spread over more tasks
- Same for labor: $L/(1-\beta_t)$ means given L concentrated on fewer tasks, raising "effective labor"

Rewriting in classic CES form

- Collecting the β terms into factor-augmenting form:

$$Y_t = F(B_t K_t, A_t L_t)$$

where

$$B_t = \left(\frac{1}{\beta_t}\right)^{\frac{1}{1-\sigma}} \text{ and } A_t = \left(\frac{1}{1-\beta_t}\right)^{\frac{1}{1-\sigma}}$$

- Effect of automation: $\uparrow \beta_t \Rightarrow \downarrow B_t$ and $\uparrow A_t$

Intuition: dilution effects just get magnified since $\sigma < 1$

Automation

- Suppose a constant fraction of non-automated tasks get automated every period:

$$\dot{\beta}_t = \theta(1 - \beta_t)$$

$$\Rightarrow \quad \beta_t \to 1$$

- What happens to $1 - \beta_t =: m_t$?

$$\frac{\dot{m}_t}{m_t} = -\theta$$

The fraction of labor-tasks falls at a constant exponential rate

Putting it all together

$$Y_t = F(B_t K_t, A_t L_t) \text{ where } B_t = \left(\frac{1}{\beta_t}\right)^{\frac{1}{1-\sigma}} \text{ and } A_t = \left(\frac{1}{1-\beta_t}\right)^{\frac{1}{1-\sigma}}$$

- $\beta_t \to 1 \Rightarrow B_t \to 1$
- But A_t grows at a constant exponential rate!

$$\frac{\dot{A}_t}{A_t} = -\frac{1}{1-\sigma}\frac{\dot{m}_t}{m_t} = \frac{\theta}{1-\sigma}$$

- When a constant fraction of remaining goods get automated and $\sigma < 1$, the automation model features an asymptotic BGP that satisfies Uzawa

$$\alpha_{Kt} \equiv \frac{F_K K}{Y} = \beta_t^{\frac{1}{\sigma}} \left(\frac{K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} \to \left(\frac{\bar{s}_K}{g_Y + \delta}\right)^{\frac{\sigma-1}{\sigma}} < 1$$

Intuition for AJJ result

- Why does automation lead to balanced growth and satisfy Uzawa?
  - $\beta_t \to 1$ so the KATC piece "ends" eventually
  - Labor per task: $L/(1-\beta_t)$ rises exponentially over time!
  - Constant population, but concentrated on an exponentially shrinking set of goods
    $\Rightarrow$ exponential growth in "effective" labor
- Limitation
  - An asymptotic result
  - Only occurs as $\beta_t \to 1$, so unclear if relevant for U.S. or other modern economies

Interesting question: What fraction of tasks automated today? β_{2022}

B. Jones and Liu Contribution

- BGP can occur "today" with $\beta_t < 1$, not asymptotically
  - Might describe modern economies like the U.S. / Europe / Japan

- Automation and KATC (Z_t) coexist along the BGP
  - The economic environment that achieves this is novel and interesting

- Empirics

Jones-Liu Economic Environment

Final good $$Y_t = \left(\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where} \quad \sigma < 1$$

Tasks $$y_{it} = \begin{cases} z_{it}^{\frac{1}{1-\sigma}} K_{it} & \text{if automated} \quad i \in [0, \beta_t] \\ L_{it} & \text{if not automated} \quad i \in [\beta_t, 1] \end{cases}$$

Familiar $$\dot{K}_t = I_t - \delta K_t, \quad \int_0^1 K_{it} di = K_t, \quad \int_0^1 L_{it} di = L$$

Resource constraint (Y) $$Y_t = C_t + I_t + \int_0^{\beta_t} d_{it}^v di + \int_{\beta_t}^1 d_{it}^h di$$

Innovation: increasing z_i Arrival rate $q_{it}^v = \zeta^v \left(\frac{z_{it} d_{it}^v}{Y_t} \right)^\alpha$, Step size ϕ

Innovation: automation Arrival rate $q_{it}^h = \zeta^h \left(\frac{z_{it} d_{it}^h}{Y_t} \right)^\alpha$, $z_{it} = \bar{h} \cdot (1 - \beta_t)$ for $i = \beta_t$

Combining equations

$$Y_t = F(B_t K_t, A_t L_t) \text{ where } B_t = \left(\frac{Z_t}{\beta_t}\right)^{\frac{1}{1-\sigma}} \text{ and } A_t = \left(\frac{1}{1-\beta_t}\right)^{\frac{1}{1-\sigma}}$$

and

$$Z_t \equiv \left(\frac{1}{\beta_t}\int_0^{\beta_t} z_{it}^{-1} di\right)^{-1} \quad \text{(harmonic mean)}$$

- Same "engine" of growth as AJJ via A_t
- Automation: Constant fraction q^h of remaining goods automated: $\dot{\beta}_t = q^h(1-\beta_t)$
  - But starting productivity of newly automated good is $z_0 = \bar{h}(1-\beta_t)$
  - declines over time (harder to automate goods start out further behind)
- $\beta_t \to 1$ as before. What happens with Z_t?

