AA 284a
Advanced Rocket Propulsion

Lecture 8
Hybrid Rocket Propulsion Fundamentals

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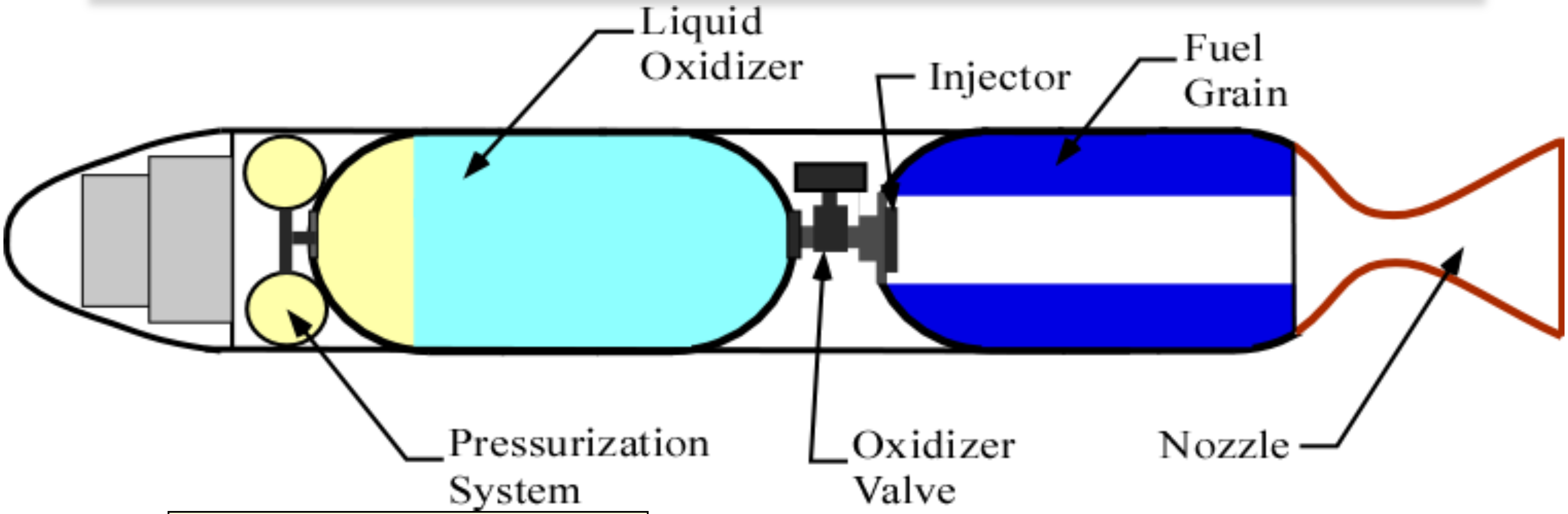
Hybrid Rocket Configuration

Fuel and oxidizer are physically separated

One of the two is in solid phase



Most Hybrids:

Oxidizer: Liquid

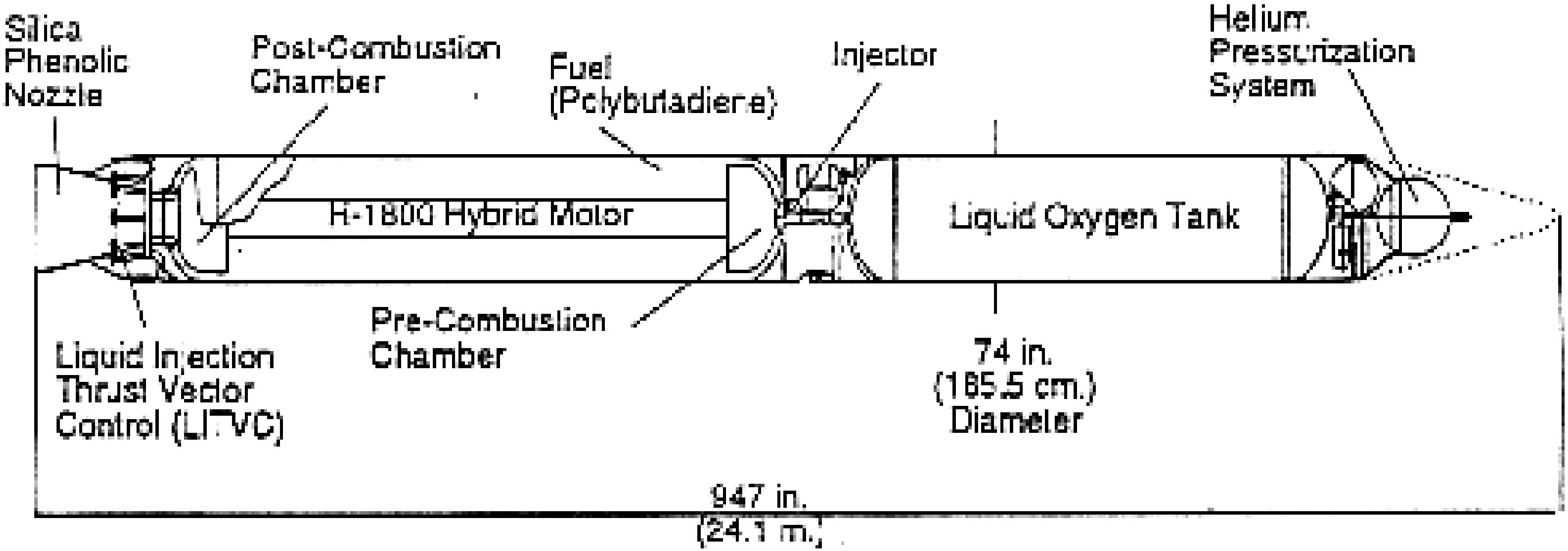
Fuel: Solid

Reverse Hybrids:

