AA 284a
Advanced Rocket Propulsion

Lecture 10
Hybrid Rocket Propulsion
Design Issues

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Axial Flow Hybrid Rocket

- For an axial flow hybrid the oxidizer flows parallel to the axis of the propulsion system through a cylindrical cavity(s) inside fuel grain
- The cross sectional shape of the cylindrical opening (port) can be
  – Circle, Triangle, D-shape or any other complex form
- There could be multiple cylindrical openings (ports), multi-port hybrid
  – Two ports: Double-D
  – Four ports: Quad
  – Five ports: Quad+1
  – Larger number of ports utilizes single or double row wagon wheel configuration
  – AMROC motor :15+1 ports
- Fuel utilization (minimizing the sliver fraction) dictates the shapes of the ports once the number of ports and overall port configuration is selected
- The fuel sliver fraction increases with the increasing number of corners
- Note that the hydraulic diameters of the ports must be matched for even burning
- Axial variation of the port geometry is also possible
- Most simple port design is a single circular geometry. This is the most efficient shape for fuel utilization. No corners
- We will limit the discussions to single port hybrids

Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- The local instantaneous regression rate expression:

$$\dot{r}(x,t) = a\ G^n x^m$$

- Note that classical theory predicts *n=0.8* and *m=-0.2*
- First consider the axial variation of regression rate for a given instant in time. For simplicity assume that the port shape and hydraulic diameter is independent of *x*.
- The axial mass balance in the fuel port yields (C_p is the circumference)

$$\dot{m}(x) = \dot{m}_{ox} + \int_0^x \rho_f C_p \dot{r}(x')\ dx'$$

- Convert to flux (A_p is the port area) and substitute the regression rate expression

$$G(x) = G_{ox} + \frac{a\rho_f C_p}{A_p} \int_0^x G^n x'^m dx'$$

Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- Convert the integral equation to a differential equation

$$\frac{dG}{dx} = a\rho_f \frac{C_p}{A_p} G^n x^m$$

- Integrate by the separation of variables

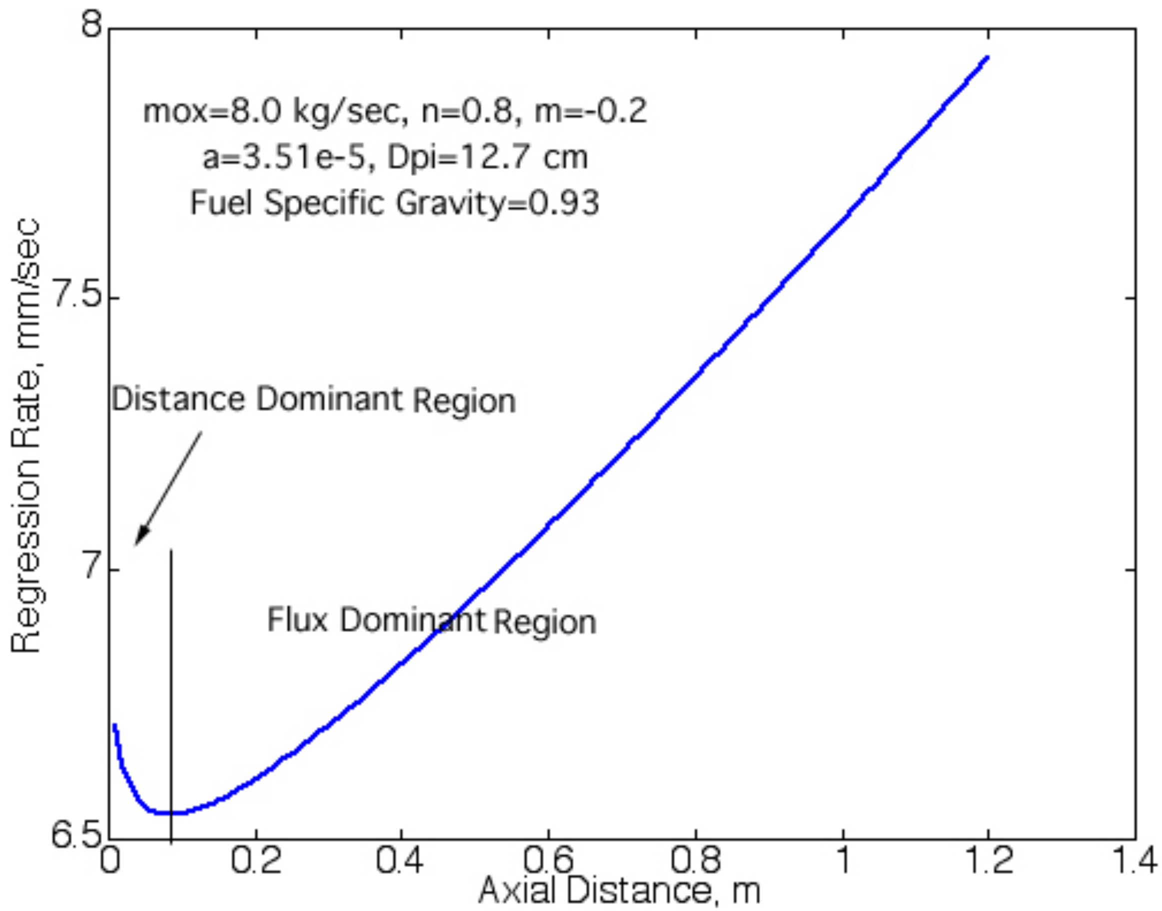
$$G(x) = \left(G_{ox} + a\ \rho_f \left(\frac{1-n}{1+m} \right) \frac{C_p}{A_p} x^{1+m} \right)^{1/1-n}$$

- Note that $G_{ox} = \dfrac{\dot{m}_{ox}}{A_p}$

- For circular port $\dfrac{C_p}{A_p} = \dfrac{4}{D_p}$

- To obtain regression rate substitute the flux expression into the regression rate equation

Axial Flow Hybrid Rocket – Axial Variation of Regression Rate



- Constant port area assumption implies that the derived formula for the axial variation of the regression rate is only valid at t=0
- The regression rate variation can be more that 20%
- Axial change in port diameter more than 10% is rarely observed
- This due to the self correcting behavior: As one part of the port opens up more, the local flux decreases, resulting in a decrease in the regression rate
- Typically space averaged port diameter is used in the calculations

Axial Flow Hybrid Rocket – Derivation of the Design Equations

- As discussed in the previous section, the axial variation of the port diameter and the regression rate will be ignored. Introduce the space averaged port diameter and the regression rate
- Use the simplified space averaged regression rate expression which is assumed to be valid at any instant of the hybrid operation

$$\dot{r}(t) = a\ G_{ox}^{n}$$

- Using the definition of the regression rate the dynamic equation for the space averaged port diameter can be written as

$$\frac{dD_p}{dt} = 2\ \dot{r} = 2\ a\ G_{ox}^{n}$$

- For a circular port hybrid

$$\frac{dD_p}{dt} = \frac{2^{2n+1} a}{\pi^n} \frac{\dot{m}_{ox}^{n}}{D_p^{2n}}$$

Axial Flow Hybrid Rocket – Derivation of the Design Equations

- For constant oxidizer flow rate, this expression can be integrated to obtain the port diameter as a function of time

$$D_p(t)=\left[D_{pi}^{2n+1}+\frac{(2n+1)2^{2n+1}a}{\pi^n}\dot{m}_{ox}^n t\right]^{1/(2n+1)}$$

