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COMBUSTION INSTABILITIES IN LIQUID-FUELLED PROPULSION SYSTEMS

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COMBUSTION INSTABILITIES IN LIQUID-FUELED PROPULSION SYSTEMS – AN OVERVIEW

by

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Abstract

Combustion of reactants in a confined volume favors excitation of unsteady motions over a broad range of frequencies. A relatively small conversion of the energy released will produce both random fluctuations or noise, and, under many circumstances, organized oscillations generically called combustion instabilities. Owing to the high energy densities and low losses in combustion chambers designed for propulsion systems, the likelihood of combustion instabilities is high. The accompanying heat transfer to exposed surfaces, and structural vibrations are often unacceptable, causing failure in extreme cases. This paper is a brief review of combustion instabilities in liquid-fueled propulsion systems—rockets, ramjets, and thrust augmentors—with emphasis on work accomplished during the past decade. To provide a common framework for discussing the wide range of works, a theory of two-phase flow is reviewed as the basis for an approximate analysis of combustion instabilities. The analysis is directed primarily to treatment of linear stability; it is sufficiently general to accommodate all processes occurring in actual systems. A new result has been obtained for an extended form of Rayleigh's criterion and its relation to the growth constant for unstable waves. The chief mechanisms for combustion instabilities in liquid-fueled systems are reviewed, followed by a summary of the common methods of analysis and applications to the three classes of propulsion systems. Control of instabilities by passive and active means is examined briefly.

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1. INTRODUCTION

Chemical propulsion systems depend fundamentally on the conversion of energy stored in molecular bonds to mechanical energy of a vehicle. The first stage of that process is combustion of fuel and oxidizer. Burning takes place at relatively high pressure in a vessel open only to admit reactants and to exhaust the hot products. Liquid-fueled rockets, ramjets and thrust augmentors (afterburners) are intended to operate under conditions that vary little during the time required for a disturbance to propagate across the combustion chamber. Normal design considerations for such systems commonly do not include thorough consideration of truly unsteady combustion and flow.

Yet unsteady motions are always present, as random fluctuations or noise, and often as organized vibratory motions generically termed "combustion instabilities". Noisy motions cause structural vibrations over a broad frequency band, usually requiring only routine qualification of the vehicle and equipment. The amount of energy contained in the noise field is a negligibly small part of the total energy available and causes no measurable reduction in the performance of the machine.

Likewise, combustion instabilities even at the highest amplitudes observed consume a small fraction of the available chemical energy. The oscillations do not directly affect the steady thrust produced by the systems. Serious problems may nevertheless arise due to structural vibrations generated by oscillatory pressures within the chamber, or induced by fluctuations of the thrust. In extreme cases internal surface heat transfer rates may be amplified ten-fold or more, causing excessive erosion of chamber walls.

Special practical problems have arisen in the three classes of liquid-fueled systems. Strong coupling between chamber pressure oscillations, low frequency structural vibrations and the propellant feed system produces the common POGO instability in liquid rockets. Axial oscillations in ramjet engines have recently become troublesome; their influence on the shock system in the inlet diffuser can produce a reduction of the inlet stability margin. Because of their light construction, thrust augmentors are susceptible to failure of flameholders or of the basic structure when combustion instabilities become severe. Many augmentors must therefore be operated with reduced performance in portions of the flight envelope.

Combustion instabilities may be regarded as the unsteady motions of a dynamical system capable of sustaining oscillations over a broad range of frequencies. The term 'combustion instability' is usefully descriptive but slightly misleading. In most instances the combustion processes themselves are stable - uncontrolled explosions or other intrinsic instabilities are not usually at work. The presence of an instability in a combustion chamber is established by observing either the gas pressure or accelerations of the enclosure. Excitation and sustenance of an oscillation occurs because of coupling between the combustion processes and the gas dynamical motions, both of which alone may be stable. If the fluctuation of energy release responding to a pressure disturbance causes a further change of pressure in phase with the initial disturbance, then the result may be an instability. Thus one may view the behavior as that of a stable open-loop system (the gasdynamics) made unstable by a positive feedback loop, the gain being associated with the combustion processes.

Owing to the internal coupling between combustion processes and unsteady motions, an observer perceives an unstable motion as "self-excited". The amplitude of the motion grows out of the noise without the need for an external influence. Two fundamental reasons explain the prevalence of instabilities in combustion systems:

i.) an exceedingly small part of the available energy is sufficient to produce unacceptably large unsteady motions;

ii.) the processes tending to attenuate unsteady motions are weak, chiefly because combustion chambers are nearly closed.