Oxidizer: Solid

Fuel: Liquid

Hybrid Rocket Configuration-AMROC Booster



- AMROC H-1800 (Test motor DM-01): 250 klb thrust

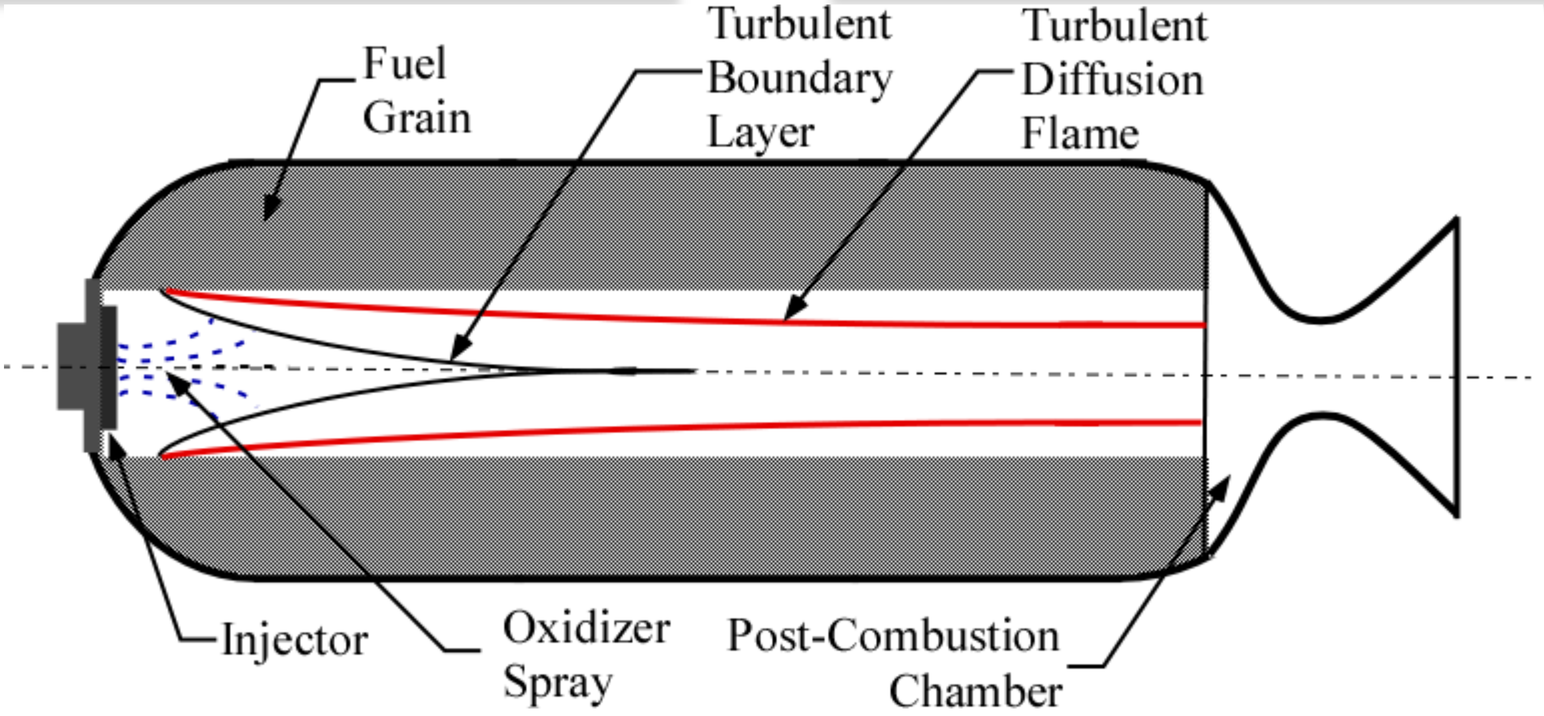
Hybrid Rocket System

Solid Fuel

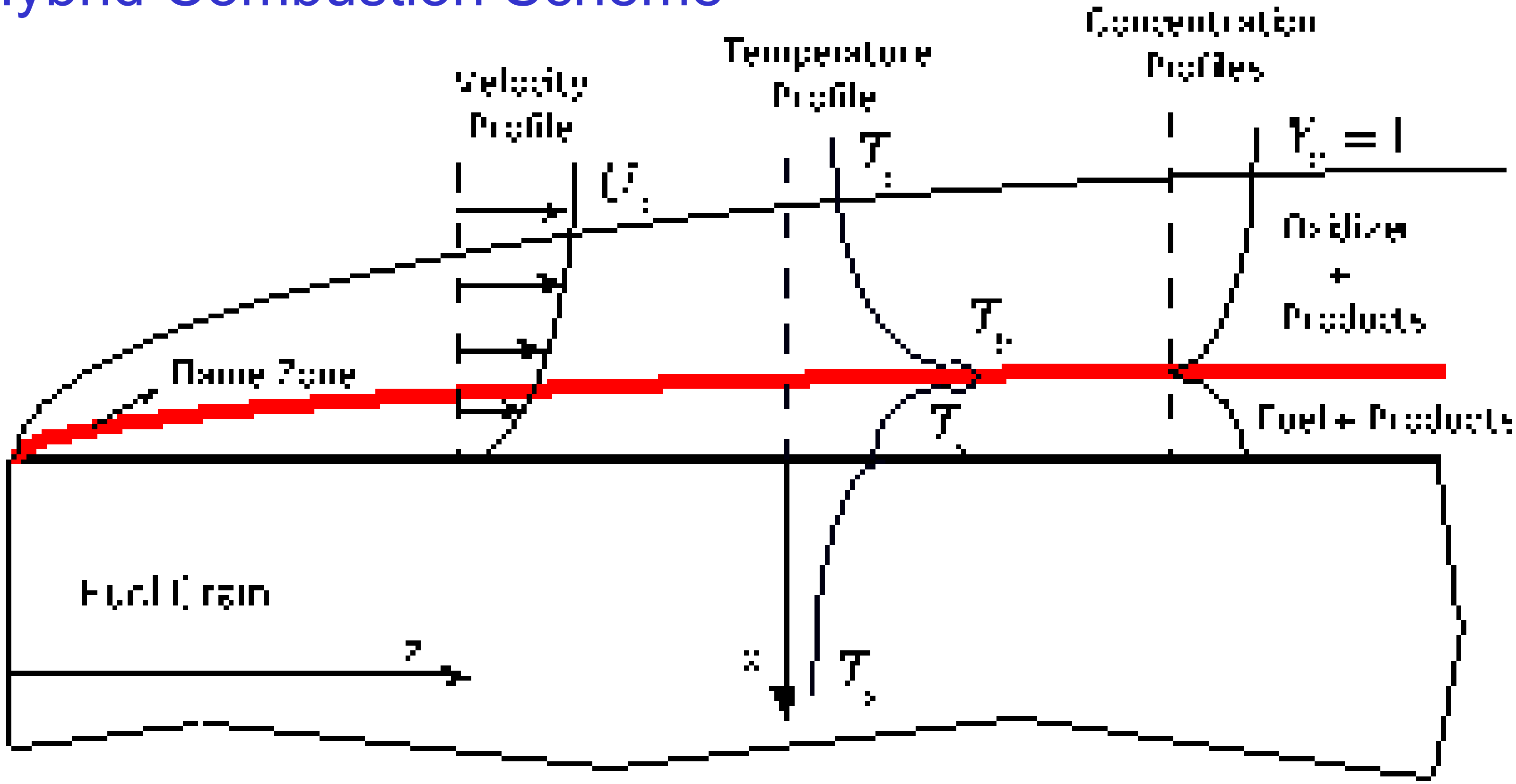
- Polymers: Thermoplastics, (Polyethylene, Plexiglas), Rubbers (HTPB)
- Wood, Trash, Wax

Liquid Oxidizer

- Cryogenic: LO_2
- Storable: H_2O_2, N_2O, N_2O_4, IRFNA



Hybrid Combustion Scheme



- Diffusion limited combustion
  - Burning Rate Law: independent of pressure (flux dependent)

$$\dot{r} = a\, G_{ox}^{n}$$

- Flame zone away from surface and blocking effect
  - Low regression rate

Comparison of Hybrid and Solid Rocket Combustion Schemes

Solid

- Active grain
- Premixed flame (globally).
- Heteregeneous rxns.
- Pressure dependent burning rate.
- High burning rates (high thrust density).

Hybrid