- The instantaneous flux, regression rate, fuel mass flow rate, *O/F* can be calculated from

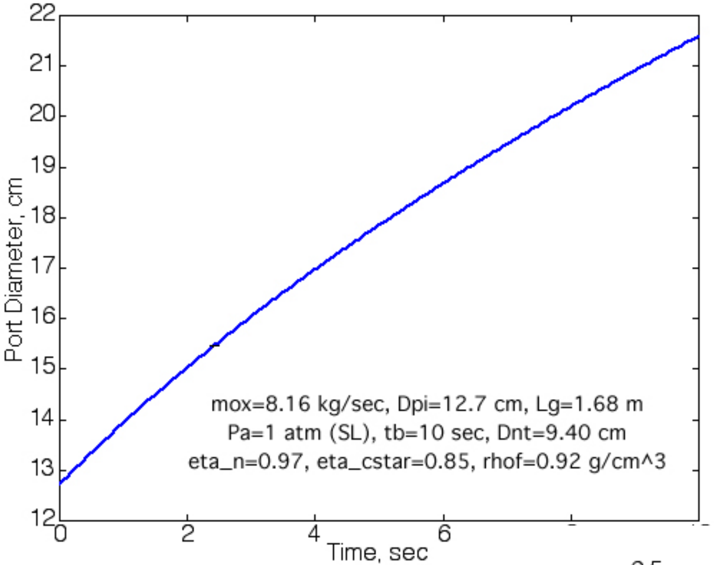
$$G_{ox}(t)=\frac{4\dot{m}_{ox}}{\pi D_p^2} \qquad \dot{r}(t)=aG_{ox}^n \qquad \dot{m}_f(t)=\rho_f \pi D_p L\,\dot{r} \qquad \frac{O}{F}=\frac{\dot{m}_{ox}}{\dot{m}_f}$$

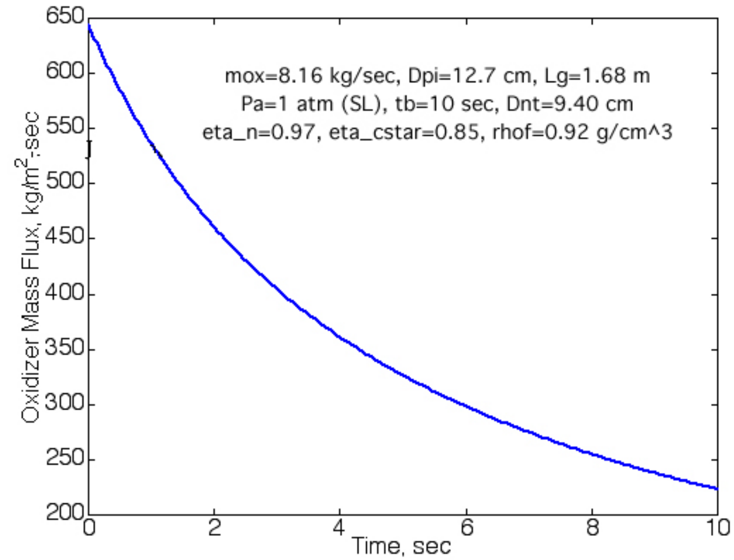
- Based on *O/F*, total mass flow rate and nozzle geometry, one can determine the chamber pressure and thrust as a function of time

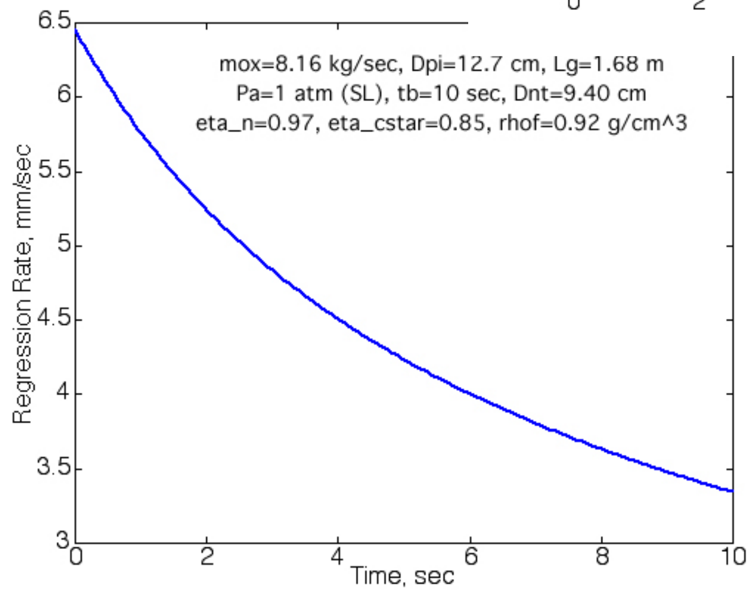
$$P(t)=\frac{(\dot{m}_{ox}+\dot{m}_f)\,c^*_{theo}\eta_c}{A_{nt}C_d} \qquad T(t)=(\dot{m}_{ox}+\dot{m}_f)\,c^*_{theo}\eta_c C_{Ftheo}\eta_n$$

- Numerical integration is required for complex oxidizer flow rate schedules

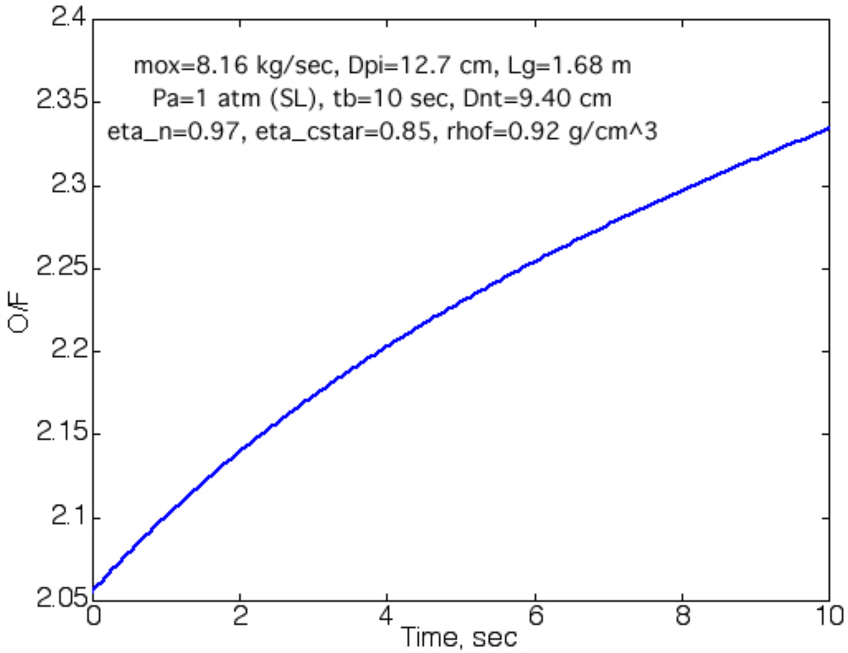
Axial Flow Hybrid Rocket –Design Example





