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CONTENTS AND COMMENTARY ON WILLIAM MOORE'S *A TREATISE ON THE MOTION OF ROCKETS AND AN ESSAY ON NAVAL GUNNERY*

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Summary—William Moore's known biography and his contribution to mechanics are summarized, and his acquaintanceship with Charles Hutton's work examined. G. K. Mikhailov's summary of workers on this subject before Moore, e.g. Daniel Bernoulli, is referred to. The contents of Moore's treatise on rocketry and naval gunnery, published in 1813, reputedly the first publication on the motion of rockets, are described in some detail and then related to his four slightly earlier papers on the same subject in *Nicholson's* journal. As well, his mathematical competence and particularly his successful use of the fluxional calculus, conflict somewhat with Charles Babbage's cry for d'ism rather than dottage. Comment on Moore's *Naval Gunnery* with reference to remarks of Howard Douglas in his classic work of 1855 on the same topic, is made. A tactic advocated by both was that of firing a cannon ball at speed just sufficient to penetrate a wooden-sided ship in order to secure maximum damage; this is a phenomenon still analytically unaccounted for.

INTRODUCTION

Finding the little known book of 1813 by William Moore, principally on rocket motion and reputedly the first on that subject, brought the author to attempt a short, useful and comprehensive description of it; this, as completed, is presented below. The task proved more difficult than the author had anticipated because few personal details about Moore are available, and sight of his papers and book is not easily obtained, see Note 1. Besides reducing my descripton to a useful length, yet giving, or referencing, available biographical details, I have tried to compare and identify to what extent he built on the work of Robins and Hutton.

My hope is that an appreciation of Moore's writings, his mode of approach to his subject and his contemporary style as well as the contents of his volume and its relation to his papers, has been conveyed.

I need incidentally also to draw attention to the paper by G. K. Mikhailov on *Early Stages in the Development of Rocket Propulsion* ..., [1]; this gives a fairly thorough survey of early attempts to understand "problems of the reaction of an outflowing stream as worked on by Daniel Bernoulli, John Bernoulli, Euler and others". Especially Mikhailov has given and discussed the elements of Moore's Propositions so that there is some overlap between our works at this point. My presentation here refers more-or-less to *all* that Moore then treated.

I believe too that a challenge implicit in this article (and in others) is that it demonstrates that the nearly contemporary *cri de coeur* of Charles Babbage and his colleagues about the urgent need to substitute Leibnizian d'ism for Newtonian dottage, when using it as an instrument of analytical research, was excessive. I remind myself that in reading Brown's translation of Euler's work of 1777, [2], on Robins' *Gunnery* and Colson's rendering into English of Agnesi's *Analytical Institutions* of 1749, [3], I saw no new impediment created when the translators substituted English dottage for continental d'ism.

MOORE'S BIOGRAPHY

We shall assume that most of the biography of Moore as given in [1] is easily available to the reader and therefore need not be repeated here.

William Moore's vital dates are not presently known to us—only that he was appointed as sixth mathematical master at the R.M.A., Woolwich in 1806. (A critical year in England's essay into rocketry, being that of the attack on Boulogne.) He survived there until he was "retired" in 1822. The former date is given in *Records of the R.M.A., 1741–1892*, in the preliminary Appointments, apparently simply as "Mr William Moore, 6th Mathematical Assistant". Under a date of 1 October 1806 there is a transcript by an Examining Board (Charles Hutton and two others), to the effect that "... Peter Barlow and William Moore had talents capable of making them very useful Masters".¶

Of rockets, Moore himself has said that it was only through the Academy of Copenhagen's proposed prize question, to calculate the curve a rocket describes when projected in any oblique direction, *in vacuo*, that he was led to consider rockets in different mediums (*sic*). Peter Barlow later became well known in British scientific circles for his writings in engineering science and he was somewhat a strong competitor of Hutton. (He published his *Mathematical Dictionary* a year or two before the 1815 edition of Hutton's work appeared.) For some of his works in Strength of Materials, Barlow has come under strong criticism from J. F. Bell†. Barlow long remained very influential with engineers and clearly his work was highly thought of by these practical men.

Papers by Moore, other than those on rockets, are cited in Refs [1,8)‡. The references dealing with the penetration of balls into uniform resisting substances, of 1812, and of an enemy's fleet at sea by artillery, 1811, obviously reflect his first work that later gave rise to the original article on Naval Gunnery in our title.

F. H. Winter has written a helpful article on Moore in the *Dictionary of Scientific Biography*, see pp. 504/5, which embraces many of the facts adumbrated here. There is of course no entry about Moore in the British D.N.B. Winter wrote of Moore, that his book was the "world's first mathematical treatise on rocket design ... but it had many shortcomings ... and lack of data hindered his calculations. Never-the-less he correctly recognized and demonstrated that Newton's third law explained the rocket principle".

Judging by Moore's mathematical proficiency in his rocket papers and book, I supposed he might be a graduate from Oxford or Cambridge. (In the early years of the 19th century there was a distinct tendency to recruit Cambridge mathematicians, the self-taught line of professors/teachers coming to an end.) Accordingly, I consulted the lists of *Alumni Oxonienses* and the *Alumni Cantabrigienses* but found no entry with which I could associate William Moore. There seems no accounting for Moore at this stage other than to observe that the development of his ability was mostly a matter of his being largely self-taught, as with his distinguished predecessors, Robins and Hutton.

(I found in Hermon's *Scloppetaria*, (1808), p. 118, a remarkable footnote which records that, Mr W. Moore, "a very ingenious workman, has found by several experiments", ways of ensuring uniformity of shooting from a rifle. (Results at the target are shown in the latter book.) This Mr William Moore worked at No. 8, White Chapel, London, a district close to Woolwich. I cannot but wonder if this refers to our original William Moore or to a member of his family. I cannot avoid commenting, however, that *the* Moore never seemed to demonstrate experimental inclination and he is wholly a theoretician, contrasting sharply with this mechanician Moore.)

† In *Mechanics of Solids*, Vol. 1, pp. 813, Springer-Verlag (1984).

‡ Mrs Ruth Wallis wrote (26 Aug., 1994) that from the R.M.A., "Moore contributed propositions and answers to the *Ladies Diary* in the year 1807/10; mostly they were of a geometrical nature".

¶ Added in proof: See F. G. Guggisberg's, "The Shop," Cassell and Co., 280 pp, 1902, where on p. 263 in *Appendix* IX, *The Mathematics' Staff, R. M. A.* We find,"1806–12(?). W Moore, Esq." In 1806 there was an increase in the number of cadets from 100 to 186.

A TREATISE ON THE MOTION OF ROCKETS AND NAVAL GUNNERY

The title page of this book by William Moore appears as in Fig. 1. It is a book of some 150 pages, about 47 000 words, 6 pages of Tables and includes several pages of logarithms at the end. The essay on Naval Gunnery is about 20% of the length of the book and the material quite different from the matter on rocketry. Following the title is a short Preface of about 2000 words which is summarized immediately below and was written whilst the author was at the Royal Military Academy, Woolwich.

Preface

Moore opens by stating that it was the Academy of Copenhagen in 1810 proposing a prize question that induced him to consider rocket motion in different media. He writes however that he published in *The Philosophical Journal* [4], some short and incomplete papers on this latter topic which prompted him to publish as a treatise the results of investigations he had made which were of considerable length. As always, Moore has the teaching of military and other students in mind. He also resolved, in the same volume to publish his new theory on Naval Gunnery in a "volume collectively".

A compilation of the Contents of the book is seen in Fig. 2, the chapter numbers being my insertion. The first three chapters of the volume, in effect, lay down the foundations on which the principles of rocketry are to be based and extend from pp. 2 to 23. However, we do not propose to examine these, since they have no special value for the modern reader. The fourth chapter briefly describes the physical machine, the rocket, and next the generation of motion from the burning of its composition or fuel.

The fifth chapter is the essence of the book to which we pay most attention; it appears in five sections, the subject matter of each being seen in Fig. 2.

Section I is concerned with general motion in a non-resisting medium and starts with a treatment of vertical ascent only. Section II embraces Moore's theory for determining the resistance to planes, cones, spheres and cylinders moving through fluids in any direction including the non-axial; this was considered necessary, by Moore, for realistic rocket theory and he believed that his work was "new". He thought that he was here formulating a method of separate solution to a general problem but unfortunately his understanding of the motion of fluids around moving solid objects was too simplistic.

Section III explains rocket motion in a resisting medium but the difficulties he encountered gave rise to problems "of no small labour". As well, in this Section he tries to determine "the effects of the wind upon the rocket in deflecting it from the plane curve of projection". Similarly, the computation of lateral errors concerning "bomb-shells and cannon-balls" are partly addressed.

Section IV pertains to the "motion of wheels on fixed horizontal axes impelled by a circumferential force", whilst Section V treats of the theory of the ballistic pendulum so that "an estimation of the arc through which the pendulum is urged by a rocket during its combustion provides an easy and correct method for finding the strength of the composition".

The essay on *Naval Gunnery* turns on, "the charge of gunpowder for any given piece of ordnance to cause its shots to produce the greatest possible damage to any splintering object of a given thickness ... wood ...". Moore continues, "... I have seen in his Majesty's dockyard at Woolwich, prize-men† of war having many shot holes in them, almost wholly closed by the wood's own efforts and that required nothing more than a small wooden peg, or a piece of cork to stop them up perfectly ... it is evident that the charges were much too great and gave to the shot an improper force ...". Illustrating this fascinating subject, Moore continues by asserting that it was not Admiral Nelson's double-shotting of his guns which produced so extensive a damage on the *Santissima Trinidada* (an opposing admiral's flag ship) at the battle of Trafalgar; rather it was "so dreadfully disabled chiefly

† Prize-men: enemy ships captured in victory became the property of the victors and when sold, the sum received was divided up between officers and crew according to strict rules.

