AA 284a
Advanced Rocket Propulsion

Lecture 13
Component Design Issues

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Mass Model

- Propulsion System
  - Tanks (estimate)
  - Feed system
    - Turbo pump (assign a value)
    - Pressurization system (estimate)
    - Shut off and throttling valves (assign a value)
    - Other components (assign a value)
  - Combustion chamber (estimate)
  - Nozzle (estimate)
  - Ignition system (assign a value)
- Rocket structures (assign percentage)
- Attitude control system (assign a value or estimate)
- Avionics (assign a value)
- Other systems (assign a value)
- Payload interface (Percent of payload mass)
- Mass margin (Percent of the estimated structural mass)

Tank Design

- Storage of liquid oxidizer and fuel in hybrid and liquid rockets
- Large component of the structural mass fraction especially for pressure fed systems
- Factors that influence the design
  - Liquid mass-overall volume
  - Geometrical constraints –tank shape and configuration
  - Tank weight – tank material selection
  - Pump fed vs. pressure fed – internal pressure (MEOP)
  - Cryogenic vs storable – insulation
  - Corrosiveness of the liquid – tank material selection
  - Chemical stability of the liquid – tank material selection
  - Gravitational environment - diaphragms for zero g
  - Anti-Slosh - Baffles

Tank Design

- Structural design of the tanks
- Loads
  - Internal pressure
  - Acceleration
  - Point loads
- Primary failure modes
  - Yield/Rupture under internal pressure
  - Buckling (especially for thin walled tanks of pump fed systems)
- Structural materials
  - Metals: aluminum, steel, titanium
  - Composite: Carbon/Epoxy
- For cryogenic oxidizers such as LOX, composite technology is still in the R&D Phase

Tank Design

- Failure envelope for metals
- Yield stress: Significant plastic deformation starts
- Ultimate stress: Material breaks
- Failure criteria based on yield for isotropic ductile materials (i.e. metals)
  - Tresca (maximum shear stress)

$$\tau_{\max} = \max\left[\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}\right] < \frac{\sigma_y}{2}$$

  - Von Mises (maximum strain energy)

$$\sigma_{\max} = \sqrt{\left((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right)/2} < \sigma_y$$

- Tresca is more conservative compared to von Mises

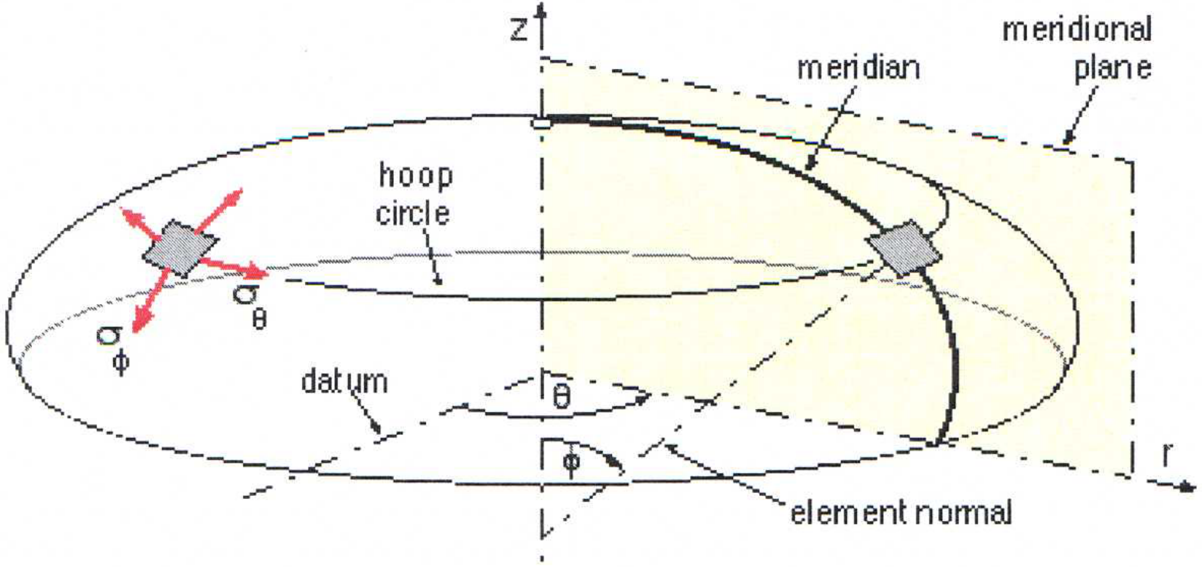
Tank Design

- Important material properties
  - Strength/density
  - Ductility
  - Cost
  - Ease of manufacturing (welding, machining, forming)
  - Low temperature characteristics
  - Liquid compatibility
- Note that welding with no post heat treatment reduces yield strength
- Aluminum 2219 is widely used in cryogenic tank fabrication

Structural Material	Tensile Yield Strength, ksi	Specific Density
Aluminum 2219	60.0 (31.0 welded)	2.7
Graphite/Epoxy	130.0	1.55
Steel 4130	125.0	7.83
Aluminum Lithium	~80	2.5

Tank Design

- Tanks used in propulsion applications are thin walled shells. Use shell theory for structural design
- For preliminary design, the bending moments can be ignored and the shell equations can be reduced to membrane equations



- For axisymmetric geometries the membrane stresses are
  - Meridional stress: $$\sigma_\phi = \frac{P\, r_\theta}{2\ t}$$
  - Circumferential stress (Hoop stress): $$\sigma_\theta = \sigma_\phi \left(2 - \frac{r_\theta}{r_\phi} \right)$$

Tank Design

- Tanks can be fabricated from combination of the following special geometries:
  - Cylinder: $r_\theta = r \qquad r_\phi \to \infty$

$$\sigma_\phi = \frac{P\, r}{2\ t} \qquad \sigma_\theta = \frac{P\, r}{t}$$

  - Sphere: $r_\theta = r_\phi = r$

$$\sigma_\phi = \sigma_\theta = \frac{P\, r}{2\ t}$$

  - Ellipsoid: (semi-major axis: a, semi-minor axis: b)

$$\sigma_\phi = \frac{P\, a^2}{2\ b\ t} u \qquad \sigma_\theta = \sigma_\phi \left(2 - \frac{1}{u} \right)^2$$

$$u = \sqrt{1 - \varepsilon^2 (r/a)^2} \qquad \varepsilon = \sqrt{1 - (b/a)^2}$$

Tank Design

- Spherical tank: (with radius *r*)
  - Stress field: $$\sigma_1 = \sigma_2 = \frac{P\,r}{2\;t}$$
  - From Tresca criterion: $$\tau_{\max} = \frac{\sigma_1}{2} = \frac{\sigma_y}{2}$$
  - Tank wall thickness: $$t_{\min} = k\frac{P\,r}{2\;\sigma_y}$$
  - Tank mass: $$M_t = S_t t_{\min}\rho_t = 4\pi\rho_t r^3\frac{P}{\sigma_y}$$
  - Liquid mass: $$M_l = V_t\rho_l = \beta\frac{4\pi}{3}r^3\rho_l$$
  - Define the tank efficiency: $$\eta_t \equiv \frac{M_l}{M_l + M_t} = \frac{1}{1+\frac{3k}{2}\frac{\rho_t}{\rho_l\beta}\frac{P}{\sigma_y}}$$

Tank Design

- Cylindrical tank:
- Different designs are possible based on various head geometries
  - Hemisphere:
    - Ideal end closure to minimize stress concentration
    - Expensive to manufacture, Ends are too long
  - Ellipsoidal:
    - Typical a/b is 2
    - Hoop stress is compressive for the outside 20% of the end closure
    - Bending moments are introduced around the ellipsoid cylinder juncture
    - The stress concentration factor and yield criterion is

$$K = \left[2 + (a/b)^2\right]/3 \qquad \frac{K\ P\,r}{2\,t} < \sigma_y$$

  - Torisphere:
    - Spherical central portion with radius *R* and a toroidal knuckle of radius *r*

$$K = \left[3 + (R/r)^{1/2}\right]/4$$

    - Higher stress concentration, but less expensive to manufacture
  - Flat Plate:
    - No membrane stresses, large stresses due to bending moments
    - Simple fabrication
- Typically 2:1 ellipsoidal design or hemispherical design is adapted for propulsion system tanks and combustion chambers

