AA 284a
Advanced Rocket Propulsion

Lecture 14
Stability of Chemical Rockets

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Fall 2019

Stability of Chemical Propulsion Systems

- **Instability Definition:**
  - Chamber pressure and/or thrust oscillations
  - Smooth combustion: Peak to peak < 5% of the mean (Sutton)
  - Rough combustion: Pressure oscillations in completely random intervals
  - Unstable combustion: More or less organized - activity at discrete frequencies
- **Importance:**
  - Mechanical and thermal loading on the components-may result in the destruction of the propulsion system
  - Performance variations (i.e. regression rate variations due to DC shift) - May result in mission failure
  - Vibration loads on the structures and payload –May result in mission failure
- **Classification of Instabilities Based on Frequency:**
  - Low Frequency Oscillations: 1-400 Hz, L* (solids), chugging, chuffing
  - Intermediate Frequency Oscillations: 400-1000 Hz, Buzzing
  - High Frequency Oscillations: > 1000 Hz, Screaming, Screeching

Stability of Liquid Propulsion Systems

- **Low Frequency:**
  - Chuffing, chugging: Systems coupling
  - Pogo: Vehicle acceleration propellant mass flow rate coupling. Eliminated by inserting gas accumulators into the propellant feed system
- **Intermediate Frequency:**
  - Buzzing: Feed system combustion chamber coupling
  - Not as destructive as the high frequency
- **High Frequency:**
  - Screaming, screeching
  - Related to the acoustic modes of the chamber
    - Longitudinal
    - Traverse: Radial or tangential
  - Most common and most destructive
  - Increases heat transfer rates up to a factor of 10. Causes metal walls to melt
  - Typically the tangential mode is the most destructive one

Stability of Liquid Propulsion Systems

- **Rating:**
  - Introduce a disturbance and check the time required to return to normal operation or if stays unstable check the amplitude of the oscillations
  - Non-directional bomb, directional explosive pulse, directed flow of inert gas
- **Control of instabilities:**
  - Most stability tests must be done in full scale
  - For chugging instabilities: decouple chamber from the feed system by increasing the injector pressure drop
  - For high frequency instabilities
    - Injector face baffles
    - Acoustic energy absorption cavities (Helmholtz resonators)
    - Combustion chamber liners (Helmholtz resonators)
    - Change injector design
    - Injector end is critical in the production of instabilities

Chamber Gas Dynamic Model

- Transient mass balance in the combustion chamber

$$\frac{d\rho V}{dt} = \dot{m}_p - \dot{m}_n$$

- For constant volume

$$V\frac{d\rho}{dt} = \dot{m}_p - \frac{P_c A_{nt}}{c^*}$$

- Ideal gas and isothermal process

$$\frac{V}{RT}\frac{dP_c}{dt} = \dot{m}_p - \frac{P_c A_{nt}}{c^*}$$

- Can be rearranged

$$\frac{dP_c}{dt} + \frac{A_{nt}}{V}\frac{RT}{c^*}P_c = \frac{RT}{V}\dot{m}_p \qquad \frac{dP_c}{dt} + \frac{1}{\tau_c}P_c = \frac{RT}{V}\dot{m}_p$$

- Characteristic chamber filling/emptying time and L* are defined as

$$\tau_c \equiv \frac{L^*}{c^* f(\gamma)} \qquad L^* \equiv \frac{V}{A_{nt}}$$

- Note that if the propellant gas generation rate is pressure dependent, coupling is possible (Solid rocket L* instability)
- Note that this coupling is not possible in a hybrid rocket
- Bulk mode instability – Pressure oscillates uniformly in the chamber

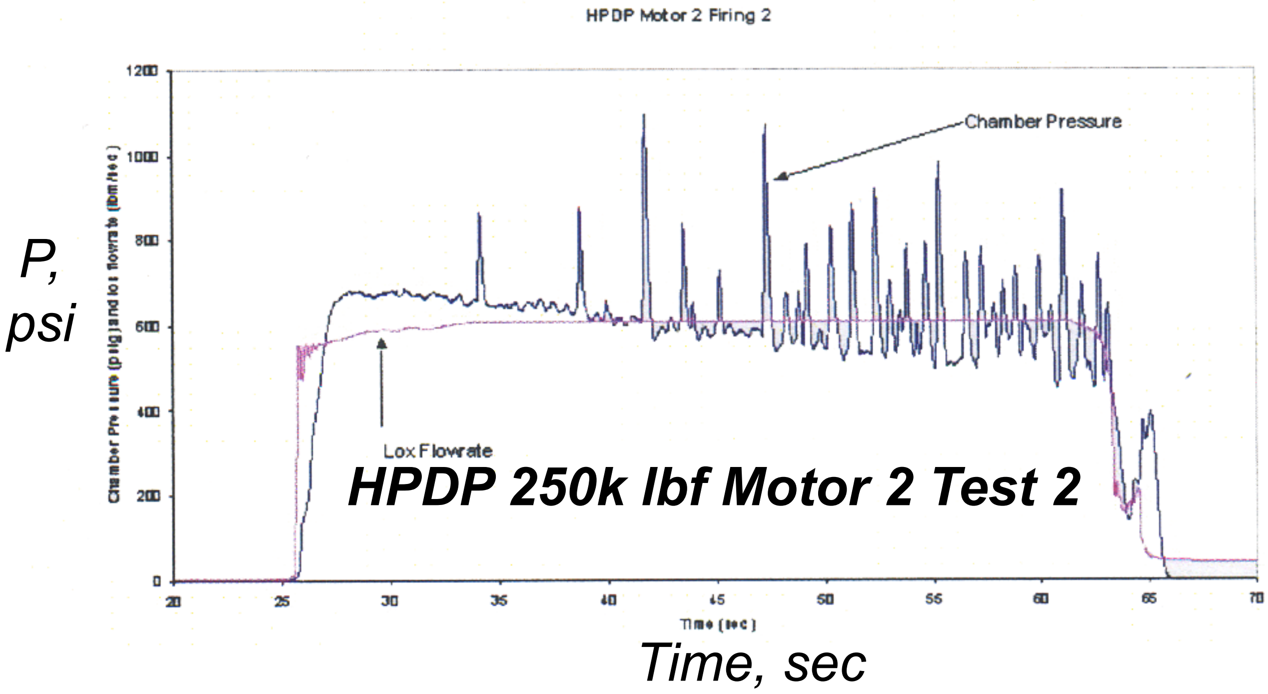
Stability of Solid Propulsion Systems

- Instabilities increase the regression rate ("DC Shift") and reduce the burn time. This may lead to mission failure
- Motor instability is not as frequent as it is in liquid engines
- Rarely cause a sudden motor failure and disintegration
- Low Frequency:
  - Bulk mode: Helmholtz mode or L* mode or chuffing mode
  - Frequencies less than 150 Hz
  - Due to a coupling between the chamber gas dynamics and thermal lags in the solid (the phase lead character of the thermal lag system is key to instability)
- Intermediate/High Frequency:
  - Acoustic modes: longitudinal or traverse
- Many plausible trigger sources (i.e. a propellant chunk flying through the nozzle)
- Amplifying factors
  - Coupling with combustion. Response function: Transfer function between the regression rate and the chamber pressure.
  - Vortex shedding
  - Flow instabilities