G. W. Miller — 1825

A

TREATISE

ON THE

MOTION OF ROCKETS:

TO WHICH IS ADDED,

AN ESSAY ON NAVAL GUNNERY,

IN

THEORY AND PRACTICE;

DESIGNED FOR THE USE OF THE

ARMY AND NAVY,

AND ALL PLACES OF

MILITARY, NAVAL, AND SCIENTIFIC INSTRUCTION.

BY WILLIAM MOORE

OF THE ROYAL MILITARY ACADEMY, WOOLWICH.

LONDON:

PRINTED FOR G. AND S. ROBINSON, PATERNOSTER-ROW.

1813

Fig. 1. The title paper of Moore's Treatise

*A different set of Propositions, 1 to 16, shown either in Arabic or Roman numerals occurs also in this chapter.

Fig. 2. Contents of Moore's Treatise

from ... the nicety of charge of gunpowder that was employed; ..., double or triple shot ... would have added but little to its destruction, had [the shot] not passed through with a proper motion".

After acknowledging the careful works of those engaged in testing at Woolwich, Moore passes on to draw attention to his subjoined table of hyperbolic logarithms for use in expediting calculations.

Lastly, the author points to the continuous exemplification of his results both in the text and by means of set examples, because the theory is then "never so well understood by a learner".

THE THEORY OF THE MOTION OF ROCKETS, IN NON-RESISTING AND RESISTING MEDIUMS (*Sic*)

Article 16, p. 26 has the same title as that of an article published by Moore in 1810 [4]. It should also be added that Moore was familiar with the work of Robins, Hutton and Euler and thus to some degree based his work on theirs.

This chapter starts on p. 26 by pointing out that it is imperative to know the strength of a rocket composition as a basis for useful theory. However, not having performed experiments, Moore asserts that he presumes that the force of the composition of the rockets used by the English in bombardment did *not differ significantly from one half that of gunpowder, which was supposed to be nearly 2000 times as great* (*when converted into fluid*) *as the elastic force of the atmosphere.* He takes for the initial force of gunpowder that which Dr Hutton proposed after much experiment and accurate computation. A further supposition about the composition was that the laminae of its fire was both uniform and parallel to the rocket's base.

SECTION I: ON THE MOTION OF ROCKETS IN A NON-RESISTING MEDIUM

Following on the introduction above, Moore remarks that Robins computed the propelling force as one half that which Hutton determined it to be but attributed this somewhat to the particularity of the enquiries which were involved.

Article 17, p. 27, Prop. I aims to find the perpendicular height ascended and the rocket velocity developed after the propellant composition has burned uniformly and parallel to the rocket's base for a certain time. We shall adhere to Moore's notation in which w denotes the weight of the case of the rocket and head, c the weight of the composition [so that m may stand for $(w+c)$, a the time of its consumption, d the rocket base diameter and x the vertical distance covered by the rocket in time t.]

Atmospheric pressure is $230\,\text{oz/in}^2$ ($14.4\,\text{lb/in}^2$) and s is 1000 times as great, being that of the "inflamed composition"; this value is used too because it was assumed so by Robins in his *Principles of Gunnery*, 1742.

Many of Moore's analytical investigations are, today, elementary and his results easily reproduced. However, there are a number of features in Moore's derivation which are, just for once, worth noting, particularly his use of the fluxional calculus. We note that this work was performed about a decade before the clamour by Babbage and his colleagues for the substitution of d'ism for dottage or the Leibnizian notation for Newtoniain fluxions.

The upward accelerating force on the rocket base, area $\pi d^2/4$ at time t after ignition is

$$\left[\left(\frac{\pi}{4}d^2\right)ns-\left(w+c-\frac{ct}{a}\right)\right]\Big/\left(w+c-\frac{ct}{a}\right)=\frac{ans\left(\dfrac{\pi}{4}d^2\right)}{am-ct}-1, \tag{1}$$

the counter-acting gravitational mass being included.

The equation for vertical motion Moore writes as,

$$\dot{v}=\frac{2agns\left(\dfrac{\pi}{4}d^2\right)}{c\left(\dfrac{a}{c}m-t\right)}\dot{t}-2g\dot{t}. \tag{2}$$

(For $\pi/4$ Moore writes throughout e and he uses $2g$ where we use only g today. This equation is written in terms of the fluxional calculus. Recall, fluxion $\equiv$ differential and fluent $\equiv$ integral.)

The fluent of (2), with respect to t is,

$$v=-\frac{2agsned^2}{c}\,\text{hyp.log}\left(\frac{am}{c}-t\right)-2gt. \tag{3}$$

This fluent is now "corrected" (Moore's term) since for indefinite integration he does not include in (3) an undetermined constant. The latter is found by recognizing that $v=0$ when $t=0$. Hence (3) becomes,

$$v=-\frac{2agsned^2}{c}\,\text{hyp.log}\left(\frac{am}{am-ct}\right)-2ga. \tag{4}$$

At the instant combustion ceases, $t=a$, and the speed is given by,

$$v=\frac{2agsned^2}{c}\,\text{hyp.log}\left(\frac{w+c}{w}\right)-2ga, \tag{5}$$

where $m=w+c$.

Moore exemplifies his results with regard to w, an 18 lb rocket, for which $c=10\,\text{lb}$, $a=3\,\text{sec}$, $d=3\,\text{in}=\frac{1}{4}\,\text{ft}$,

$g = 16$ ft/s, and $e = \pi/4$. The result he finds for $v = 2896.9895$† ft/s, at the instant of exhaustion of the "composition" (or "wildfire", p. 280 [4(i)]).

Using (5), Moore proceeds to find an expression for "the space" or distance travelled (using a previous 9th theorem, see p. 30, [4(i)]), at the end of time a, through $\dot{x} = v\dot{t}$. The fluent of x, duly "corrected", is found as,

$$x = \frac{ab}{c} + \left[(m-c)\text{hyp.log}\left(\frac{m-c}{m}\right) + c - \frac{acg}{b} \right], \tag{6}$$

where $b = 2agsned^2/c$. For the example given, $x = 4015.9827735$ ft traversed after the expiry of 3 s burning of the rocket fuel.*À propos* of Eqn (5) above we recall Mikhailov's remarks (p. 16, [6]). He writes, " ... the theory of rocket motion under a constant fuel burn-out rate, and accordingly under a constant thrust developed by the rocket on its active path, was first developed by William Moore... he did not however relate the constant jet reaction to $V(dm/dt)$ but believed it could be found by some other coefficient representing the experimentally determined characteristic of the reactive force of the exhaust jet of combustion products". Mikhailov, using the treatise by W. H. Besant (1828–1917), Chapter X, mentions an Example 22 in it, which yields the greatest rocket velocity,

$$V.\log\frac{M}{M'} - \frac{g}{e}\left(1 - \frac{M'}{M}\right).$$

(The rocket of mass M throws off per unit time a mass eV with relative velocity V and M' is the weight of the case. It cannot rise at once unless $Ve > g$, not at all unless $MVe/M' > g$.) This would appear to be the same as Eqn (5) above; the corresponding terms to which he is referring, would seem to be the V in the equation above, and the b given immediately below Eqn (6). The experimentally determined material which Moore has used derives of course from Robins'/Hutton's work.

Article 18, p. 31 provides, on the supposition that the gravitational force remains constant and independent of distance above the earth's surface, that the rocket further ascends,

$$v^2/4gf = (2896.9895)^2/64 \times 0.9993709 = 131261.131 \text{ ft}.$$

Thus the total height to which the rocket ascends above the surface of the earth is 135 277.1137735 ft or, "it has just lost all its motion" at a height "which is nearly equal to 27 miles".

If however, "the height ... be demanded on the true principle, that gravity varies inversely as the square of the distance from the earth's centre" then, Moore calculates that the total height of the rocket above the earth's centre, taking the earth's radius, r, to be 3979 miles, as

$$x = 4agr^2/(4gr^2 - ac^2) = 21\,145\,143.65521 \text{ ft}. \tag{7}$$

The total height of the rocket from the earth's centre is thus,

$$21\,145\,143.6552 - r = 136\,023.65521 \text{ ft}.$$

Thus the height to which the rocket rises from the point where the impelling force of the composition ceases is 132 007.67221 ft, and it ascends nearly 746.54121 ft higher from a point 4230.609 ft above the earth's surface with a velocity of 2896.9895 ft/s, than it would if the same force had remained constant.

If the rocket has a velocity of 2 896.9895 ft/s upwards when at a height from the earth's surface equal to $(4gr^2/c^2) - r$, Moore shows that it would never return. He states, "the velocity of projection to cause a body to move to an infinite distance is 39 450.2377 ft/s or 7.471767 miles/s"‡.

Proposition I in Moore's book is almost a word-by-word repeat of pp. 278–285 of Moore's first article, [4(i)], in *Nicholson*'s Journal for 1810.

Article 20, p. 34, Prop. II, reads, "*To find the period of the rocket's motion; or the time from its first going off to the time of its return to the earth*". With the same values as those in Prop. I, the rocket's time of ascent is shown to be 94.32307 s, and of descent 92.69881 s, so that the whole time of motion is 3 min, 7 s.

All the analyses used for this proposition are performed using the fluxional calculus, but are algebraically more complex than those in Prop. I.