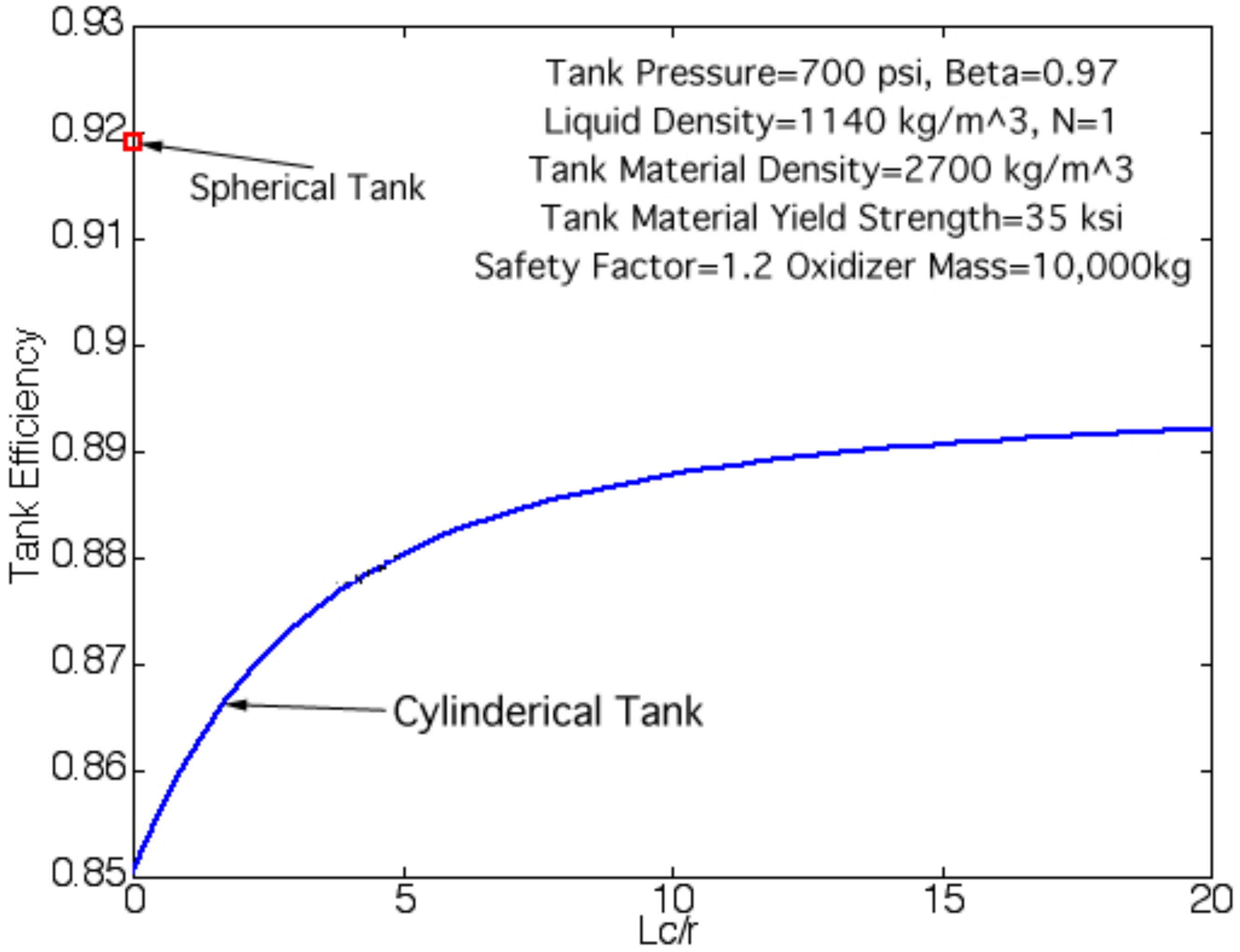
Tank Design

- Cylindrical tank design with hemispherical ends
- Assume uniform wall thickness
- Tank radius: *r*, Length of the cylindrical portion: *Lc*
- Maximum shear stress:
  - Sphere: $\tau_{\max} = \frac{P\,r}{4\,t}$
  - Cylinder: $\tau_{\max} = \frac{P\,r}{2\,t}$
- Minimum thickness: $t_{\min} = k\frac{P}{\sigma_y}r$
- Tank mass: $M_t = S_t t_{\min}\rho_t = 2\pi\rho_t k r^2\left(L_c + 2\ r\right)\frac{P}{\sigma_y}$
- Liquid mass: $M_l = V_t\rho_l = \beta\pi\ \rho_l r^2\left(L_c + \frac{4}{3}r\right)$
- Tank efficiency: $\eta_t \equiv \frac{M_l}{M_l + M_t} = \frac{1}{1+2\ k\frac{\rho_t}{\rho_l\beta}\frac{P}{\sigma_y}\left(\frac{L_c/r+2}{L_c/r+4/3}\right)}$
- Tank length: $L = 2\ r + L_c$