Understanding Z_t

$$Z_t \equiv \left(\frac{1}{\beta_t} \int_0^{\beta_t} z_{it}^{-1} di \right)^{-1} \quad \text{(harmonic mean)}$$

- Already automated goods improve at rate $q^v \phi$ over time, raising Z_t
- Newly automated goods come in with very low productivity $z = \bar{h}(1 - \beta_t)$
  - Harmonic mean is dragged down by these low additions
- Surprise! Z_t aggregates as if

$$\dot{Z}_t = \kappa_t (1 - \beta_t) \quad \text{with} \quad \kappa_t \to \kappa^*$$

- Just like β_t!

$$\Rightarrow \; Z_t / \beta_t \quad \text{constant along BGP}$$

Remarks

- BGP even with $\beta_t < 1$. Automation and KATC along BGP
- Requires the equivalent of $\dot{Z}_t = \kappa(1 - \beta_t)$
  - Why should this be?
  - On the one hand, standard growth models have Z growing exponentially
  - Cool structure with newly-automated goods having lower productivity in just the right way.
  - But it's a very specific assumption.
  - Parallels Acemoglu (2003) in that very special structure required
- Paper should do a better job of clarifying that this is the contribution

Empirics

- What does β_t look like over time?

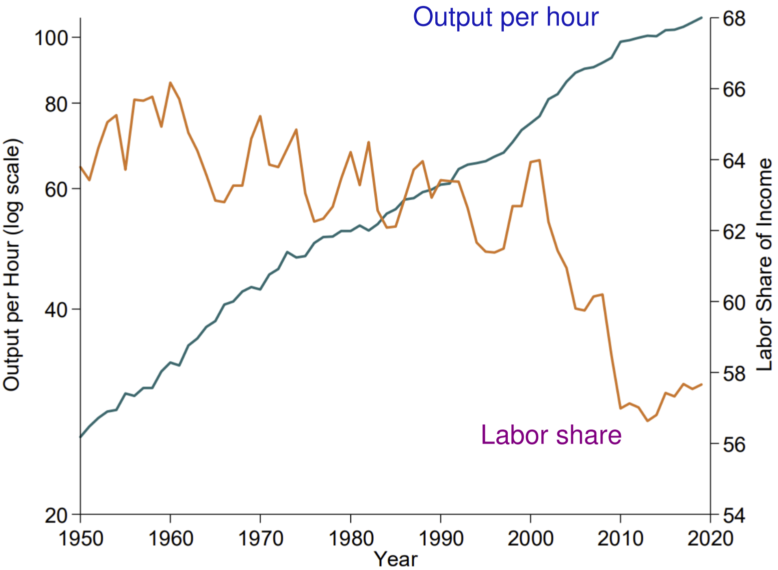
- Two equations in two unknowns

$$\alpha_{Kt} = \beta_t \,/\, Z_t$$

$$\frac{Y_t}{L_t} = (1 - \alpha_{Kt})^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{1 - \beta_t} \right)^{\frac{1}{1-\sigma}}$$

- Get β_t from labor productivity and Z_t from capital share

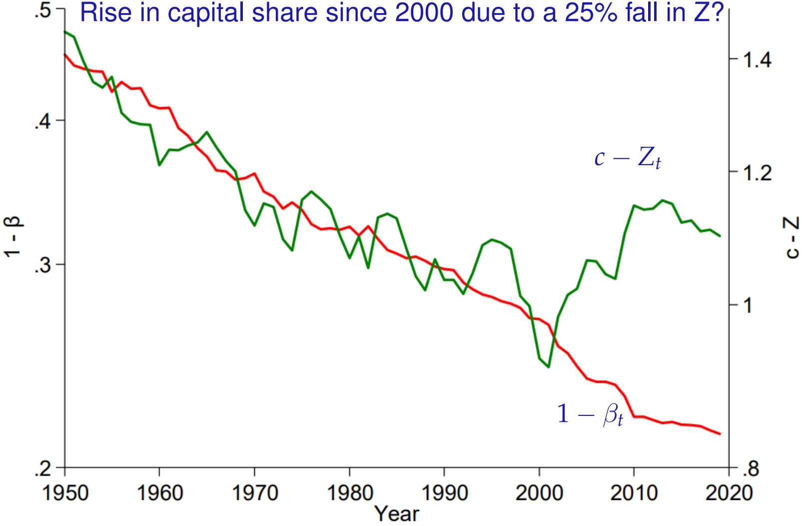
Falling Labor Share and Growth Slowdown since 2000



Estimates of β_t and Z_t

Share automated has risen from 0.5 to 0.75

Rise in capital share since 2000 due to a 25% fall in Z?



Would be nice to show β_t and Z_t directly

Remarks on Empirics

- Share automated has risen from $\beta_{1950} = 0.5$ to $\beta_{2020} = 0.75$
  - Do we believe this? I don't know. Lots of automation!
  - What other evidence? Unclear, but model nicely points to α_K and Y/L
- Rise in capital share since 2000 due to a 25% fall in Z_t?
  - Not a burst of automation b/c automation should **increase** growth (temporarily)
  - Model cannot help us understand a **decline** in Z_t
- Likely other forces contributing to growth that would change the calibration?
  - Educational attainment, LATC apart from automation, markups
  - Exponential declines in the relative price of information technology

Final Thoughts

Very interesting, provocative, and fun to read!