These two characteristics are common to all combustion systems and imply that the possibility of instabilities occurring during development of a new device must be recognized and anticipated. Treating combustion instabilities is part of the necessary price to be paid for high performance chemical propulsion systems.

The fact that only a small part of the total power produced is involved suggests that the existence and severity of combustion instabilities may be sensitive to apparently minor changes in the system. That conclusion is supported by experience. Moreover, the complicated chemical and flow processes prohibit construction of a complete theory developed from first principles. It is therefore essential that theoretical work always be closely allied with experimental results. No single analysis will encompass all possible instabilities in the various practical systems. There are nevertheless many features common to the three types of combustion chambers discussed in this paper. While it is not possible to predict accurately the occurrence or details of instabilities, a framework does exist for understanding their general behavior and for formulating statements summarizing the chief characteristics. For practical purposes, theory serves mainly to analyze, understand, and predict trends of behavior. Experimental data are always required to deduce quantitative results.

All combustion instabilities are unsteady motions of the compressible gases within the chamber. If the amplitude is small, the instability is closely related to classical acoustical behavior occurring in the absence of combustion and mean flow. The geometry of the chamber is therefore a dominant influence. Corresponding to classical results, traveling and standing waves are found at frequencies approximated quite well by familiar formulas depending only on the speed of sound and the dimensions of the chamber.

Such wave motions comprise the majority of instabilities observed in the three types of systems discussed here. They are driven by the energy released by the combustion processes and influenced by the mean flow as well as by the conditions at the inlet and exhaust. Under suitable circumstances the flow of energy to the waves may so dominate the losses that nonlinear behavior becomes significant; in extreme cases shock waves may form.

Nevertheless, because the propagation speed of disturbances is a weak function of the amplitude, the frequencies don't differ greatly from classical values computed for the same geometry. Hence mere prediction of frequency is no test of a theory of combustion instability.

Owing to the presence of combustion and mean flow, other kinds of motion are possible, having frequencies below that of the fundamental wave mode. For any chamber, the lowest frequency is associated with a "bulk" mode, in which the pressure is nearly uniform throughout, but pulsating in time. The velocity fluctuation is nearly zero. This mode corresponds to the vibrations of a Helmholtz resonator obtained, for example, by blowing over the open end of a bottle. In a combustion chamber, the driving source may be directly the burning processes; or it may be due to oscillations in the supply of reactants, caused in turn by the variations of pressure in the chamber. In a liquid rocket, structural oscillations of the vehicle or the feed system may also participate, producing the POGO instability.

Motions in the range of frequencies between those of the bulk mode and the wave motions are also possible. Instabilities in this intermediate frequency range have been observed in liquid rockets for which the influence of unsteady behavior of the propellant supply system may extend over a broad frequency range. The shift of frequency upward from the value for the bulk mode is due to the combined action of boundary conditions (including the fuel system) and the coupling between combustion processes and the gasdynamics. In afterburners, the inlet flow is not choked, so disturbances in the chamber can propagate upstream. As a result, it is possible that much of the engine may participate in oscillation, producing frequencies quite different from those one would estimate from the geometry of the afterburner itself. This interpretation emphasizes the view that one may conveniently regard a combustion instability as an oscillation in the gasdynamic medium perturbed by other processes in the system, mainly the burning, the mean flow, and the boundary conditions. General features of the possible oscillations and formulation of the theoretical framework are discussed further in Section 2, partly as a means of classifying observed instabilities. A new result given in Section 2.5 is a form of Rayleigh's criterion accounting for all linear processes. That leads naturally to direct connection with the growth constant, Section 2.6. Thus two general ways of assessing unstable behavior are shown to be equivalent.

Section 2 contains lengthy, though abbreviated, calculations included here chiefly for two reasons. First, it is important to realize that combustion instabilities in all liquid-fueled systems can be accommodated within a common theoretical framework. The analysis given here certainly is not unique, but it is a convenient form encompassing at least the essential ideas of most previous works. The second reason is that the calculations produce several general results that we use in later discussions of specific problems. However, it is unnecessary to know the details of the analysis to understand the applications of the results in the remainder of the paper.

