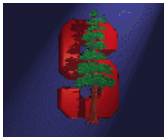

AA 284a
Advanced Rocket Propulsion

Lecture 7
Launch Trajectories

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KOC UNIVERSITY

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Orbital Mechanics - Review

- Newton's law of gravitation:

$$F_g = \frac{mMG}{r^2}$$

- M, m : Mass of the bodies
 - r : Distance between the center of masses of the two bodies
 - F_g : Gravitational attraction force between the two bodies
 - G : Universal gravitational constant
- Assume that m is the mass of the spacecraft and M is the mass of the celestial body. Arrange the force expression as (Note that $m \ll M$)

$$F_g = \frac{\mu m}{r^2} = g m \quad \mu = M G \quad g = \mu / r^2$$

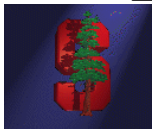
- Here the gravitational parameter μ has been introduced for convenience. It is a constant for a given celestial mass. For Earth

$$\mu_{\oplus} = 398,600 \text{ km}^3 / \text{sec}^2$$

- For circular orbit: centrifugal force balancing the gravitational force acting on the satellite

- Orbital Velocity: $V_{co} = \sqrt{\frac{\mu}{r}}$

- Orbital Period: $P = 2\pi \sqrt{\frac{r^3}{\mu}}$



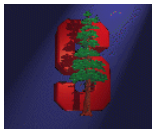
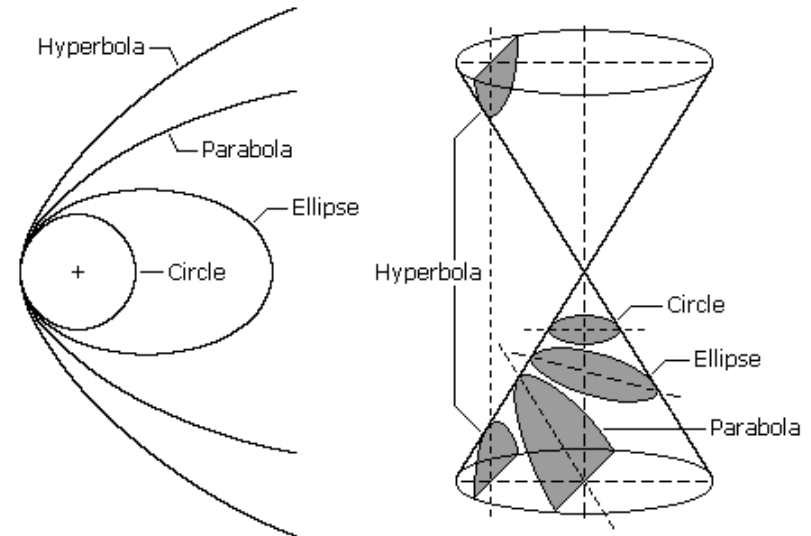
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Orbital Mechanics - Review

- Fundamental Assumptions:
 - Two body assumption
 - Motion of the spacecraft is only affected by a single central body
 - The mass of the spacecraft is negligible compared to the mass of the celestial body
 - The bodies are spherically symmetric with the masses concentrated at the center of the sphere
 - No forces other than gravity (and inertial forces)

$$\ddot{\vec{r}} = -k \frac{\vec{r}}{r^3}$$

- Solution:
 - Sections of a cone



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Orbital Mechanics - Review

- Solution:

- Orbits of any conic section, elliptic, parabolic, hyperbolic
- Energy is conserved in the conservative force field. Specific potential + kinetic energy of the orbiting body is a constant

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{r}$$

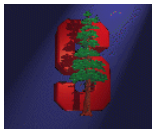
- Note that potential energy increases with increasing r . Asymptotes to zero at infinite distance
- The general expression for the velocity can be given as

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

- Here a is the semimajor axis
 - Circular orbit: $a = r$
 - Elliptic orbit $a > 0$
 - Parabolic orbit $a = \infty$
 - Hyperbolic orbit $a < 0$

- Note that $a = \frac{r_a + r_p}{2}$

- Apogee: r_a Perigee: r_p



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Orbital Mechanics - Review

- Escape velocity:
 - Parabolic orbit: $a = \infty$

$$V_{es} = \sqrt{\frac{2\mu}{r}}$$

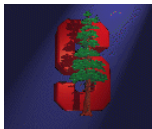
- Note that the escape velocity is

$$V_{es} = \sqrt{2} V_{co}$$

- Eccentricity of an elliptic orbit:

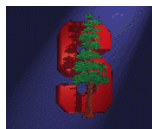
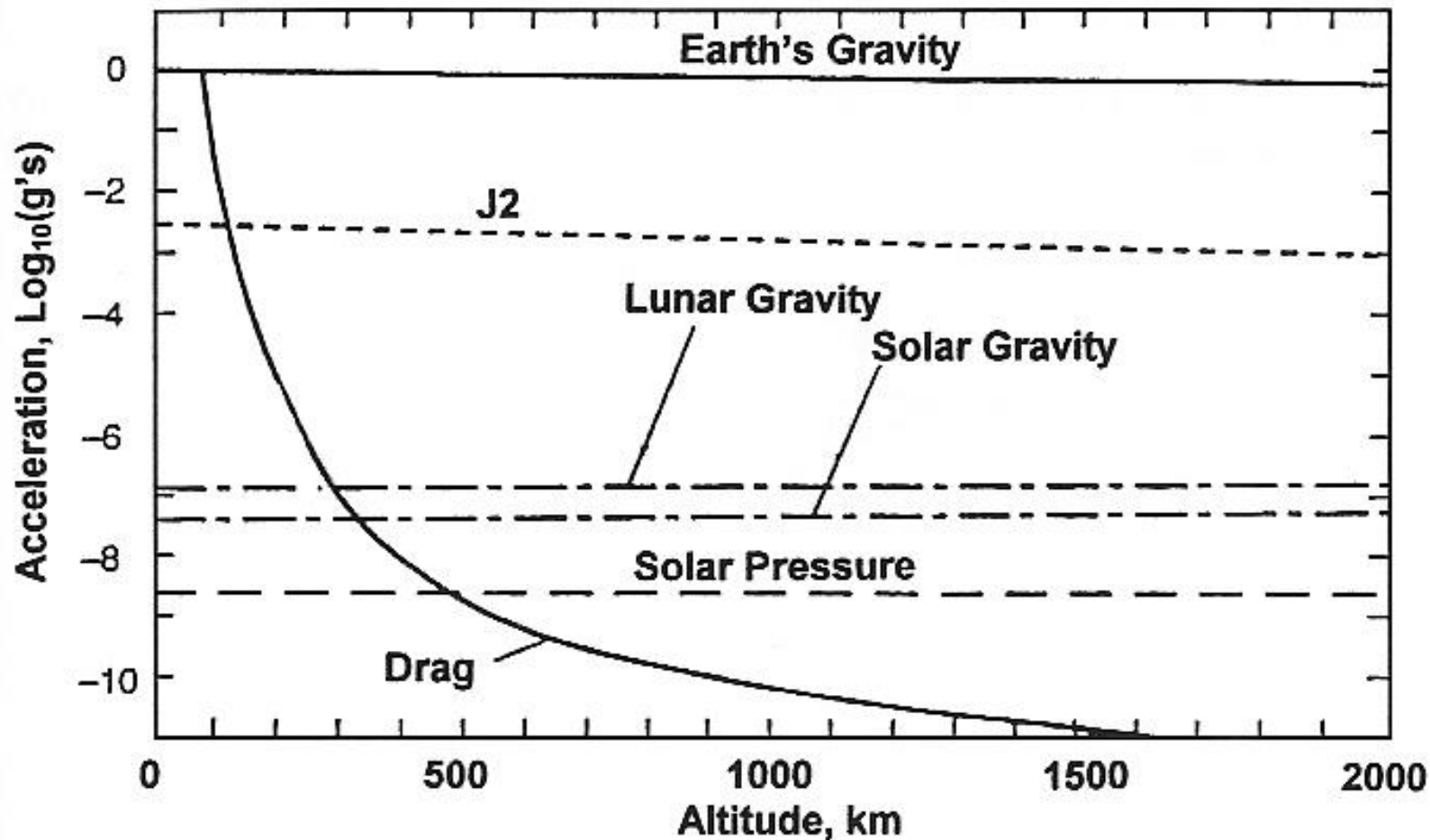
$$e = \frac{r_a}{a} - 1 = \frac{r_a - r_p}{r_a + r_p}$$

- r_a r_p : apogee and perigee of the orbit
- Note that the eccentricity for a circular orbit is zero



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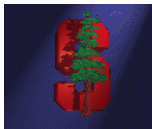
Orbital Perturbations



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List of Common Earth Orbits

- Geosynchronous Orbit (GEO):
 - Period equal to one sidereal day
 - Spacecraft stationary as observed from earth
 - Distance from the center of the earth: 42,164.2 km
 - Orbital Velocity: 3.07 km/sec
 - Telecommunication satellites
 - Difficult orbit to launch a payload
 - High energy orbit
 - Requires plane change to 0 degree inclination
 - Parking orbit (Hohmann transfer) is used to achieve the mission
- Geosynchronous Transfer Orbit (GTO):
- Low Earth Orbit (LEO):
 - Lower limit 200 km due to drag considerations
 - Manned spacecraft: 200 km to 600 km
 - Atmospheric/Earth research, surveillance, astronomy
 - Altitudes up to 1000 km can be achieved without a Hohmann transfer
- Sun-Synchronous Orbit:
 - Provides constant sun angle to observe Earth
 - Critical for surveillance satellites
 - Retro orbits
 - For example: the orbital inclination for a 500 km orbit is 97.4 degrees.