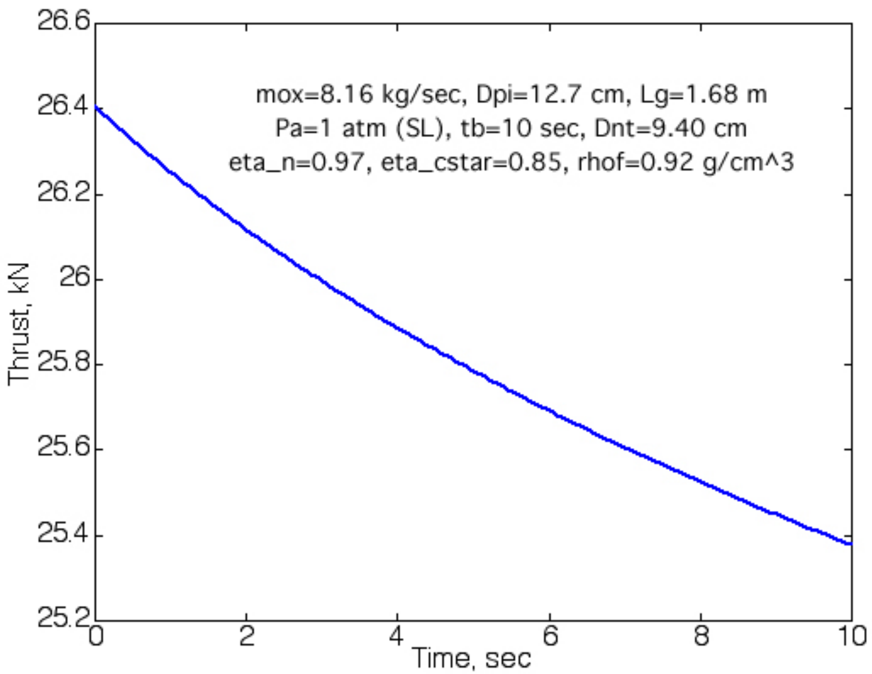


Axial Flow Hybrid Rocket –Design Example



- *O/F* Shift: *O/F* in a hybrid rocket changes in time
  - Shift due to port opening
  - Shift due to throttling

- Thrust change is due to
  - Change in the fuel mass flow rate
  - *O/F* shift
  - Chamber pressure drop



Axial Flow Hybrid Rocket – Scaling Laws

- For constant oxidizer flow rate, the following formula relates the initial and final port diameters

$$\left(\frac{D_{pf}}{D_{pi}}\right)^{2n+1} - 1 = \frac{(2n+1)2^{2n+1}a}{D_{pi}^{2n+1}\pi^n}\dot{m}_{ox}^n t_b$$

- The final port diameter can be solved in terms of the total oxidizer mass M_{ox}

$$D_{pf} = \left[\left(\frac{(2n+1)\ 2^{2n+1}a}{\pi^n}\right)\frac{M_{ox}^n t_b^{1-n}}{1-\left(D_{pi}/D_{pf}\right)^{2n+1}}\right]^{\frac{1}{2n+1}}$$

- The fuel grain length can be related to the O/F

$$L = \frac{4M_{ox}}{\pi\rho_f (O/F)\left(D_{pf}^2 - D_{pi}^2\right)}$$

Axial Flow Hybrid Rocket – Design Process

- Select
  - Propellants
  - Port diameter ratio
    - Structural design constraint
    - Bore stress increases with increasing diameter ratio
    - A typical value is 2
    - Note that the diameter ratio is related to the volumetric loading

$$VL = 1 - \left(\frac{D_{pi}}{D_{pf}} \right)^2$$

    - The diameter ratio also determines the flux ratio during the burn
  - *O/F* (from optimal Isp)
  - Burn time
    - from optimal trajectory and constraints
    - Minimize the gravity loss under the acceleration and Qmax constraints
  - Propellant mass (from mission requirement)
  - Chamber pressure
  - Nozzle area ratio
- Use the design equations to determine the geometrical parameters: grain dimensions, nozzle throat and exit areas

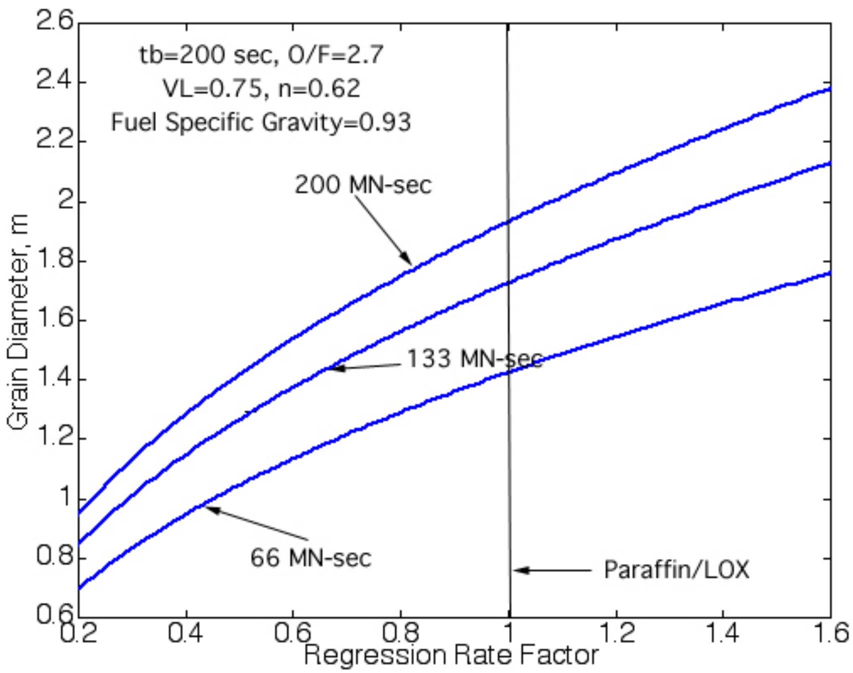
Axial Flow Hybrid Rocket – Design Process

- The grain geometry is critical in achieving an efficient packing in a hybrid rocket system
- For liquid systems packing is relatively easy since the tank geometries can be selected freely
- For solids packing is less of an issue
  - Solids are denser
  - A wide range of fuel grain geometries are possible (fuel generation rate is proportional to the fuel surface area)
- For most applications small *L/D* values are desirable. Maximize the grain diameter (Written in terms of the total impulse (I_{tot}), volumetric loading (VL) and burn time (t_b))

$$D_{pf} = \left[\left(\frac{(2n+1)\; 2^{2n+1} a}{I_{sp}^n g_o^n \pi^n} \right) \frac{\left(\frac{O/F}{1+O/F} \right)^n I_{tot}^n \; t_b^{1-n}}{1-(1-VL)^{(2n+1)/2}} \right]^{\frac{1}{2n+1}}$$