Stability of Solid Propulsion Systems

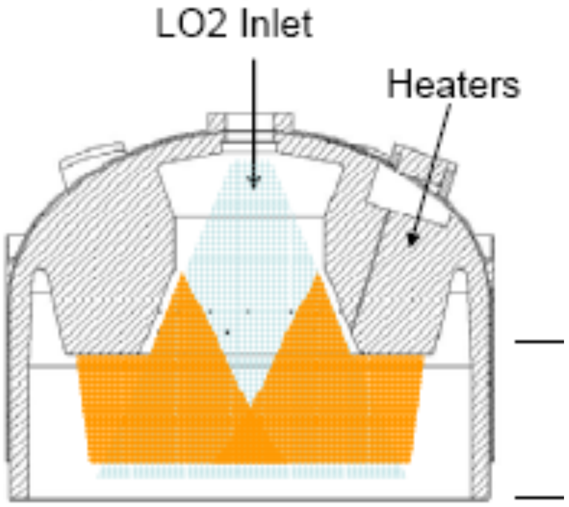
- **Attenuating factors:**
  - Viscous damping
  - Particle or droplet damping: due to drag induced by relative velocity. There exists an optimum particle size for a given frequency
  - Nozzle
  - Viscoelastic character of the propellant
- Intrinsic instability of a solid propellant charge: Due to thermal lags and combustion coupling
- Use T-burners to determine the response function of a solid propellant
- **Stability Fixes:**
  - Change grain geometry
  - Change propellant formulation
    - Al addition helps. Optimal particle size for a given motor size
  - Add mechanical devices to attenuate the unsteady gas motion or alter the natural frequency of the chamber

Low Frequency Instabilities



- *Hybrids are prone to low frequency instabilities (2-100 Hz)*
- *High amplitude spiky combustion*
- *Especially common in liquid oxygen (LOX) based systems*
- *A number of feasible mechanisms exist*

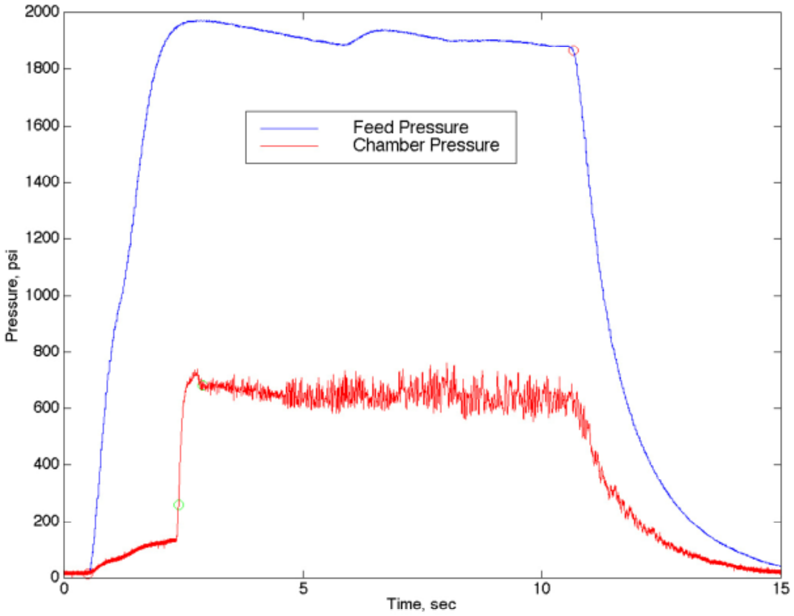
Low Frequency Instabilities - Remedies



- Solutions used in the field
  - Lockheed Martin –Michoud and HPDP used hybrid heaters to vaporize LO_2
  - AMROC injected TEA (triethylaluminum) to vaporize LOX
- Both solutions introduce complexity minimizing the simplicity advantage of hybrids
  - Heaters- extra plumbing
  - TEA – extra liquid, hazardous material

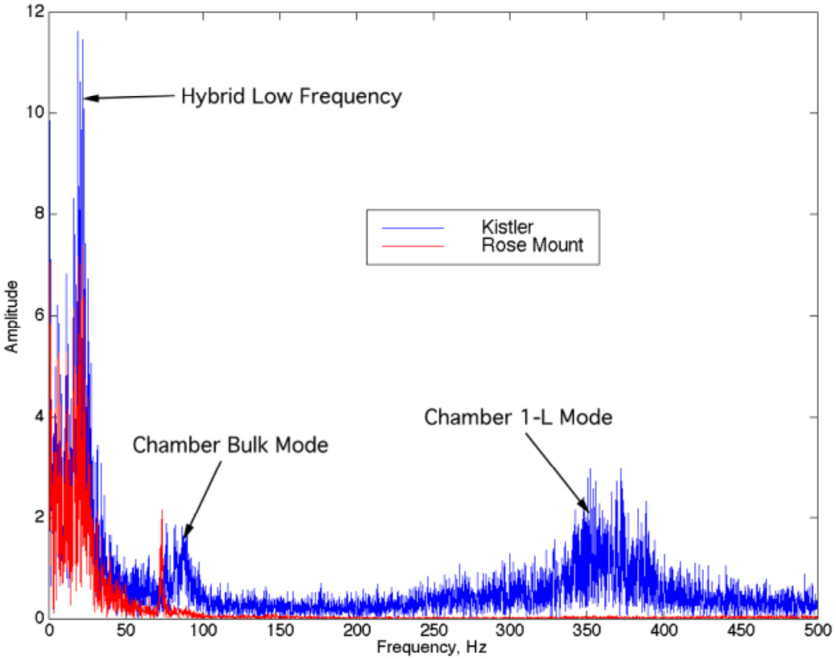
- We believe that a LOX motor can be made stable
  - Without the use of heaters or TEA injection
  - By advanced injector and combustion chamber design

Stability of Hybrid Propulsion Systems



- Pressure-time history for NASA Ames motor test 4L-05.
- Paraffin-based/GOX

- FFT for test 4L-05
- Three modes are observed:
  - Hybrid low frequency
  - Bulk mode
  - 1-L mode