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Orbital Launch Mission Delta V

- Propulsion performance can be conveniently represented by the ideal Delta V delivered.

- Rocket equation:

$$\Delta V = I_{sp} g_o \cos(\alpha_{TVC}) \ln\left(\frac{M_i}{M_{bo}}\right)$$

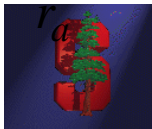
α_{TVC} : Thrust Vector Control Angle (Thrust velocity vector)

M_i : Initial Mass

M_{bo} : Burn out Mass

- Thus it is convenient to express the mission requirement in terms of Delta V. We call this the required Delta V. In practice the delivered and required Delta Vs should be matched.
- Consider the mission of launching a payload into a circular orbit. Here are the important mission requirements
 - Orbital altitude, h
 - Orbital inclination, i
 - Equatorial orbit: zero degrees of inclination
 - Polar orbit: 90 degrees of inclination
 - Launch latitude
- Let's consider the components of the Delta V requirement
 - Orbital velocity (Kinetic energy component) (reversible)

$$V_{co} = \sqrt{\frac{\mu}{R_o + h}}$$



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Orbital Launch Mission Delta V

- Orbital potential energy (reversible)
- Must convert the potential energy into a Delta V number
- Use the following relation (Derivation involves energy arguments)

$$\Delta V_{poten} = \sqrt{\frac{\mu}{R_o}} \sqrt{2 - \frac{R_o}{R_o + h}} - \sqrt{\frac{\mu}{R_o + h}}$$

- In the linear form (for $h \ll R_o$)

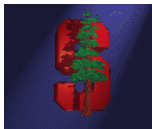
$$\Delta V_{poten} \cong \frac{h}{R_o} \sqrt{\frac{\mu}{R_o}}$$

- Gravity Loss (irreversible)

$$\Delta V_{grav} = \left(\int_0^{t_b} g \sin(\gamma) dt \right)_{irrev}$$

- Drag loss (irreversible)

$$\Delta V_{drag} = \int_0^{t_f} \frac{D}{M} dt$$



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Derivation of the Delta V Due to Potential Energy

- Goal: Calculate the Delta V requirement difference between two orbits at different altitudes
- Define a reference mission with a circular orbit at an altitude of r_{ref}

$$V_{ref} = \sqrt{\frac{\mu}{r_{ref}}} \quad \varepsilon_{ref} = \frac{V_{ref}^2}{2} - \frac{\mu}{r_{ref}}$$

- Different altitude mission with circular orbit:

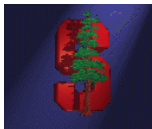
$$V_2 = \sqrt{\frac{\mu}{r_2}} \quad \varepsilon_2 = \frac{V_2^2}{2} - \frac{\mu}{r_2}$$

- Fly to the reference altitude attain a transitional velocity such that (match energies)

$$\varepsilon_2 = \varepsilon_{trans} \quad \frac{V_{trans}^2}{2} - \frac{\mu}{r_{ref}} = \frac{V_2^2}{2} - \frac{\mu}{r_2}$$

- Solve for the orbital velocity V_{trans}

$$\frac{V_{trans}^2}{2} = \frac{\mu}{r_{ref}} + \frac{V_2^2}{2} - \frac{\mu}{r_2} = \frac{\mu}{r_{ref}} - \frac{\mu}{2r_2}$$



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Derivation of the Delta V Due to Potential Energy

- The transfer velocity becomes

$$V_{trans} = \sqrt{\frac{2\mu}{r_{ref}} - \frac{\mu}{r_2}}$$

- If the altitude of the launch site is taken to be the reference orbit (h_{laun}) and the orbit 2 as the target orbit (h)

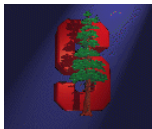
$$V_{trans} = \sqrt{\frac{2\mu}{R_o + h_{laun}} - \frac{\mu}{R_o + h}}$$

- The Delta V component due to potential energy can be written as

$$\Delta V_{poten} = V_{trans} - V_{co} = \sqrt{\frac{2\mu}{R_o + h_{laun}} - \frac{\mu}{R_o + h}} - \sqrt{\frac{\mu}{R_o + h}}$$

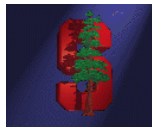
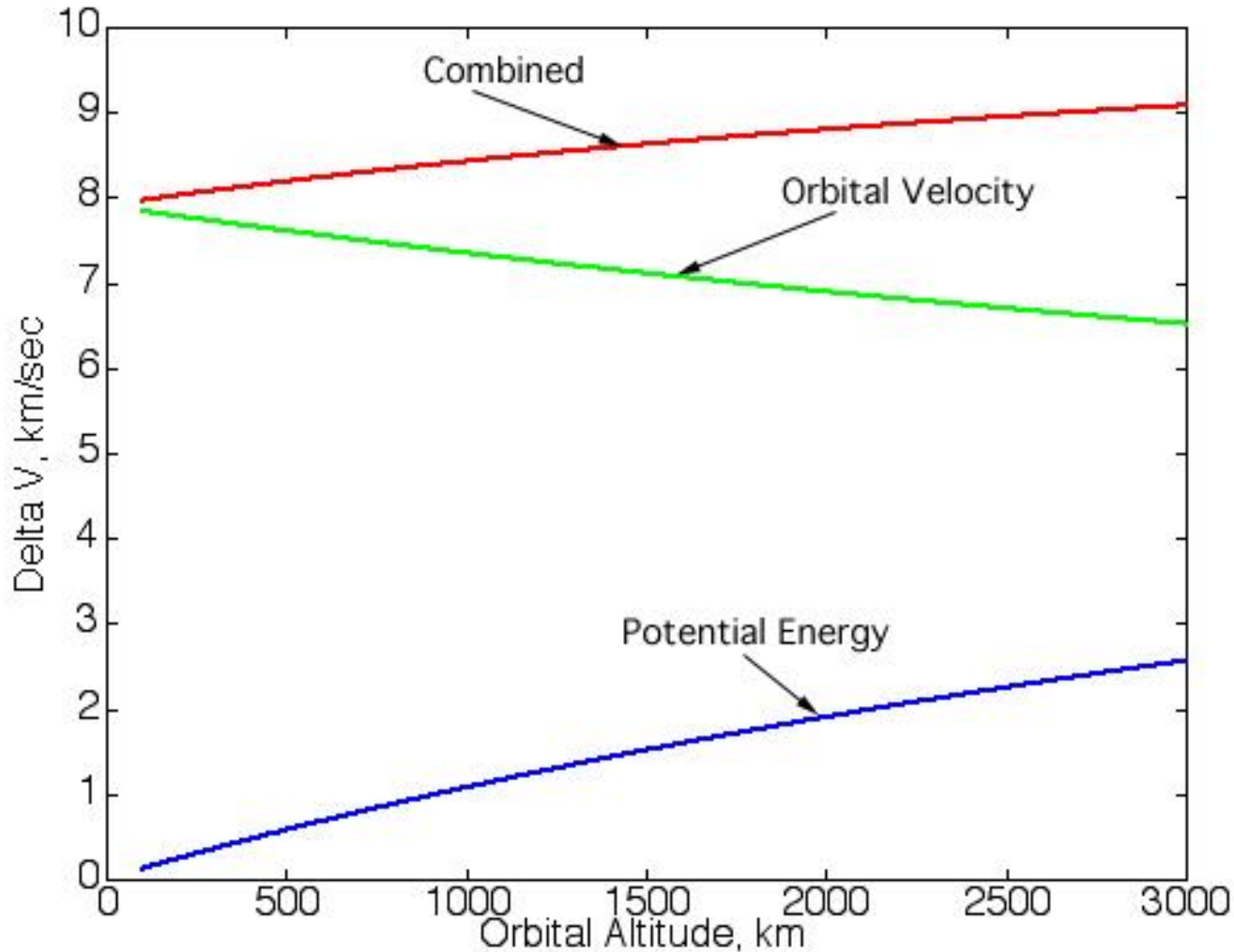
- If the launch altitude is small

$$\Delta V_{poten} = V_{trans} - V_{co} = \sqrt{\frac{\mu}{R_o}} \sqrt{2 - \frac{R_o}{R_o + h}} - \sqrt{\frac{\mu}{R_o + h}}$$



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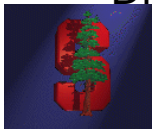
Orbital Launch Mission Delta V – Reversible Terms