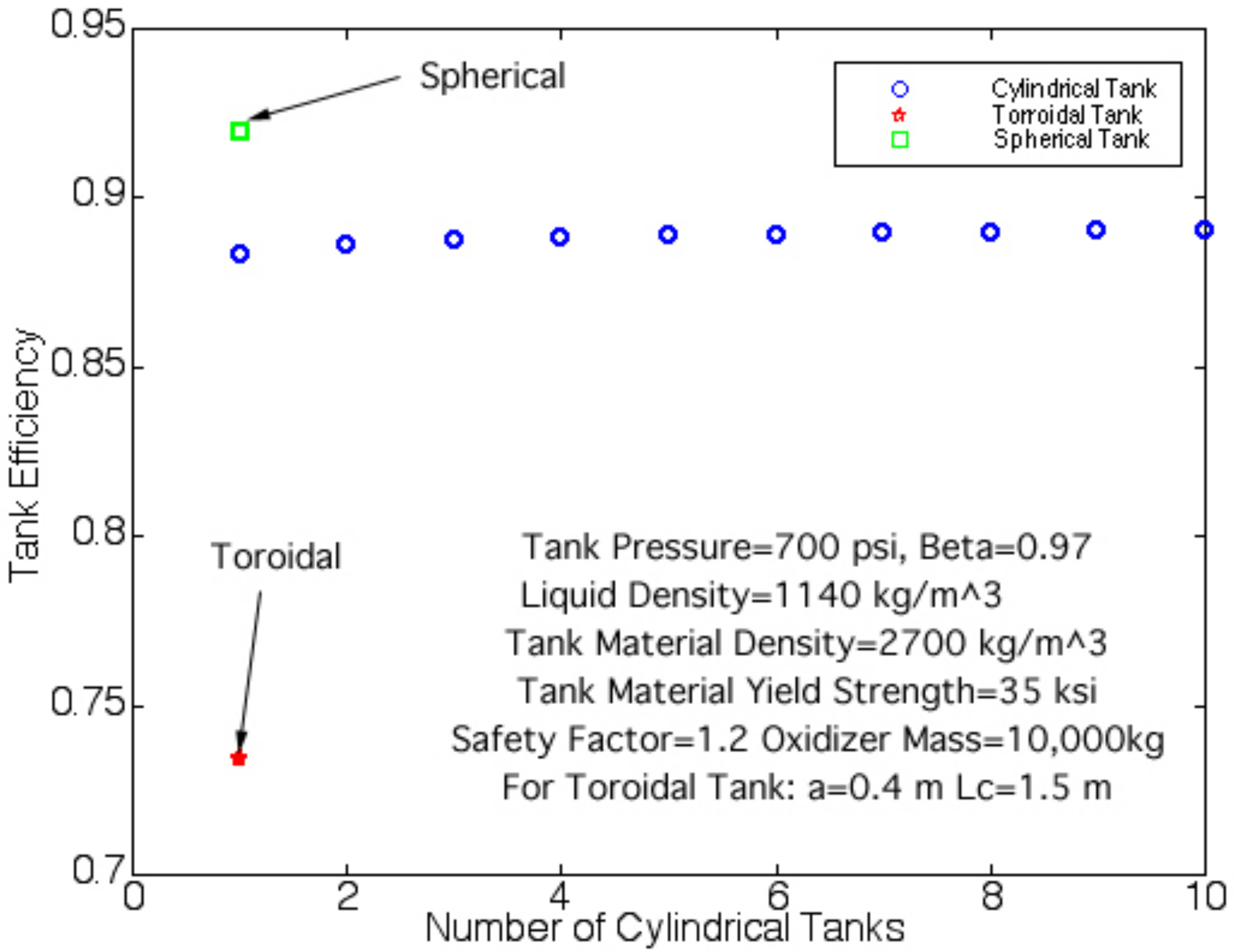
Tank Design

- Toroidal Tank:
- Assume uniform thickness based on most critical stress
- Tank inside radius: *a*, Tank outside radius: *b*, Length of the cylindrical portion: *Lc*, Radius of toroidal head: *r*
- Maximum shear stress:
  - Inside cylinder: $\tau_{\max} = \frac{P\, r}{2\ t}\left[\left(r/2a+1\right)+a/r\right]$
- Minimum thickness: $t_{\min} = k\,\frac{P}{\sigma_y}\,\frac{r}{4}\left[\left(r/2a+1\right)+a/r\right]$
- Tank mass: $M_t = S_t t_{\min} \rho_t = 4\ \pi\ k\ \rho_t \frac{P}{\sigma_y} r\ \left(r+a\right)\ \left(L+\pi\ r\right)\ \left[\left(r/2a+1\right)+a/r\right]$
- Liquid mass: $M_l = V_t \rho_l = \beta\ 4\ \pi\ \rho_l r\ \left(r+a\right)\left(L_c+\frac{\pi}{4}r\right)$
- Tank efficiency: $\eta_t \equiv \frac{M_l}{M_l+M_t} = \frac{1}{1+k\,\frac{\rho_t}{\rho_l \beta}\,\frac{P}{\sigma_y}\left(\frac{L_c/r+\pi}{L_c/r+\pi/4}\right)\left(\frac{r}{2\ a}+\frac{a}{r}+1\right)}$
- Tank length: $L = 2\ r + L_c$

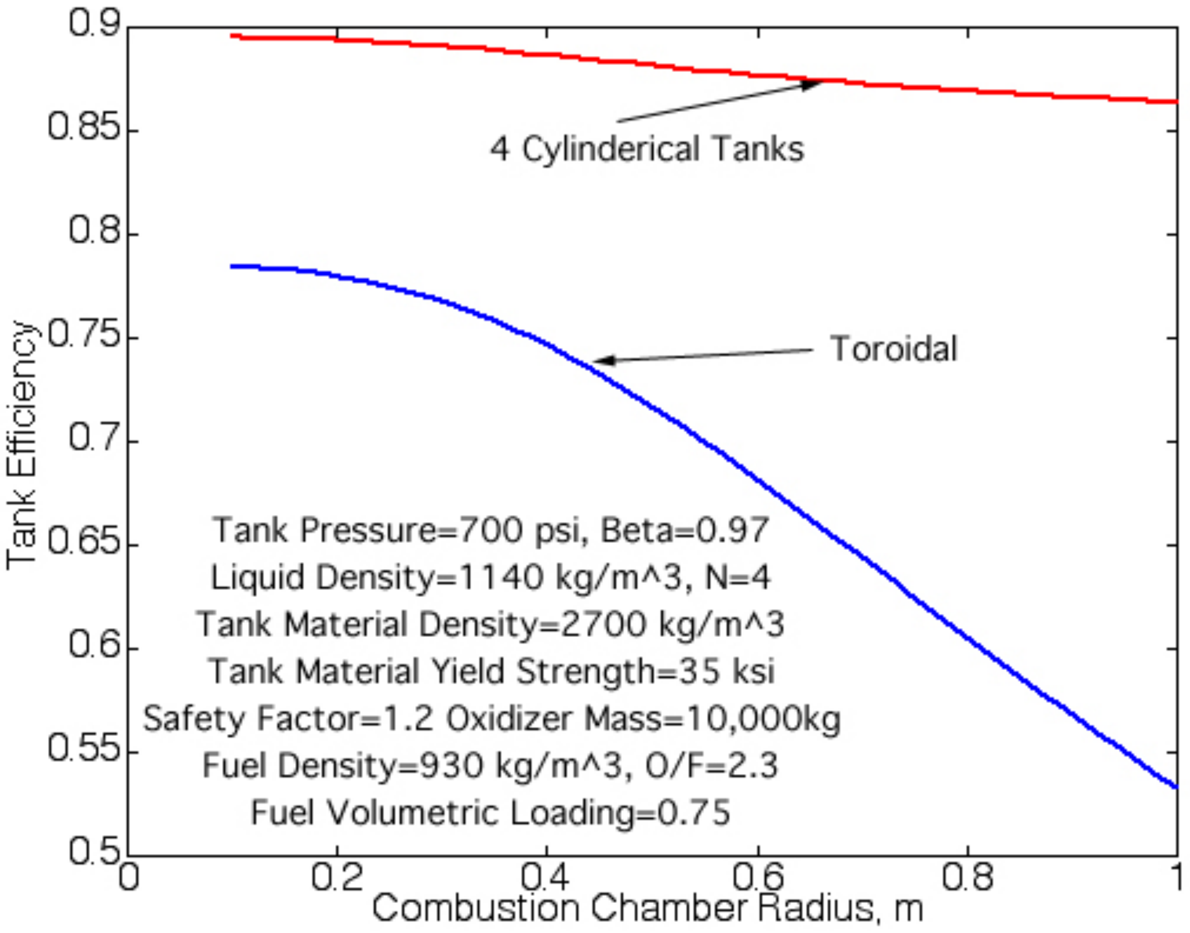
Tank Design-Effect of *Lc/r* for Cylindrical Tanks



Tank Design-Effect of Number of Cylindrical Tanks



Tank Design-Effect of Combustion Chamber Radius



- Combustion chamber inside the tank
- No common wall for the combustion chamber and tank
- Combustion chamber volume is estimated using

$$V_{cc} = \frac{M_f}{\rho_f V L}$$

- Length can be estimated from the volume and assumed radius

$$L_c = \frac{V_{cc}}{\pi a^2}$$

- As the combustion chamber radius increases toroidal tank becomes very inefficient

Other Tank Design Issues

- Include the mass of the other tank components into the mass budget
  - Baffles
  - Diaphragms
  - Mounting supports
  - Insulation (for cryogenic liquids)
- The first three items are difficult to estimate in the preliminary design phase. Account for them by increasing the safety factor.
- Insulation can be estimated based on the total surface area of the tank
- For certain cases the stiffness of the tank may become critical
- For pump fed systems, the tanks are designed for a small but finite pressure (50-75 psi)
  - For pump fed systems check that the calculated wall thickness is more than the minimum acceptable material thickness (minimum gauge)
- Note that the yield stress for metals increases with decreasing temperature. Useful feature for cryogenic liquids