Article 21, p. 36, Prop. III, reads, "*To determine the path of a rocket near the earth's surface, neglecting the resistance of the atmosphere*".

Specifically, we briefly outline Moore's approach thus. In Fig. 3, AC is the *initial* direction of the rocket and AD the curve it actually pursues, CDB being perpendicular to the horizontal line AB. After time t the rocket would arrive at C if gravity did not act; and if AC $= x$, then the value of x using (6) above, is,

$$x = \left(bt - \frac{abm}{c}\right)\text{hyp.log}(am) + \frac{b}{c}(am - ct)\text{hyp.log}(am - ct) + bt, \tag{8}$$

† Many authors in these years quote an excessive number of significant figures as seen here! (Newton used even more than are evident here.) That using so many was valueless seems never to have been contemplated.

‡ The /s is omitted by Moore.

and therefore,

$$\dot{x} = b\dot{t}\,\text{hyp.log.}am/(am-ct).$$

Moore proceeds to find the fluent of the latter after first putting log $(am/(am-ct))$ into fluxions and then finding its fluent in a series and "wanting no correction" as,

$$x = \frac{bc}{2am} \cdot \left(t^2 + \frac{ct^3}{3am} + \frac{c^2t^4}{6a^2m^2} + \dots \right). \tag{9}$$

Thus the time of describing space x along AC from the commencement of motion is obtainable.

CD, or y, is the distance descended due to gravity in the same time. Evidently, with the latter two elements, x and y, the track of the rocket may now be drawn.

Article 22, p. 39, Prop. IV reads "*To find the velocity of the rocket in the curve at any given instant*".

In Fig. 4 we put $AC = x$ and let $AD = z$, which is the distance described by the rocket in time t. If the speed at C is V then that at D is $\dot{z}V/\dot{x}$. However, $CD = \boldsymbol{g}t^2$, $AB = lx$, $CB = kx$ and $DB = kx - \boldsymbol{g}t^2$, where k and l are the sine and cosine of $\widehat{CAB}$.

Thus,

$$\dot{z} = [(k\dot{x} - 2\boldsymbol{g}t\dot{t})^2 + l^2\dot{x}^2]^{\frac{1}{2}}$$

and

$$v = \frac{\dot{z}V}{\dot{x}} = \frac{[l^2\dot{x}^2 + (k\dot{x} - 2\boldsymbol{g}t\dot{t})^2]^{\frac{1}{2}}}{\dot{x}} \cdot V$$

Also,

$$\dot{x} = V\dot{t},$$

and consequently,

$$\begin{aligned} v &= [l^2V^2\dot{t}^2 + (kv\dot{t} - 2\boldsymbol{g}t\dot{t})^2]^{\frac{1}{2}}/(V\dot{t}) \\ &= [V^2l^2 + (kV - 2\boldsymbol{g}t)^2]^{\frac{1}{2}} \\ &= \left[l^2b^2.\text{hyp.log}^2\left(\frac{am}{am-ct}\right) + \left(kb.\text{hyp.log}\left(\frac{am}{am-ct}\right) - 2\boldsymbol{g}t \right)^2 \right]^{\frac{1}{2}}. \end{aligned} \tag{10}$$

The latter v needs no correction, since $v = 0$ at $t = 0$. Moore finally verifies the correctness of (10) by inserting values of projection angle 90°, 0°, 30° and 60°.

Article 23, p. 41, Prop. V reads, "*To find the horizontal range of the rocket, having the angle of elevation of the engine, and the time the rocket is on fire, given*".

Figure 1 reproduces Moore's own sketch. Line DI represents the (tangential) direction of motion of a rocket at point D on trajectory ADI, when all the composition is just exhausted. It is then shown that,

$$\sin \widehat{Cnm} \equiv \cos \widehat{CAI} = \frac{\text{vel. at C}}{\text{vel. at D}} \cdot \cos \widehat{CAB} = \frac{V \cdot \cos \cdot \widehat{CAB}}{v}.$$

The speed at C, V has the same meaning as in Prop. IV, whilst v is the speed at D. Also,

$$\widehat{Cnm} = \widehat{IDB};\ \text{thus, } \sin \widehat{IDB} = V \cdot \cos \widehat{CAB}/v.$$

After D, the trajectory is a parabola, so $DH = (v^2/\boldsymbol{g})\sin \widehat{IDH} \cdot \cos \widehat{IDH}$, and $VE = v^2.\sin^2 \widehat{IDH}/4\boldsymbol{g}$. Next,

$$\begin{aligned} VF &= VE + EF = VE + DB \\ &= (v^2.\sin^2 \widehat{IDH}/4\boldsymbol{g}) + (x \cos \widehat{IDH}) - \boldsymbol{g}t^2. \end{aligned}$$

Now,

$$VE{:}VF{::}EH^2{:}FL^2 = \frac{v^2\cos^2 \widehat{IDH}}{\boldsymbol{g}} \left(\frac{v^2.\sin^2 \widehat{IDH}}{4\boldsymbol{g}} + kx - \boldsymbol{g}t^2 \right).$$

Hence FL and then,

$$AL = \frac{v.\cos \widehat{IDH}}{4} \left(\frac{v^2.\sin^2 \widehat{IDH}}{4\boldsymbol{g}} + kx - gt^2 \right)^{\frac{1}{2}} + \left(\frac{v^2 \sin \widehat{IDH}.\cos \widehat{IDH}}{2\boldsymbol{g}} + lx \right). \tag{11}$$

which is the entire range of the rocket.

The example Moore gives is for an initial angle of projection of 45° using the same values as those of the previous Proposition. The magnitude of v at the end of its burning time is,

$$\left\{l^2b^2.\text{hyp.log}^2\left(\frac{m}{m-c}\right)+\left(kb.\text{hyp.log}\left(\frac{m}{m-c}\right)-6g\right)^2\right\}^{\frac{1}{2}}=2925.6\text{ ft/s}.$$

Angle IDB turns out to be 134°6′ or angle $\widehat{\text{IDH}}=44°6'$. Thus the numerical values for the range are,

$$\sin\widehat{\text{IDY}}=0.696,\qquad k=0.7071,\qquad g=16\text{ ft/s}^2,$$

$$u=0.718,\qquad v=2925.6\text{ ft/s},\qquad x=4159\text{ ft},\qquad t=3\text{ s}.$$

The range is found to be 272, 116.29 ft, or 51.73 miles.

The Section ends with nine numerical Examples For Practice.

SECTION II: ON THE RESISTANCE OF BODIES MOVING IN FLUIDS WITH GIVEN VELOCITY

This Section II, pp. 44–55, is wholly devoted to calculating the resistance to bodies moving in fluids with given velocities, especially when a centre of gravity does not follow an axis of symmetry. Moore attempts to establish the air resistance encountered by planes, cones, spheres and cylinders that may have some relevance to rockets. He asserts that, "no book extant ... contains the principal part of the information ... to which reference could ... be made". In all his considerations of the fluid (air) he neglects to remark that it is an elastic fluid with gaseous properties which is being examined.

This section comprises four Propositions and seven samples; treatment of the topics is unsophisticated.

Article 25, p. 45, Prop. VI is where Moore gives Fig. 5 in which AB is a given plane and CA the direction of the fluid moving against it. BC is perpendicular to AB. Moore conceives of AC representing the *full* force of the fluid against AB. It is resolved into components parallel to AB and perpendicular to CB; the first can have no effect in tending to move AB and only the second could do so, which is the force CB. The velocity of impact is CA.sin$\widehat{\text{CBD}}$; or $V\sin\alpha$, where α is angle CBD. Thus with the area of the plane BD "rotated" to AB, the resistance on it becomes proportional to $\sin^3\alpha$. The actual resistance force on the plane is thus $AnV^2\sin^3\alpha/4g$; here, $g=16\frac{1}{12}$ ft/s^2; A is the area of the given plane, and n the density of the fluid. The force conceived to apply then, was thought of as "equal to the weight of a column of such fluid" falling through "a height" sufficient "to acquire its velocity of motion", V; this accounts for the $4g$.

Articles 26 to, and including Art. 32 (incorporating Props VII, VIII and IX) are effectively algebraic exercises including some use of fluxions for various bodies set at different angles to impinging uniform flow.

Some of the cases described by Moore pertain to,

(i) A cylinder of radius r moving in a fluid in the direction of its axis: the resistance is, $\pi r^2nV^2/4g$.
(ii) A cone, moving in the direction of its axis, apex foremost, the resistance is, $\pi r^2nV^2\sin^2\alpha/4g$, where α is the cone's semi-angle.
(iii) And for a circular plane moving such that its inclination is at angle α to the direction of motion is, $\pi r^2nV^2\sin^3\alpha/4g$.
(iv) A sphere or cylinder with hemispheric end, moving in the direction of its axis; the resistance is, $\pi r^2nV^2/8g$.
(v) A spherical segment, where y is the radius of its base and r the "spherical" radius; the resistance for motion forward, but perpendicular to its base, is $\pi nV^2y^4/8gr^2$.
(vi) For a cylinder of height h meeting a fluid by moving perpendicular to its axis; the resistance is, $nV^2rh/3g$.
(vii) For a sphere; resistance is, $\pi nV^2r^2/8g$.
(viii) The ratio of the resistances for a sphere to that of its circumscribing cylinder is thus $nV^2r^2/8g$ to $2nV^2r^2/3g$ or as 1 to $16/3\pi$ (1.698).