- Inert grain
- Diffusion flame in a TBL.
- No heteregeneous rxns.
- Mass flux dependent burning rate.
- Low burning rates (low thrust density).

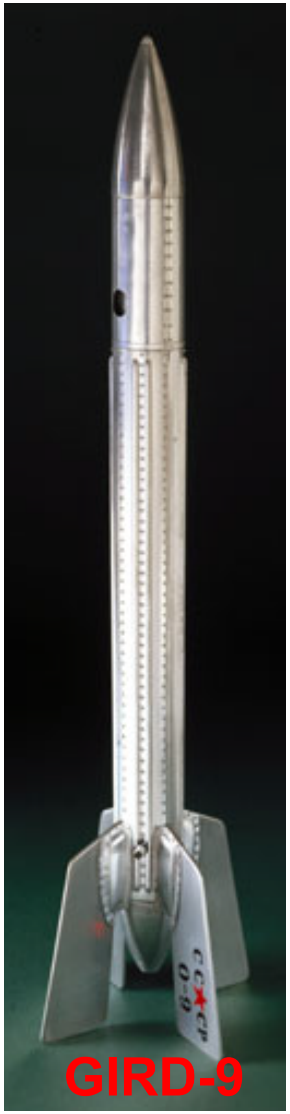
Advantages of Hybrids

Compared to	**Solids**	**Liquids**
Simplicity	- Chemically simpler - Tolerant to processing errors	- Mechanically simpler - Tolerant to fabrication errors
Safety	- Reduced chemical explosion hazard - Thrust termination and abort possibility	- Reduced fire hazard - Less prone to hard starts
Performance Related	- Better Isp performance - Throttling/restart capability	- Higher fuel density - Easy inclusion of solid performance additives (Al, Be)
Other	- Reduced environmental impact	- Reduced number and mass of liquids
Cost	- Reduced development costs are expected - Reduced recurring costs are expected	

Hybrid Rocket History

Early History (1932-1960)