While it is surely true that combustion is the ultimate source of the energy for the unsteady motions, and therefore in some sense the 'mechanism'for instabilities, this observation is broad and offers little help in understanding or curing the problem in practice. It is essential to identify more precisely the specific mechanism causing the particular instability at hand. In liquid-fueled rockets the most important mechanisms are associated with the formation of liquid drops (or droplets, since they are quite small!) from the injected streams and vaporization. Chemical kinetics is of course fundamental to the reactive processes but generally occur on time scales much shorter than the periods of unsteady motions. The must successful analyses of instabilities in liquid-fueled rockets have been based on mechanisms involving droplet formation, vaporization and combustion.

Similar processes must take place in ramjets and afterburners but there is considerable evidence that flow separation and the formation of vortex structures may be more significant. The chief reason is due to the different geometry. Both kinds of devices are commonly designed with rearward facing steps or bluff bodies to anchor the mixing and combustion zones. The associated shear layers tend to be unstable, shedding vortices in the frequency range of acoustic modes, for the chamber. Coupling with the acoustic field encourages this resonance. The vortex motions then cause periodic entrainment of unburnt reactants and, subsequently, periodic combustion. The unsteady energy release is coupled to the field, closing the feedback loop.

Whatever the mechanism, energy must be supplied to the oscillating flow field at a suitable location in space and time during a cycle of motion. If the energy addition is improperly timed, the oscillation may in fact be attenuated. This condition led early in the history of the subject to the notion of time lag as a means of interpreting the mechanisms for causing instabilities. Apparently von Karman in 1941 suggested introducing a time lag in the theoretical description as a means of explaining combustion instabilities. Whatever may have been the first proposal, it was Crocco and his co-workers and students at Princeton who developed and applied the idea during the 1950's and 1960's. It remains as the basis of one of the two or three standard methods for studying instabilities.

The time lag is usually defined as the interval from the instant at which an element of liquid reactant enters the chambers to the time at which combustion of that mass is completed. Following Crocco's early work virtually all formulations assume that the conversion processes acting during the lag period are dependent on pressure only. Some work has accounted for dependence on velocity. For the most common case, the result of reasoning in this basis produces a representation of the unsteady energy release in the chamber, a formula containing two parameters, the time lag τ and the pressure index n. Such a description is commonly referred to as the $n-\tau$ model, meaning almost always the model constructed by Crocco.

In practice, the $n-\tau$ model has been used chiefly in what might fairly be called indirect fashion. An analysis for the stability of small amplitude oscillations is carried out with the $n-\tau$ model representing the unsteady source acting as the feedback mechanism. Such an analysis produces two results: a formula for the frequencies of oscillations, and a formula for the rate at which those oscillations will grow or decay. If all input variables and parameters are known, the result can be used to predict, for a specified operating condition, whether the oscillations are stable or not. The difficulty is that the values of n and τ are not known: they are really present only as parameters characterizing the unsteady combustion processes in some global sense.

Hence the usual procedure is to assume that everything else is known and calculate the loci n versus τ for neutral stability (i.e. no growth or decay) of the various modes. Those results can be used, for example, to correlate data and to deduce the values of n and τ associated with, say, various injector designs. Unless the detailed processes are analyzed, theoretical values of n and τ are not known and the $n-\tau$ model cannot be used for predicting actual behavior. Nevertheless, this approach seems to have been the most common strategy for studying instabilities in liquid-fuel rockets.

Following a survey of various mechanisms is Section 3, we discuss the chief analytical schemes in Section 4. Theories and computational procedures for combustion instabilities may be classified in two ways: linear or nonlinear behavior and analytical or numerical. Whether or not nonlinear calculations are to be done rests in the first instance on decisions regarding the physical behavior. At least as a preliminary, a linear theory should be worked out first, but the limitations must be understood. Whatever the content of a linear theory, only two results are obtained: the frequencies of allowed oscillations in the system, and the rate at which small amplitude distrubances will grow or decay. If a disturbance is unstable in an actual system, the amplitude grows without limit unless one or more nonlinear processes act. That is a fundamental characteristic of self-excited systems. Moreover, full understanding of the response of a combustion system to finite initial disturbances can be gained only with a nonlinear theory. By far most of the theoretical work on combustion instabilities has been based on linear behavior. We do not review nonlinear theory in this paper.

Ultimately for applications in design of actual systems, there seems little doubt that elaborate numerical calculations are required. The complications presented by the gas dynamics, chemical processes and geometrical configurations block solution to the governing differential conservation equations in any but the simplest cases: nothing like closed form solutions can be expected. When the $n - \tau$ model is used, it is possible to progress further in that direction, mainly because much of the complicated physical behavior is effectively swept under the rug and presented in global approximation. When the details of, say, droplet dynamics are treated, only numerical results can be obtained.