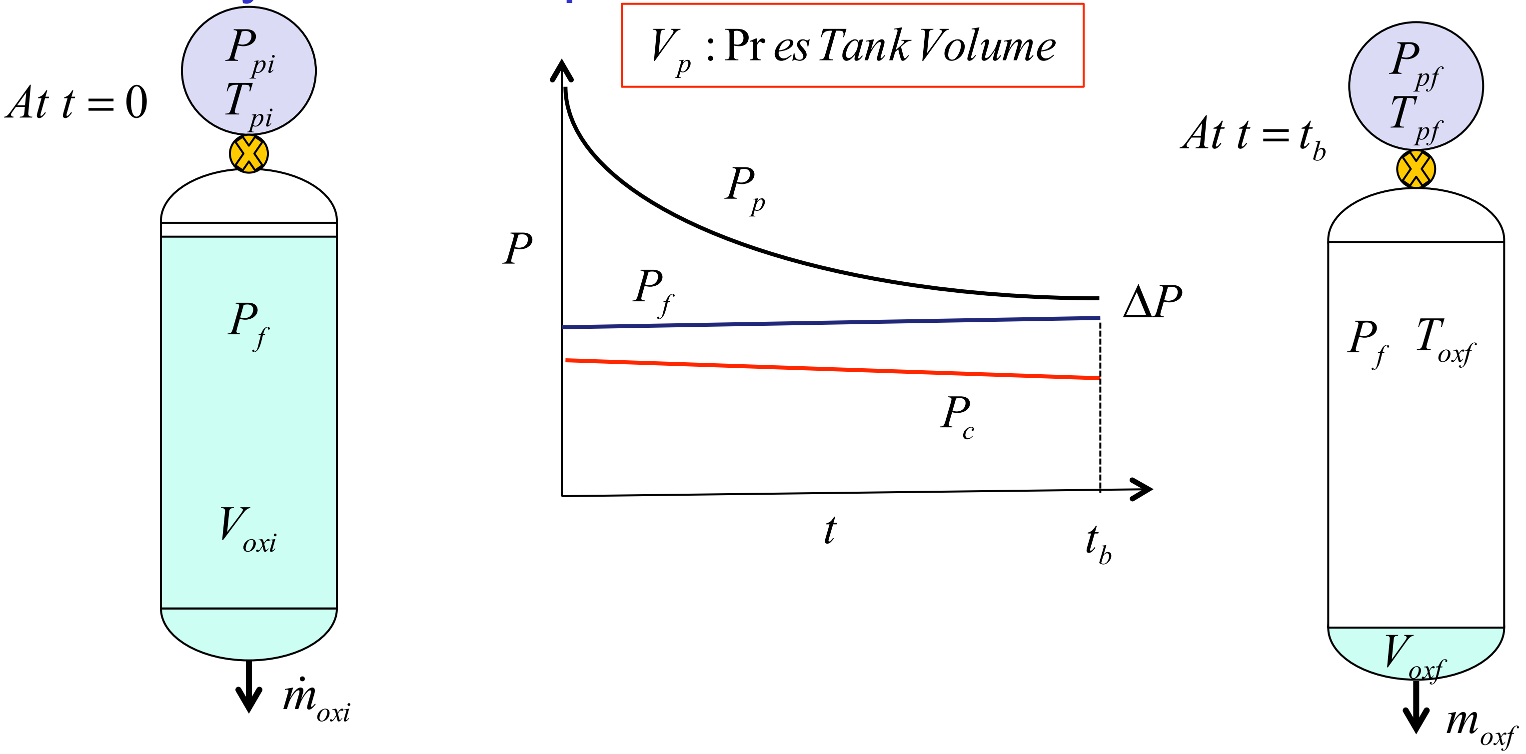
Combustion Chamber Design

- For hybrids and solids fuel/propellant storage volume also serves as the combustion chamber
- Pressure vessel design equations derived for tanks are valid
- Design to Maximum Expected Operating Pressure (MEOP)
- In most cases combustion chambers are cylindrical vessels with 2:1 ellipsoid end caps or hemispherical end caps
- Common combustion chamber case materials are
  - Carbon fiber composite, Kevlar
  - Alloy steel
  - Aluminum
  - Titanium
- Must include the following items in the mass budget for the combustion chamber
  - Fuel sliver mass/web support material
  - Insulation material
  - Injector for hybrids
  - Igniter system
- For liquid systems combustion chambers are small and typically made out of metals

Feed System Components

- Feed system components
  - Oxidizer (and fuel) pumps or pressurization system
  - Main shut off valves for oxidizer (and fuel)
  - Other components (i.e. pipes etc)
- Turbo pump weight and cost are difficult to estimate
  - Typically pump fed systems are lighter but more complex and expensive
  - Pump weight and cost increases with increasing chamber pressure and liquid mass flow rate
  - Another system is needed to derive the turbine (H2O2 or solid/hybrid gas generators)
  - Assume a reasonable weight value for the preliminary design. Base the guess on the existing pump systems with similar operational characteristics
- The weight of the pressure fed systems can easily be estimated

Feed System Components



V_p : Pres Tank Volume

- The mass of pressurant gas in the oxidizer tank at burn out

$$M_g = \Delta V_{ox} \frac{P_f}{R_p T_{oxf}} = \frac{\beta M_{ox}}{\rho_{ox}} \frac{P_f}{R_p T_{oxf}}$$

- The mass change in the pressurization tank

$$\Delta M_{pt} = V_p \left(\frac{P_{pi}}{R_p T_{pi}} - \frac{P_f + \Delta P}{R_p T_{pf}} \right)$$

Pressurization System

- From mass balance $M_g = \Delta M_{pt}$

$$V_p = \frac{\dfrac{\beta M_{ox}}{\rho_{ox}} \dfrac{P_f}{T_{oxf}}}{\dfrac{P_{pi}}{T_{pi}} - \dfrac{P_f + \Delta P}{T_{pf}}}$$

- Tank weight and geometry can be calculated from the volume requirement
- Pressurization gas mass in the tank (assume ideal gas)

$$M_{pg} = V_p \frac{P_{pi}}{R_p T_{pi}} = \frac{\dfrac{\beta M_{ox}}{\rho_{ox}} \dfrac{P_f}{T_{oxf}} \dfrac{P_{pi}}{R_p T_{pi}}}{\dfrac{P_{pi}}{T_{pi}} - \dfrac{(P_f + \Delta P)}{T_{pf}}}$$

- In order to minimize the total pressurant gas mass use light gases (i.e. He)
- Total mass of the pressurization system

$$M = M_{pg} + M_{p\tan k} + M_{pother}$$

Pressurization System

- Mass of regulators valves and other small components must be included in M_{pother}
- The initial and final pressurization tank temperatures are related according to the polytropic relation

$$\frac{T_{pi}}{T_{pf}} = \left(\frac{P_{pi}}{P_f + \Delta P} \right)^n$$

- For cold gas pressurization systems, exponent *n* is in the range of 0.1-0.28 for most cases (*n=0* corresponds to isothermal process)
- The oxidizer temperature in the tank at burn out can be calculated as