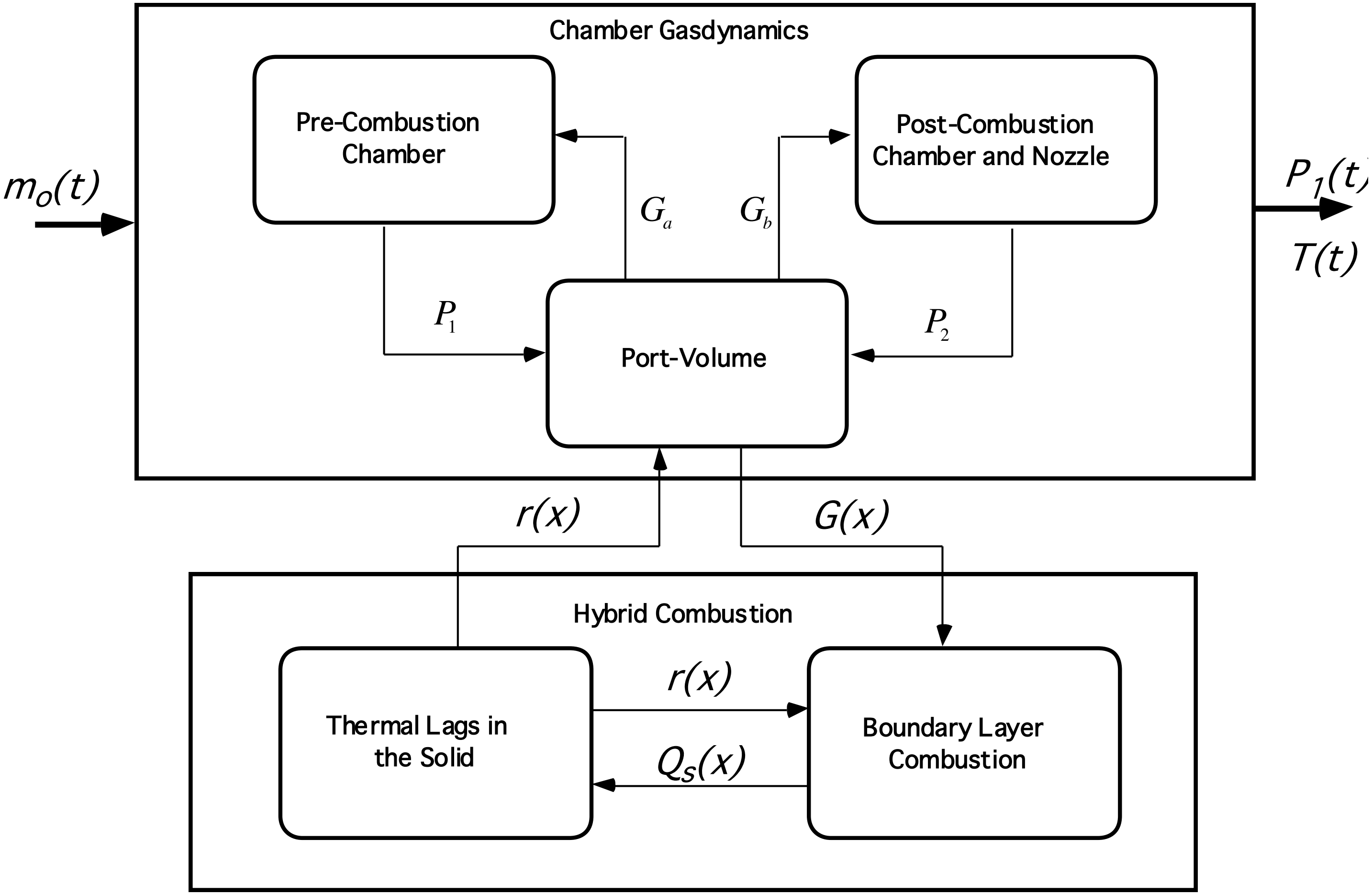
Stability of Hybrid Propulsion Systems

- Most hybrids show the "Low Frequency" mode which typically dominates the other modes.
- Observed in all hybrid development/research programs to our knowledge.
- The exact frequency is case dependent but ranges in the 2-100 Hz for most practical hybrids.
- Oscillations are in the limit cycle form. Amplitudes are typically in the range of 2-30 % rms of the mean.
- The low frequency mode is typically accompanied by acoustic modes.
- The fore end configuration/volume effects the amplitude. (i.e. axial injection is more stable compared to radial injection).
- The low frequency mode is encountered in both liquid and gaseous oxidizer systems.
- Few theories exist-None of them are based on a mathematical formalism that one commonly encounters in solid/liquid fields.
- TCG coupled theory: Develop transient mathematical models of hybrid subsystems and couple these subsystems to search for instabilities.