In one form or other much of the physical behavior is common to all liquid-fueled systems. The different kinds of devices are distinguished either by geometrical configuration or by the ways in which reactants are introduced in the chamber. Thus the material covered in Sections 2-4 has a strongly unifying character; specific examples are called upon mainly to clarify ideas or to show typical results. In Section 5, special cases of combustion instabilities are discussed at some length to emphasize both the similarities and differences among the three classes of systems.

The prevalence of combustion instabilities has motivated efforts to develop "cures" or methods of limiting the amplitudes to acceptable values. It seems that all successful applications to operational systems have been based on passive devices. The use of baffles, acoustic liners or resonant cavities - occasionally all three in the same engine - has become commonplace, especially in liquid rockets. If the geometry permits installation, adding a device of this sort can be an effective strategy. There are mainly two reasons they work: 1) by shifting the frequencies of permitted oscillations out of the range where unsteady energy transfer to the nodes is strongest; and 2) by direct attenuation of the motions, primarily due to the action of viscous stresses.

During the past several years, interest has grown in the possibilities for active control of combustion instabilities. The idea is not new, dating back to the late 1940's at least, but developments of lightweight fast computers and better sensors make active control an attractive alternative to passive control. Both subjects are discussed briefly in Section 6.

This cannot be a thorough review of the subject. In particular we do not explicitly treat design issues such as types of injectors, rationale for choices of particular passive damping devices and stability rating of chambers. Some aspects are quite well known from experience with liquid rockets especially but much work remains to treat satisfactorily contemporary systems operating at higher pressures with hydrocarbon fuels.

The list of references is divided for convenience into four groups dealing with rockets; ramjet engines; thrust augmentors; and passive and active control. I do not claim completeness and there is unavoidably some overlap, but each reference is entered in only one group. Also, not all references in the list are cited in the text. That is not a matter of value judgement, but follows from the need to curb the length of the text.

I am especially indebted to the following people who aided me early in this effort by providing me with lists of references, and in some cases copies of papers and reports: Professor A. Acosta, Caltech; Dr. Paul Kuentzmann, ONERA; Dr. P.V. Liang, Rockwell International, Rocketdyne Division; Professor C.E. Mitchell, Colorado State University; Dr. T.V. Nguyen, Aerojet TechSystems Company; Professor F.H. Reardon, Sacramento State University; Dr. Klaus Schadow of the Naval Weapons Center; Mr. A.A. Shabayek, Sverdrup Technology, Inc.; Professor V. Yang, Pennsylvania State University; and Professor B.T. Zinn of Georgia Tech. Mms. Jean Anderson and Pat Gladson of the Caltech Aeronautical Library have been most helpful locating references and making copies.

I have tried to give a fair coverage of the subject of combustion instabilities in liquid-fueled combustion systems based on the literature available to be me. I shall greatly appreciate receiving copies of works that I have not included in this survey.

2. SOME GENERAL FEATURES OF COMBUSTION INSTABILITIES AND THE THEORETICAL FOUNDATIONS

Probably the most important fundamental characteristic of combustion instabilities is that in first approximation they may be viewed as perturbations of classical acoustical motions. The chief perturbations are due to the combustion processes; the associated mean flow; and the boundary conditions imposed at the inlet and exhaust. It is a very robust approximation indeed. Often one may ignore apparently significant processes and still obtain remarkably good results for some of the dominant features.

For example, if the average temperature, and therefore speed of sound a of the chamber gases is known, close estimates for frequencies of allowable wave modes may often be had by simply dividing a by integral multiples of the length or diameter. Values within 10% or so of observed oscillations, and good approximations to the mode structures, can be obtained by numerical solution to the unperturbed classical acoustics problem solved for the same geometry as the combustion chamber.*That conclusion often remains valid even when subtantial amounts of a condensed phase are present in the chamber or when the mean flow field is highly non-uniform as happens when flow separation occurs. Large spatial variations of temperature can be more significant because the speed of sound then is non-uniform; even so, an averaged value may serve quite well. The approximation of classical acoustics also deteriorates if the Mach number of the average flow is larger than roughly 0.4, for then the Doppler effect and refraction may cause substantial distortions of the acoustic field.

One reason for emphasizing the surprising confidence that one may place in the classical acoustics approximation is that the idea seems to extend to many aspects of nonlinear behavior as well. For example, recent work, mainly for application to liquid-fueled ramjets and solid propellent rockets, suggests that under broad conditions the existence of limit cycles can be explained on the basis of nonlinear gasdynamics, with other processes affecting mainly details.