- 1932-1933: GIRD-9 (Soviet)
  - LO_2/Gellified gasoline 60 lbf thrust motor
  - Firsts
    - Hybrid rocket
    - Soviet rocket using a liquid propellant
    - First fast burning liquefying fuel
  - Tikhonravov and Korolev are designers
  - Maximum altitude: 1,500 m
- 1937: Coal/Gaseous N_2O hybrid motor 2,500 lbf thrust (Germany)
- 1938-1939: LOX/Graphite by H. Oberth (Germany)
- 1938-1941: Coal/GOX by California Rocket Society (US).
- 1947: Douglas Fir/LOX by Pacific Rocket Society (US)
- 1951-1956: GE initiated the investigations in hybrids. H_2O_2/Polyethylene. (US)


GIRD-9

Hybrid Rocket History

Era of Enlightenment (1960-1980)

- 1960's: Extensive research at various companies.
  - Chemical Systems Division of UTC
    - Modeling (Altman, Marxman, Ordahl, Wooldridge, Muzzy etc…)
    - Motor testing (up to 40,000 lb thrust level)
  - LPC: Lockheed Propulsion Company, SRI: Stanford Research Institute, ONERA (France)
- 1964-1984: Flight System Development
  - Target drone programs by Chemical Systems Division of UTC
    - Sandpiper, HAST, Firebolt
  - LEX Sounding Rocket (ONERA, France)
  - FLGMOTOR Sounding Rocket (Sweeden)

CSD's Li/LiH/PBAN-F_2/O_2 Hybrid
Measured Isp=400 sec

Firebolt Target Drone

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Hybrid History

Recent History (1981-Present)

- 1981-1985: Starstruck company developed and sea launched the Dolphin sounding rocket (35 klb thrust)
- 1985-1995: AMROC continuation of Starstruck
  - Tested 10, 33, 75 klb thrust subscale motors.
  - Developed and tested the H-1800, a 250 klb LO_2/HTPB motor.
- 1990's: Hybrid Propulsion Development Program (HPDP)
  - Successfully launched a small sounding rocket.
  - Developed and tested 250 klb thrust LO_2/HTPB motors.
- 2002: Lockheed developed and flight tested a 24 inch LO_2/HTPB hybrid sounding rocket (HYSR). (60 klb thrust)
- 2003: Scaled Composites and SpaceDev have developed a N_2O/HTPB hybrid for the sub-orbital vehicle SpaceShipOne. (20 klb thrust)

Hybrid Combustion Theory (Diffusion Limited Model)

- Developed by G. Marxman in early 1960's.
- The purpose is to predict the regression rate.
- Assumptions:
    - Steady-state operation.
    - Simple grain configuration (flat plate).
    - No exothermic reactions in the solid grain (No oxidizer in solid phase).
    - Oxidizer enters the port as a uniform gas.
    - *Le*=*Pr*=1 $(Le = \kappa / D)$
    - No heat transfer to the ambient air through the walls of the rocket.
    - All kinetic effects are neglected (Characteristic times for all chemical rxns << characteristic times for diffusion processes).
    - Flame zone is infinitely thin. (Flame sheet). No oxidizer beneath the flame.
    - Boundary layer is turbulent.

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Energy balance at the fuel surface: (Steady-state)

$$\dot{Q}_w = \dot{m}_f h_v = \rho_f \dot{r} h_v = (\rho v)_w h_v$$

$\dot{Q}_w$ = Total heat flux to the wall

h_v = Effective heat of gasification (Heating of the solid fuel grain + Heat of evaporation and melting + Heat of reaction for degradation of the polymer)

- Conductive Heat Transfer Only (No radiation)

$$\dot{Q}_w = \dot{Q}_c = -\left(\frac{k}{c_p} \frac{\partial h}{\partial y} \right)_w$$

- Define a Stanton number as

$$C_H \equiv \frac{\dot{Q}_c}{\rho_b u_b \Delta h} \qquad \Delta h = h_b - h_w$$

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Combine to find mass flux from the wall

$$\rho_f \dot{r} = G_f = \frac{\dot{Q}_c}{h_v} = C_H \rho_b u_b \frac{\Delta h}{h_v}$$

- Why did we introduce C_H?
  - *Because we can relate C_H to C_f*
  - There is a extensive amount of data on C_f *for boundary layer literature*
- Reynolds analogy between the flame and the wall.

$$\frac{\dot{Q}_c}{\Delta h} = \frac{\tau_w}{u_b}$$

  - Assumption: No chemical rxns beneath the flame

- Skin friction is related to the friction coefficient

$$\tau_w = 0.5 C_f \rho_e u_e^{2}$$

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Combine to find mass flux from the wall

$$C_H = 0.5 C_f \frac{\rho_e u_e^2}{\rho_b u_b^2} \qquad \dot{r} = \frac{C_f \rho_e u_e B}{2\rho_f}$$

- Here the blowing parameter is defined as

$$B \equiv \underbrace{\frac{2(\rho v)_w}{\rho_e u_e C_f}}_{\text{Aerodynamic}} = \frac{u_e}{u_b} \underbrace{\frac{\Delta h}{h_v}}_{\text{Thermochemical}}$$

- Skin Friction coefficient over a flat plate for TBL with no blowing

$$C_{f0} = 0.06 \,\mathrm{Re}_x^{-0.2} \qquad \mathrm{Re}_x = \frac{\rho_e u_e x}{\mu_e}$$

- Substitute to obtain:

$$\dot{r} = 0.03 \frac{\rho_e}{\rho_f} u_e \,\mathrm{Re}_x^{-0.2} \left(\frac{C_f}{C_{fo}} \right) B$$