$$T_{oxf} / T_{pi} \equiv a_2$$

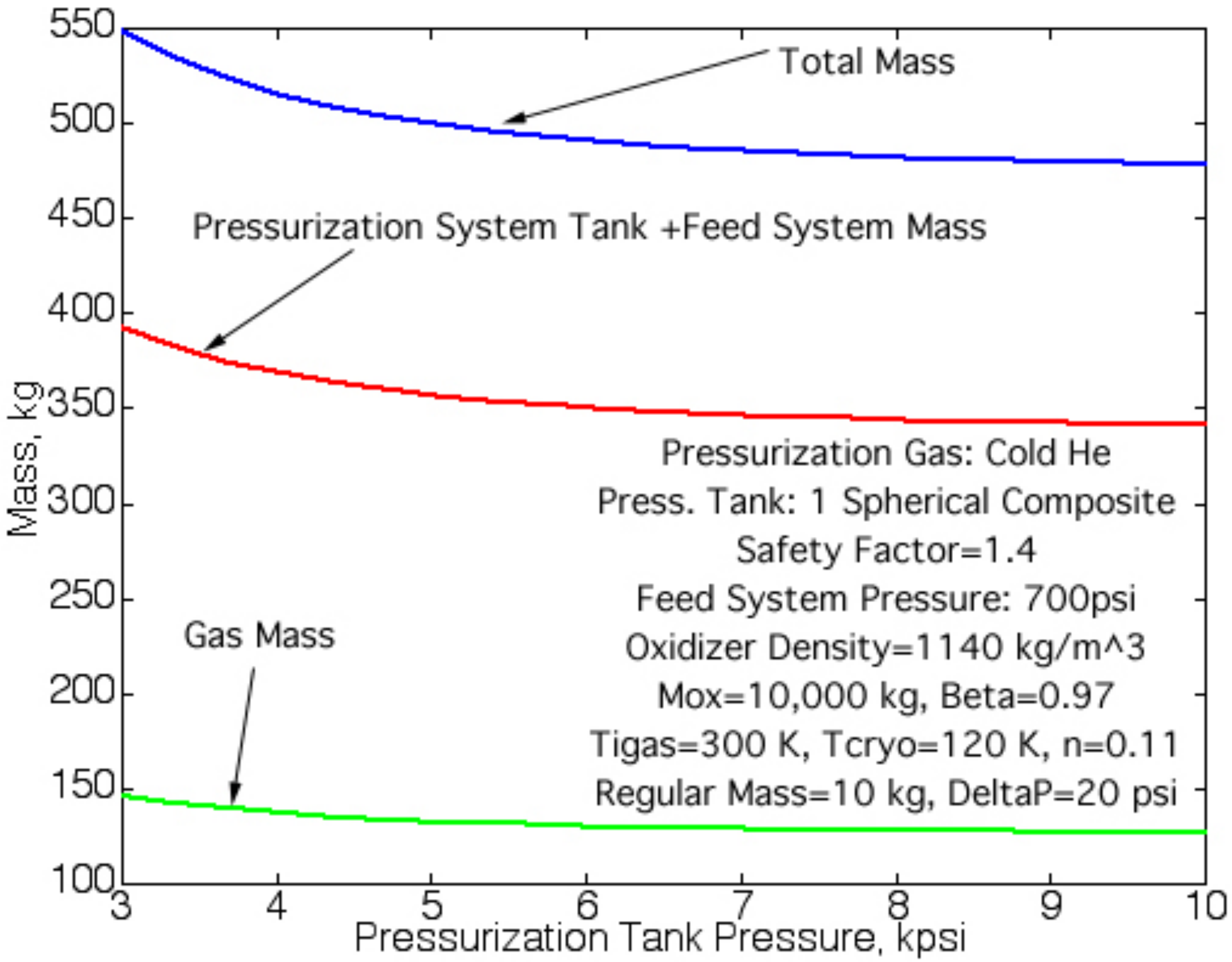
Where

Ppi/Ppf	10	7	4	2
a_2	0.75	0.80	0.87	0.90

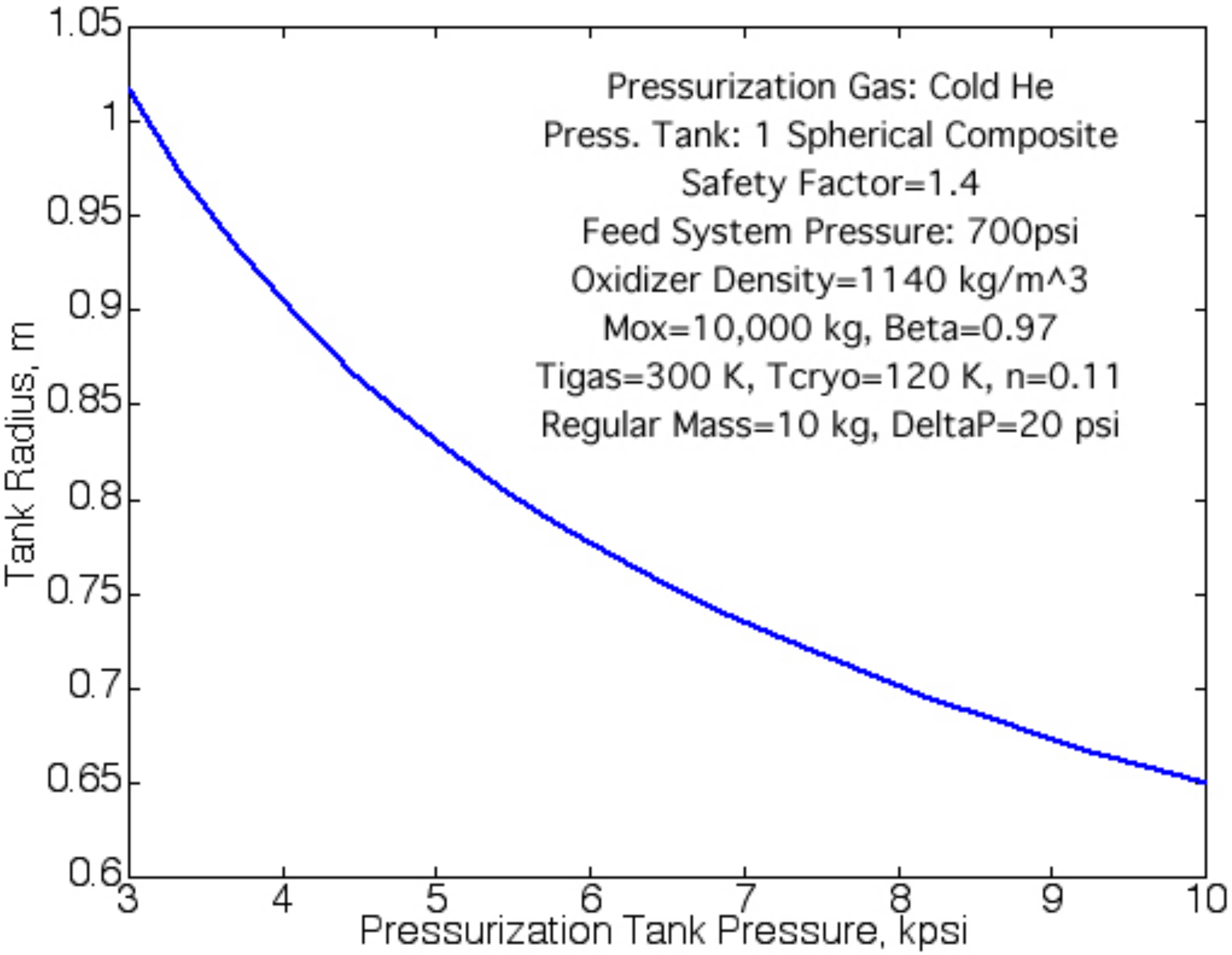
- Note that the pressurization system mass has the following general variation with the pressurization system pressure