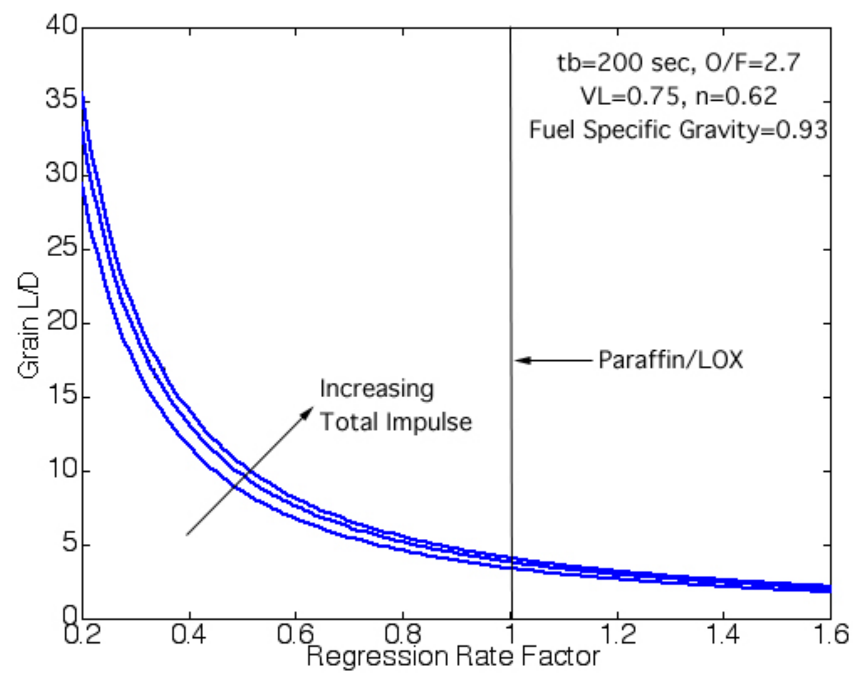
- Note that grain length is $L = \frac{4}{\pi \rho_f VL} \frac{I_{tot}}{I_{sp} g_o} \frac{1}{(1+O/F) D_{pf}^2}$

Axial Flow Hybrid Rocket – Scaling Trends

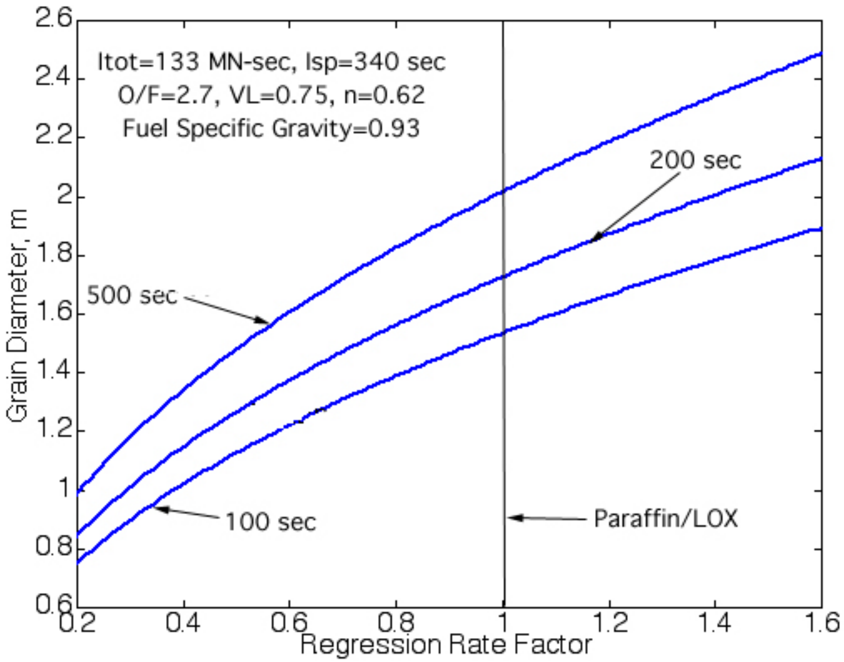


- Grain Diameter increases with increasing regression rate coefficient
- Grain L/D decreases with increasing regression rate coefficient

- Grain Diameter increases with increasing impulse
- Grain L/D slightly increases with increasing total impulse

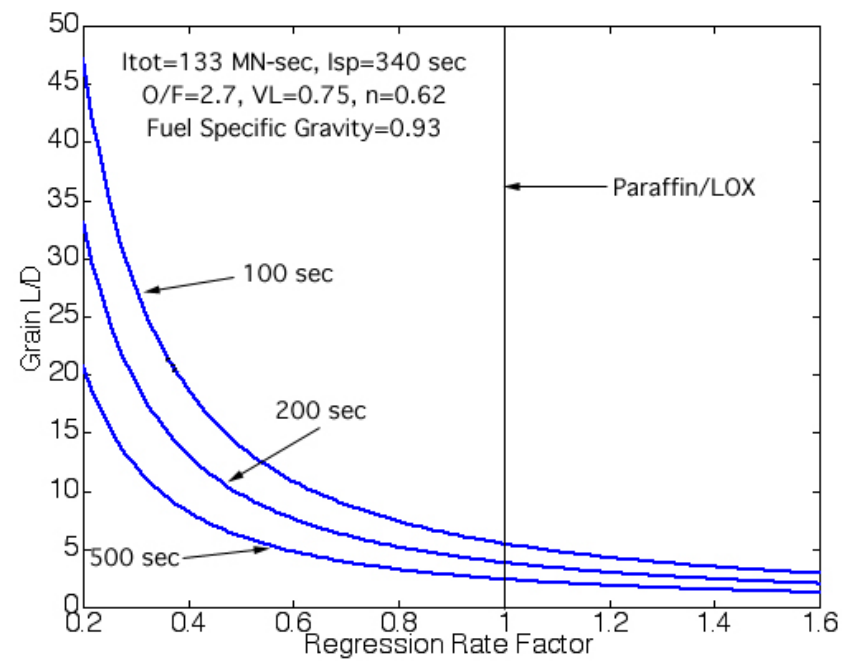


Axial Flow Hybrid Rocket – Scaling Trends



- Grain Diameter increases with increasing burn time (for n < 0.5)
- Grain L/D decreases with increasing burn time

- For slow burning fuels, L/D for a single port system is unacceptably large
- This is the driving force for multi-port designs