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Correction for blowing C_f/C_{fo} :
  - Lee's film theory (valid for $B<5$)

$$C_f \Big/ C_{f0} = \ln\left[\frac{(1+B)}{B}\right]$$

  - Marxman's formula based on mixing length arguments (valid for 5<B<100)

$$C_f / C_{f0} = 1.2 B^{-0.77}$$

  - Introduce the mass flux $G \equiv \rho_e u_e$ and rearrange to obtain the hybrid regression rate expression

$$\dot{r}(x) = \frac{0.036}{\rho_f \mu_e^{-0.2}} G^{0.8} x^{-0.2} B^{0.23}$$

  - The only parameter that is hard to estimate is B

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Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Correction for blowing C_f/C_{fo} (Blocking effect: $C_H/C_{Ho} < 1$):
  - Lee's film theory (valid for B<5)

$$C_f \Big/ C_{f0} = \ln\left[\frac{(1+B)}{B}\right]$$

  - Marxman's formula based on mixing length arguments (valid for 5<B<100)
  -

$$C_f / C_{f0} = 1.2 B^{-0.77}$$

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$$\dot{r}(x) = \frac{0.036}{\rho_f \mu_e^{-0.2}} G^{0.8} x^{-0.2} B^{0.23}$$

  - The only parameter that is hard to estimate is *B*

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Calculation of *B*:
  - Combustion model is required.
  - General solution is obtained by solving the gas-phase field equations with regression rate equation as one of the boundary conditions. (Difficult problem)
  - Marxman obtained an approximate solution using the mixing length concept.
  - For *L/D* < *25 B* does not change significantly with *x*.
  - (*L*: Length of the grain, *D*: Hydraulic diameter) Thus threat *B* as a constant for a given oxidizer/fuel selection.
  - *B* is a dual parameter:
    - Thermochemical property of the selected propellant
    - Aerodynamic property (Similarity parameter of the TBL profile)
- Regression rate law in the nondimensional form:

$$\dot{r}_{nd} = \frac{G_f}{G}\frac{2}{C_{fo}} = 1.2\,B^{0.23} = cons.$$

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Regression rate is not a strong function of *B*. Mass flux has the most significant effect on the burning rate.

$$\boxed{\dot{r}(x) = A\,G^{0.8}x^{-0.2}}$$

$$A = \frac{0.036}{\rho_f \mu_e^{-0.2}} B^{0.23}$$

- *A* can be assumed to be constant as a first order approximation
- For purely convective systems, regression rate is not a function of pressure.

where

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Effect of Radiative Heat Transfer:
  - Simple model: Grain as gray body, flame zone as the radiative continuum.
  - The radiative heat transfer can be written as

$$\dot{Q}_r = \sigma \varepsilon_w \left(\varepsilon_g T_r^4 - T_w^4 \right) \qquad \varepsilon_g = 1 - e^{-\alpha N z} = F(P_c)$$

  - Here ε_w and T_r (Effective radiation temperature) depend on the propellant combination.
- Combined Heat Transfer:
  - Superposition is not possible since radiative and convective heat transfers are coupled through the blocking effect.
  - In the coupled case the following formula can be derived

$$\dot{r} = \frac{\dot{Q}_c}{\rho_f h_v} \left[e^{-\dot{Q}_r / \dot{Q}_c} + \frac{\dot{Q}_r}{\dot{Q}_c} \right]$$

  - For $\dot{Q}_r << \dot{Q}_c$

$$\dot{r}(x) = A G^{0.8} x^{-0.2} + \frac{\dot{Q}_r}{\rho_f h_v}$$

  - For most fuels without metal additives, the radiative contribution can be ignored.

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Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Hybrid Burning Rate Law:
  - If we take an average of the regression rate expression over the grain length and firing period, we obtain the space-time averaged regression rate expression, namely, the burning rate law for hybrids.

$$\boxed{\bar{\dot{r}} = a\, G_o^n}$$

$$n \approx 0.5 - 0.8$$

$\dot{m}_{ox}$: *Oxidizer Flow Rate*

A_p : *Average Fuel Port Area*

  - Here oxidizer mass flux is defined as $G_o = \dot{m}_{ox} / A_p$
  - This is the most commonly used form in the design of hybrid rocket systems
  - Note that the burning rate law for solid rockets has the form

$$\bar{\dot{r}} = c\, P_c^n$$

- Typical conditions:

$$B \approx 7 - 15 \qquad {}^{O/F}\!\Big/_{(O/F)_{stoic}} < 1 \qquad T_w = 600 - 800\,K \qquad T_b = 1500\,K$$