The emphasis on classical acoustics has been successful and will be followed throughout this work, because the main departures, while crucial in defining the real problems, are often small perturbations in some sense. The mean flow Mach number is generally not large over most of the combustion chamber. Even though the mass fraction of liquid may be substantial, the volume fraction remains small on average. As a result, the elasticity of the multi-phase mixture is dominated by the gases, while the inertia is a mass-weighted average of the gas and liquid phases. The boundary conditions provided by the propellant supply system and the exhaust nozzle may be significantly different from the condition for a rigid wall but those influences are easily accounted for by introducing appropriate impedance or admittance functions. And as we remarked in the introduction, the amount of energy possessed by the unsteady motions is a small part of the total chemical energy released and converted to average mechanical energy of the flow.

We therefore always seek a theoretical formulation that in an obvious fashion, when all perturbations vanish, reduces to a representation of classical waves in an enclosure. This strategy allows construction of a theoretical framework accommodating all types of propulsion systems.

It is of course the geometry and the perturbations that define the actual physical situations and distinguish one system from another. The geometry causes technical difficulties-solving equations in peculiarly-shaped volumes - and sets the spectrum of allowed oscillations. But geometry alone does not pose any fundamental problems, nor does it contain explanations of the instabilities. By far the greater part of research on combustion instabilities must therefore be spent on the perturbations of classical linear acoustics, especially directed to understanding the physical mechanisms responsible for the instabilities and their nonlinear behavior.

It is a consequence of this point of view that combustion instabilities in the three kinds of propulsion systems are profitably considered together. Much is to be gained by regarding individual examples as special cases of a general formulation such as that summarized in the following sections.

2.1 Conservation Equations

The state-of-the-art for experiment and analysis of combustion instabilities in liquid rockets as of (roughly) 1970 was summarized in the thorough reference volume edited by Harrje and Reardon (1972). A computer program, AUTOCOM [Reichel et al (1973, 1974)] essentially captured the main procedures commonly used at that time in the U.S. for treating combustion instabilities in liquid rockets. That program was re-written, for running on later computers, by Nickerson and Nguyen (1984a, 1984b), but the formulation and physical basis remained without change. Currently, extensive work is being carried out in the U.S. by the Aerojet Tactical Systems Co. [Fang(1984a, 1984b, 1987); Fang and Jones (1984); Muss and Pieper (1987, 1988); Nguyen (1988); Nguyen and Muss (1987); and Pieper and Fang (1986)]; and by Rockwell International [Liang et al (1986, 1987, 1988)]. Philippart (1987) and Philippart and Moser(1988) have reported some numerical results obtained with those computer programs, but without discussion of the physical basis for the formulations.

Following the failure during flight of the Viking motor, due to combustion instability (Souchier et al (1982)], a continuing research program has been in progress for seven years in France. Some of the results have been reported by Schmitt and Lourme (1982), Habiballah et al (1984, 1985, 1988) and by Lourme et al (1983, 1984, 1985, 1986).

All of those works begin with essentially the same physical basis, a compressible gaseous medium containing a liquid phase. Major differences arise in the representation of the mass and energy sources and in the analysis followed to obtain solutions. We shall discuss those matters later. Here we are concerned only with the broad character of the general formulation. In analysis of liquid rockets, particular emphasis is placed on the behavior of the liquid phase and the interactions between the liquid and gas phases, matters that are common to all liquid-fueled systems. The average flow field is relatively simple compared with the circumstances in ramjet engines and augmentors. Because the flow of oxidizing gas and flow separation are important features in those systems, analysis of combustion instabilities has additional complications. We defer discussion of those phenomena and begin here with a simplified formulation of the problem for liquid rockets; most of the results will be applicable to ramjet engines and thrust augmentors as well.

We assume that the medium in the chamber consists of reactant gases, liquid oxidizer and fuel, and gaseous products of combustion. Condensed products of combustion (e.g. soot) may also form and can be accommodated. For analysis of unsteady motions many of the details necessary to a successful description of the steady flow can

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* Some care is required with the boundary condition at the exhaust plane. A poor choice may lead to unacceptable results.

be approximated or neglected. To simplify, the representation, we treat a two-phase mixture, a mass-average gas comprising all species, identified by subscript $(\ )_g$ and a single mass-averaged liquid phase identified by subscript $(\ )_l$. A proper analysis must account for the differences between fuel and oxidizer and for the broad range of sizes of liquid drops, streams or sheets. We shall not treat those matters here.