$$M = \frac{A\,P_{pi}}{P_{pi} - C} + M_{pother}$$

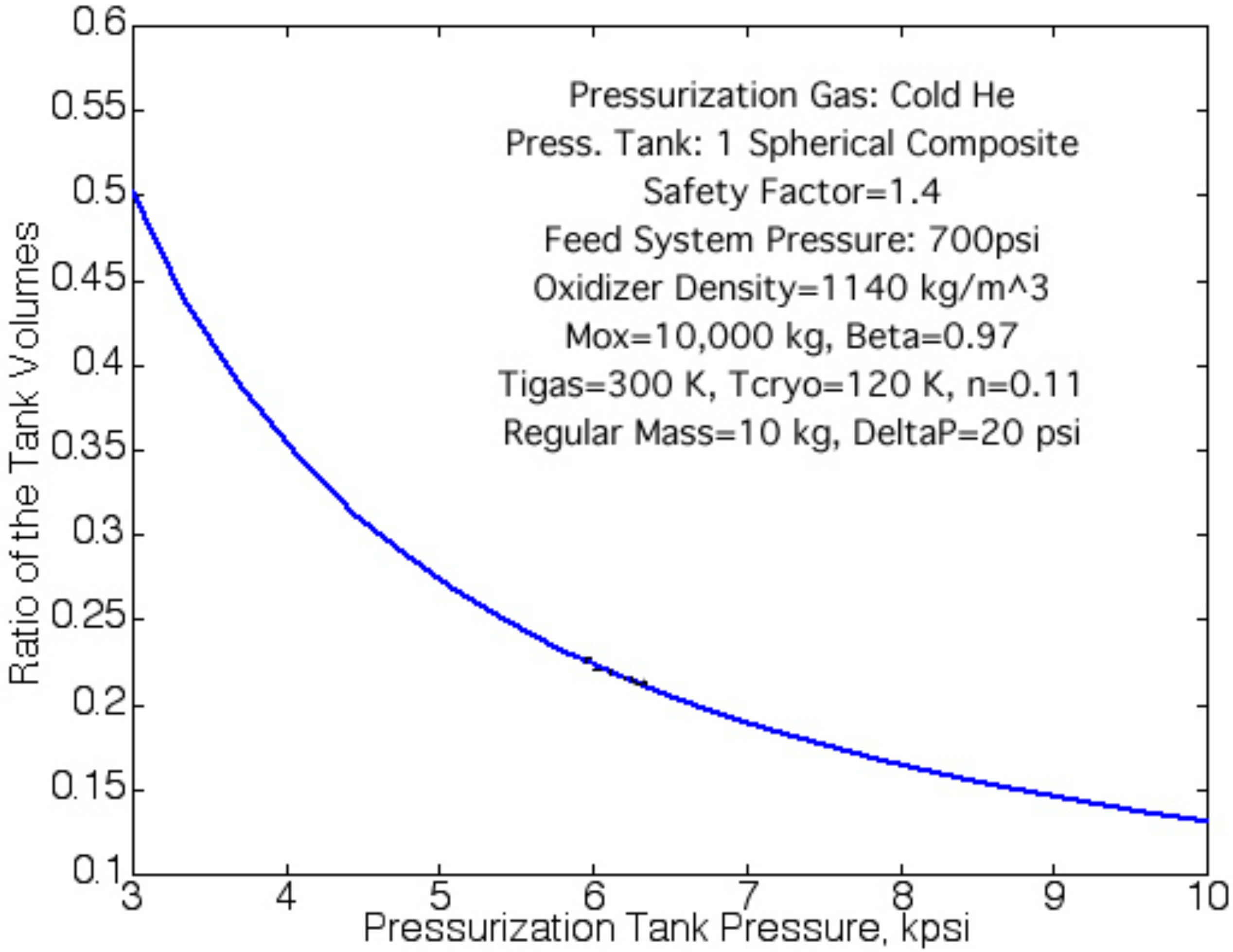
Pressurization System



Pressurization System



Pressurization System



- Large pressures are desirable to minimize the system size
- Practical considerations such as tank availability determine the design pressure
- For typical systems pressure is 4-10 ksi
- Heating the pressurant gas reduces the mass and volume requirements

Nozzle Design – Rao's Method

- Ideal nozzle has zero 3D flow losses
- Ideal nozzle length, fit to Rao's curve for $\gamma = 1.23$

$$\frac{L_{ni}}{D_{nt}} = 2.231\ (AR - 1.8055)^{0.556}$$

- Use the following correction on C_F for the non-ideal nozzle $\eta_{n3D} = C_F / C_{Fi}$

Ln/Lni	1	0.9	0.7	0.6	0.5	0.45	0.40
CF/CFi	1	1	0.9975	0.9950	0.990	0.985	0.970

- Average cone angle for the non-ideal nozzle

$$\tan(\theta_c) = \frac{\sqrt{AR} - 1}{2} \frac{L_{ni}}{L_n} \frac{1}{2.231(AR - 1.8055)^{0.556}}$$

Nozzle Design

- Estimate the throat area from the c* equation

$$\frac{\pi D_{nt}^2}{4} = A_t = \frac{\dot{m}_p c_{theo}^* \eta_c}{C_d P_c}$$

- Select a nozzle area ratio. Estimate the nozzle exit area from

$$D_{ne} = \sqrt{AR}\ D_{nt}$$

- If the nozzle exit diameter is matched to the chamber or tank diameter, increasing chamber pressure allows for higher area ratio (better Isp)
- Estimate the ideal nozzle length from Rao’s expression
- Select a 3D nozzle efficiency. Estimate the nozzle length for the selected 3D nozzle loss.

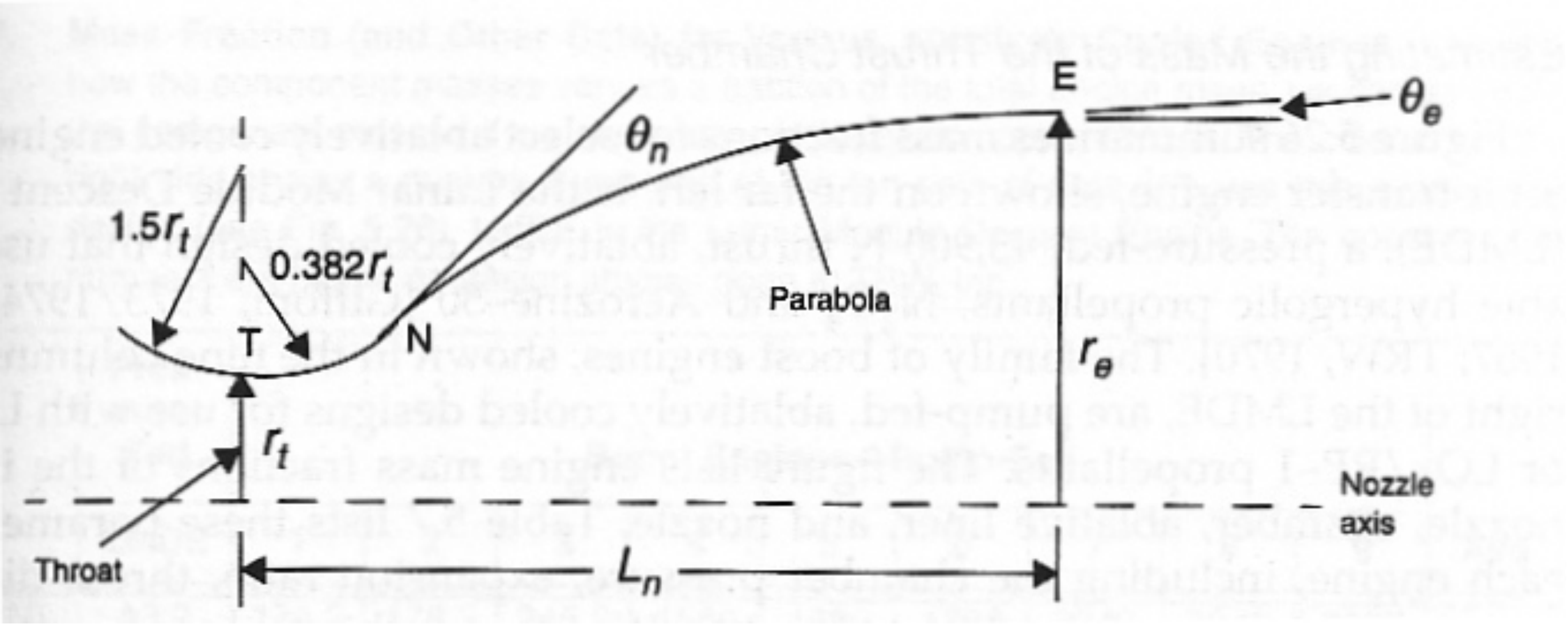
$$L_n = L_{ni} f\left(\eta_{n3D}\right)$$

- Estimate the total nozzle loss (kinetic losses + 2 phase flow losses + 3D flow losses)

$$\eta_n = \eta_{n3D}\eta_{nk}\eta_{n2P}$$

- Estimate the nozzle mass
- Iterate on area ratio and nozzle 3D loss selection for optimum condition
- A good value for the 3D nozzle efficiency is 0.985