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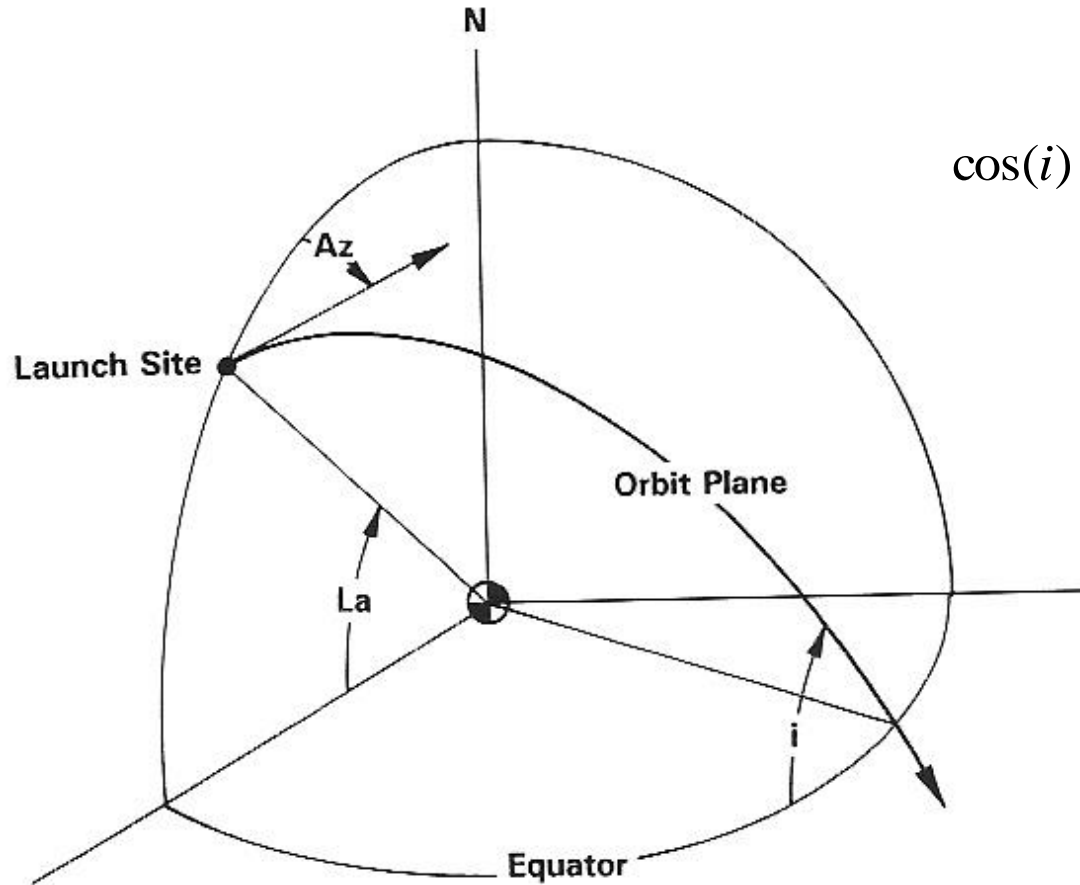
Orbital Launch Mission Delta V

- Typical values for a LEO mission (Low Earth Orbit)
 - Assume an orbital altitude of 400 km
 - Assume polar launch to polar orbit (no effect of Earth's rotation)
 - Note that mean equatorial radius of earth is 6378.14 km
 - Orbital velocity: 7.67 km/sec
 - Potential Energy: 0.47 km/sec
 - Gravity Loss: 2.2 km/sec
 - Drag Loss: 0.1 km/sec
 - Total Delta V requirement: **10.44 km/sec**
- Note that the important components of the delta V are the actual orbital velocity and the gravity loss.
- The drag loss is negligible
- The combination of the orbital velocity and orbital potential energy (which is the orbital specific energy) is a reversible component (conservative force field)
 - For high orbits potential energy is important
 - For LEO type orbits kinetic energy dominates
- Gravity loss is different from the potential energy gain. It is an irreversible term. This term has paramount importance in the mechanics of launching a rocket into the orbit and will be discussed in detail
- The exact value of the gravity loss depends on the specifics of the trajectory flown.
- Drag loss is also irreversible. Exact value also depends on the trajectory and vehicle.

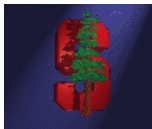


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Effects of Earth's Rotation



$$\cos(i) = \cos(La) \sin(Az)$$



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Effects of Earth's Rotation

- From trigonometry

$$\cos(i) = \cos(La) \sin(Az)$$

- Velocity increment due to Earth's rotation

$$\Delta V_{rot} \cong V_E \sin(Az) = V_E \frac{\cos(i)}{\cos(La)}$$

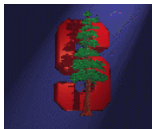
- Earth's rotation

$$V_E = V_E^e \cos(La) \quad V_E^e = 483.8 \text{ m/sec}$$

- Results in

$$\Delta V_{rot} \cong V_E^e \cos(i)$$

- Note that $i \geq La$
- For launching (easterly) into an orbit with an inclination of 40 degrees the delta V boost due to the rotation of the Earth is 0.36 km/sec ($La < 40$ degrees)
- This reduces the mission Delta V requirement for the example case to 10.08 km/sec (a 3.5 % reduction)
- For best boost from the rotation effect, launch easterly from a low latitude launch site
- The retro (westerly) orbits are more difficult due to the rotation effect

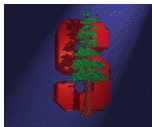


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Launch Capability of US Launch Sites



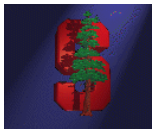
Range Safety Determines the Allowable Range of Angles



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A List of Launch Sites

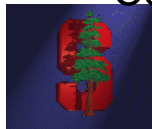
Launch site	Country	Latitude	Longitude
ETR	United States	28° 30' N	80° 33' W
WTR	United States	34° 36' N	120° 36' W
Wallops Island	United States	37° 51' N	75° 28' W
Kourou	Europe, ESA	5° 32' N	52° 46' W
San Marco	Italy	2° 56' S	40° 12' E
Plesetsk	Soviet Union	62° 48' N	40° 24' E
Kapustin Yar	Soviet Union	48° 24' N	45° 48' E
Tyuratam (Baikonur)	Soviet Union	45° 54' N	63° 18' E
Thumba	UN/India	8° 35' N	76° 52' E
Sriharikota	India	13° 47' N	80° 15' E
Shuang-Ch'Eng-Tzu	China	40° 25' N	99° 50' E
Xichang	China	28° 06' N	102° 18' E
Tai-yuan	China	37° 46' N	112° 30' E
Wuzhai	China	38° 35' N	111° 27' E
Kagoshima	Japan	31° 14' N	131° 05' E
Osaki	Japan	30° 24' N	130° 59' E
Takesaki	Japan	30° 23' N	130° 58' E
Woomera	Australia/U.S.	31° 07' S	136° 32' E
Yavne	Israel	31° 31' N	34° 27' E



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Ideal Delta V

- Mass categories in a rocket
 - Payload mass: Mass that must be delivered to the orbit
 - Structural mass: Mass of propulsion system components (other than propellant), rocket vehicle structures, control systems, avionics, electric system etc (some of this mass is also inserted into the orbit)
 - Propellant mass: Mass expelled to produce the thrust force
- Mass definitions:
 - Gross mass: Combined mass before firing
 - Burn out mass: Vehicle mass at the time of burn out
- Ideal Delta V delivered by the propulsion system must be matched to the required Delta V for the specific mission.
- For a single stage rocket
 - Delta V: 10.08 km/sec
 - Delivered Isp: 350 sec
 - Mass ratio can be calculated for the rocket equation to be 18.8
 - This allows for only 5.3% for payload and structures
 - Very difficult to get a positive payload with today's rocket technology
- Staging is universally used in all orbital launch systems
- Staging reduces the energy spent to accelerate the structural mass
- Unused structural mass is dropped at certain stages of the flight
- Continuously staged rocket is the most efficient solution, but not feasible



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Staging Analysis -Serial Staging

- Stages fired sequentially
- Consider a n stage rocket
- Define the normalized stage mass, m_i , based on the gross mass of the i^{th} stage (M_i) and the payload mass, P

$$m_i = M_i / P$$

- Define the non-dimensional stack mass, s_i . This is the combined mass of all the stages stacked on stage $i-1$.

$$s_i = 1 + \sum_{j=i}^n m_j$$

- Define the structural mass fraction, β_i for stage i

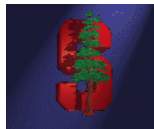
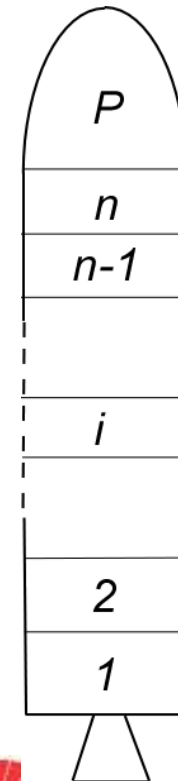
$$\beta_i \equiv \frac{M_i - M_{pi}}{M_i}$$

- Define mass ratio, μ_i for stage i

$$\mu_i = \frac{s_{i+1} + m_i}{s_{i+1} + \beta_i m_i}$$

- Based on these definitions the total delivered delta V becomes

$$\Delta V_{del} = \sum_{i=1}^n I_{spi} g_o \ln(\mu_i)$$



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Staging Analysis

- The stack mass for stage $i+1$ can be solved from the definition of mass ratio

$$s_{i+1} = \frac{m_i [1 - \beta_i \mu_i]}{\mu_i - 1}$$

- Note that

$$s_i = s_{i+1} + m_i$$

- The ratio of stack mass for the adjacent stages can be written as

$$\frac{s_i}{s_{i+1}} = \frac{\mu_i [1 - \beta_i]}{1 - \beta_i \mu_i}$$

- Also note that

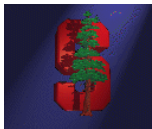
$$s_1 = \prod_{i=1}^n \frac{s_i}{s_{i+1}}$$