Hybrid Combustion Time Scales

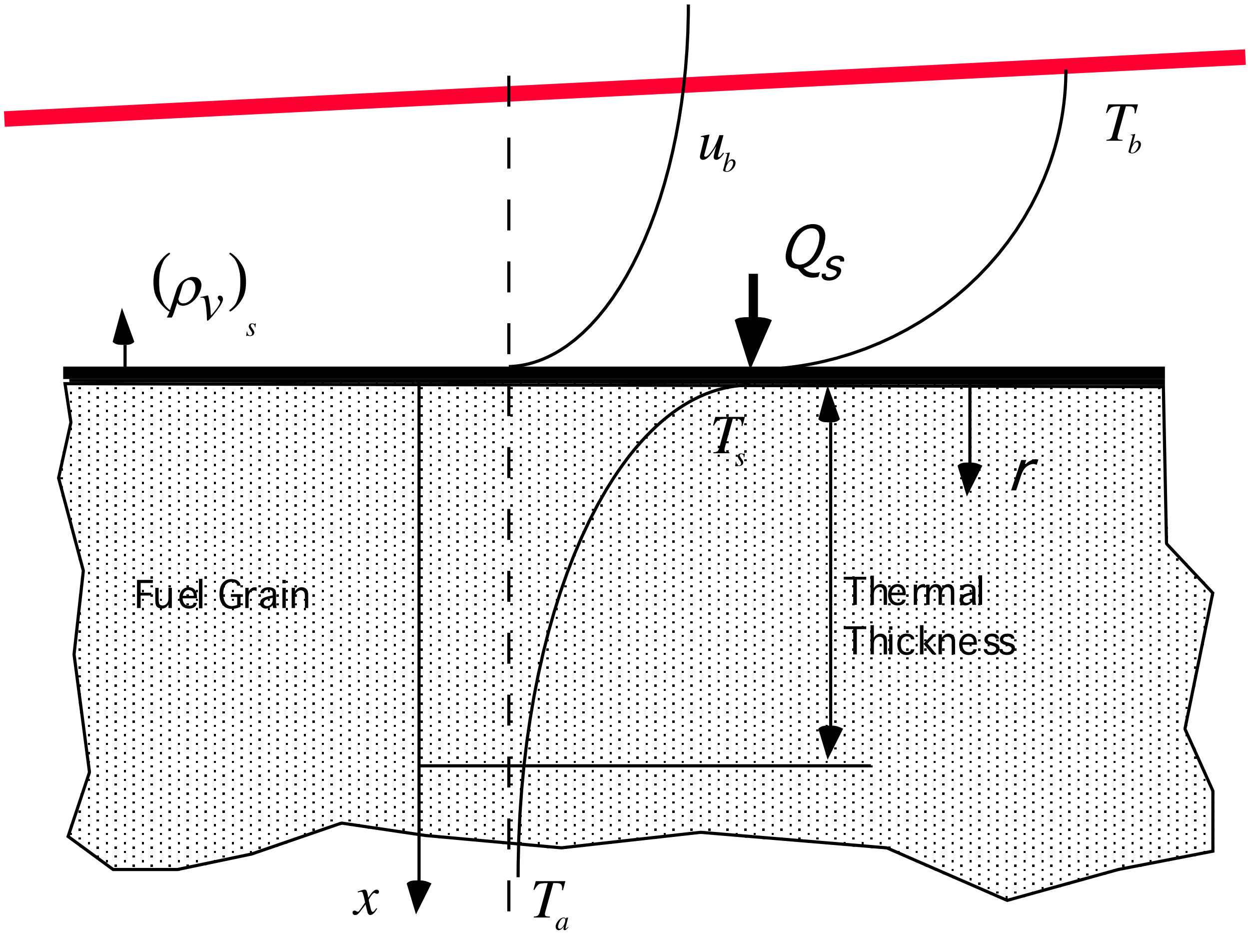
Physical Phenomenon:	Time Scale:	Explanation:
1) Solid phase kinetic times	$\tau_{sp} < 10^{-3}$ sec	Degradation mechanisms of the polymer
2) Gas phase kinetic times	$\tau_{gp} < 10^{-3}$ sec	Hydrocarbon combustion mechanisms
3) Feed system response times	(Varies greatly from system to system)	Response time of the feed system
4) Evaporation times	$\tau_{evap} = f(U_o, T_1, \Delta P)$	Evaporation process of the liquid oxidizer
5) Thermal lags in solid	$\tau_{tl} \propto \kappa / r^2 \approx 10^{-1}$ sec	Thermal profile changes in the solid grain
6) Boundary layer diffusion times	$\tau_{bl} \propto L / u_e \approx 10^{-1}$ sec (Varies greatly form case to case)	Turbulent boundary layer diffusion processes
3) Acoustic times (longitudinal)	$\tau_a \propto L / c \approx 10^{-3}$ sec (Varies greatly form case to case)	Propagation of the acoustic waves
7) Gas dynamic filling times	$\tau_{fill} \propto L^* / c^* \approx 10^{-1}$ sec (Varies greatly form case to case)	Global mass flow balance

Hybrid Transient Model Schematic



Gaseous oxidizer, no feed system dynamics

Thermal Lag Model



- Input: Wall heat flux schedule
- Output: Regression rate variation in time
- Surface Model:

$$\dot{r} = A\ e^{-E_a / R_u T_s}$$

Thermal Lag Model Transfer Function

- First perturbation solution around a nominal operating point generates the transfer function

$$F_T(s) = \frac{R_L(s)}{Q_L(s)} = \frac{2\mathrm{E}_{E_a} s}{\left(1+\sqrt{4s+1}\right)\left(s+\mathrm{E}_{E_a}\right) - 2\mathrm{E}_{E_a} + 2\mathrm{E}_L \mathrm{E}_{E_a} s}$$

- Stability character of the thermal lag system:
  - No poles, just a zero at (0, 0)
  - No instabilities can be generated by this system alone
  - No intrinsic instability of an inert fuel (no heterogeneous rxns are permitted)
- The square root terms represents the diffusive character of the system
- Phase lead behavior at low frequencies
- This subsystem is key in the generation of solid rocket intrinsic and L* instabilities
  - In hybrids L* instability is not possible since regression rate is only a weak function of the chamber pressure.
  - In hybrids intrinsic instability is not possible since no heterogeneous rxns are expected.

Gas Phase Combustion Model: Quasi-Steady Case

- Initially assume that gas phase is fast compared to solid phase
- Ignore the radiative component of the wall heat flux.
- Modify the classical theory (by Marxman in the early 60’s).
- Relation between the wall flux, oxidizer mass flux and the fuel regression rate:

$$\overline{\dot{Q}}_c(\bar{t}) = E_L \overline{G}_o^{\,n/(1-k)} R^{-k/(1-k)}$$

- The regression dependency of the flux comes from a phenomenon called the “Blocking Effect”. k is the blocking exponent.

$$C_H / C_{Ho} = B^k$$

- This effect produces a cross coupling mechanism between the gas phase and the solid phase.

Gas Phase Combustion Model: Transient Case

- Boundary layer can not respond to changes in the oxidizer mass flux and the fuel regression rate.
- Introduce this transient as a time lag:

$$Q_1(\bar{t}) = \left(E_L + 1\right)\left(\left(\frac{n}{1-k}\right)G_1(\bar{t} - \bar{\tau}_{bl1}) - \left(\frac{k}{1-k}\right)R_1(\bar{t} - \bar{\tau}_{bl2})\right)$$

- From literature for turbulent boundary layers with no combustion and no blowing this delay can be written as

$$\tau_{bl} = c'\frac{z}{u_e}$$

- We have estimated the constant to be 0.55.
- Even though this looks like a time flight characteristic time scale, the origin of the formula is based on the radial diffusion period across the boundary layer thickness.