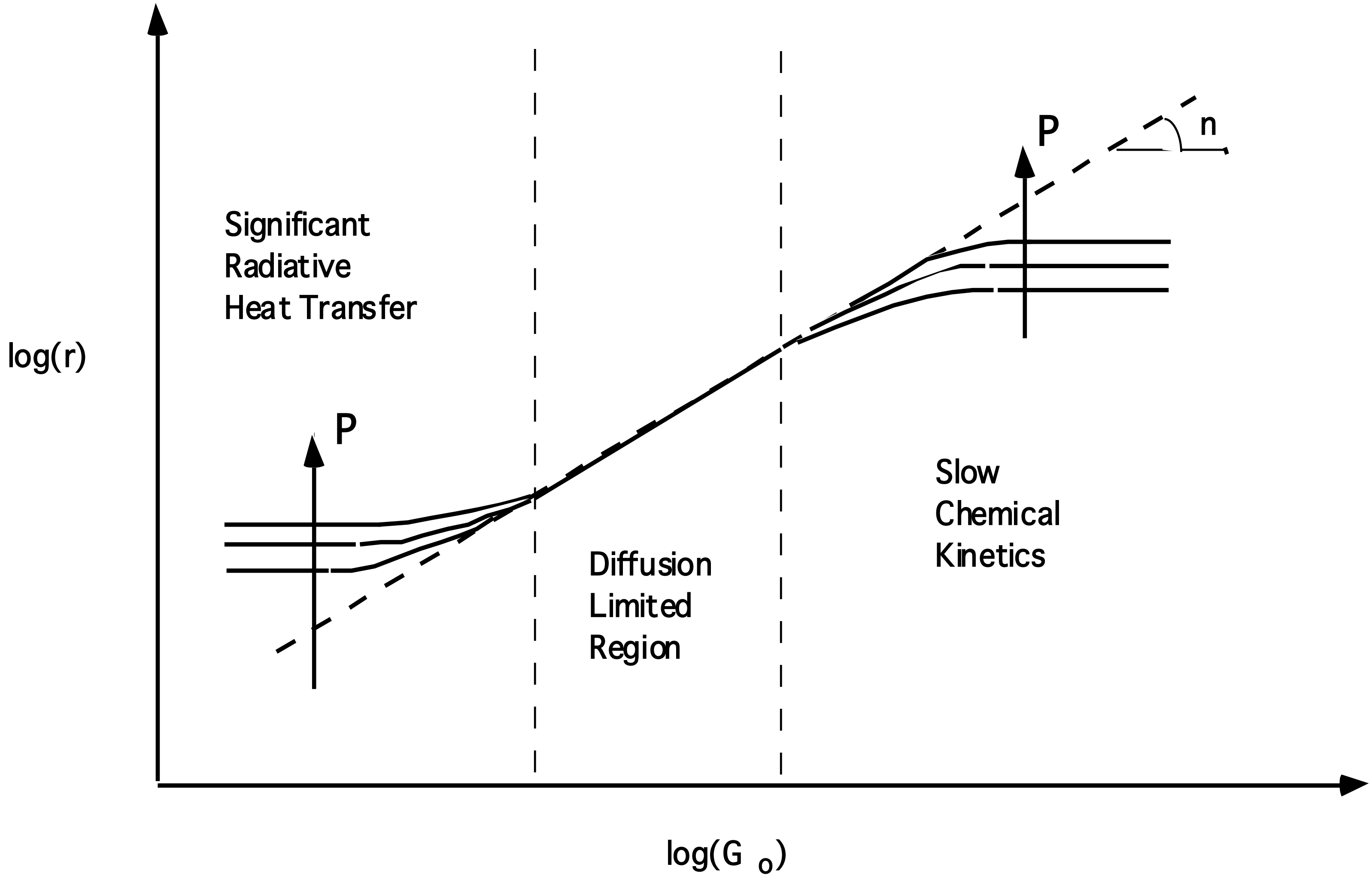
Flame height: 0.15- 0.2 BL thickness

Flame thickness: 0.1 BL thickness

Hybrid Combustion Theory (Diffusion Limited Model)-Cont.

- Limitations of the Theory:
  - Each propellant combination has an upper and lower limit for the mass flux beyond which the model is not applicable.
    - High mass fluxes → Kinetic effects (Pressure dependency via the gas phase rxn rates)
    - Low mass fluxes → Radiation effects (Pressure dependency via the radiation effects)
  - Transition to laminar boundary layer
  - Cooking of the propellant (at very low regression rates)
  - Dilution of the oxidizer

Effect of Pressure on the Regression Rate



Thermal Layer Thickness in the Fuel

- The heat conduction equation in the solid in reference of frame fixed to the regressing surface

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \dot{r}(t)\frac{\partial T}{\partial x}$$

- Here the heat diffusivity is defined as $\kappa \equiv \lambda_f / \rho_f C_{pf}$
- During Steady state operating this expression can be integrated to yield

$$T(x) = (T_s - T_a)e^{-x/\delta_T} + T_a$$

- Here the characteristic thermal thickness can be given as

$$\delta_T = \kappa / \dot{r}$$

- Similarly the characteristics time is

$$\tau_T = \kappa / \dot{r}^2$$

- During typical operation of a polymeric hybrid fuel

$$\delta_T = 10^{-6} / 10^{-3} = 10^{-3}\ m \qquad \tau_T = 10^{-6} / 10^{-6} = 1 \sec$$