No such simple rules truly apply to these kinds of situation; the consequences of vortex formation, form drag, edge effects and compressibility, etc., then being unknown and unsuspected. The introduction in the mid-19th century of ogival shells led to profounder considertions of the resistance offered by air to high-velocity, elongated projectiles and ultimately progress is well reflected in the long Ch. II of say, Cranz and Becker's *Exterior Ballistics*, Vol. I, H.M.S.O., 1921. The emergence of computational fluid mechanic techniques has yielded highly realistic results, something unimaginable in Moore's time; this is not of course to belittle Moore's early efforts.

SECTION III: ON THE MOTION OF A ROCKET IN RESISTING MEDIUMS

Article 33, p. 56, Prop. IX, aims to find the height in vertical ascent and the velocity acquired at the end of the time of burning of a rocket, when the resistance encountered is as the square of the speed.

In addition to the notation earlier used we also have,

c: the weight of the whole quantity of material which fills the rocket,
a: the time in which c in consumed uniformly.
x: the space the rocket describes in time t where
b: is its speed, R: is the air resistance at b ft/s and $g\sim16$ ft/s^2.

Moore, using fluxional methods, finds the rocket's equation of motion as,

$$\dot{v} = [sAld^2b^2 - Rv^2)/(am - ct)b^2]a2g\dot{t} - 2g\dot{t} \tag{12}$$

or,

$$l\dot{v} - pt\dot{v} - q\dot{t} + kv^2\dot{t} - 2gpt\dot{t} = 0,$$

where coefficients, $l = amb^2$, $p = cb^2$, $q = h - 2gl$, $h = 2agsned^2b^2$, $k = 2agR$ and $h - 2gl = q$, so that v may follow in terms of t. Moore next assumes that

$$v = At + Bt^2 + Ct^3 + Dt^4 + Et^5 \text{ etc.}$$

Making $\dot{t} = 1$,

$$\dot{v} = A + 2Bt + 3Ct^2 \text{ etc.},$$

and after substituting in the previous equation and equating the coefficients of equal powers of t, A, B, C, D and E can be found. The fluent is then taken and an expression for v derived; note that the first five terms only of the series were taken.

The value of R for velocity b is $psr^2b^2\int^2/4g$. (By the long ess $\int$, Moore here denotes the sine of the angle which the slant side of the conical rocket head makes with the rocket axis.) For $b = 1$ ft/s, $s = 1/2(30°)$ $R = 0.0002343$ oz and velocity, $v = 2733$ ft/s.

The space described by the rocket x is the fluent of $v\dot{t}$, a two-line equation at the foot of p. 58; with Moore's previous constants it is here 3910 ft, being the height at the end of burning.

The author seems at this point to have some doubts about the series he uses and looks to "observe a proper law".

Article 34, p. 59, shows the further rocket height achieved from the acquired speed is, numerically, 7914 ft which combined with the earlier figure of 3910 ft gives 11 824 ft, ($2\frac{1}{4}$ miles) from earth.

This seems to complete the work set up in Prop. IX. From Art. 35, p. 61 to the end of Art. 37, p. 65, the concern is to determine various times. As Moore shows no heading for Prop. X, I assume it covers these pages.

Article 35, p. 61, is a short matter of integration to find the time for the latter distance to be totally covered and numerically it is 14.2 s. Thus the whole ascending flight time is 17 s.

Article 36, p. 61, shows that the time required to descend to earth from the maximum rocket height achieved (with resistance operating) is 48.2984 s. Thus the total flight time is (17.2 s + 48.3 s) = 1 min, $5\frac{1}{2}$ s. The calculated speed of arrival at the earth's surface is 350 ft/s. (Moore gives 350 ft, so here, as occasionally elsewhere, sometimes he has incorrect units.)

Article 37, p. 64 begins by commenting on the validity of the resistance law used, noting that gravity and density do change with height. Experiments are said to disprove the square law for cannon balls but Moore asserts hardly so for rockets, because the heat of the exhaust gases rarifies ambient air. He points out, however, that an nth power of velocity resistance law brings no substantial change in the analysis.

Article 38, p. 65, Prop. XI is short and shows that rocket speed cannot become uniform under any law of resistance.

Article 39, p. 66, Prop. XII treats of finding the speed and distance described by a rocket due solely to the composition and the medium resistance—gravity not acting. For typical rocket dimensions the author determines a velocity little different than that calculated including gravity, and dubs it remarkable.

Articles 40 and 41, p. 70, pursue the lines of Arts 38 and 39 without new conclusions. It is not at all clear why Moore devotes so much time and space to this topic.

Article 42, p. 72, opens with Prop. XIII and seeks to ascertain the magnitude of the deflection of a vertically ascending and descending rocket when it returns to earth from the point of its initial projection, due to the effect of a horizontal wind. Introducing the article, its author observes that a body moving from rest, put into motion by a fluid, cannot acquire a speed greater than that of the fluid itself: the force on a body is proportional to the difference between that of the fluid and that of the body. Similarly, Moore asserts that in a perpendicular cross wind, the "sideral" (side-ways?) motion cannot exceed the velocity of the wind. Thus the rocket suffers no resistance from the medium in its deflection from the original line of projection. Moore assumes that the wind force, to begin with, moves the cylindrical rocket sideways, its quantity being $nV^2rh/3g$ and when it has accumulated a sideways speed of v, $nrh(V - v)^2/3g$, (see Art. 31, p. 49). After three pages of calculation the rocket deflection at the end of its burning is found and given on p. 76.

Article 43, p. 77, shows that the rocket is further deflected during the remainder of the time, see p. 78. Typical values are, for an 18 lb rocket, $c = 160$ oz, $h = 3$ ft, $V = 22$ ft/s, $m = 448$ oz, $n = 1$, $r = 6$ in, and at the end of the burning time the sideways movement is 7.1 ft; at the end of the second part of the flight the distance moved is 674.76 ft, or the whole lateral movement is 681 ft.

Article 46, p. 80, is where Moore enquires into the lateral deviation from the vertical plane of fire of a shell when driven by the wind at 22 ft/s, (or 15 m.p.h.) during an ascent and descent. For a spherical shell of 13-in diameter and weight 228 lb, he gives an algebraic result on p. 80 which is, numerically for the above constants, 70.4 ft.

A twelve-pounder cannon ball is calculated using Moore's theoretical approach, to deflect 67.8 ft, for a 20 m.p.h.

transverse wind speed with a flight time of 32 s. These latter results underline the need to pay attention to cross winds and thus the need to adjust the vertical plane of flight of a ball, slightly, to take some account of lateral deflection.

Article 47, p. 82, Prop. XIV, shows the case, for a given time of flight, speed and angle in which the rocket is sent, as well as the direction and speed of the wind, and Moore assesses the expression for the distance from the plane of projection at which the rocket falls. (The effects of ball revolution—possible Magnus Effects—are neglected.)

Article 48, pp. 83–87. Scholium. The subject is that of the two rotations to which a rocket in flight is subjected; one, resulting from the action of the wind and the other the resistance of the air in its descent to the earth under gravity. The inertia of a varying mass in flight makes for difficulties; due to wind, the one end of the rocket which is less heavy than the other, causes that to move to leeward. As regards the other rotation, the heavier end because it has greater "power" to overcome any resistance, will preponderate to cause the rocket body to rotate until it reaches the vertical position.

Moore advocates firing directly into or against wind, if at all possible but realising how great an error can be made, provides knowledge which should enable rocket gunners somewhat to compensate in anticipation.

Eight problems, p. 86/7, without answers or outline solutions, complete the section.

SECTION IV: ON THE APPLICATION OF THE FORCE OF ROCKETS TO THE MOTION OF WHEELS SUSPENDED ON FIXED HORIZONTAL AXES

We are not told where the information derived from this section would find employment. There is no obvious use for it in military circumstances, and the only one for which this writer can imagine a purpose is the Catherine wheel, either as a recreational firework or an implement of torture in ancient days; see Note 2 below.

Article 49, p. 89, Lemma† 1, concerns a solid circular plane, see Fig. 6, which oscillates about a fixed horizontal axis *nSm* parallel to diameter AB, the plane being inclined to SG. The investigation's purpose is to explore the ability of a force (or impulse?) on the plane to turn it about *nSm*. Moore's calculations show that the force producing rotation about the axis is independent of the inclination of the lane of the circle.

Article 50, p. 91, Lemma 2, is illustrated by Fig. 7 which represents the oscillation of a cylinder, AB, again about the horizontal axis *nSm* to which diameter CD is parallel. Moore simply shows, via the consideration of a normal pressure *p*, on typical sections ELF distance SO ($=\mathrm{SG}+\mathrm{AB}^2/12.\mathrm{SG}+\mathrm{AM}^2/4\mathrm{SG}$) from the centre of oscillation, S is independent of whether the solid oscillates in the horizontal, vertical or any other position as long as *nsm* and CD are parallel.

Article 51, p. 92, Prop. XV, concerning the solid cylindrical wheel ABCD, suspended on an horizontal axis XY passing through its centre of gravity, has a cylindrical rocket of negligible weight, RO attached tangentially at its mid-point T, see Fig. 8. The velocities of the wheel at an instant, are to be ascertained. Moore notes that, due to the burning of the rocket composition, analyses are made complicated because the decreasing quantity of the fuel with time, alters the overall centres of gyration and gravity. The calculations proceed through nine pages, at one point its author needing to solve a cubic equation in order to effect an integration and thence determine the peripheral acceleration of the wheel—"It will therefore be proper to integrate the fluxion upon the supposition that the cubic equation involves imaginary roots" (p. 97).