- And for the non-dimensional overall vehicle gross mass, s_1

$$s_1 = \prod_{i=1}^n \frac{\mu_i [1 - \beta_i]}{1 - \beta_i \mu_i}$$

- Or in the log form

$$\ln(s_1) = \sum_{i=1}^n [\ln(\mu_i) + \ln(1 - \beta_i) - \ln(1 - \beta_i \mu_i)]$$



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Staging Analysis

- Optimization Problem:
 - Minimize gross weight (s_1) subject to the constraint of total delivered delta V.
- This can be solved using the method of Lagrange multipliers
- The solution reduces down to the solution of the following nonlinear algebraic equations ($n+1$ equations and $n+1$ unknowns) - λ is the Lagrange multiplier

$$\mu_i = \frac{1 + \lambda I_{spi}}{\lambda \beta_i I_{spi}} \quad \Delta V_{del} = \sum_{i=1}^n I_{spi} g_o \ln(\mu_i)$$

- If all of the n stages have identical specific impulses and structural mass fractions

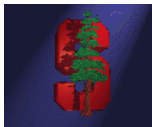
$$\frac{s_i}{s_{i+1}} = \frac{\mu [1 - \beta]}{1 - \beta \mu} \quad \mu = e^{\Delta V_{del} / n I_{sp} g_o}$$

- Example

- Three stage system, $n=3$
- Delta V: 10.08 km/sec
- Isp: 350 sec
- Structural mass fraction of 0.10

$$\mu = 2.66 \quad \frac{s_i}{s_{i+1}} = 3.26$$

- The gross mass payload ratio (s_1) for the overall system is 10.6. Compare to SSTO value of 18.87 (which also includes the structures weight)



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Two Stage System – Different Formalism

- Consider a two stage system
- The total delivered delta V can be written as

$$e^{\Delta V_{del}/I_{sp1}g_0} = \left(\frac{P + M_{g2} + M_{g1}}{P + M_{g2} + \beta_1 M_{g1}} \right) \left(\frac{P + M_{g2}}{P + \beta_2 M_{g2}} \right)^{I_{sp2}/I_{sp1}}$$

- In terms of nondimensional variables

$$e^{\Delta V_{del}/I_{sp1}g_0} = \left(\frac{1 + \alpha}{\alpha + \beta_1 + \theta (1 - \beta_1)} \right) \left(\frac{\alpha + \theta}{\theta + \alpha \beta_2 + \alpha \theta (1 - \beta_2)} \right)^{I_{sp2}/I_{sp1}}$$

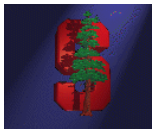
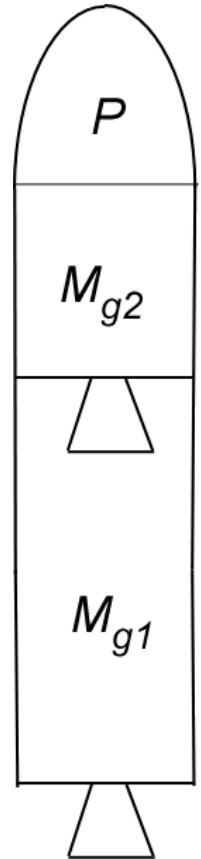
- Where

$$\alpha \equiv \frac{M_{g2}}{M_{g1}} \quad \theta \equiv \frac{P}{P + M_{g2} + M_{g1}}$$

- Optimization problem becomes

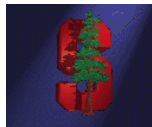
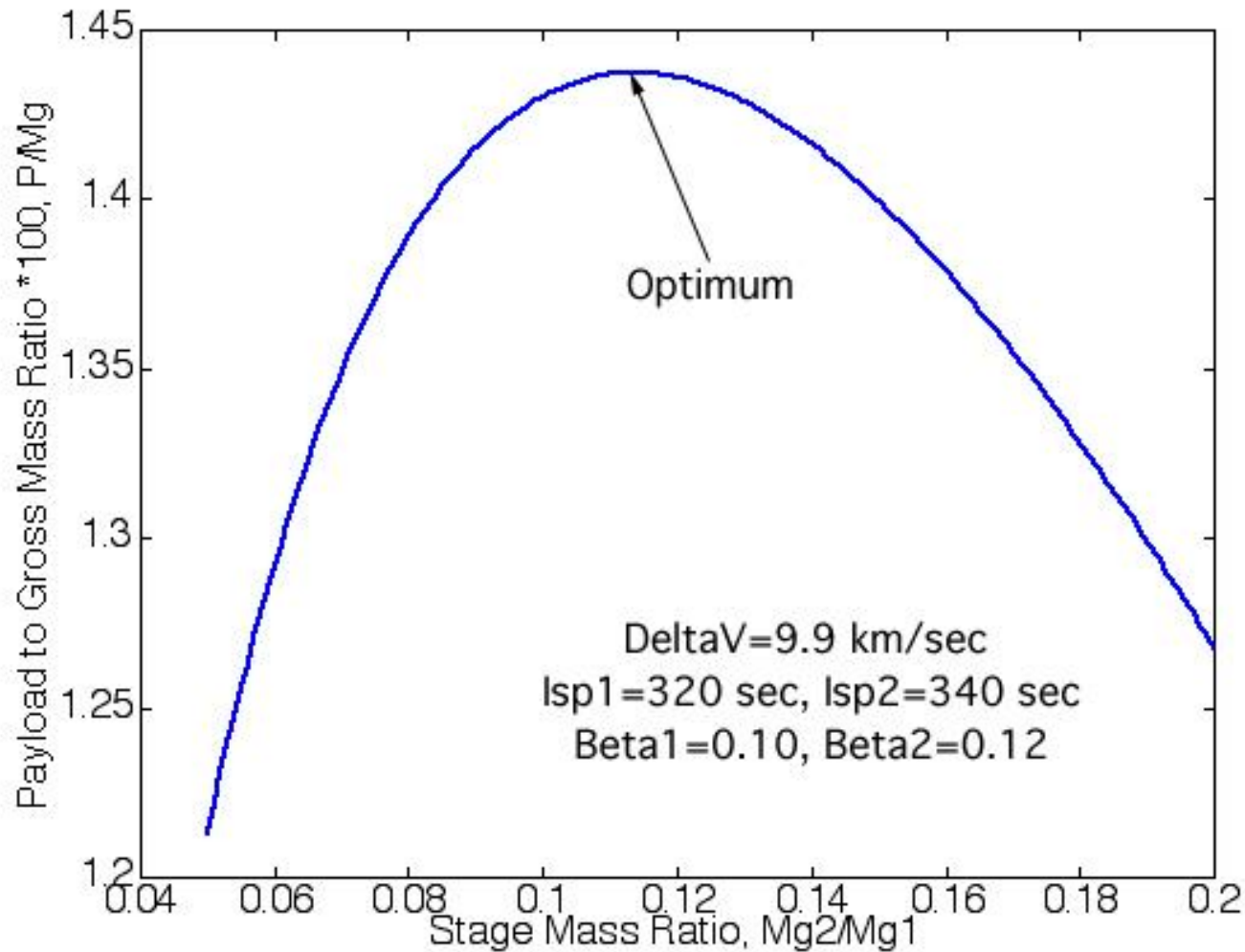
$$\text{Max } \theta \quad \text{subject to } \Delta V_{del} = \text{cons.}$$

α



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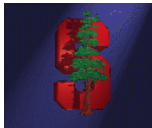
Two Stage System Optimization (Delta V=9.90 km/sec)



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Staging –Parallel Stages

- Stages burned simultaneously. Most commonly used in the form of strap on boosters
- Examples:
 - Systems with large segmented solid boosters
 - Titan IV, Shuttle, Ariene V
 - Systems using smaller solid rockets
 - Delta and Atlas Heavy
 - ATK (GEM series), Aerojet
- Strap on boosters allow for the capability of changing the payload without modifying the core of the vehicle
- Core is designed for high efficiency operation at high altitudes. It cannot produce the large thrust forces needed for the lift off
- Use the relatively inefficient boosters to lift off the pad
- Boosters typically have low expansion ratio nozzles tailored for low altitudes
- High Isp systems are used for upper stages
- Structural Mass Fraction
 - Typically decreases with increasing stage mass
 - 10% is a typical number for most modern systems
 - In general, solids have better structural mass fraction values compared to liquids (especially H₂/LOX systems), 9 % is a typical value
 - Structural mass fraction for liquid and hybrid systems depends primarily on
 - Feed system: pump fed or pressure fed
 - Chamber pressure



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Delta V Budget-Preliminary Design Logic

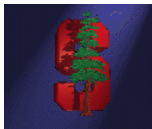
- It is useful to estimate the Delta V's before starting with the trajectory integration
- Define the mission (Final state)
 - Target orbit altitude
 - Target orbit inclination
 - Target orbit eccentricity
 - Payload to the orbit
- Launch conditions (Initial state)
 - Launch latitude
 - Launch azimuth angle
 - Launch altitude
- Estimate the required delta V

$$\Delta V_{req} = \Delta V_{co} + \Delta V_{poten} + \Delta V_{grav} + \Delta V_{drag} - \Delta V_{rot}$$

- Match the required delta V to the ideal delivered delta V to predict the vehicle gross mass

$$\Delta V_{req} = \Delta V_{del} = \sum_{i=1}^n I_{spi} g_o \cos(\alpha_{TVCi}) \ln(\mu_i)$$

- Note that the stage mass distribution results from the optimization process based on assumed structural mass fractions and Isp's for all stages.