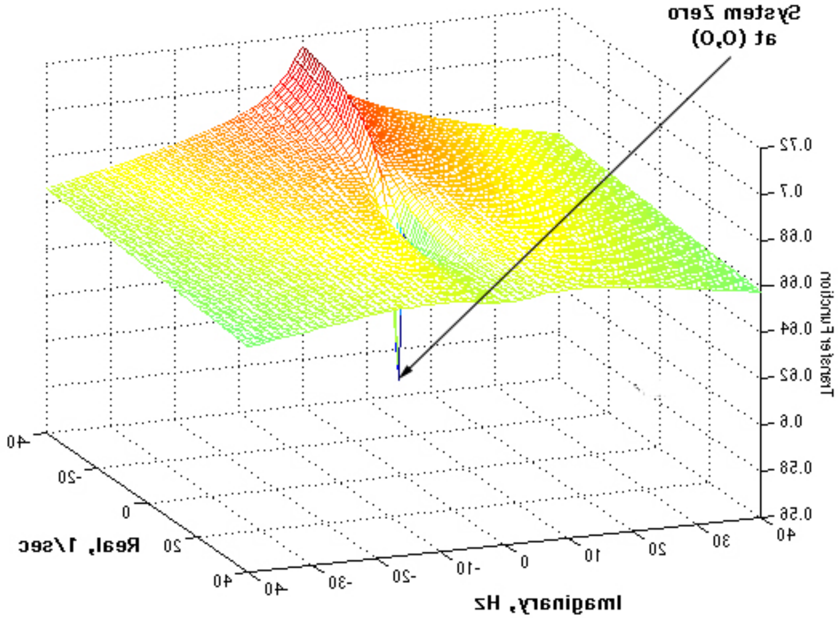
Thermal-Combustion (TC) Coupled System

- Transfer function for the coupled system (Input: Oxidizer mass flux. Output: Regression Rate)

$$F_{TC}(s)=\frac{R_{1L}(s)}{I(s)}=\frac{2E_{E_a}\sigma_2 e^{-\bar{\tau}_{bl1}s}s}{\left(1+\sqrt{1+4s}\right)\left(s+E_{E_a}\right)-2E_{E_a}+2E_{E_a}s\left(E_L+\sigma_1 e^{-\bar{\tau}_{bl2}s}\right)}$$

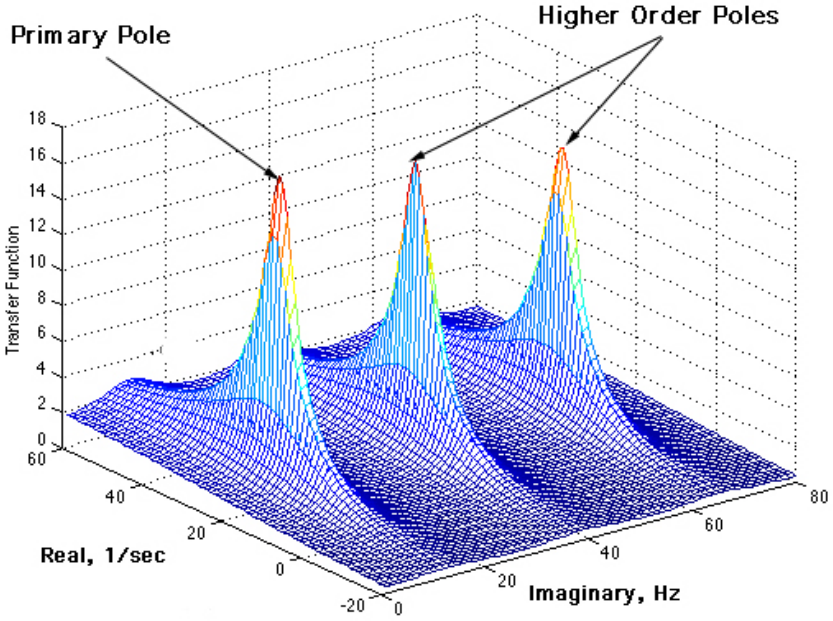
- Stability Character:
  - τ_{bl1}: No effect on stability
  - If $\tau_{bl2}=0$ no poles
  - If $\tau_{bl2}>0$ a series of poles in the positive real half of the s plane-unstable system
  - We only consider the fundamental mode (pole with the lowest frequency)

Thermal-Combustion (TC) Coupled System



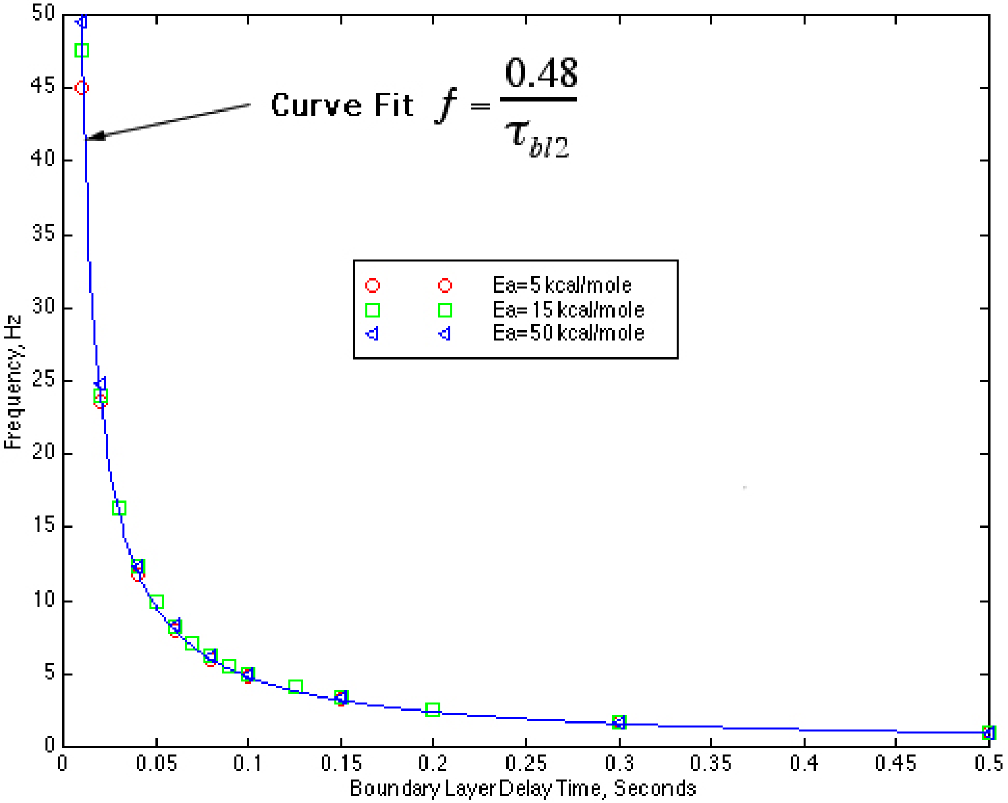
$$\tau_{bl2} = 0$$

$$\tau_{bl2} = 38\ m\sec$$



Effect of System Parameters on the TC Coupled Instabilities

- Effect of all the system parameters other than τ_{bl2} on the oscillation frequency and the amplification rate is negligible for the range of these parameters commonly encountered in hybrid applications.