Nozzle Design – Parabolic Nozzles

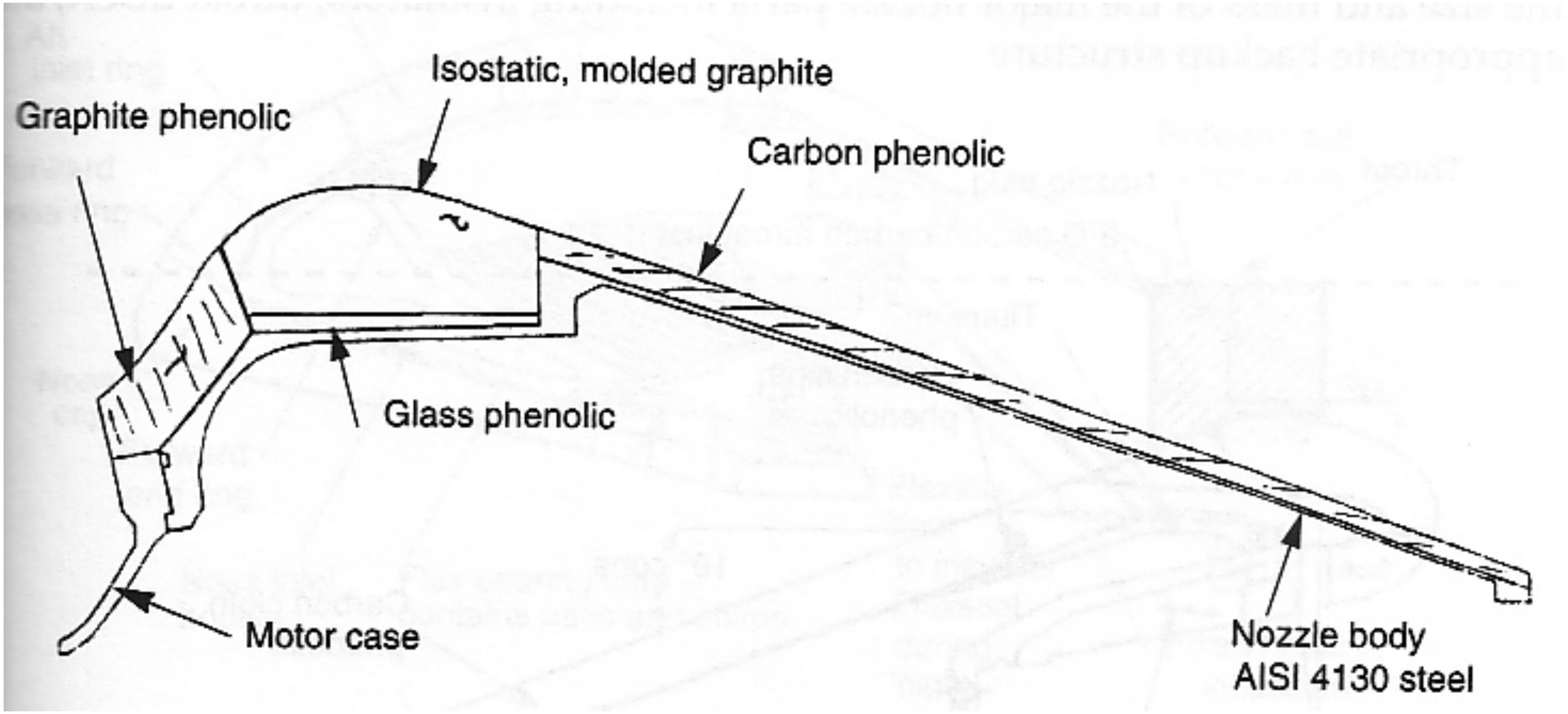


- The length and 3D flow efficiency for a parabolic nozzle can be written as

$$\frac{L_n}{D_{nt}} = \frac{\sqrt{AR}-1}{4}\left[\frac{1}{\tan(\theta_n)} + \frac{1}{\tan(\theta_e)}\right] \qquad \eta_{n3D} \cong \frac{1+\cos(\theta_e)}{2}$$

$$\frac{2}{\tan(\theta_c)} = \left[\frac{1}{\tan(\theta_n)} + \frac{1}{\tan(\theta_e)}\right]$$

Ablative Nozzle Design



- Simple, effective, generally light
- All solids/hybrids and some liquids
- Ablative inner shell (Thickness based on ablation rate x burn time)
- Structural outer shell (Thickness based on internal pressure + other loads)

Nozzle Erosion

- Ablative nozzle surface slowly recesses. The effective heat of gasification protects the structure of the nozzle from heat
- The heat transfer is typically diffusion limited (as in a hybrid rocket system)
- The nozzle regression rate can be written in terms of the local flux

$$\dot{r}_n = a_n (O/F)\ G_n^m$$

- Note that

$$G_n = \frac{\dot{m}_p}{A_n}$$

- Using the c* equation one can write

$$G_n = \frac{P_c C_d}{c^*_{theo} \eta_c} \frac{A_{nt}}{A_n}$$

- Combine to yield

$$\dot{r}_n = a_n (O/F) \left(\frac{C_D}{c^*_{theo} \eta_c} \right)^m \left(\frac{A_{nt}}{A_n} \right)^m P_c^m = B(O/F) \left(\frac{A_{nt}}{A_n} \right)^m P_c^m$$

Nozzle Erosion

- Nozzle erosion dynamics

$$\frac{dD_n}{dt} = 2\dot{r}_n = 2B_n\left(O/F\right)\ P_c^m$$

- Or

$$D_n^{2m} dD_n = \frac{2^{2m+1}}{\pi^m} a_n \dot{m}_p^m dt$$

- This ODE can be integrated to find the change in the nozzle area ratio at any point in the nozzle at any instant
- For a linearly throttled rocket, the relation between the initial and final area ratios is (exit plane erosion is ignored)

$$\frac{AR_i}{AR_f} = \left[1 + \frac{2m+1}{m+1} 2^{0.5} a_n \left(\frac{C_d}{c_{theo}^{*}\eta_c}\right)^{m+0.5} \frac{P_{ci}^{m+0.5} t_b^{1.5}}{M_p^{0.5}} \frac{\left(1+TR\right)^{0.5}\left(1-TR^{m+1}\right)}{1-TR}\right]^{2/(2m+1)}$$