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Trajectory Equations

- An accurate analysis requires the integration of the trajectory equations
 - The gravity and drag losses depend on the details of the trajectory
 - Launch constraints can only be introduced when you fly the system into the orbit
 - Maximum acceleration
 - Maximum dynamic pressure
 - Maximum heating
- The trajectory equations
 - For a 2 DoF system
 - Flat Earth
 - Non rotating Earth (polar launch)

$$\frac{dV}{dt} = \frac{T \cos(\alpha_{TVC})}{M} - \frac{D}{M} - \frac{\mu \sin(\gamma)}{r^2}$$

$$\frac{d\gamma}{dt} = \frac{V \cos(\gamma)}{r} + \frac{T \sin(\alpha_{TVC})}{VM} + \frac{L}{VM} - \frac{\mu \cos(\gamma)}{V r^2}$$

$$\frac{dy}{dt} = V \sin(\gamma) \quad \frac{dx}{dt} = V \cos(\gamma)$$

$$\frac{dM}{dt} = - \frac{T}{I_{sp} g_o}$$

V : Velocity

M : Mass

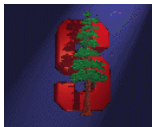
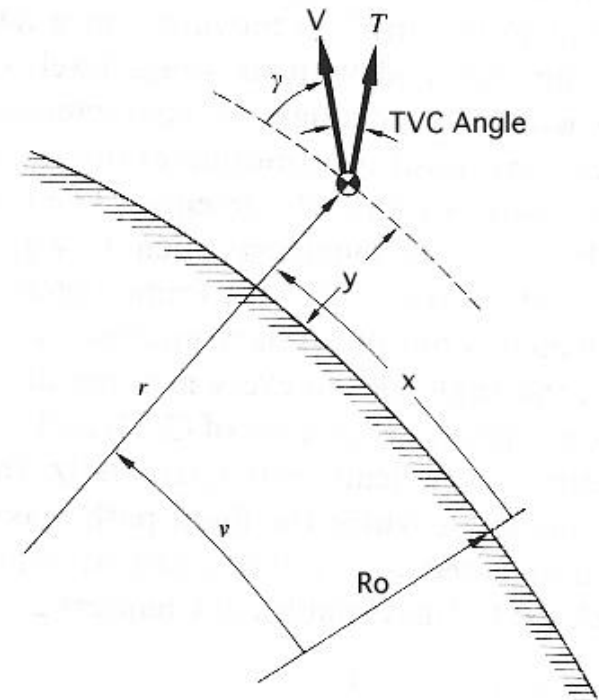
r : Distance from Center of Planet

γ : Trajectory angle (wrt local horizon)

T : Thrust Force

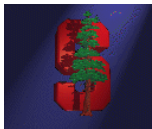
D, L : Drag and Lift Forces

y, x : Altitude and Down Range



Trajectory Equations

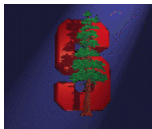
- Note that the state vector includes
 - Velocity, trajectory angle, altitude, range, vehicle mass
- The force components are
 - Thrust, Drag, Lift
 - Lift and drag are calculated from instantaneous dynamic pressure and aerodynamic properties of the vehicle along with the angle of attack
 - The thrust variable is fed from the propulsion system calculation module
- The input variables are
 - TVC angle, thrust profile, angle of attack (for the atmospheric part of the flight)
- Integration process
 - Start from a specified initial condition
 - Initial velocity, trajectory angle, altitude, range and vehicle mass
 - Stop at the target orbit
 - Velocity: orbital velocity at the specified altitude
 - Altitude: Target orbit altitude
 - Trajectory angle: 0.00 degrees
 - Vehicle Mass: No propellants left
 - Flying into the target orbit requires
 - Neighboring optimal control/Dynamic programming
 - Iterative process



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Trajectory Integration Process

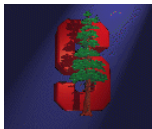
- No general analytical solutions for the trajectory equations with the following exceptions
 - Constant acceleration solution
 - Small velocity solution
 - Specified trajectory angle variation
- Must integrate numerically
 - Behaves very well other than the region around zero velocity
 - Use explicit time marching techniques such as RK4
- Integrate the trajectory equation for each segment. For example
 - Flight before firing
 - Stage one powered flight
 - Coast phase
 - Stage two powered flight into orbit
- Match the state vectors at segment interfaces
 - Note that mass is discontinued due to staging
- Iteration to converge on the desired orbital state
 - Orbital altitude and velocity (orbital energy): Total propellant mass from the Delta V match
 - Eccentricity (finite trajectory angle and rate at orbital altitude): TVC of stages, burn times and initial launch angle



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Launch Trajectory Design Issues

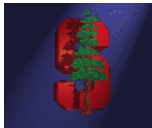
- The goal is to minimize the Delta V requirement for the mission
- The Delta V's due to orbital velocity and potential energy can not be changed
- Only the gravity loss and the drag loss can be altered
- Since the drag loss is fairly small, gravity loss is the only component that can be altered by tailoring the trajectory.
- Problem becomes the minimization of the gravity loss subject to the constraints
- To minimize the gravity loss
 - Burn the propellant in a very short period of time early in the flight
 - Acceleration and Q_{max} constraints are violated
 - Rocket systems become too heavy (large flow rates-large valves nozzles etc)
 - Accelerate parallel to the gravitational force field (Parallel to the surface of the planet)
 - Not feasible if there exists an atmosphere
 - Also must clear the obstacles
- Launch from an airless body
 - In the initial launch segment have a vertical component to gain altitude to clear the mountains
 - When the required altitude is reached (perigee) accelerate horizontally to the required perigee orbital velocity such that apogee has the altitude of the target circular orbit
 - Burn again to circularize at the apogee.



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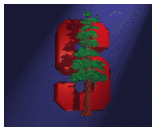
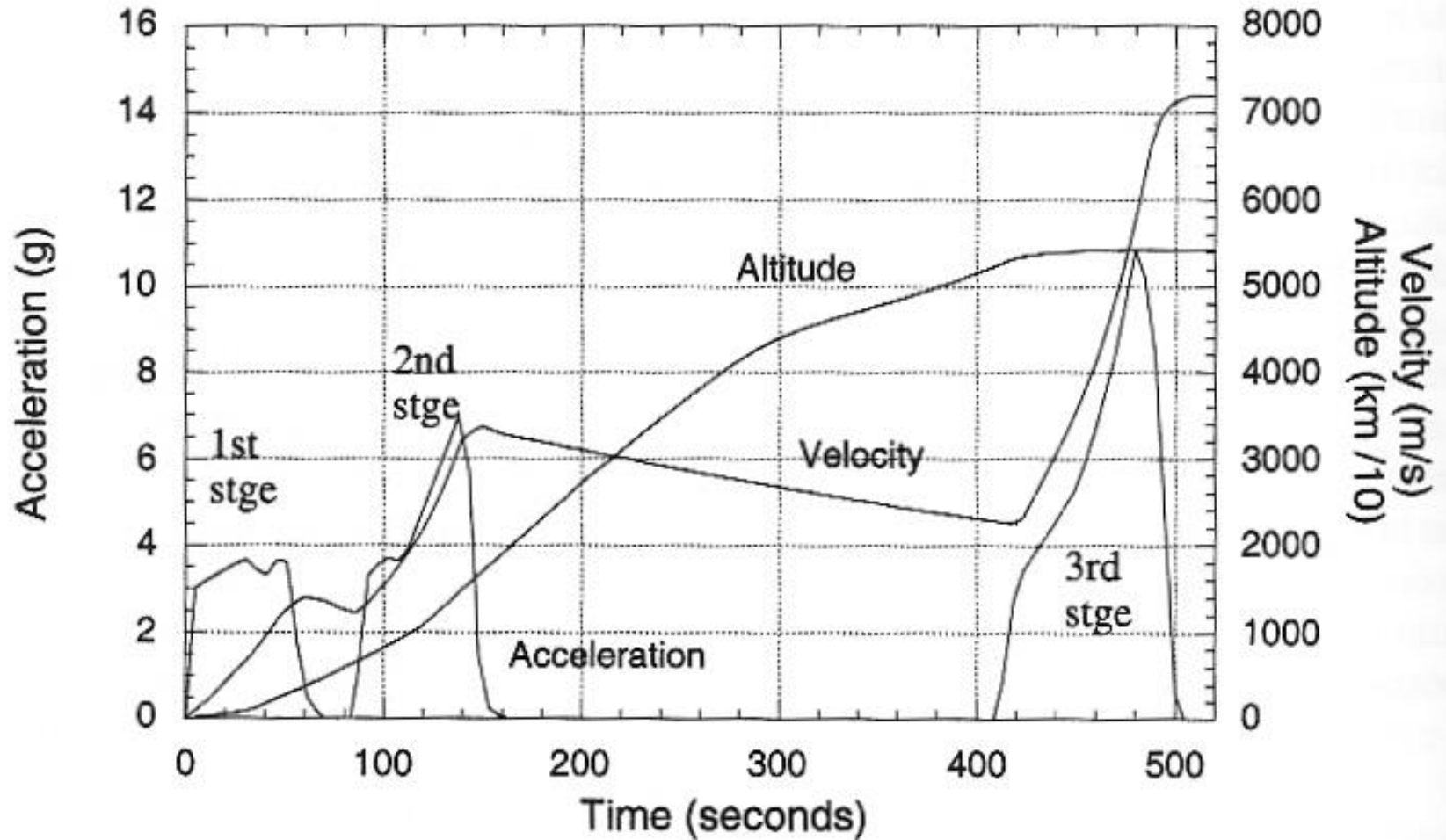
Launch Trajectory Design Issues