A numerical illustration, p. 101, completes this Article; it is for a wheel "of sound, dry oak of thickness 6 in, of radius $2\frac{1}{2}$ ft". The weight of the composition is 160 oz, and its time of burning 4 s; the base diameter of the cylinder is 6 in, and IG, the centre of gyration of the wheel from its centre of gravity. It is found that the circumferential speed is 3237 ft/s and the angular wheel speed 1295 rad/s or 206 r.p.s.

Article 52, p. 102, Prop. 15‡ here aims "to find the number of revolutions the wheel makes during the time of the rocket's combustion". Moore commences by writing that he will "confine (him)self to the most difficult and laborious case ..., ... of the preceding proposition ...," obtained on the supposition that the denominator (a cubic equation) of its fluxion (sic) contains two impossible and one real, root. His analysis is in "solid" algebra and calculus, and proceeds for more than six pages. However, lack of space, time and general interest preclude reviewing it here in any detail. It almost suffices to note only Moore's ready and efficient approach in handling the fluxional or non-Leibnizian calculus; the date is 1813, only about five years before the vocal clamours of Babbage and colleagues for European d'ism.

Friction between the wheel and axis, and the wheel and air, were neglected in the above analysis; but Moore indicates how calculations incorporating them can proceed, and also of rocket wind resistance, $R(v^2)$ if so desired.

SECTION V: OF THE APPLICATION OF THE FORCE OF ROCKETS TO THE MOTION OF OF PENDULUMS

Article 53, p. 110, refers to the acquaintanceship with ballistic pendulums, see Fig. 9, "as invented by our late ingenious countryman, Mr Benjamin Robins", being presumed, Moore describes simple methods for finding experimentally its centre of gravity and of oscillation, the former by vibrating the pendulum and the latter by suspending and raising it to a horizontal position and balancing it with a suspended vertical weight, see Fig. 10.

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† An argument or subject prefixed as a heading or title. (Shorter O.E.D.)

‡ Why this should be Prop. 15 (Arabic) (Art. 52) after writing Prop. XV (Roman) (Art. 51), is not clear.

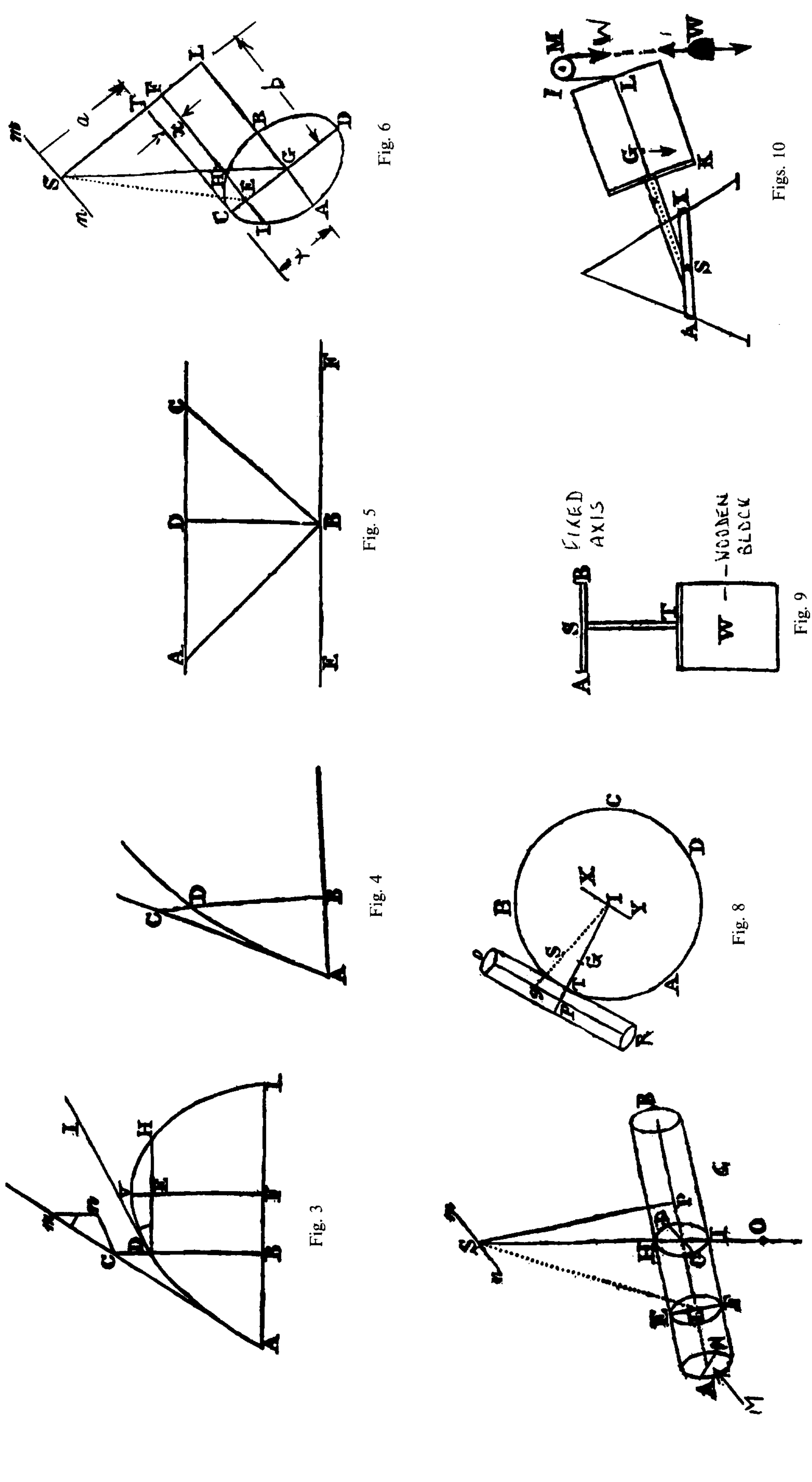

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Figs. 10

Figs 3–10.

Article 54, p. 116, Prop. 16, treats of the measurement of the deflection of a pendulum from the vertical, when a rocket is attached to a pendulum face. This—"the greatest ascent"—occurs at the moment there is complete exhaustion of the rocket composition: the pendulum continues to ascend until the end of firing because there is continuing rocket fuel diminution†. It is argued by Moore that

x, the sine of the angle made by the pendulum axis with the vertical at its greatest altitude, is $\dfrac{snpr^2i}{gow}$.

w: wt. of pendulum (wt. of rocket case neglected)

g: distance of centre of gravity from axis of suspension

o: distance of centre of oscillation

$(go)^{\frac{1}{2}}$: distance of centre of gyration

i: distance of rocket axis

r: radius of rocket

n: atmospheric pressure, 230 oz

s: 1000

p: π.

Typical values for exemplification are,

w: 570 lb or 9120 oz	i: 60 in	o: $84\frac{7}{9}$ in
g: $7.8\frac{1}{2}$ in	r: 1 in	

which yield, $x=0.7143$ or an angle of 45°35′.

If x is given, then $s=gowx/npr^2i$.

Simple experiments to determine the precise height of a pendulum at the instant of the completion of burning of the rocket composition (or the sine of its angle with the vertical) is all that is needed to satisfactorily complete any investigation.

On p. 115, Moore discusses errors which can be considered possible using the pendulum arrangement described, i.e. axis friction and air resistance behind the pendulum block, but as the effect of these is self-cancelling, they are neglected.

That the fixing of a rocket with axis perpendicular to the line of suspension or rotation of the pendulum always involves some degree of error is not noted.

Adjustments to his analysis are considered by Moore, to take into account the weight of a rocket casing attached to a pendulum face. By measuring sine x, values for p the rocket burning pressure can be calculated, at something of the order of 1000 atm.

Moore does not, incidentally, report himself as having carried out any experiments to test his analyses, which is perhaps both surprising and disappointing, since Charles Hutton was then an older colleague in a centre where experimentation surely was possible.

ON NAVAL GUNNERY

(i) ... advance towards perfection ... of warfare for the uses of the *navy or army* (is) entitled to every attention ...
(ii) ... charges made use of are not always the most eligible for producing the greatest destruction ... *owing to their being too great* (in) practice, as well *military as naval.*

Article 55, p. 117 begins with the above extracts. This topic is covered in a short 23-page "treatise" on the penetration of wood by cannon balls, one very different from that on rockets in the first 116 pages. It would seem that Moore had completed this article and placed it in this basically rocket treatise—for ease of final dissemination.

The essential aims and emphases are to calculate the charges for use in actions at sea and they are set to rest upon experiments. It is however disappointing to find that there is virtually no explicit mention of experimental results. Not merely are such results not recorded (the limits of calculation would then be apparent, to name only one advantage) but neither are there any sketches of damage inflicted (on wooden walls, masonry walls and the like. We also have in mind the processes of partial penetration or reflection of shot and the means of damage limitation). A further notable deficiency, though one common in those days, is the lack of systematic reference to the work of earlier authors.

It should be remembered specifically that Moore is dealing with pre-1859 naval gunnery—it is still the age of wood, not iron or steel, of cannon ball, not ogival projectiles from rifled guns, of wind sailing ships and not steam power.

Articles 56 and 57, pp. 118/9 and Lemma I pertain to simple statements about the penetration of uniform materials, see below.

Article 57, p. 120, Prop. I, discusses the destruction of an enemy fleet at sea by artillery. Of course the discussion of the subject is nowise as "wide" as that adopted by Douglas a generation later in

† Since the mass attaching to the pendulum is reducing over a period (typically) or 3 s of firing, that we are examining an impact process rather than a quasi-static one would seem to be moot.

his *Naval Gunnery* [7]. It is here supposed only a matter of the calculation of numerical quantities—and there is nothing about naval tactics and seamanship.