Hybrid Rocket Design

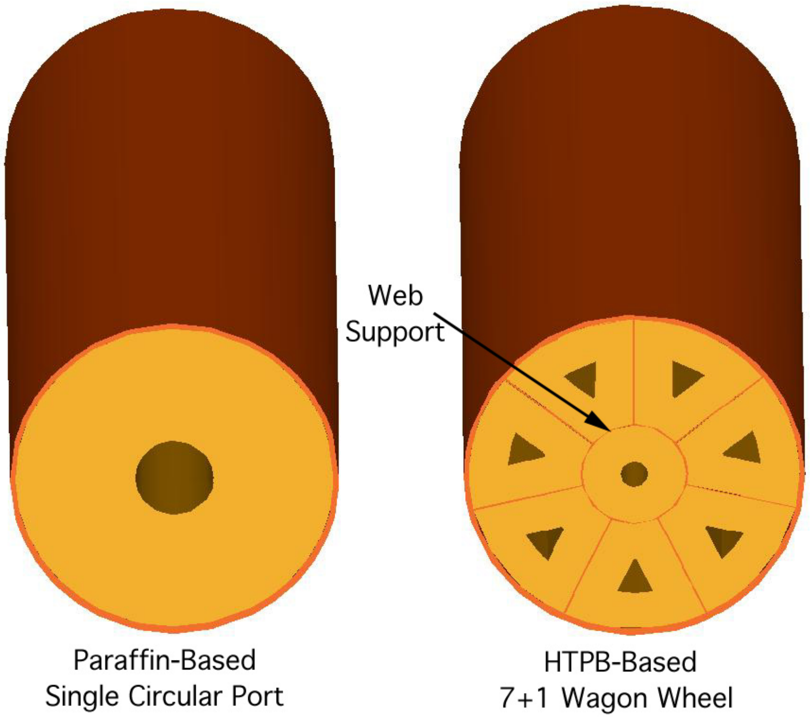
- Regression rate: $\bar{\dot{r}} = a G_o^n$
- Oxygen mass flow rate: $\dot{m}_o = A_p G_o$
- Fuel mass flow rate: $\dot{m}_f = A_b \rho_f \bar{\dot{r}}$
- (A_b :Burning surface area)
- Global O/F ratio: $O/F = \dot{m}_o / \dot{m}_f$
- Combustion products properties: $T_c\,,\ M_c, c^* = f(O/F)$

(Initially ignore the effect of pressure)

- Total mass flow rate: $\dot{m} = \dot{m}_o + \dot{m}_f$
- Chamber pressure: (Mass flow relation) $P_c = \dfrac{\dot{m} c^* \eta}{A_n}$
- Thrust: $F = C_F P_c A_n$
- Specific impulse: $Isp = F \big/ \dot{m} g_o$

Disadvantage of Classical Hybrids

- Low Burning Rates --> Multi-port design

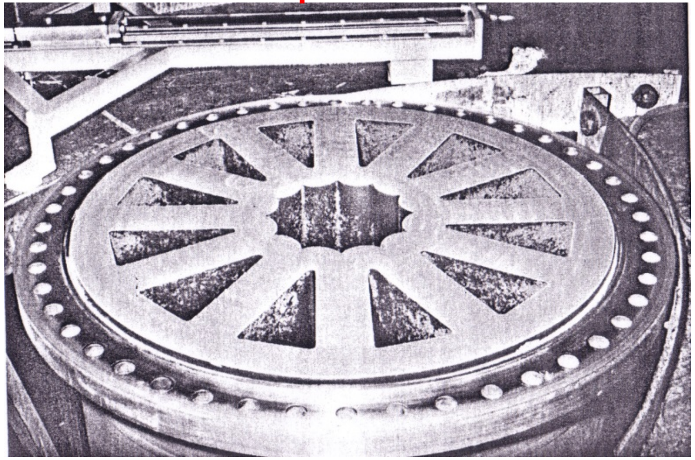


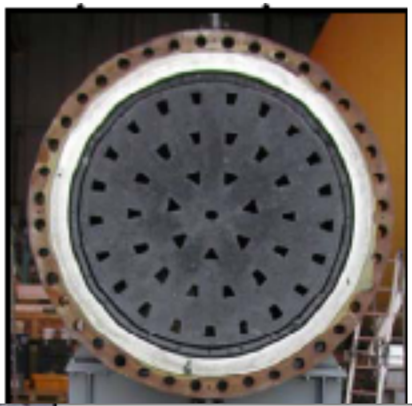
Issues with multi-port design

- Excessive unburned mass fraction (i.e. typically in the 5% to 10% range).
- Complex design/fabrication, requirement for a web support structure.
- Compromised grain structural integrity, especially towards the end of the burn.
- Uneven burning of individual ports.
- Requirement for a substantial pre-combustion chamber or individual injectors for each port.