- Earth Launch Trajectories:
 - Must clear the atmosphere (Q_{max})
 - Segment 1: Vertical Flight
 - Clear the dense part of the atmosphere
 - Large gravity loss due to large trajectory angle
 - Balance Q_{max} and gravity loss to determine the duration of this segment
 - Segment 2: Gravity turn
 - The vehicle naturally turns due to the effect of gravity (even with zero TVC and zero lift)
 - Or controlled turn: apply the optimal amount of TVC and/or lift
 - Segment 3: Vacuum trajectory
 - Accelerate horizontally
 - Maximize velocity under the acceleration constraint
 - Stage 4: Orbital Injection
 - Once the orbital altitude is reached horizontal acceleration to the orbital velocity
 - Injection is not final in many cases: Go into a “Parking Orbit”
- “Dog-Leg” Maneuver
 - Over shoot your target altitude with a short burn to minimize the gravity loss
 - Accelerate down due to gravity and thrust
- Coasting:
 - Thrust free flight between stage firings
 - Constant energy surface (trading altitude for velocity)
 - Improves the gravity loss significantly especially for high altitude orbits
 - Reduces the burn time for stages and also reduces the trajectory angle for the upper stage



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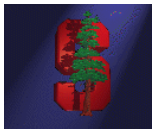
Trajectory – Mu-3-S II Launch System (Japan)



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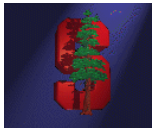
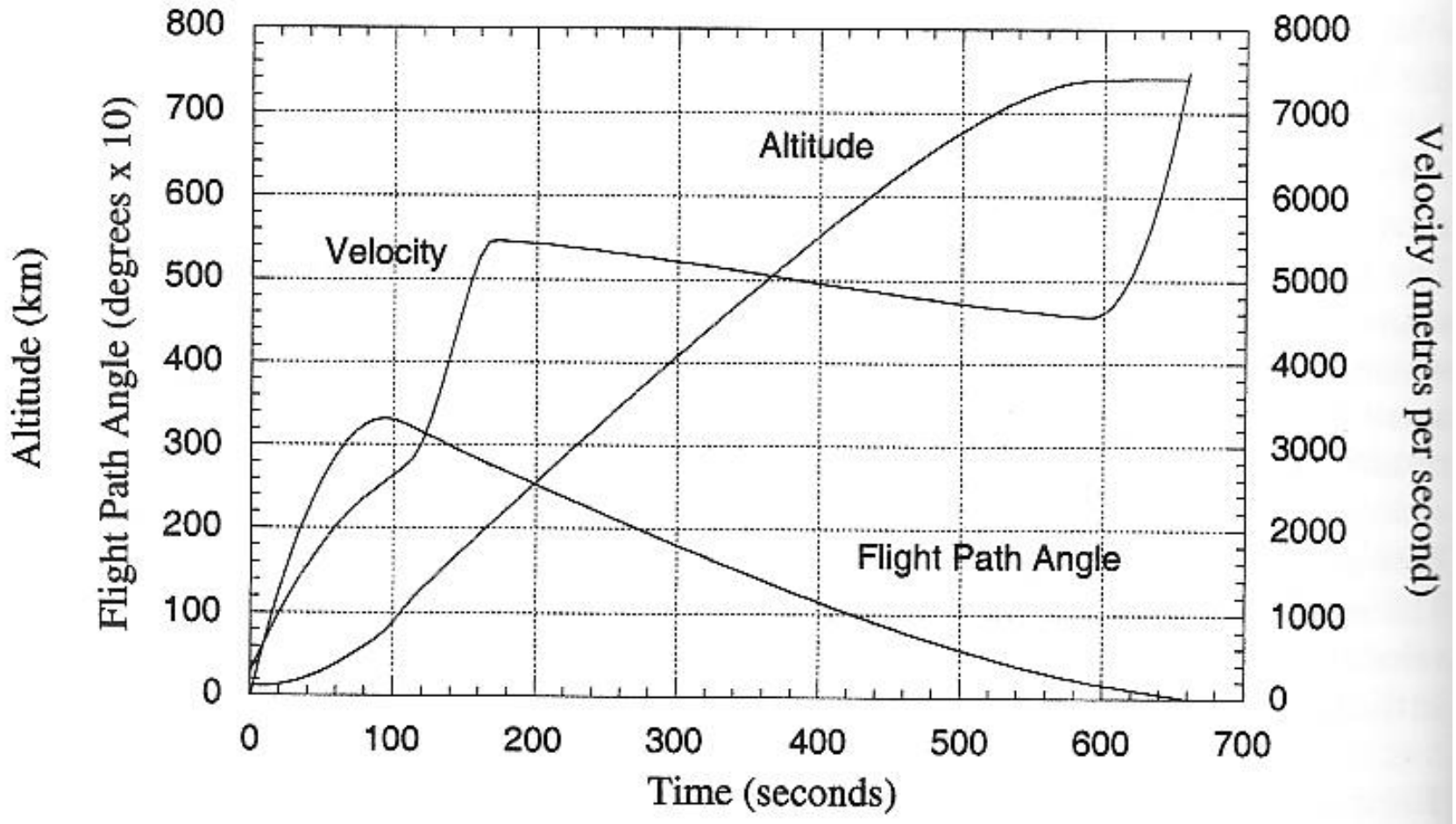
Air Launch

- Use an aircraft or balloon to reach a high altitude before the orbital rocket vehicle is launched
- Advantages of the Air Launch
 - Low air density
 - Reduced dynamic pressure and heat loading. One can minimize the gravity loss more effectively
 - Flexibility in selecting an overall vehicle architecture. Vehicle configuration can be less streamlined
 - Low atmospheric pressure allows for higher nozzle area ratios and better Isp performance
 - Reduced delta V requirement. Start at a higher energy state. Gains up to 10 % with subsonic systems
 - Mobile launch platform. Enhanced flexibility in launch latitude and angle (relaxed range safety issues). Increases the range of achievable orbital inclinations
 - Simplified abort modes. Increased safe distance from a populated area.
- Disadvantages of Air Launch
 - High development cost for the aircraft
 - If using an already existing aircraft, the weight and geometrical envelope constraints can be hard to meet
 - For large systems, aircraft operation can be complex and expensive
 - Need high launch frequency to make economical sense. Large flat costs



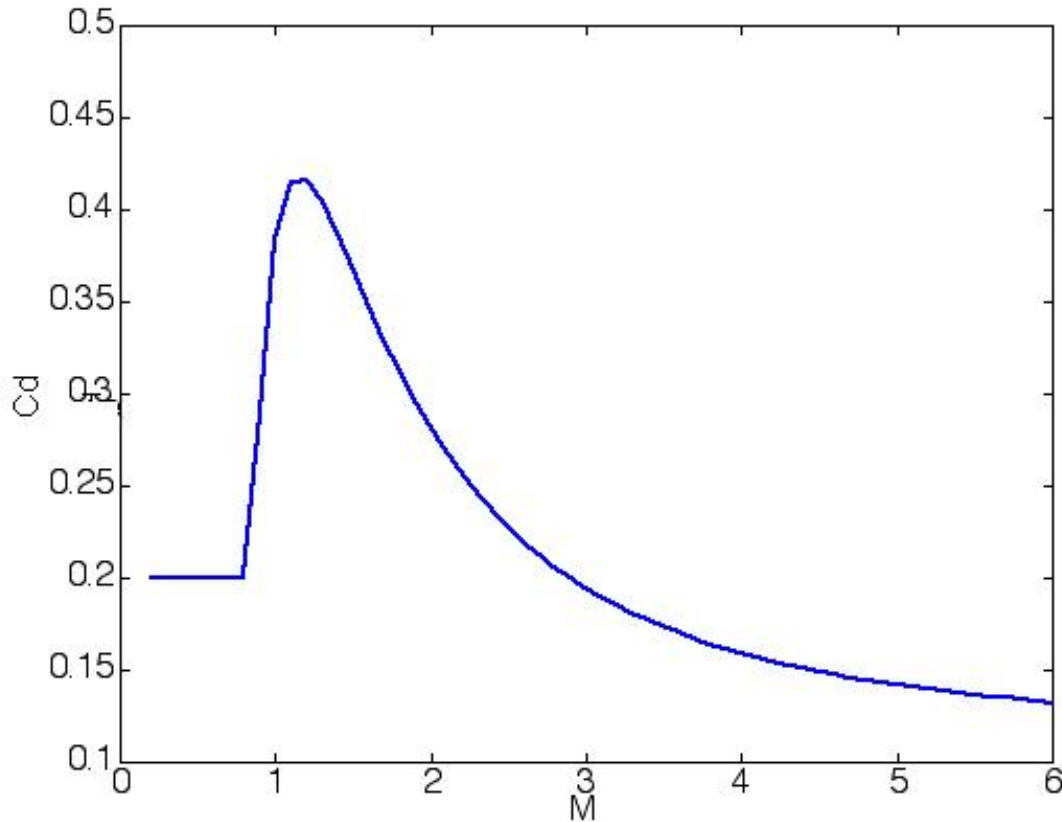
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Trajectory – Pegasus Launch System