Reverting to Art. 56, p. 118, Lemma 1, it is maintained by elementary argument p. 119, that the "forces retarding (two) spheres (this notion is common in Moore's technical arguments) penetrating (perpendicularly) uniformly resisting substances, (F & f), are as the absolute strength of the fibres of the two substances directly, (R & r), the diameters (D & d) and the specific gravity of the spheres (N & n) inversely"; i.e.

$$\frac{F}{f}=\frac{R}{r}\cdot\frac{d}{D}\cdot\frac{n}{N}.$$

It is held that though the forces are not constant, until after the spheres have penetrated to the depth of their radii, it is alleged that it does not materially affect conclusions.

In Lemma 2, p. 120, it is proven that,

$$\frac{S}{s}=\frac{V^2}{v^2}\cdot\frac{D}{d}\cdot\frac{N}{n}\cdot\frac{r}{R}.$$

S & s refer to the depth of penetration of spheres and V & v their velocities.

Article 57, Prop. I, p. 120 aims to find a general formula for the charge of gunpowder for a given gun, to deliver the greatest degree of destruction in oak of given thickness; it is considered that shot velocity in close encounters is the same at the moment of projection and impact against the enemy.

From the last two lemmata above it is deduced that,

$$V^2=\left(\frac{S\cdot d\cdot n\cdot R}{s\cdot D\cdot N\cdot r}\right)v^2.$$

It is stated that the charges of powder vary as the squares of velocity and weight of the ball.

Moore makes use of an experimental result in which $\frac{1}{2}$ lb of powder could generate in a shot of weight 1 lb, a speed of 1600 ft/s, so that for any vessel the required charge to penetrate a given thickness of wooden side S is,

$$\frac{S\cdot R\cdot d\cdot n\cdot v^2\cdot w}{2D\cdot N\cdot r\cdot s\times 1600^2}.$$

To introduce more experimental data, Moore turns to p. 273 of Robins' *Mathematical Tracts*, Vol. 1, as edited by J. Wilson (p. 761)†. An 18 lb cast iron ball (0.42 ft dia.) penetrated a block of "well seasoned" oak to a depth of $3\frac{1}{2}$ in, (measured to the top, the bottom or where of the cannon ball?) when fired at 400 ft/s. The charge, generally, is said to be,

$$\frac{400^2\times 0.42}{2\times 1600^2\times 7/24}\times\frac{w\cdot S\cdot R\cdot n}{D\cdot N\cdot r};$$

and since $R=r$ and $N=n$ for the two cases considered, the expression for the charge is,

$$\frac{0.045\times S\cdot w}{D};\tag{A}$$

S is the thickness of the side of the ship and w the weight of the impinging ball.

No allowance is made for the splitting of timber when a ball has reached its far side. Air resistance is noted as reducing ball speed in passing from ship to ship and no note is taken of any non-normality when firing. These latter three considerations are held to counteract one another.

On p. 124, two examples are provided. In the first, a "74" vessel, $1\frac{3}{4}$ ft thick, is to be attacked with a 42 lb cast iron shot of diameter 0.557 ft. Then the charge required using (A), is $0.045\times 7/4\times 42/0.557$ or 5.93 lb. Secondly, a 24-pounder fired at 1300 ft/s, penetrates a bank of soil to 15 ft. The shot is to destroy a fortification of dry earth 2 yards thick, bound on both sides by oak planks $\frac{3}{4}$ ft thick, using a 44 pounder. For the bank, the charge to penetrate is,

$$\text{(see Art. 55)},\ s\cdot d\cdot v^2 w/2(S\cdot D\times 1600^2)=\frac{6\times 0.46\times 1300^2\times 42}{2\times 0.15\times 557\times 1600^2}=4.58\text{ lb}.$$

To penetrate the two planks $\frac{3}{4}$ ft thick, requires 5.08 lb, so that the total charge is $4.58+5.08=9.66$ lb.

Two pages of Tables are included (pp. 125–127) giving the charge, in pounds, required to penetrate oak of specific thickness for ordnance firing balls of different weight/poundage; Fig. 11 is an extract. The author writes

† Moore's cited reference here, p. 121 [5], is incorrect.

Nature of Ordnance.	Thickness of the side of the Vessel.		
	5 ft. 10 in.	5 ft. 11 in.	6 ft. 0 in.
Pounder.	lbs.	lbs.	lbs.
12	8·585918	8·708574	8·831230
18	11·249983	11·410697	11·571411
24	13·629277	13·823981	14·018685
32	16·509566	16·745417	16·981268
36	17·859380	18·114514	18·369648
42	19·781933	20·064532	20·347131

Fig. 11

that his results are also useful for the navy and artillerists ashore, to break open the gates of besieged towns or to destroy wooden fortificiations. Moore underlines the fact that a ball which just penetrates a plank does more damage than does one which quits it with speed. In the latter case, the hole made in the wood may almost totally close up due to the "springiness" of the wood, whilst a shot almost totally exhausted, will cause splintering and carry away large pieces. (See Fig. 13.) Advantage from firing with smaller charges is beneficial in that the gun does not heat up so quickly and to an extreme degree, and therefore does not need to be out of service whilst it cools down.

Interestingly, the penetration or forcing of gates by using the recoil of a relatively massive gun placed adjacent to them and then firing with the muzzle pointed away from the gate is recommended. (How an enemy would allow this situation to arise, is not addressed!)

Article 61, p. 131 chooses elm as the material to be attacked and Moore deduces, instead of (A) above, the formula,

$$0.0676 \times sw/D; \qquad \textbf{(B)}$$

this is derived by using data from the experimental work of Hutton, [9], of 1783/5. In the Lemma on p. 133, Moore proposes to find the speed of a cannon ball after passing through air and experiencing resistance proportional to the square of its speed. (Researchers from Robins onwards were aware that this one law was not obeyed at all velocities.) The force retarding a ball of speed v at time t, after firing, he writes as, $3nv^2/16gNd$. By using the fluxional calculus Moore easily finds, $v = a \exp(-bs)$, where a is the initial speed of projection and $b = 3n/8Nd$. Our author also finds the time of flight of the ball.

In the earlier Problem it was supposed that the ships are close when shots are exchanged but in Problem 2, p. 136, he considers the situation to be one where they are at "any considerable distance": the author concludes it with, "there is an impossibility of solving the problem rightly", because the circumstances of encounter at wide distances apart cannot be accurately specified, e.g. the amount of rolling of the ships and the degree of obliquity at impact.

Problem 4, p. 138, is devoted to the penetration of oak, "to any given depth not exceeding its radius". This however seems to be examined in a quite unsatisfactory manner by not attending properly to the shallow depth proposed.

The book concludes with eight practical examples though without solutions.

MOORE'S ORIGINAL PAPERS [4] AND THE CONTENTS OF HIS BOOK [5]

The two purposes of this section are first, to help the reader gain an impression of the degree to which Moore's papers became his book and second, to seek out from them other material relevant to understanding the subject development.

There are four papers by Moore in *Nicholson*'s journal but the fourth and last paper is only two pages in length and carries nothing of reportable significance. The first three papers (the introductory material of the first excepted) are effectively continuous, Papers 1 and 2 being absorbed into Section I. Section II of the book consists of two lemmae from Paper 3 preceded by new Props VI and VII. Section III includes two Propositions from Paper 3, preceded by two new Props, IX and X and succeeded by a Prop. XIII. There is no account of, or reference in the papers to material which constitutes Sections IV and V of the book.

Worthy of reproduction here are extracts from the first paper, (pp. 276/8) as Fig. 12; this introductory material is not included in the book. Figure 12 also shows Moore's letter

IV.

On the Motion of Rockets both in Nonresisting and Resisting Mediums. By W. MOORE, *Esq.; communicated by the Author.*

To Mr. NICHOLSON.

SIR,

SHOULD the following Essay on the Motion &c. of Rockets be considered sufficiently interesting for your valuable and well conducted Journal, you are at liberty to make use of it.

I am, Sir,
Yours very respectfully,
W. MOORE.

Royal Military Academy, Woolwich,
November 3, 1810.

The theory of rockets not investigated by mathematicians.

The theory of rockets is a subject, which has never yet engaged the attention of mathematicians; a circumstance which perhaps is partly to be ascribed to their not having been used until very recently as implements of warfare. The practice however of throwing them into besieged places, to cause their surrender, is now nearly universal among the English, and indeed is almost confined to them.

Their military use.

The invention of the military rockets* (as they are now called) as it regards the exemption of our troops from the enemy's power of annoyance, is to be esteemed as valuable. By the help of these machines the capital of Denmark and the well fortified town of Flushing, together with much of the French navy have within a few years been taken and destroyed with scarcely the loss of a single man: on which account, it is a matter of no small moment to bring the rules for discharging them and the methods of estimating their effects under various circumstances into one general

and complete system; especially as their use is likely to become greater, and the improvements in making them extended.

------ the force of the fluid generated from firing it cannot differ very much from that from fired gunpowder, which is about 1000 times as great, (according to Robins,) as the pressure of the atmosphere; and until I am able to convince myself otherwise, I shall adopt this as the measure of the strength of the rocket composition.

The rockets described.