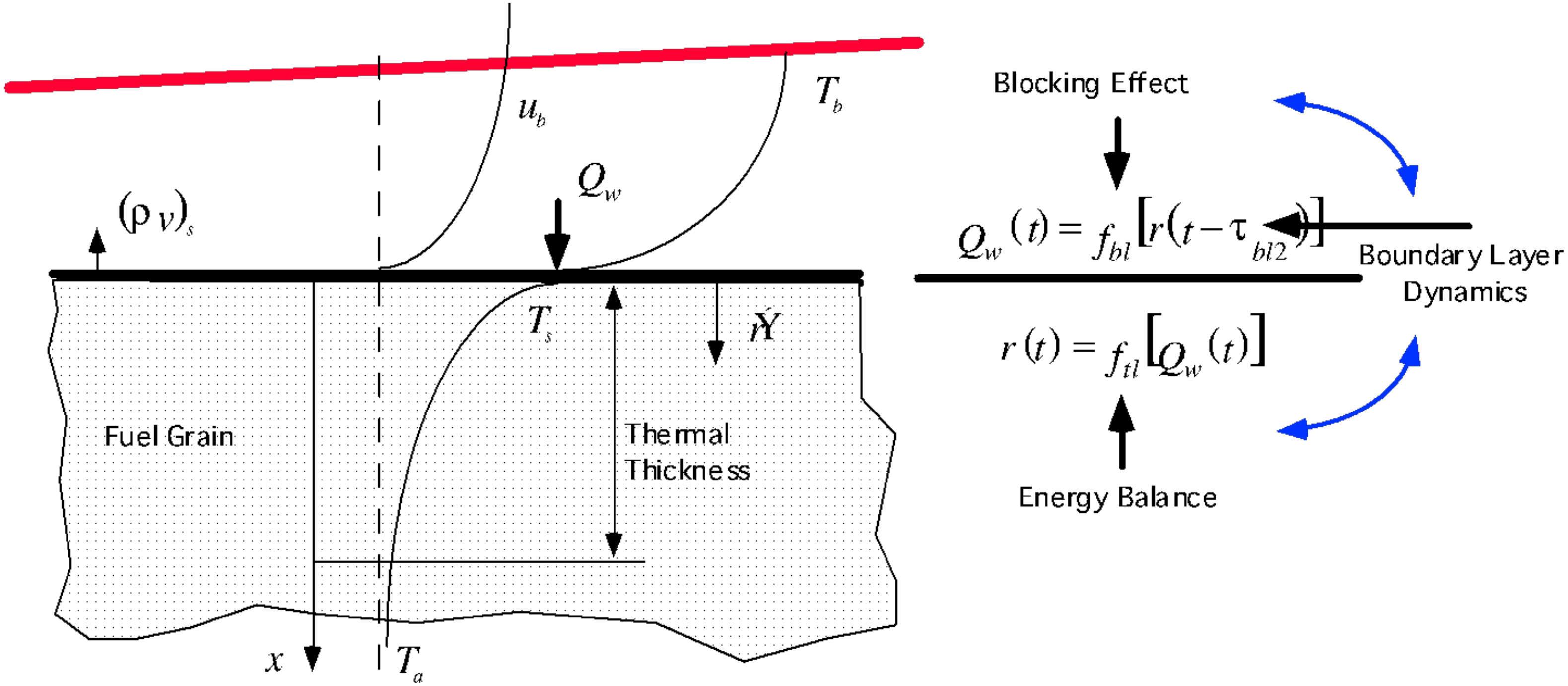


TC Coupled Theory Prediction:

$$f = \frac{0.48}{\tau_{bl2}}$$

Hybrid Low Frequency Instability Coupling Mechanism

- The cross coupling of three important phenomena generates the TC coupled instabilities:
  - Wall transfer blocking effect, k
  - Heat transfer in the solid
  - Boundary layer dynamics, τ_{bl2}