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Drag Model



- For systems that are not producing significant lift, the following drag model can be used in the preliminary design process

- For $M < 0.85$

$$C_d = 0.2$$

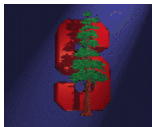
- For $M > 0.85$

$$C_d = 0.11 + \frac{0.82}{M^2} - \frac{0.55}{M^4}$$

- Note that the drag force is based on the maximum cross sectional area and can be calculated using

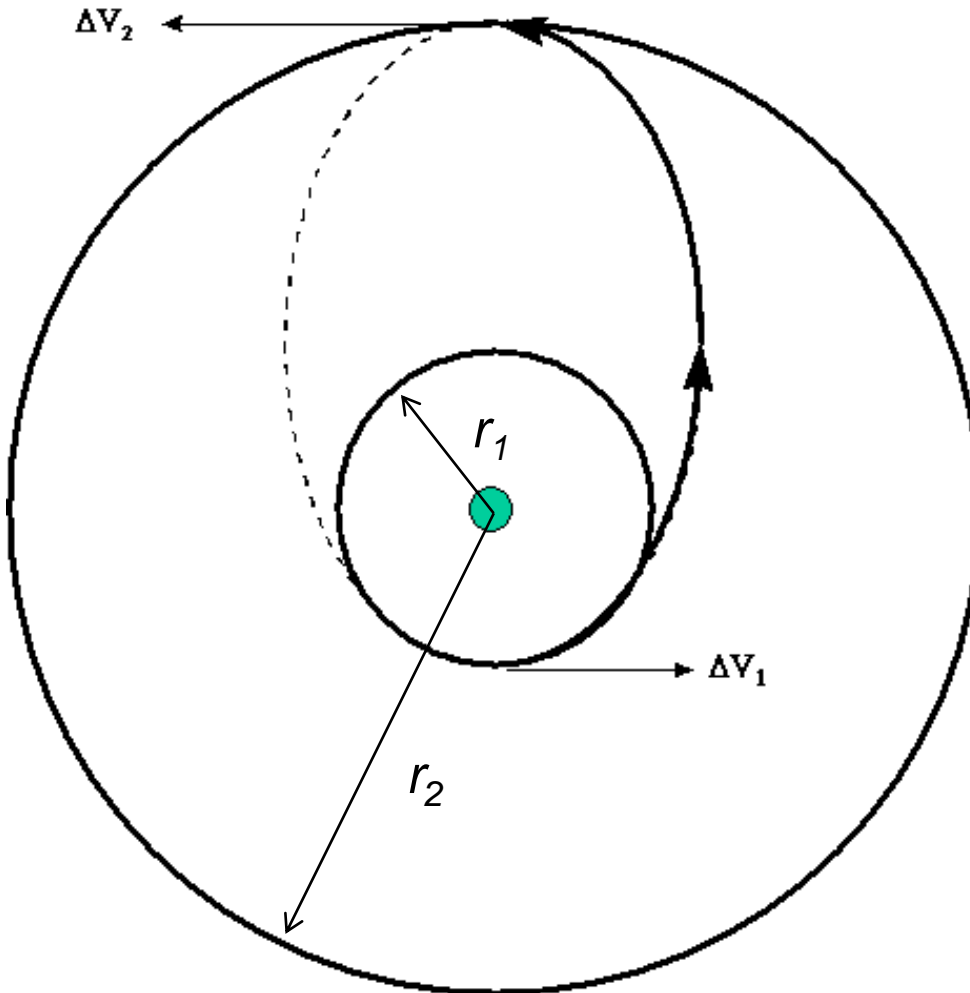
$$D = \frac{1}{2} \rho V^2 C_d S_{ref}$$

- For most launch systems the drag loss is very small compared to the gravity loss



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Orbit Raising - Hohmann Transfer



- Lowest energy transfer (no gravitational losses)
- Total Delta V

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

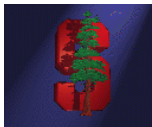
- For circular orbits (initial and final)

$$\Delta V_1 = V_{Tp} - V_{col} = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right)$$

$$\Delta V_2 = V_{co2} - V_{Ta} = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right)$$

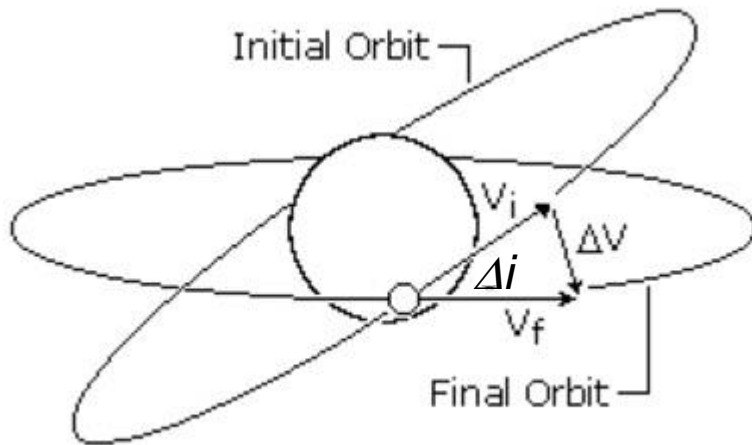
- Similar formulas for transfer between elliptical orbits
- Transfer time

$$t_{trans} = \pi \sqrt{\frac{(r_1 + r_2)^3}{8\mu}}$$



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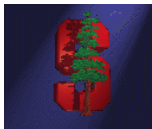
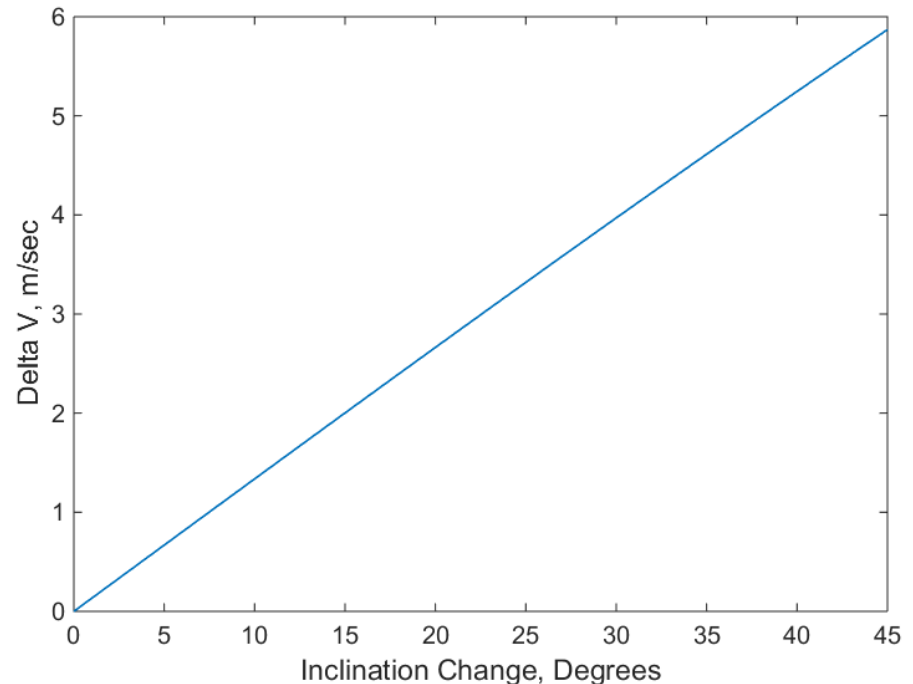
Orbital Inclination Change