- The throttling ratio is defined as $TR = \dot{m}_{pf} \big/ \dot{m}_{pi}$

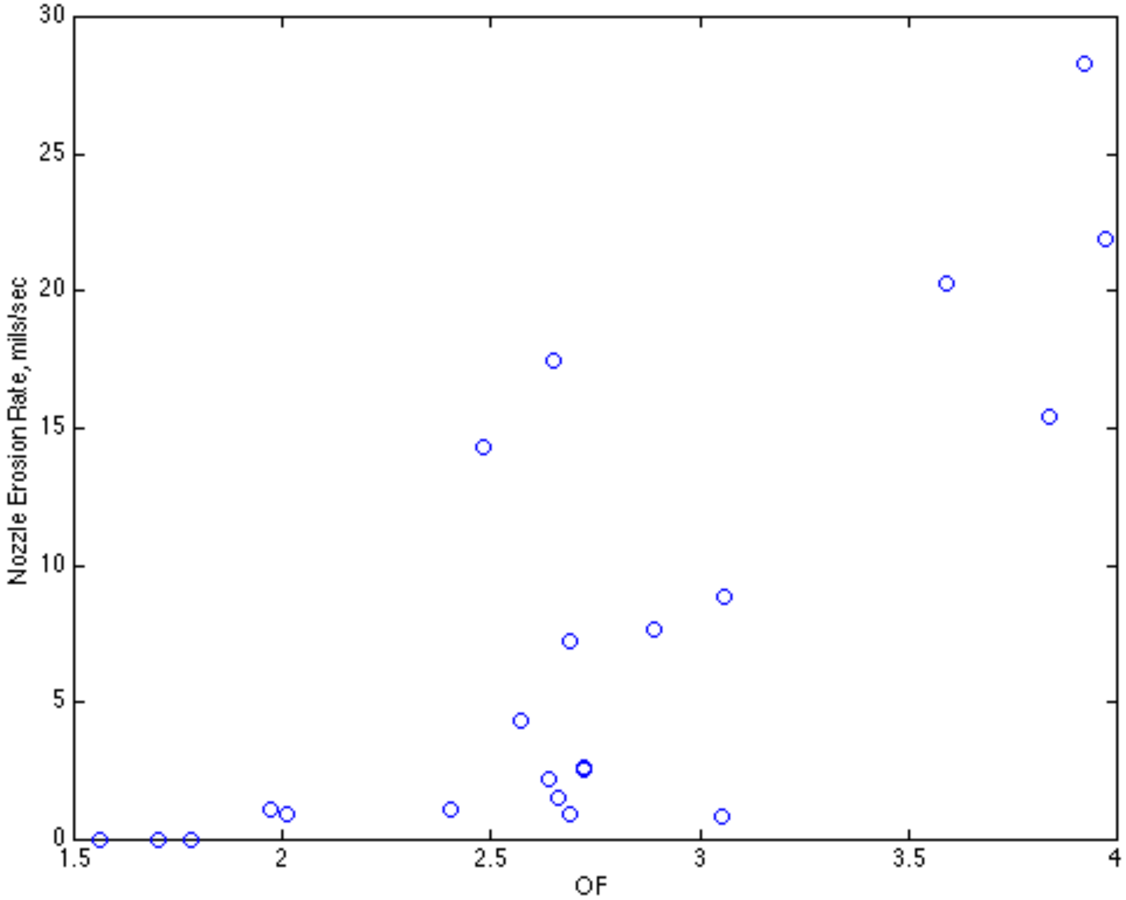
Nozzle Erosion

- Define a reference pressure for which the nozzle erosion rate is known (for a selected average O/F for the motor). Solve for the unknown a_n

$$(\dot{r}_n)_{ref} = B_n \, P_{cref}^m = a_n \left(\frac{C_d}{c_{theo}^* \eta} \right)^m P_{cref}^m$$

- For most systems it is reasonable to assume that *m=1*
- The erosion rate increases with the increasing mass fraction of the oxidizing agents in the nozzle exhaust.
  - CO, HO, H2O, 2 x O2, O
- This value is high in hybrid and liquid systems resulting in high erosion rates
- In hybrids and liquids the erosion rate is a strong function of the O/F of the motor. For high O/F the erosion rates can be quite high.
- Aluminum addition typically reduces the erosion rate for hybrids since Al2O3 formation decreases the mass fraction of oxidizing agents
- In solid rockets the nozzle throat erosion rate for various nozzle throat insert materials are
  - ATJ Graphite: 0.004-0.006 in/sec
  - 3D Carbon-Carbon: 0.0005-0.001 in/sec
- 2D carbon/carbon or 3D carbon/carbon nozzle inserts are not suitable for liquid or hybrid rockets due to the oxidizer attack to the surface

Nozzle Erosion Data – ATJ Graphite

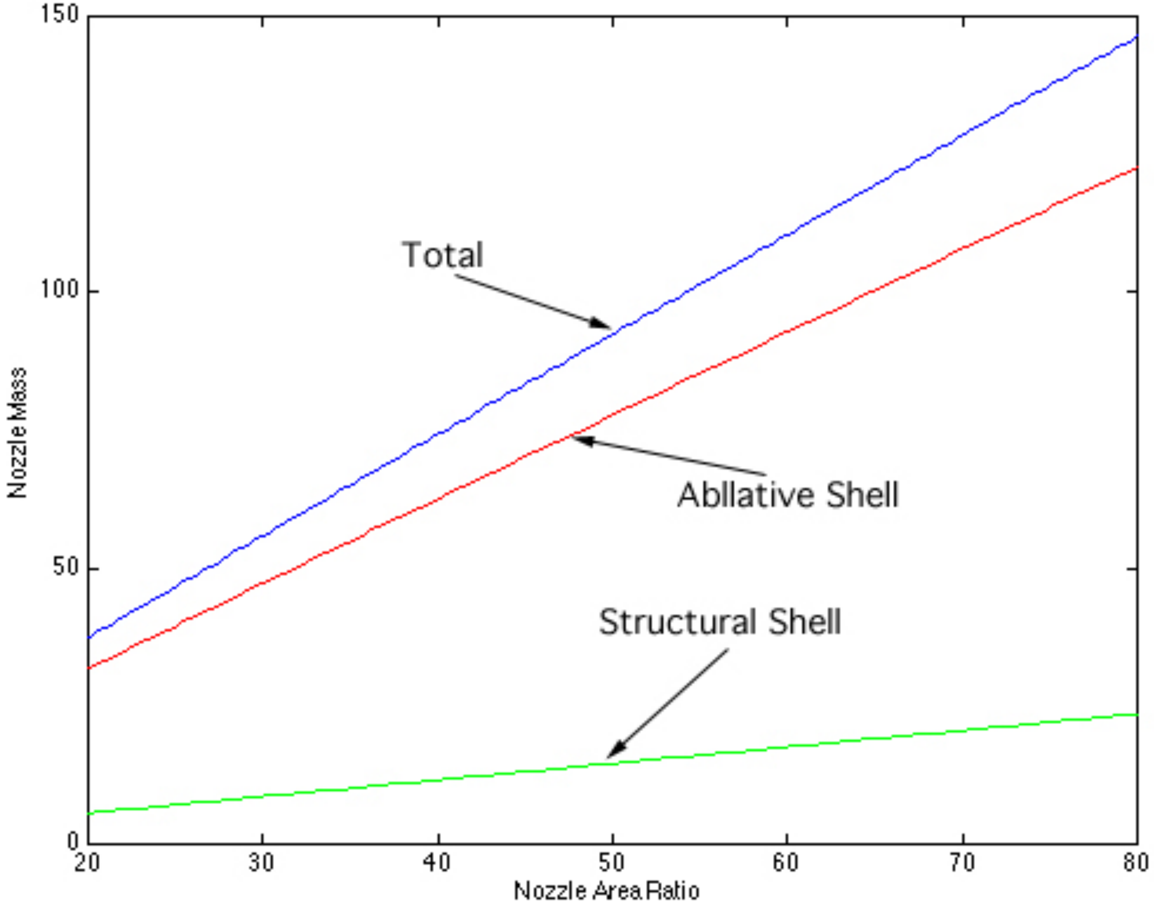


- Propellants: GOX/ Paraffin
- Nozzle material: ATJ graphite
- Burn time: 8 sec nominal
- Chamber pressure: 800 psi nominal
- Nozzle throat diameter: 2” nominal

Nozzle Design Issues

- Nozzle erosion effects the performance adversely due to
  - Reduction of the nozzle area ratio in time, Isp loss
  - Increase in nozzle weight, structural mass fraction increase
- Note that the erosion (or the effect of nozzle erosion) can be minimized by
  - Keeping the chamber pressure low (reduce the nozzle mass flux)
  - Running at low O/F
  - Formulating the propellants to minimize the mass fraction of the oxidizing agents (can use the results of the Isp code)
  - Selecting a suitable nozzle material
  - Introducing a cool film on the surface of the nozzle
- Nozzle weight can be estimated from the nozzle erosion rate equation by estimating the required thickness of the ablative material. Use a safety factor (i.e. 1.5). The weight of the structural shell can be calculated using the hoop stress induced by the pressure inside the nozzle
- For hybrids silica phenolic is commonly used as the ablative nozzle material over the entire nozzle surface. Silica phenolic is resistant to oxidizer attack.
- For small inexpensive hybrid systems ATJ graphite is also commonly used. Note that ATJ is a brittle material. One must minimize the stress concentration areas.
- Use the following reference erosion rate for preliminary design purposes (LOX/HC hybrids running at O/F less than 2.5)
  - Erosion rate: 0.007 in/sec at 500 psi ($m=1$)

Nozzle Mass Area Ratio Variation Example



- LOX/paraffin Hybrid
- Ablative shell: silica phenolic
- Structural shell: glass phenolic
- Increase in area ratio improves Isp but increases the structural mass fraction