The conservation equations for three-dimensional motions are

$$\text{mass(gas)} \quad \frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \vec{u}_g) = w_l \tag{2.1}$$

$$\text{mass(liquid)} \quad \frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_l \vec{u}_l) = -w_l \tag{2.2}$$

$$\text{momentum} \quad \frac{\partial}{\partial t}(\rho_g \vec{u}_g + \rho_l \vec{u}_l) + \nabla \cdot (\rho_g \vec{u}_g \vec{u}_g + \rho_l \vec{u}_l \vec{u}_l) + \nabla p = \nabla \cdot \overleftrightarrow{\tau}_v \tag{2.3}$$

$$\text{energy} \quad \frac{\partial}{\partial t}(\rho_g e_{go} + \rho_l h_{lo}) + \nabla \cdot (\rho_g \vec{u}_g e_{go} + \rho_l \vec{u}_l h_{l0}) + \nabla \cdot (p \vec{u}_g) = Q + \nabla \cdot \vec{q} \tag{2.4}$$

The viscous stress tensor and heat flux vector (conduction) are represented by $\overleftrightarrow{\tau}_v$ and $\vec{q}$; Q is the energy released by homogeneous reactions in the gas phase (energy/sec.-vol.) and w_l is the rate of conversion, liquid to gas (mass/sec.-vol). Note that ρ_l is the liquid density, mass per unit volume of chamber, not the material density. The velocities $\vec{u}_g$, $\vec{u}_l$ are mass-averaged values.

Equations (2.1) – (2.4) form essentially the system used in early work, e.g. Crocco and Cheng (1956), Crocco (1965), and Harrje and Reardon (1972), and as the basis for most of the analysis incorporated in the computer program AUTOCOM. With the notable exception of the French work referred to above, most current work still does not account for different species explicitly in the conservation equations or for the manner in which liquid may be distributed among streams, sheets and droplet sizes. That does not mean that such complications are simply ignored. It has long been common practice to accommodate such details in analysis of the motion of the liquid and in the representations of the steady and unsteady sources of mass and energy. Thus rather fine distinctions between systems may appear in numerical results. For the purposes here a more blurred view is adequate to understand the main global behavior.

The momentum and energy equations (2.3) and (2.4) are correctly written in the first instance for the combined gas and liquid phases. Separate momentum equations are obtained by defining the force of interaction, $\vec{F}_l$, between the gas and liquid,

$$\vec{F}_l = -\rho_l [\frac{\partial \vec{u}_l}{\partial t} + \vec{u}_l \cdot \nabla \vec{u}_l] \tag{2.5}$$

and the heat added to the gas by heat transfer between the phases, vaporization, and chemical reaction,

$$Q_l = -\rho_l [\frac{\partial h_l}{\partial t} + \vec{u}_l \cdot \nabla h_l] \tag{2.6}$$

The enthalpy, h_l of the liquid includes the heat release associated with the transformation of liquid to gas. Note that $\vec{F}_l$ and Q_l are force and energy per unit volume of chamber. Thus F_l is the drag force acting on the average droplet times the number droplets per unit volume of chamber. With some rearrangement, the momentum and energy equations can be written

$$\rho_g \frac{\partial \vec{u}_g}{\partial t} + \rho_g \vec{u}_g \cdot \nabla \vec{u}_g + \nabla p = \nabla \cdot \overleftrightarrow{\tau}_v + \vec{F}_l - (\vec{u}_g - \vec{u}_l) w_l \tag{2.7}$$

$$\begin{aligned} \rho_g C_v \frac{\partial T_g}{\partial t} + \rho_g C_v \vec{u}_g \cdot \nabla T_g + p \nabla \cdot \vec{u}_g = {} & Q + Q_l + \nabla \cdot \vec{q} + \Phi \\ & + (h_{l_o} - e_o) w_l \\ & + \vec{u}_g \cdot (\vec{u}_g - \vec{u}_l) w_l + (\vec{u}_l - \vec{u}_g) \cdot \vec{F}_l \end{aligned} \tag{2.8}$$

where Φ is the dissipation function; its specific definition is unimportant here.

Equations (2.1), (2.7) and (2.8) are the equations of motion for the gas phase; and (2.2) (2.5) and (2.6) are those for the liquid phase. Within this formulation, equations (2.5) and (2.6) govern the motion of the "average droplet". It is difficult to use this approach in regions where the liquid is still moving as a stream or sheet; in practice, calculations are carried out only for droplet clouds. To determine the motions, one must specify $\vec{F}_l$ and Q_l. That's a fairly easy problem for slow motions of nonreactive particles; solution gives a familiar result for the attenuation of acoustic waves by gas/particle interactions [see, for example, Epstein and Carhart (1955), Temkin and Dobbins (1966); and Marble (1970); Culick (1975, 1976)].