A C D B (Plate VIII, fig. 3,) is the case of the rocket of cylindrical figure and made of sheet iron; *a* the place where the rocket is fired at the base of it A B; and C G D is the head of the rocket in the form of a right cone, and filled with inflammable matter, that consumes much more slowly than that with which the case or body is filled. This head is made also of sheet iron, and is quite solid near the apex G, in order that it may the better enter any object of penetrable substance, as ships of war, and all buildings composed of combustible and yielding materials. The white spots in the head denote holes, through which the fire and flame rush and fire the building into which the rocket penetrates.

The head is about nine inches in length; the length of the case A C being about $2\frac{1}{2}$ feet.

When the head of the rocket is not used, as in the case of firing it among troops; then the machine is simply in the form of that represented by *Fig.* 4; having about $\frac{1}{4}$ of a foot of its length from C to E filled with grape and canister shot; at the extremity of which, is a quantity of gunpowder E F n m to give them an additional impetus after the consuming of the wild fire; and by this means causing them to kill and terrify the enemy at a very great distance.

Nicholson's Philos. Journal Pl. VIII.

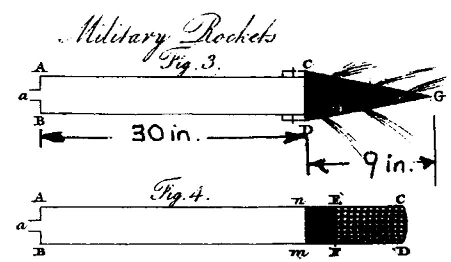

Fig. 12.

introducing himself to the journal editor and generally remarking on what was then being discussed on rocketry.

Paper 1 is identical with what appears in Section I of the book being a word-for-word adoption of Prop. I of that paper, apart from some arithmetical confusion and error (p. 280, [4(i)] and p. 31, [5]), and with two other small changes.

Paper 2 is also substantially absorbed into Section I except that there is a new Prop. 2. The Prop. 2 of the paper has become Prop. 3 in the book but with some alteration made in the second paragraph of the former.

(We have to insert that it is confusing throughout these works to find that Moore seems to use Arabic and Roman numerals inconsistently.)

Prop. III becomes Prop. IV in the book and also squeezes in minor results for launch elevations of 30° and 60°.

Prop. IV becomes book Prop. V but there is nothing to correspond to paper Prop. V. This paper ends with a Scholium and the book with nine examples but no solutions.

Paper 3 contains Props 6 (p. 242) and 7 (p. 243/8) which become Props XI (p. 65/6) and XII (p. 66/7) in the book except for p. 242 where there is arithmetical error; a rocket range by "the end of burning", (p. 241), is given as 53 miles (!) but corrected to 4 miles in the book, (p. 70). Book Section 2 includes Props VI and VII whilst Props VIII and IX are Lemmas 1 and 2, (pp. 248/50 and pp. 251/4) of Paper 3. There are many discrepancies/changes between book and paper regarding Prop. IX.

Props XIII and XIV occur in Section III followed by a Scholium and eight examples.

Moore's covering letters offering his papers to Mr Nicholson for his journal are dated respectively, for Paper I, Nov. 1810, for Paper 2, Jan. 1811 and Paper 3, June 1811; these were all sent in during the short period of eight months†. Moore's appointment to the Military Academy, Woolwich began in 1806 and was maintained during the period of the major rocket actions at Boulogne in October, 1807, Copenhagen, Sept. 1807, Flushing, Belgium, Aug. 1809 and Santanem, Spain, Nov. 1810.

The London newspapers and magazines must have reported these actions at length and on the effects of the employment of these weapons. Together within the confines of Woolwich there must have been much stimulation for a young man such as Moore to investigate these new "engines". We recall that Charles Hutton was on hand in these years, as was William Congreve.

The first two letters and the beginning of the third are on much the same topics but of Paper 3 Moore explained that it contained two Propositions preparatory to his next enquiry, namely into the effects of wind on rocket flight; he comments that he believes that this material is new and original. He writes too that he will send Mr Nicholson his findings on this topic in due course, but he does not do so and instead leaves it eventually to appear in Section III of his book. Clearly, Sections IV and V on Wheels and Pendulums respectively had not been envisaged at this time.

One understands Moore's endeavour to calculate wind effects and the results concerning pendulums (Section V), but not that on rotational problems (Catherine wheel-like "engines") in Section IV, for there appeared to be no likely use of them. But see Note 4 on *The Great Panjandrum.*

DAMAGE TO "WOODEN SIDES"

Much attention is directed by Moore to the cannon ball damage inflicted on wooden-sided vessels being greatest when the speed of a ball is just sufficient to penetrate, i.e. the speed on emerging is virtually zero; the hole created is then one which is highly splintered, splitting of the wood taking place mostly parallel to the grain. It was well known in those times that the fast-moving splinters which were created were particularly responsible for inflicting injury on crew members; loss of limbs was a very common consequence of wood fragmentation. At high speeds, holes are "cleaner"—"blanked out"—clearly a matter wholly of high speed shearing; a low speed of exit is associated with a "mixture" of first shearing

† See Note 5 at the end of paper.

169. The effect produced by shot when fired against iron steamers was remarkably exemplified on the "Lizard" during the operations which took place in the Paraná in 1846, when it was found that, on being struck, the plates of the ship bulged, and the perforations were so irregular and jagged that, for the purpose of stopping them, the common plugs were quite useless. This circumstance suggested the expedient of employing

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what has been called a *parasol plug*, which consists of an iron bolt furnished with arms of the same metal and covered with thick canvass well tarred. On being thrust through the shot-hole, and then forcibly drawn back, the head expanded, and thus, the aperture being covered, the leak was closed. In consequence also of the ship being struck, the splinters and rivets detached by the shot flew about like grape, and nearly all the men killed and wounded suffered from this cause. Grape-shot fired at a distance of 200 yards pierced the side; and persons present, who were highly capable of judging, concurred in opinion that a 32-pounder shot would have gone through the sides of three or four iron steamers, doing damage which would be successively greater in those more remote from the ship first struck, till the force were spent. A remarkable circumstance is said to have happened to the "Alecto" at the same time. An infantry soldier fired his ramrod at her, when, like a dart, it went point foremost quite through the nearest side of the funnel, but being prevented by the button from passing through the other side, it fell down in the interior.

+From H. Douglas, Ref. [7], 1855 ".. (or closing up)".

Fig. 13

followed by fracture due to bending in tension. Fast-formed holes with a tendency to the ejection of wooden frusta, have a tendency for the material to recover leaving a perforation which is smaller than that of a piercing ball; there is considerable stretching which results in thinning and recovery due to "springiness" of the material, (to use Moore's term). The same phenomenology with regard to the perforation of *iron-plated* steamers is described by Douglas in his book [7], see the interesting extract about amount of damage and speed in Fig. 13. (Of course, the possibility of the "popping off" of rivet heads as spalls is relatively well known.) Also well known today is the particular kind of injury suffered by limbs penetrated by present-day high-speed small calibre projectiles; entry can result in a relatively small hole but the exit wound is very extensive.

It is evident that the speed for maximum damage as recommended by Moore was not one of greatest speed and the use of the largest charges to rapidly propel a ball. (Aside from damage considerations, there are the advantages of conserving gunpowder and aiming to keep down the temperature of guns.) It was therefore of great value to make known the characteristics we describe, particularly to gunnery officers.

These phenomena seem hardly known to modern literature but are, surely, of great importance. With wood, in the circumstances we describe, are the associated factors, not well researched and understood, such as the role of moisture which changes both the material properties of the wood and its density.

À propos of Moore's remarks about the kind of damage suffered by the *Santissima Trinidada*, (Preface, p. viii, [5]), they must have been the result of eye witness accounts, though it is well documented that the Trafalgar engagement was fought at very close quarters—"pistol-shot distance". My enquiry to ascertain more detail of the damage she suffered, addressed to the Maritime Information Centre, London (20, July 1994), brought a reply to the effect that the ship "was scuttled and that we do not think there would have been time to assess the damage by cannon balls".

Defects in Hutton's article on rockets?

Hutton's article *Rockets* on p. 337 in Vol. II of his *Philosophical and Mathematical Dictionary* published in 1815 is disappointing. He commences it by stating some details about the different compositions for rockets of various sizes. Towards explaining the theory of the rocket he first refers to the work of Mariotte about the "resistance of the air against flame" and secondly, he describes Desaguiliers' quasi-static theory for propulsion and the function of the choke. Thirdly, the function of the stick draws some attention from Hutton by referring to the effect of the air "friction" on it and to the continuous change in the centre of gravity in the "engine" due to burning of the composition. Both these latter matters receive some attention in Moore's book. Finally, Hutton mentions the experiments of Robins (and his co-workers), referring to the latter's *Tracts* from the Proceedings of the Royal Society for 1749 and 1750, [11]. At this point he ends his article. It is very surprising to find no mention of Moore, his papers or his book, because his volume appeared two years before Hutton's Dictionary. Hutton was on the committee which appointed Moore in 1806 and there is evidence that they knew one another and William Congreve as well; it surely cannot have been otherwise, seeing that they all worked on the same site?

It is curious too that Moore nowhere acknowledged help and encouragement from Hutton and apparently was not stimulated officially from within the R.M.A. (though we note that his address for correspondence with Nicholson was the Woolwich Academy). One can only judge that Moore's initiative on this topic was entirely due to himself and the scientific interest he derived perhaps having been started by the advertised competition from Denmark [11].

One would have supposed that Hutton, as a minimum, could have simply referenced Moore's papers and book and mentioned Congreve's experiments. Of course Hutton was already aged 76 when Moore's book appeared and his own two volumes must have been about to go to press. Perhaps this inadequate article may simply be explained as a decline due to old age and fatigue.