- For circular orbits

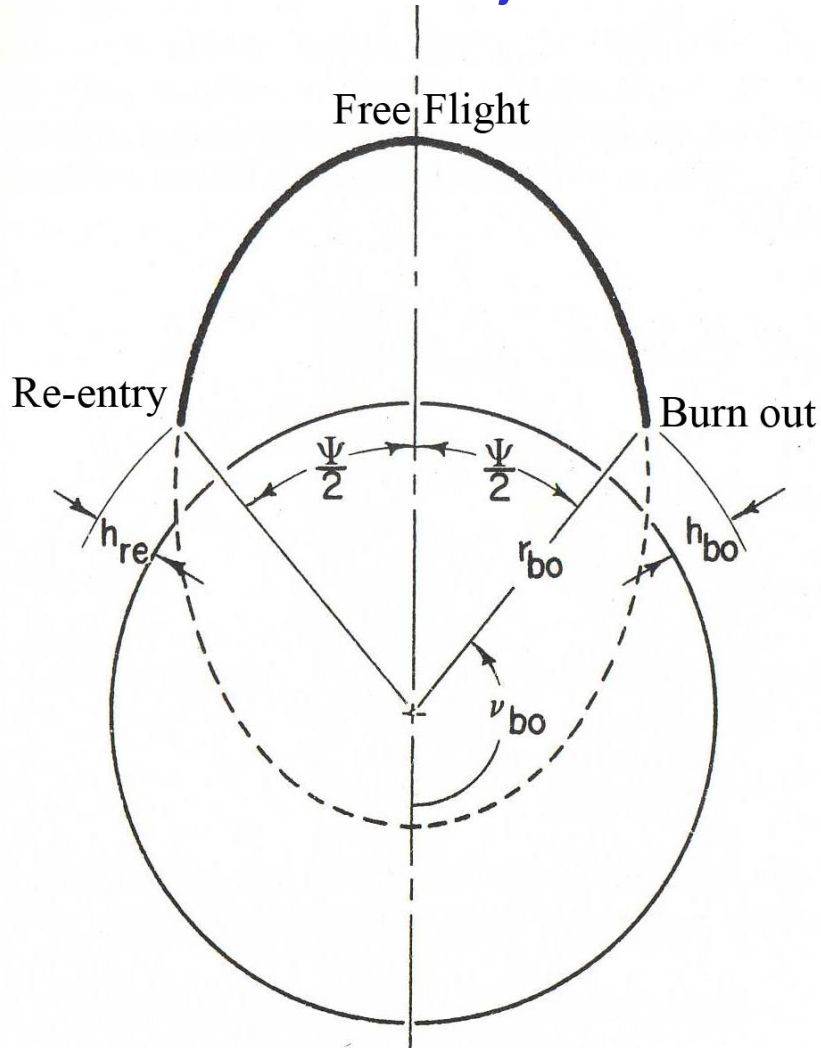
$$\Delta V_{tot} = 2V_{co} \sin\left(\frac{\Delta i}{2}\right)$$

- Very expensive mission
- For a typical LEO orbit a change of 10 degrees require an Delta V of 1.33 km/sec



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Ballistic Missile Trajectories



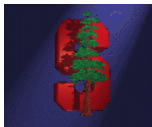
- Objective: Predict the range of a ballistic missile for a given set of burn out conditions
- Assumptions:
 - Non-rotating Earth
 - Burn out and re-entry altitudes are identical
 - Ignore all orbital perturbations
- Only consider the free flight part of the trajectory
 - No drag or lift
- The assumptions lead to the symmetrical trajectory
- Define the nondimensional parameter

$$Q \equiv \frac{V^2 r}{\mu} = \left(\frac{V}{V_{co}} \right)^2$$

- For example

$$Q_{bo} \equiv \frac{V_{bo}^2}{V_{co}^2} = \frac{V_{bo}^2 r_{bo}}{\mu}$$

Ref: Bates, Mueller and White, "Fundamentals of Astrodynamics", Dover Pub.



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Ballistic Missile Trajectories

- Specific energy and semimajor axis

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = \frac{\mu}{r} \left(\frac{Q}{2} + 1 \right) \quad a = \frac{r}{2-Q}$$

- Range equation:

$$r_{bo} = \frac{p}{1 + e \cos(\nu_{bo})} \quad \cos(\nu_{bo}) = \frac{p - r_{bo}}{e r_{bo}}$$

- From geometry

$$\nu_{bo} + \psi/2 = \pi \quad \cos(\psi/2) = -\cos(\nu_{bo})$$

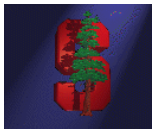
- Thus

$$\cos(\psi/2) = \frac{r_{bo} - p}{e r_{bo}}$$

- Use the definitions of p and e

$$p \equiv \frac{h^2}{\mu} = \frac{r^2 V^2 \cos^2(\phi)}{\mu} = r Q \cos^2(\phi) \quad p = r_{bo} Q_{bo} \cos^2(\phi_{bo})$$

$$e^2 = 1 - p/a \quad e = \sqrt{1 + Q(Q-2)\cos^2(\phi)}$$



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Ballistic Missile Trajectories

- The range equation becomes

$$\cos\left(\frac{\psi}{2}\right) = \frac{1 - Q_{bo} \cos^2(\phi_{bo})}{\sqrt{1 + Q_{bo}(Q_{bo} - 2)\cos^2(\phi_{bo})}}$$

- Or from geometry

$$\sin\left(2\phi_{bo} + \frac{\psi}{2}\right) = \frac{2 - Q_{bo}}{Q_{bo}} \sin\left(\frac{\psi}{2}\right)$$

- Maximum range exists for $Q_{bo} < 1$

$$2\phi_{bo} + \psi/2 = \pi/2 \qquad \phi_{bo}^* = \frac{1}{4}(\pi - \psi^*)$$

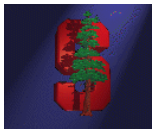
$$\sin(\psi^*/2) = \frac{Q_{bo}}{2 - Q_{bo}} \qquad Q_{bo} = \frac{2 \sin(\psi^*/2)}{1 + \sin(\psi^*/2)}$$

- Time of flight

$$t_{ff} = 2\sqrt{a^3/\mu}(\pi - E_1 + e \sin(E_1))$$

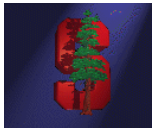
- Where eccentric anomaly is defines as

$$\cos(E_1) = \frac{e - \cos(\psi/2)}{1 - e \cos(\psi/2)}$$



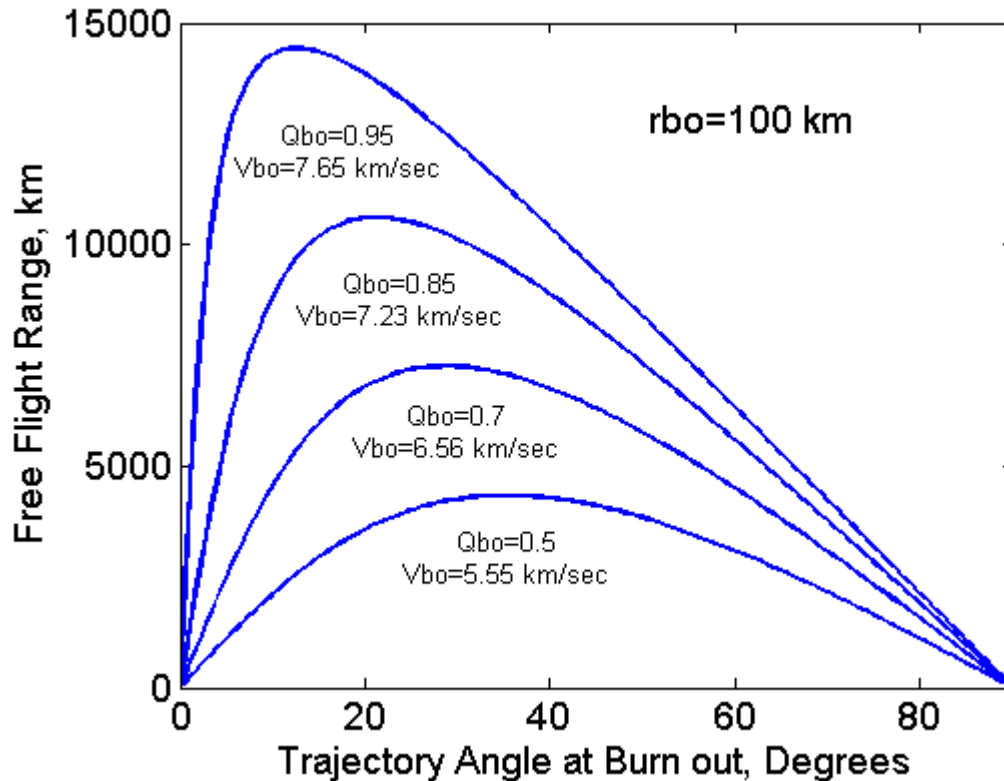
Ballistic Trajectory Possibilities

- For $Q_{bo} < 1$
 - Low energy trajectories
 - Two solutions for range less than the optimal range
 - High trajectory
 - Low trajectory
 - Range is always less than 180 degrees
 - Trajectory for $\psi > 180$ degrees crashes back to Earth
- For $Q_{bo} = 1$
 - One of the solutions is $\phi_{bo} = 0$
- For $Q_{bo} > 1$
 - High energy trajectories
 - $\psi > 180$ degrees is possible
 - Only one solution (low trajectory has negative ϕ_{bo})

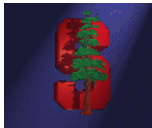


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Ballistic Missile Trajectories



- Optimum exists
- Optimal trajectory angle at burn out decreases with increasing energy
- Range increases with increasing energy
- **Long ranges require delta V's that are close to the orbital values!!!!**
- Note that the re-entry velocity is equal to the burn out velocity
- **For long range systems TPS is critical!!!!**
- Burn out altitude has a relatively small effect on the range



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Ballistic Missile Trajectories – Optimum Solution

- Optimum solution
- $h_{bo}=100$ km
- Free flight time is less than 40 minutes