The next more difficult problems involve droplets that are vaporizing or condensing. Beginning in the 1960's vaporization received serious attention as a possible contribution to driving instabilities, independently of combustion processes. We shall discuss the matter further in Sections 3 and 4.

If droplets are dispersed in the gas, the momentum and energy equations may be written in the more convenient form involving the mass-averaged properties of the two-phase mixture:

$$\rho\frac{\partial \vec{u}_g}{\partial t} + \rho\vec{u}_g\cdot\nabla\vec{u}_g + \nabla p = \nabla\cdot\overleftrightarrow{\tau}_v + \delta\vec{F}_l + \vec{u}_g\cdot\delta\vec{u}_l \tag{2.9}$$

$$\begin{aligned}\rho\bar{C}_v\Big(\frac{\partial T_g}{\partial t} + \vec{u}_g\cdot\nabla T_g\Big) + p\nabla\cdot\vec{u}_g = Q + \delta Q_l + \nabla\cdot\vec{q} + \Phi + (h_{lo} - h_o)w_l \\ - \vec{u}_g\cdot\delta\vec{u}_l w_l + \delta\vec{u}_l\cdot\vec{F}_l\end{aligned} \tag{2.10}$$

where

$$\delta\vec{F}_l = -\rho_l[\frac{\partial\delta\vec{u}_l}{\partial t} + \delta\vec{u}_l\cdot\nabla\delta\vec{u}_l + \delta\vec{u}_l\cdot\nabla\vec{u}_g + \vec{u}_g\cdot\nabla\delta\vec{u}_g] \tag{2.11}$$

$$\delta Q_l = -\rho_l[\frac{\partial\delta h_l}{\partial t} + \rho_l\delta\vec{u}_l\cdot\nabla\delta h_l + \delta\vec{u}_l\cdot\nabla(C_lT) + \vec{u}_g\cdot\nabla\delta h_l] \tag{2.12}$$

and $\delta\vec{u}_l = \vec{u}_l - \vec{u}_g$, $\delta h_l = h_l - C_lT$. The density of the mixture is

$$\rho = \rho_g + \rho_l = \rho_g(1 + C_m) \tag{2.13}$$

and $C_m = \rho_l/\rho_g$ is the mass fraction of liquid.

The mass weighted specific heats for the mixture are defined in the usual fashion [Marble(1969)]:

$$\bar{C}_v = \frac{C_v + C_mC_l}{1 + C_m};\ \bar{C}_p = \frac{C_p + C_mC_l}{1 + C_m} \tag{2.14}$$

Now add $(1 + C_m)T$ times equations (2.1) and (2.2) to $1/\bar{C}_v$ times equation (2.10) to find an equation for the pressure:

$$\begin{aligned}\frac{\partial p}{\partial t} + \vec{u}_g\cdot\nabla p + \bar{\gamma}p\nabla\cdot\vec{u}_g = \frac{\bar{R}}{\bar{C}_v}[Q + \delta Q_l + \nabla\cdot\vec{q} + \Phi + \delta\vec{u}_l\cdot\vec{F}_l \\ + \vec{u}_g\cdot(\vec{u}_g - \vec{u}_l)w_l + (h_{lo} - e_o)w_l - \bar{C}_vT_g\nabla\cdot(\rho_l\delta\vec{u}_l)\end{aligned} \tag{2.15}$$

We assume that the perfect gas law is valid, $p = \rho_gRT_g$, where R is the gas constant for the gases only; for the mixture, $\bar{R} = \bar{C}_v - \bar{C}_p$, $\bar{\gamma} = \bar{C}_v/\bar{C}_p$ and

$$p = \bar{R}\rho T_g \tag{2.16}$$

The preceding manipulations have established the forms of the equations that account for the presence of liquid but are most appropriate for conditions when the mass fraction is nearly uniform, so C_m is approximately constant throughout the chamber. That is true in a solid propellant rocket, for which these equations were originally derived, and certainly not true for liquid-fueled systems. But even though C_m may vary significantly, the mass averaged thermodynamic properties are not greatly affected. We shall use equations (2.9)-(2.16) without further elaboration of the possible inaccuracies. One important consequence is that the linearized form will give a good first approximation to the speed of sound accounting for the presence of liquid,

$$a = \sqrt{\bar{\gamma}\bar{R}T_g} = [\frac{\bar{\gamma}}{1 + C_m}\frac{p}{\rho_g}]^{\frac{1}{2}} \tag{2.17}$$

This formula explicitly shows the characteristic remarked upon earlier, that the propagation of disturbances is governed by the elasticity of the gas (the pressure), and by inertia modified by the condensed material, represented in the factor $(1 + C_m)\rho_g$.