O/F Shift

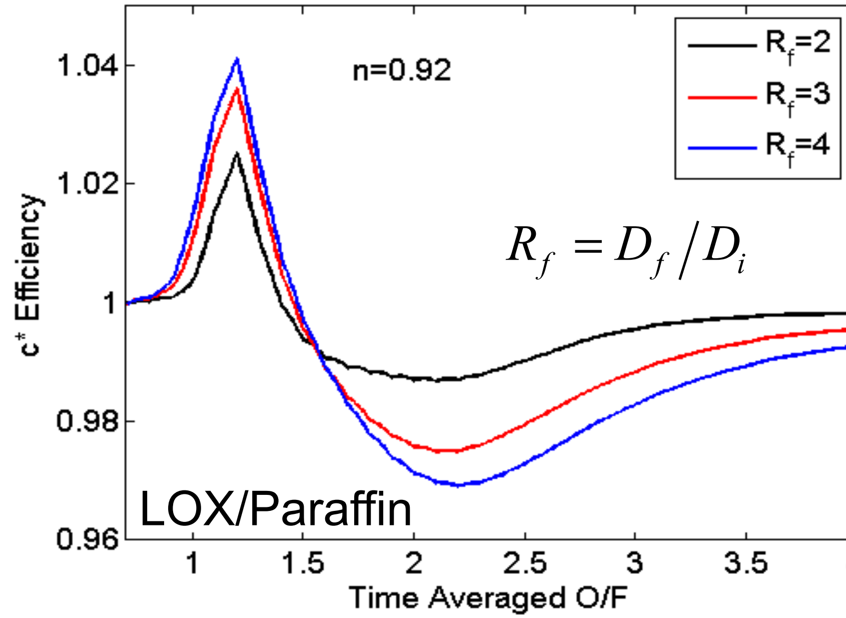
- The oxidizer to fuel ratio of a liquid rocket is directly controlled by adjusting the oxidizer and fuel mass flow rates
- The O/F of a solid system is constant (or passively programmed) since fuel and oxidizer are premixed in the solid phase
- The O/F of a hybrid rocket shifts during the operation since the fuel generation is determined by the physics and chemistry of the combustion process
- The O/F for a single circular hybrid can be written as

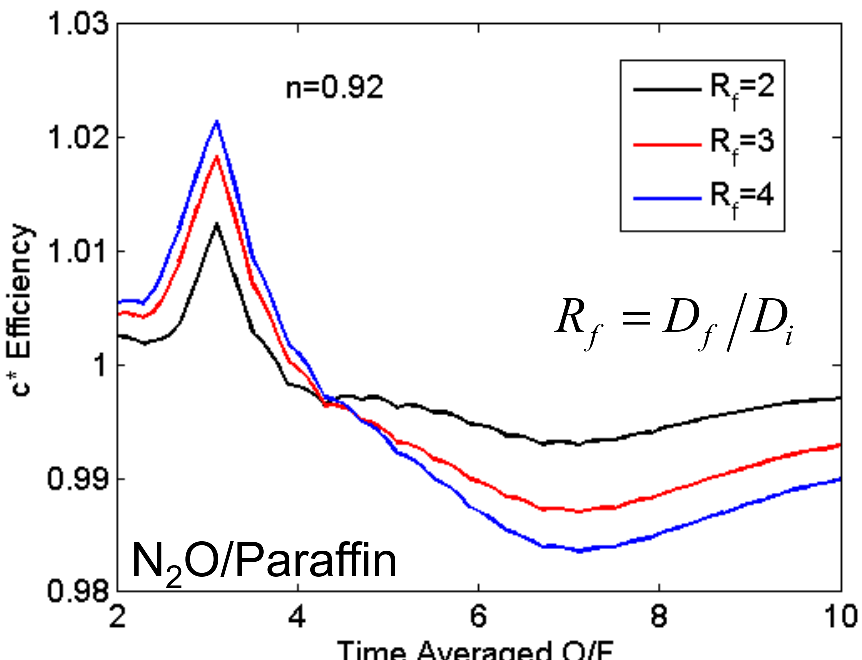
$$\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{\pi^n D_p^{2n} \dot{m}_{ox}}{\rho_f \pi \ D_p L \ a \ 4^n \dot{m}_{ox}^n} = \frac{D_p^{2n-1} \dot{m}_{ox}^{1-n}}{\rho_f \pi^{1-n} a \ 4^n \rho_f L}$$

- For typical hybrids the port exponent *2n-1* and the oxidizer mass flow rate exponent *1-n* are both positive (*0.5<n<0.8*)
- Thus O/F increases with time (diameter effect) for constant oxidizer mass flow rate. For *Dpf/Dpi=2* and *n=0.62*, O/F shifts by a factor of 1.18. Effect of this kind of a shift on the c* efficiency is small due to the flat nature of the c* curve around the optimum O/F
- Also O/F increases with increasing oxidizer mass flow rate (for *n=0.62*, a throttling ratio of 10:1 changes the O/F by a factor of 2.4)
- If *n=0.5* no shift due to port opening, if *n=1* no shift due to throttling
- Oxidizer flow rate can be programmed to keep the O/F constant $\dot{m}_{ox} \propto D_p^{(2n-1)/(1-n)}$
- In a classical hybrid both O/F and thrust can not be controlled simultaneously
- Aft oxidizer injection allows one to schedule the required thrust profile at a constant O/F

Shift Efficiency

- For an *n* exponent of 0.92 shift efficiency is still close to unity, but not negligible for $R_f > 2.0$
- For $R_f < 2.0$, shift efficiency is higher than 0.99