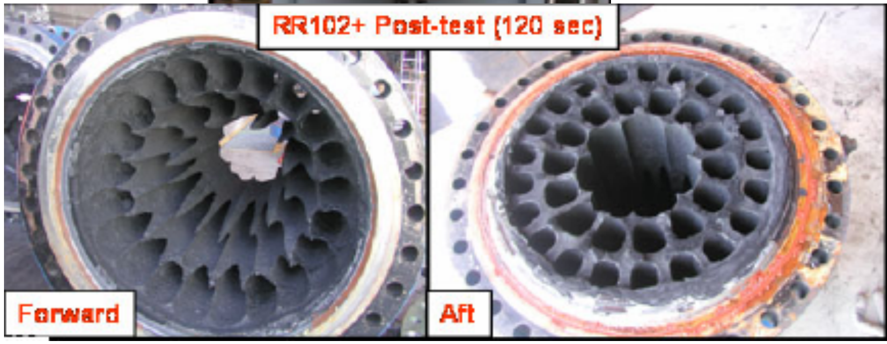
Disadvantage of Multiport Designs

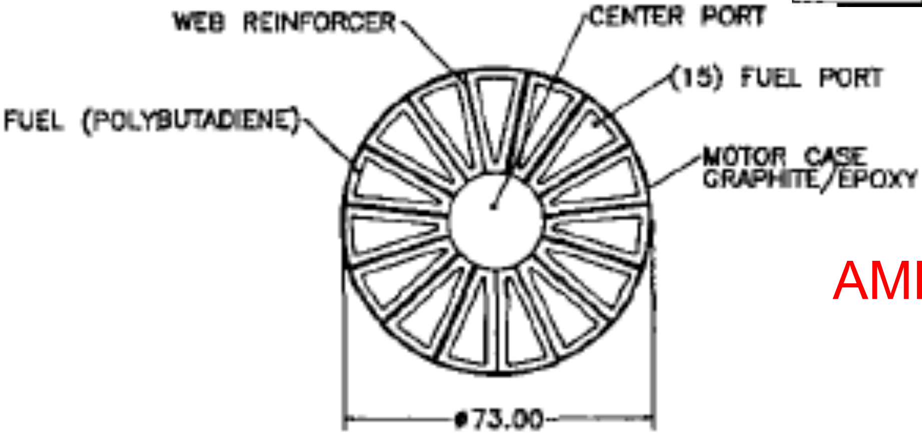
CSD (1967)
13 ports





Lockheed
Martin
(2006)
43 ports





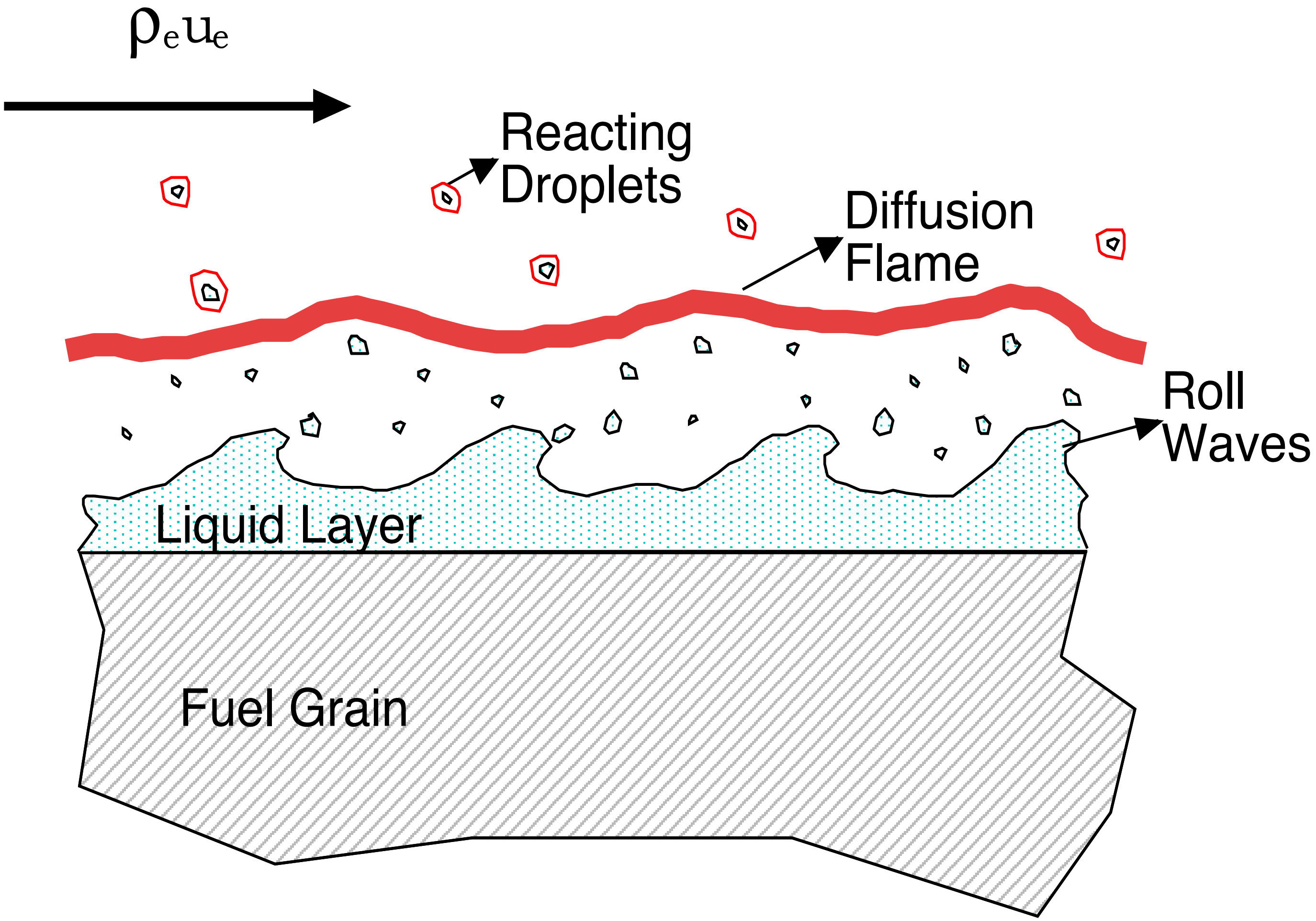
AMROC (1994)
15 ports

Approaches for High Regression Rate

Technique	Fundamental Principle	Shortcoming
Add oxidizing agents self-decomposing materials	Increase heat transfer by introducing surface reactions	• Reduced safety • Pressure dependency
Add metal particles (micron-sized)	Increased radiative heat transfer	• Limited improvement • Pressure dependency
Add metal particles (nano-sized)	Increased radiative heat transfer	• High cost • Tricky processing
Use Swirl Injection	Increased local mass flux	• Increased complexity • Scaling?

All based on increasing heat transfer to fuel surface

Entrainment Mass Transfer Mechanism



Regression Rate = Entrainment + Vaporization