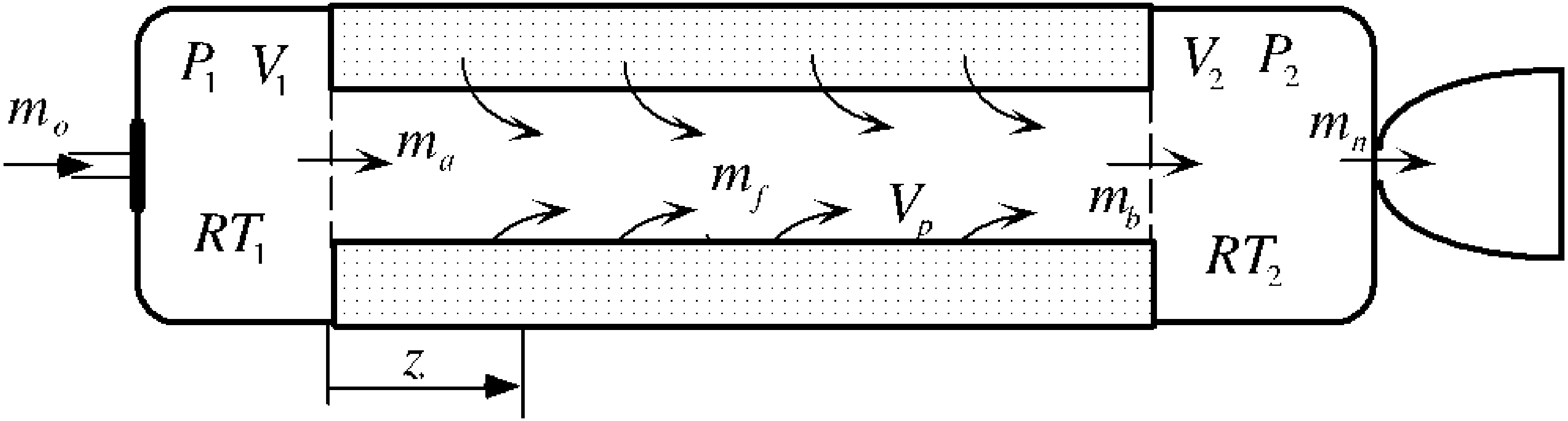


$$Q_w(t) = f_{bl}\left[r\left(t-\tau_{bl2}\right)\right]$$

$$r(t) = f_{tl}\left[Q_w(t)\right]$$

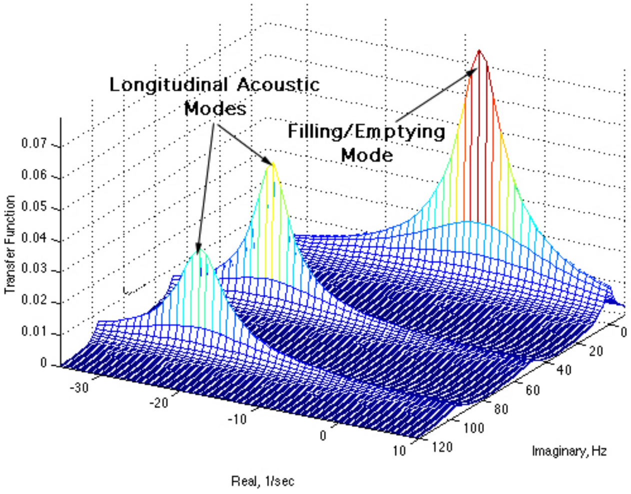
Gas Dynamic Model

- A gas dynamic component is required to complete the basic transient modeling of hybrid transients.
- Also needed to convert the regression rate oscillations into chamber pressure or thrust oscillations.
- Model: 2 Volume-Port, Isothermal



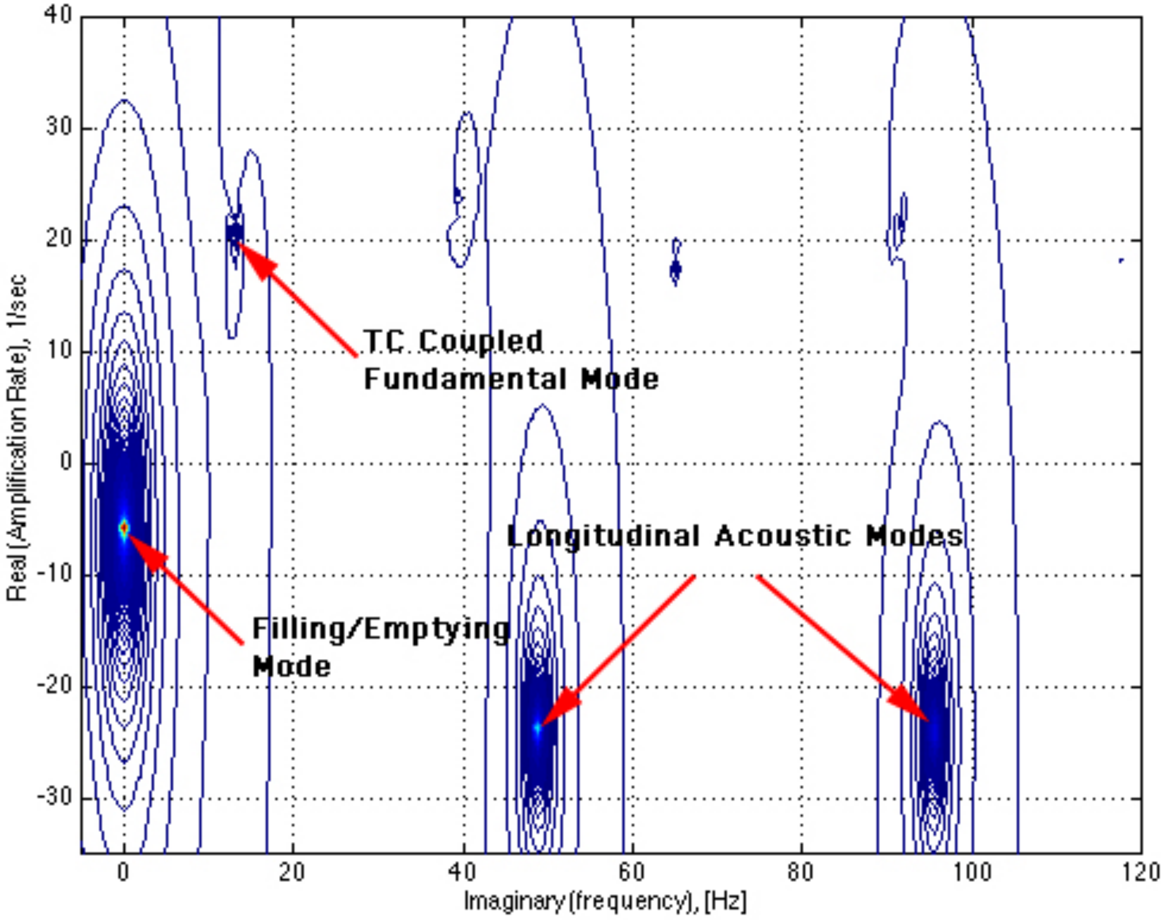
Important Results: Gas Dynamic Model

- A transfer function for the linearized gas dynamic system is derived.
- Numerical simulations on the full nonlinear system are also performed.
- Gas dynamic system by itself is stable.
- The model resolves the filling/emptying and longitudinal acoustic behavior of the chamber.
- When coupled with the combustion subsystem with delay $\tau_{bl1} > 0$, the system preserved its stability character.



Thermal-Combustion-Gas Dynamics (TCG) Coupling

- TCG coupled transfer function



- Shows the TC coupled poles
- Frequency/amplification rate are not altered
- The instabilities are now in terms of chamber pressure oscillations
- Shows most critical transient aspects of a gaseous hybrid with a decoupled feed system:
  - Low frequency oscillations
  - Filling/Emptying
  - Longitudinal acoustic behavior

Hybrid Oscillation Frequency Scaling Law

- Boundary layer delay time in terms of L*: (AMROC)

$$\tau_{bl2} = c' \frac{V_p}{V_m} \frac{\left[(1+O/F)/(1+2O/F)\right]}{RT_{ave}} L^* c^*_{\exp}$$