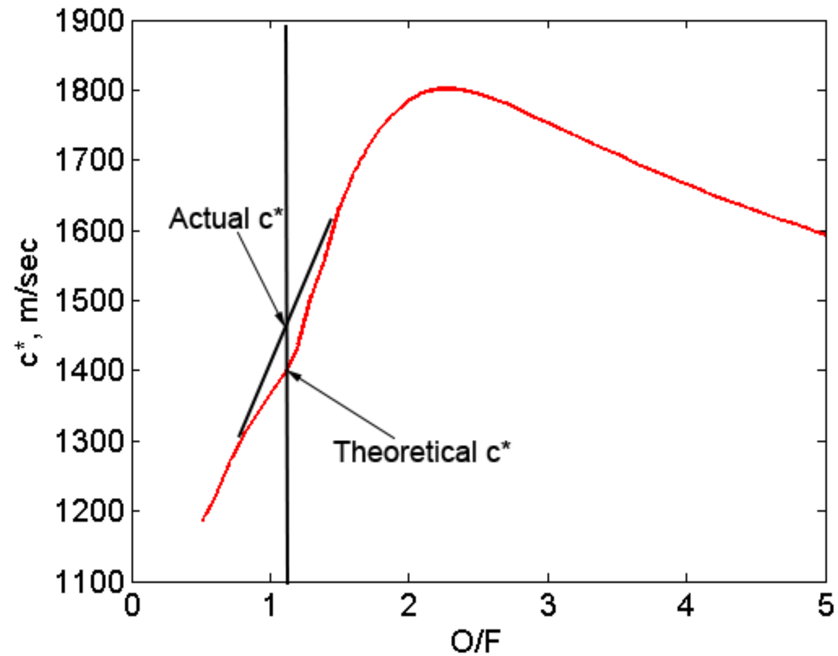


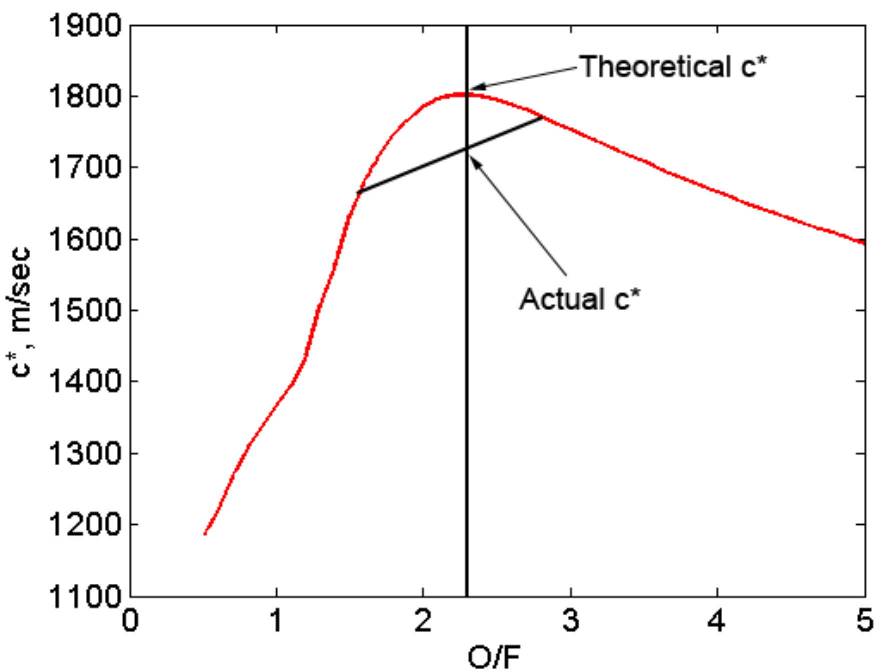


- Same conclusions can be drawn for other oxidizers such as N_2O
- The only difference is the stretching of the curves in the *O/F* axis
- Shift efficiency scale is the same for $N_2O$and LOX

Efficiency > 1.0 ?

- At low average *O/F*, the c* curve is concave
- Thus average c* experienced by the motor can be higher than the c* evaluated at the average *O/F*
- Efficiency > 1.0





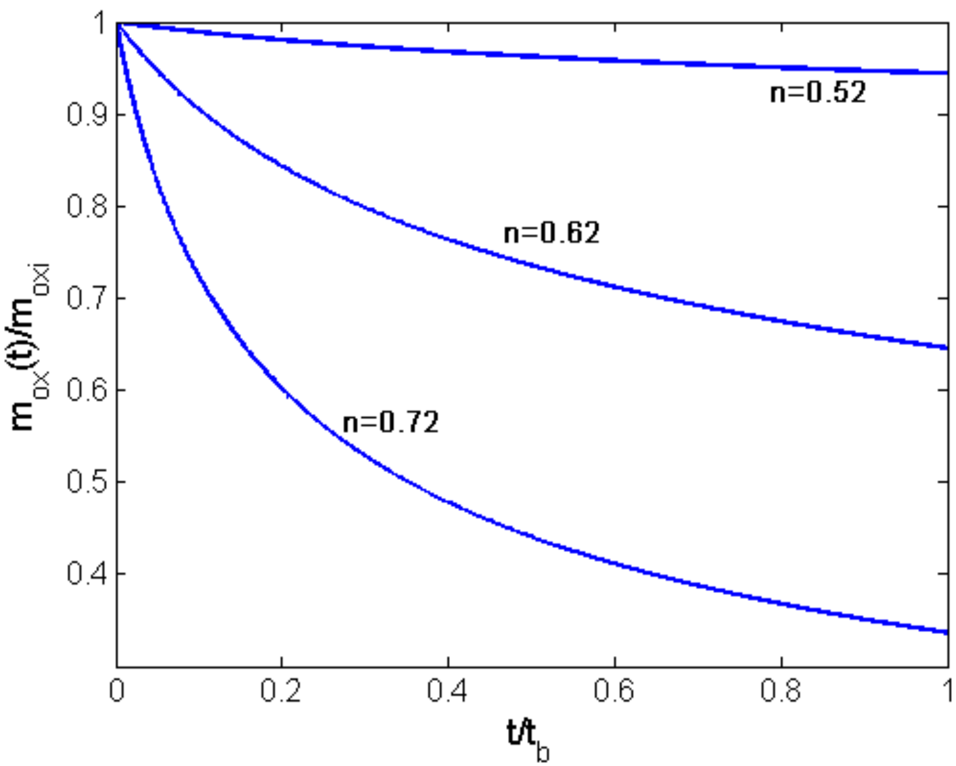
- At high average *O/F*, the c* curve is convex
- Thus average c* experienced by the motor is lower than the c* evaluated at the average *O/F*
- Efficiency < 1.0

Active O/F Shift Control – Oxidizer Flow Rate Adjustment

- The following oxidizer flow rate schedule generates neutral burning

$$\frac{\dot{m}_{ox}(t)}{\dot{m}_{oxi}} = \left(1 + \frac{2}{1-n}\frac{\dot{r}_i}{D_i}t\right)^{1-2n}$$

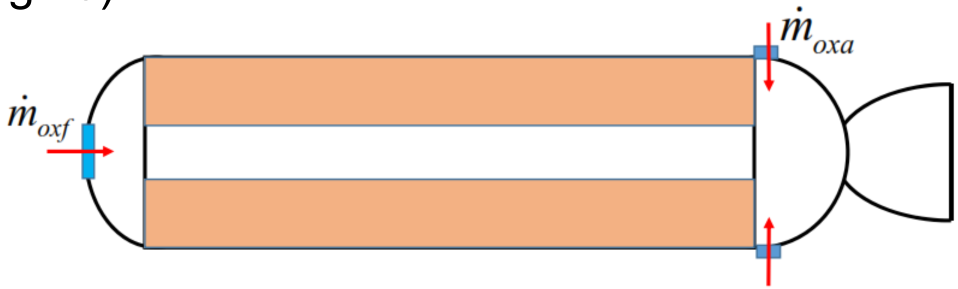
$$\dot{m}_{oxi} = \left(\frac{C_n (O/F)_{opt}}{D_i^{2n-1}}\right)^{n/(1-n)}$$



- For most propellants (n>0.5) the oxidizer flow rate variation in time is significant
- Since the influence of the shift on c* is negligible for most circular port systems, oxidizer flow rate scheduling is NOT recommended

Active O/F Shift Control – Aft Oxidizer Injection

- Oxidizer injection at two locations results in 2D control in a hybrid (as in a liquid engine)