CONCLUDING REMARKS

Nothing has anywhere been said about Hale's rockets in this paper. However the author points to an article in press which makes a valuable contribution to this topic, see [19].

In the *Sources of Invention*, [12], a short case history of the development of rockets from about 1900 up to the late 1950s is given and summarizes how, over a period of 25 years, "the high altitude rocket became a weapon of war and a useful instrument for the study of conditions of the upper air", (p. 355). Some detailed remarks are made and references given about a weapon development as stated earlier but a full treatment of the subject lies outside our orbit of interest.

It will perhaps suffice here to reflect the two sides of rocketry by referring to the work of two men who have had close involvement with it, R. V. Jones and D. G. King-Hele over a period of nearly half a century.

King-Hele worked at R.A.E., Farnborough designing a cheap rocket (indeed he records 17 attempts) in order to study upper atmosphere physics, (in the region of 100–200 km depth), to produce masses of basic data at "modest cost" and giving "great value for money". The latter story is entertainingly but accurately recorded in his book, *A Tapestry of Orbits*, published only in 1992 [13].

R. V. Jones commented especially on (German) rocket development for military purposes in his *Most Secret War*, 1978, [14] and recently in his *Reflections on Intelligence*, 1989 [15]. He has made his comments as a physicist and by speaking from experience in his sometime government post of Director of Scientific Intelligence in the U.K. Ministry of Defence, 1952–53. Postam's official volume adds detail to these latter works [16]. Interest also attaches to Jones' short paper on *Genius in Engineering*, [17], which is, in part, in the same vein as [12]. Interesting remarks on our topic will also be found in Hartcup's book [18].

Modern works about rocket evolution mostly commence after about 1880 and mention sometimes Congreve's contributions, but never, I believe, will readers encounter the name of Moore. It is therefore hoped that this essay will well demonstrate his wide theoretical contribution and thus lead to a better overall appreciation of the total history of rocketry and of one who has made a significant input to it.

NOTE 1

An initial reason for trying to produce a useful review of the contents of this book of Moore's in detail, stemmed from the information that it was rare. Opinion was that copies of it were only to be found in the British Museum Library, London, the Bodleian Library, Oxford and the Royal Military Academy, Woolwich. However, recently I found that the British Museum Library was able to produce a film copy of this small volume and this is what I have used in writing this paper.

NOTE 2

The Catherine wheel has mythical origins but none-the-less some interesting meanings for engineers. Catherine is the name of several saints in the early Christian church but the Catherine of the Catherine wheel has many stories related about her, all ultimately apocryphal. Despite this the myth associating her with Alexandria has remained. Perhaps the most favoured story is that in which, at a sacrificial feast, a certain young woman named Catherine, publicly confessed to having accepted the Christian gospel and advocated it in front of the Emperor Maximinus (or Maxentius). The era was Hellenistic when Christian theology was spreading and supplanting the gods of the failing Western Roman Empire. She was imprisoned, then bound on a spiked wheel which inexplicably fractured and she was unharmed, but later beheaded. Philosophers sent to re-convert her had themselves been converted to Christianity. With divine intervention her head was later conveyed to Mt. Sinai and the well known monastery founded there by Emperor Justinian became named after her.

There is a likelihood that aspects of this story were added to from that of Hypatia (370–415 A.D.), who was a female mathematician–philosopher of distinction, but was actually a neo-Platonist rather than a Christian. In the 18th century it was proven that the legends of Catherine of Alexandria were mythical and therafter her name declined and proceeded to disappear from the Christian hagiography.

It was in the mediaeval period that Catherine was the patron saint of Paris; she was sometimes regarded as the especial saint of learned men but also in these early days images were made of the wheel as an amulet in the cure of diseases. Particularly too, she has been the patron saint of wheelwrights and mechanics working with wheels, e.g. spinners and potters. Pythagoras' name has also been associated with her through the wheel—"Life is no more than a circle of good and evil"; the wheel is also a well known emblem of fortune.

In Siemienowicz's *The Great Art of Artillery*, five sorts of fire wheel, all recreational, are recognized and described in Book V, Part I, Ch. VI, p. 319.

Pin wheels or Catherine wheels are long paper-cases filled with composition and packing wire, wound round a wooden disc which can be set to rotate on a vertical or horizontal axis. The primed end when lighted and burning, causes the disc to spin in the opposite direction following certain simple recoil principles. *Pastiles* have the paper case wound spirally on the disc but the *fire wheel* has straight cases at the end of wheel spokes, all arranged at the same angle to them and connected by leaders, so that as each one burns out, the next one is fired. This situation is addressed in Section IV, Ch. 5 [5]. Powder can be chosen to give different colours and such wheels can indeed be combined and made to revolve in different planes and directions. There are *bisecting* wheels, *plural* wheels, *caprice* wheel and *spiral* wheels: models of the solar system are possible; (powders of various elemental materials and filings provide "the stars").

The Panjan-drum on the beach at Instow, U.K. after being launched from its landing-craft.

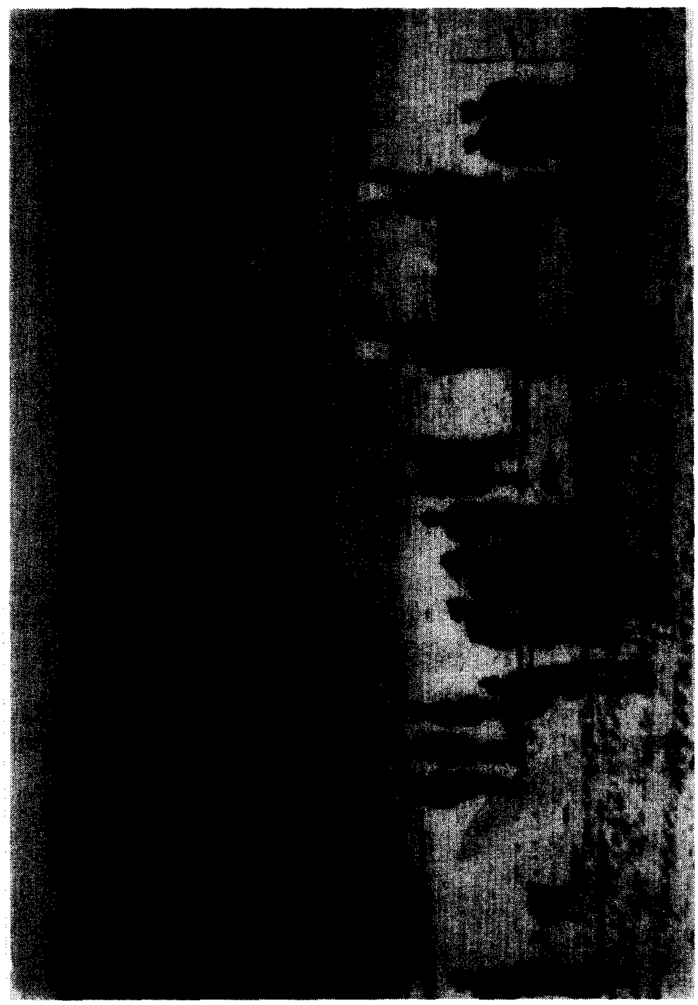

Fig. 14

NOTE 3

I made some enquiries of national naval museums and libraries seeking references to papers, books and artefacts for details and sketches of the use of plugs and fitting equipment, and of the "parasol plug", but without any success. See Fig. 13, for a description of the latter.

NOTE 4

After completing the writing of this paper I recalled reading an account of a device which underwent some development during World War II by the British "Wheelers and Dodgers" group and was called *The Great Panjandrum*, see [10]. This "machine" consisted of two steel wheels of 10 ft dia., with a tread of 1 ft; see Fig. 14, and seen here at the end of a trial. They were connected with a drum-like axle which it was intended should carry 1 ton of high explosive. A large number of slow-burning cordite rockets, at one time as many as 70, were fitted around the circumference of the wheels; all were to be started to fire at once. Carried on a tank-landing craft, this device was to be released on reaching shore where it was intended to run up a sandy beach at about 60 m.p.h. to crash into a concrete barrier (erected as a first line protection by German forces) preparatory to the Allied invasion of Europe in 1944. After impinging on the barrier, the expectation was that the deposited and then detonated explosive would "blow" a hole in the "wall". After several trials the device was abandoned, accurate control of the weapon in the circumsances of its intended operation being found to be unattainable.

The relationship of this 2-wheel rotating rocket-driven system to Ch. IV is obvious. We observe that on the one hand, the innovators knew nothing of Moore's available theoretical speculation and on the other that Moore can properly claim to have made yet another potentially useful first theoretical investigation.

The word *Panjandrum*—a pompous high-ranking person—was coined in 1755 by 18th c. English comic actor–writer Samuel Foote (1720–77), "to test the memory" of a fellow actor. It occurs in a ten-line farrago of nonsense, but none-the-less is well fitting for the occasion—" ... And there were present the Picninnies, Jobillies, the Garyulies and the Grand Panjandrum himself ... all fell to playing the game of catch-as-catch-can till *the gunpowder ran out of the heels of their boots.*"

NOTE 5

A letter, signed ZENO, p. 384, 1811 (in Nicholson's journal) pointed out that Moore in his journal treatment of "the resistance opposed to the cylinder moving in a fluid" had overlooked the fact that three particular angles in his original figures were, in fact, in different planes and not co-planar as he had supposed. Moore published a correction in the journal for 1812, on pp. 93–94. These corrected formulae do appear in his book however.

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