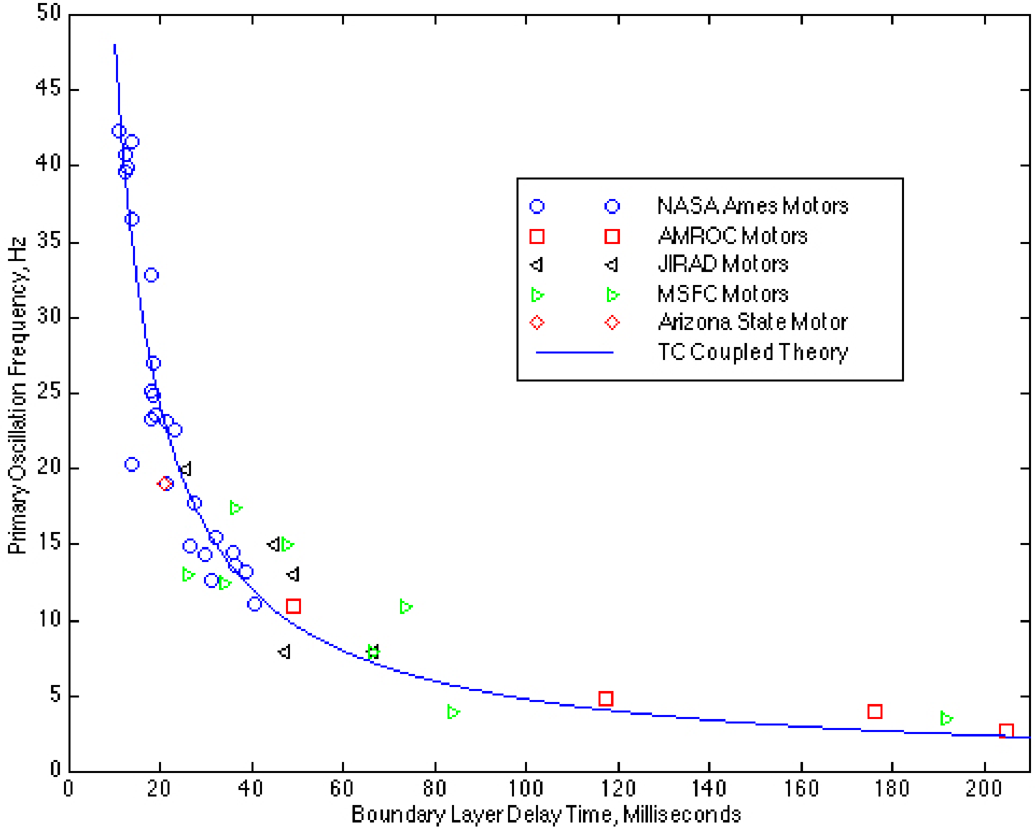
- Boundary layer delay time in terms of operational parameters group A: (HPDP, JIRAD, Arizona State)

$$\tau_{bl2} = c' \frac{LP}{(G_o + G_t)RT_{ave}}$$

- Boundary layer delay time in terms of operational parameters group B: (Ames/Stanford)

$$\tau_{bl2} = c' \frac{LP}{\left(2 + \frac{1}{O/F}\right) G_o RT_{ave}}$$

Comparison to Hybrid Motor Test Data



- 43 motor tests used in this comparison.
- c' value of 2.01 used for all calculations.
- Motor test data covers a wide range of variables:
  - 5 programs
  - Three oxidizers (LOX, GOX, N2O)
  - Wide range of motor dimensions (5" OD to 72" OD)
  - Wide range of operating conditions
  - Several fuel formulations (HTPB, HTPB/Escorez, paraffin-based)

Hybrid Low Frequency Instabilities-Overall Picture

- Linear TC coupled theory predicts indefinite growth of oscillations in time.
- The amplitudes are determined by nonlinear phenomenon and the strength/spectral content of the excitation source (Limit cycle).
- Unlike the amplitude, the frequency of the oscillation is set by the TC coupled theory.
- All hybrids have the TC coupled instability mechanism (root cause).
- Some motors show very low amplitude oscillations because TC coupled mode is not disturbed strongly at the frequency content that it prefers to amplify.
- The possible source of disturbance is the fore-end flow configuration.
- Note that it has been observed that the fore end configuration (i.e. injection, geometry) determines the amplitude of the oscillation but not the frequency.

Hybrid Low Frequency Instabilities-Analogy

- An analog system that works on the same principle is flue organ pipe.

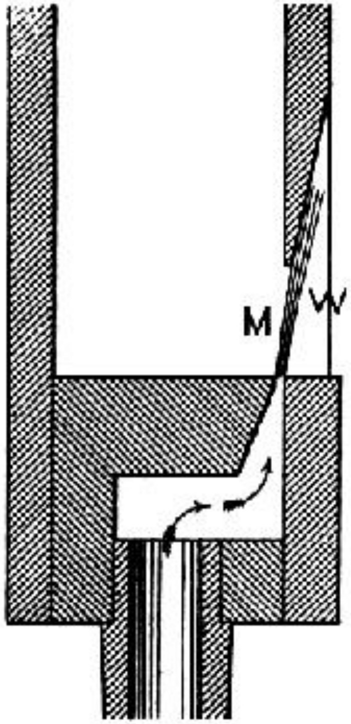


Fig. LXI.

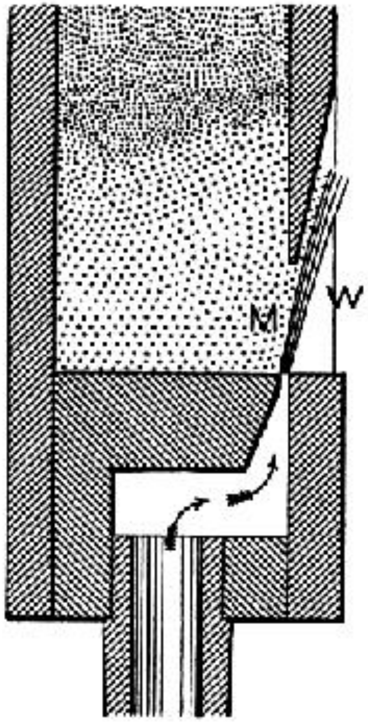


Fig. LXII.

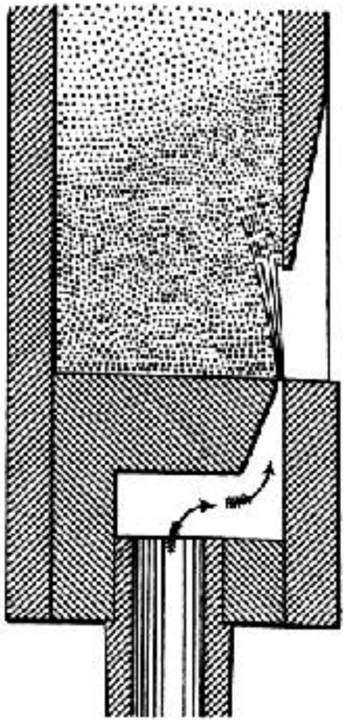


Fig. LXIII.

Conclusions- Hybrid Transient Modeling

- A plausible mechanism that generates low frequency chamber pressure oscillations is developed.
- The oscillation frequency for a hybrid system can be predicted by this universal formula:

$$f = 0.119\left(2 + \frac{1}{O/F}\right)\frac{G_o R T_{ave}}{L\ P}$$

- The amplitudes can not be predicted by this simple model.
- We believe that the stability character of a hybrid motor is determined by the spectral features of the disturbances generated at the fore-end of the motor.