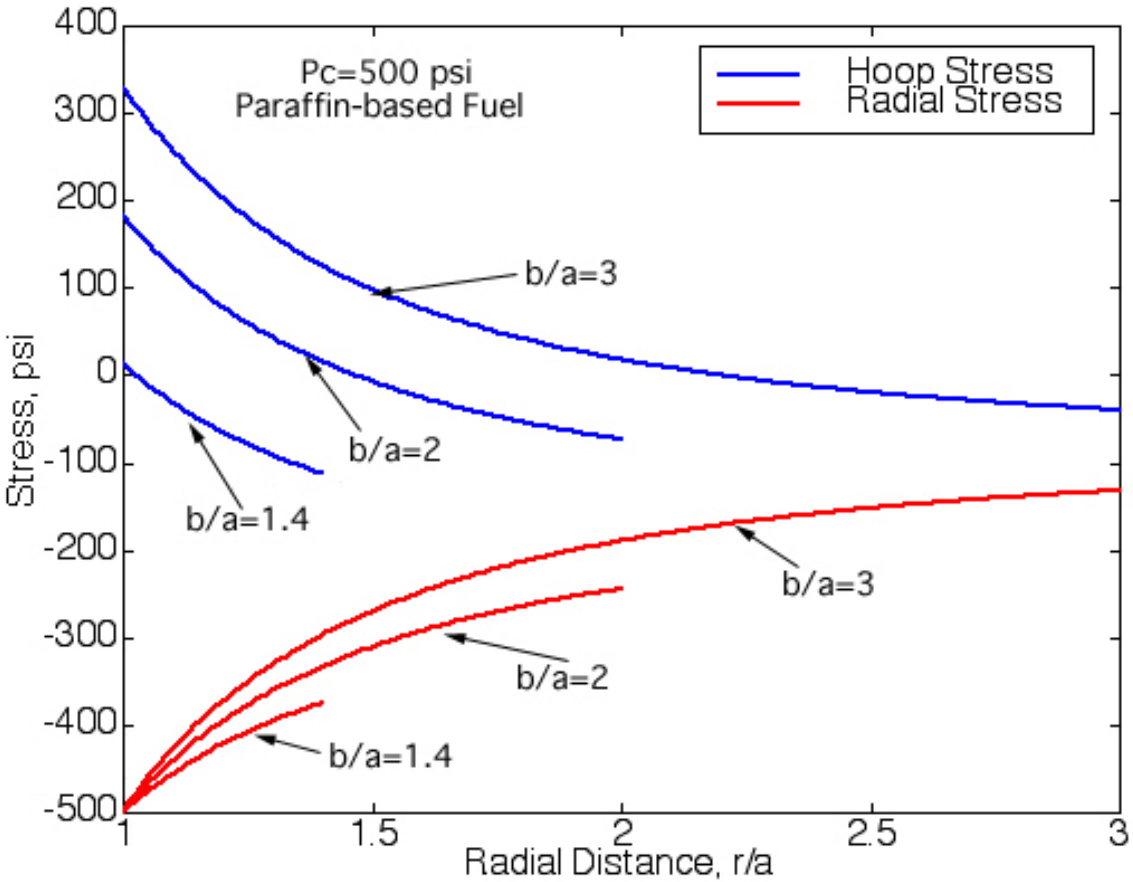
- For neutral burn the following scheduling should be implemented

$$\frac{\dot{m}_{oxfore}(t)}{\dot{m}_{ox}} = \frac{\dot{m}_{ox}^{(1-n)/n} D_i^{(2n-1)/n}}{\left[C_n (O/F)_{opt}\right]^{1/n}} \left(1 + \frac{G_{oxref}}{L\rho (O/F)_{opt}} t\right)^{(2n-1)/2n}$$

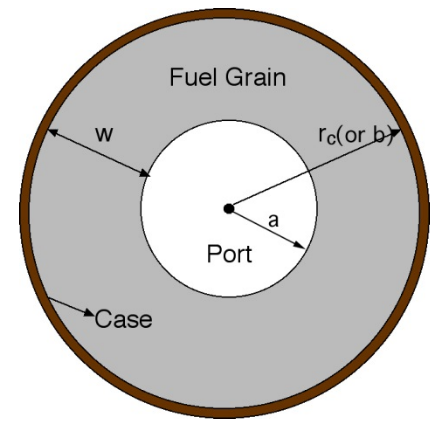
$$\frac{\dot{m}_{oxaft}(t)}{\dot{m}_{ox}} = 1 - \frac{\dot{m}_{oxfore}(t)}{\dot{m}_{ox}}$$

- For systems requiring precise control of thrust and *O/F*, aft injection is the best method
- Only shortcoming is the extra hardware (plumbing, injectors etc…)

Fuel Grain Stress Distribution-Pressure Loading



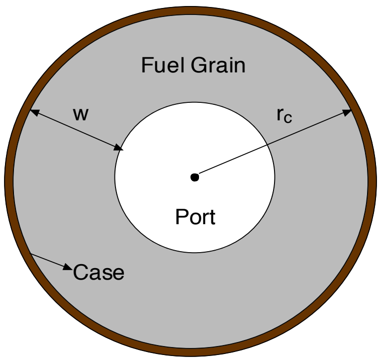
- **Stress Distribution**
- **Pressure Loading**
- **Single Circular Port**
- **Paraffin-based Fuel**



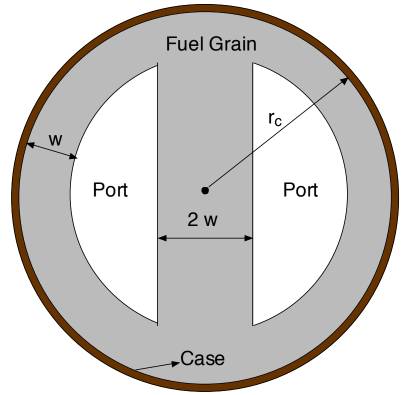
- **Material approaches failure boundary as b/a increases**

Failure Boundaries: $\tau = \frac{|\sigma_r - \sigma_\theta|}{2} < \tau_y \qquad \sigma = \max(\sigma_r, \sigma_\theta) < \sigma_f$

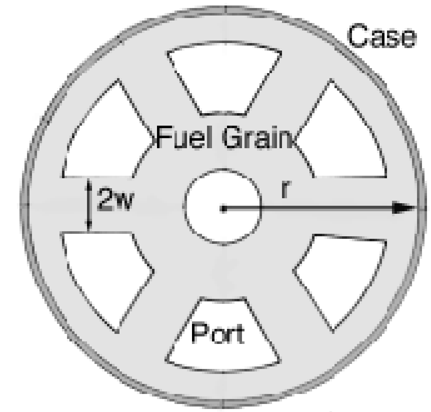
Hybrid Rocket Fuel Port Designs



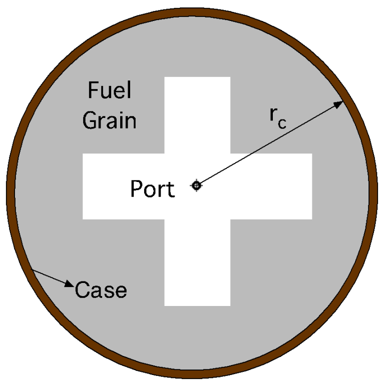
Single Circular Port



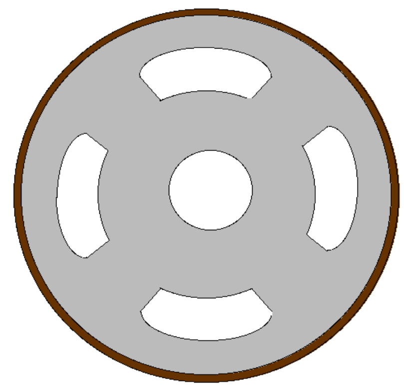
Double-D



6+1 Port Wagon Wheel



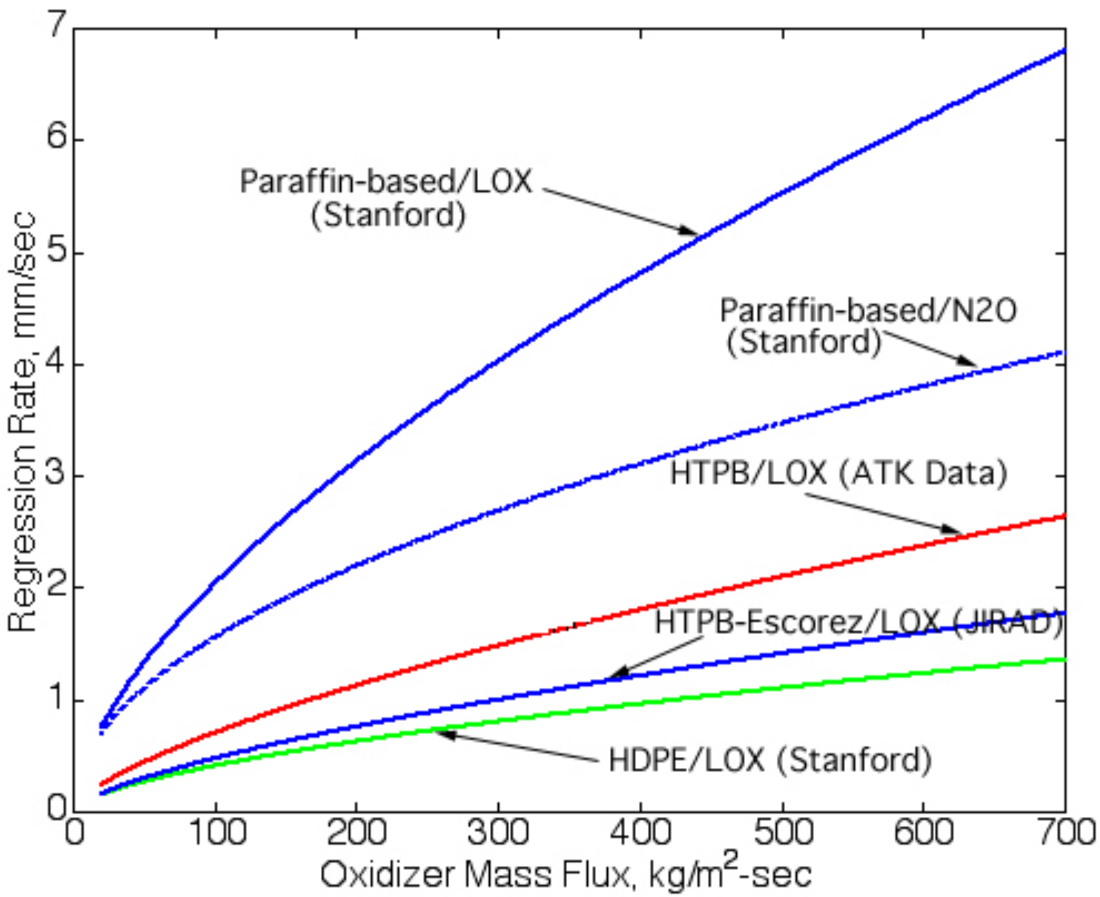
Cruciform Port



4+1 Port Design

Decreasing Regression Rate

Regression Rate Data for Various Hybrid Propellants



- **HTPB/LOX:**

  $$\dot{r} = 3.043\ 10^{-2} G_{ox}^{0.681}$$

- **HTPB/Escorez/LOX**

  $$\dot{r} = 2.061\ 10^{-2} G_{ox}^{0.68}$$

- **HDPE/LOX**

  $$\dot{r} = 2.340\ 10^{-2} G_{ox}^{0.62}$$

- **Paraffin/LOX**

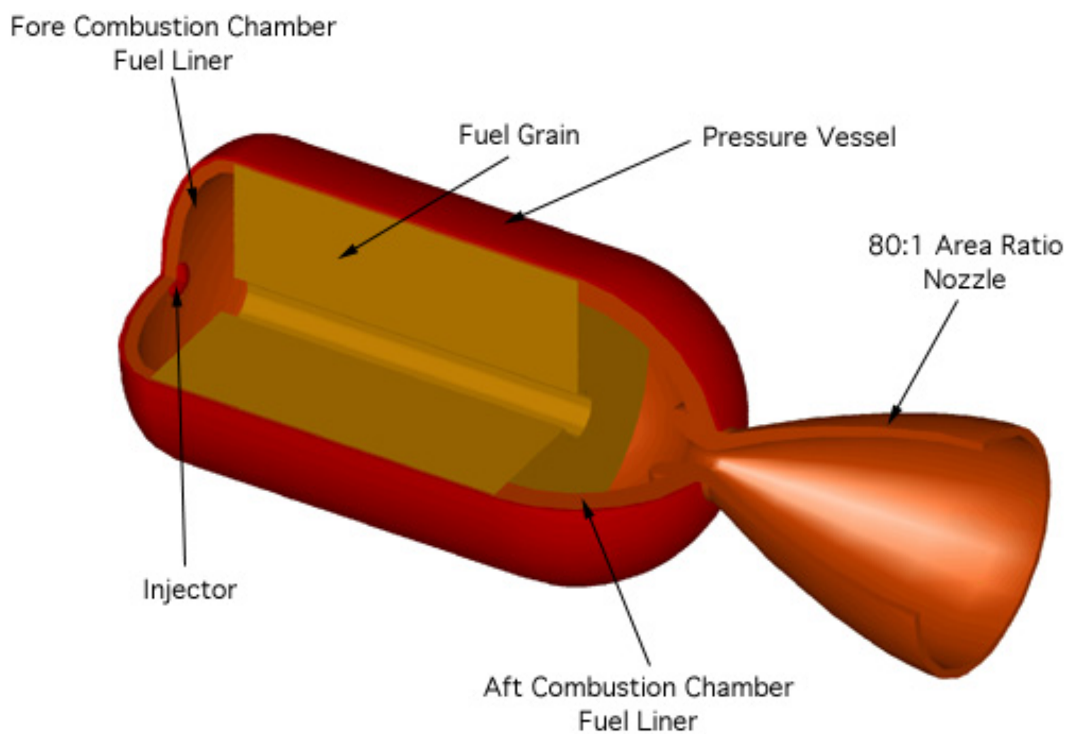
  $$\dot{r} = 11.70\ 10^{-2} G_{ox}^{0.62}$$

- **Paraffin/N2O**

  $$\dot{r} = 15.50\ 10^{-2} G_{ox}^{0.50}$$

(Units are mm/sec and kg/ m^2-sec)

Hybrid Rocket Internal Ballistic Design Example



- **Pre-combustion chamber: vaporization of the oxidizer (2:1 ellipse)**
- **Post-combustion chamber: mixing and reaction of the unburned fuel and oxidizer. Increase effective L* (hemisphere)**