We now have conservation equations in a form emphasizing the view that combustion instabilities are unsteady motions best regarded as perturbations of classical acoustics. The framework for analysis is based on the sum of the continuity equations (2.1) and (2.2); the momentum equation (2.9); and the energy equation (2.18) written with the pressure as the dependent variable:

$$\frac{\partial\rho}{\partial t} + \vec{u}_g\cdot\nabla\rho = \mathcal{W} \tag{2.18}$$

$$\rho\frac{\partial\vec{u}_g}{\partial t} + \rho\vec{u}_g\cdot\nabla\vec{u}_g = -\nabla p + \vec{\mathcal{F}} \tag{2.19}$$

$$\frac{\partial p}{\partial t} + \bar{\gamma}p\nabla\cdot\vec{u}_g = -\vec{u}_g\cdot\nabla p + \mathcal{P} \tag{2.20}$$

For the conditions treated above:

$$\mathcal{W} = -\rho\nabla\cdot\vec{u}_g - \nabla\cdot(\rho_l\delta\vec{u}_l) \tag{2.21}$$

$$\vec{\mathcal{F}} = \nabla\cdot\overleftrightarrow{\tau}_v + \delta\vec{F}_l + \delta\vec{u}_lw_l \tag{2.22}$$

$$\mathcal{P} = \frac{\bar{R}}{\bar{C}_v}[Q + \delta Q_l + \nabla \cdot \vec{q} + \delta\vec{u}_l \cdot \vec{F}_l + \{(h_l - e) + \frac{1}{2}(\delta\vec{u}_l)^2\}w_l - \bar{C}_v T_g \cdot (\rho_l \delta\vec{u}_l)] \tag{2.23}$$

Equations (2.18)-(2.20) are suitable for two-and three- dimensional problems. There are many important cases of nearly pure axial or longitudinal motions arising in all three types of systems. A perfectly serviceable analysis may then be constructed using the one-dimensional approximation obtained by replacing $\vec{u}_g$ by u_g, the axial component of velocity; $\vec{u}_g \cdot \nabla$ by $u_g \frac{\partial}{\partial z}$ and $\nabla \cdot (\ )$ by $\frac{1}{S_c}\ \frac{\partial}{\partial z}(S_c\ )$ where S_c is the cross-section area. Equations (2.18)-(2.20) then become

$$\frac{\partial \rho}{\partial t} + u_g \frac{\partial \rho}{\partial z} = \mathcal{W}_1 \tag{2.24}$$

$$\rho \frac{\partial u_g}{\partial t} + \rho u_g \frac{\partial u_g}{\partial z} = -\frac{\partial p}{\partial z} + \mathcal{F}_1 \tag{2.25}$$

$$\frac{\partial p}{\partial t} + \bar{\gamma} p \frac{1}{S_c}\frac{\partial}{\partial z}(S_c u_g) = -u_g \frac{\partial p}{\partial z} + \mathcal{P}_1 \tag{2.26}$$

where $\mathcal{W}_1$, $\mathcal{F}_1$, $\mathcal{P}_1$ are (2.21)-(2.23) written for one-dimensional motions according to the rules given above.

2.2 Formulation of an Approximate Analysis

With the recent developments in high-speed computers, serious consideration must now be given to extensive numerical analysis of internal flows and combustion instabilities based on the complete equations of motion. That is a formidable task; it appears that the most extensive program of that sort is being pursued in France. Work in the U.S., including the computer program AUTOCOM [Reichel et al (1973, 1974) and the later version written by Nickerson and Nguyen (1984a, 1984b)] has been based on approximations to the equations explicitly ignoring, for example, possible influences of turbulence on the motions of liquid drops. Especially, approximations to the crucial source terms are generally based on models that are founded on heuristic reasoning. As a result, much reliance must be placed on correlations of data. It is the nature of flows in combustion chambers that, due to the complicated gasdynamics and chemistry, all theoretical work must eventually make use of experimental results.

Thus it is fundamental to the subject that an approximate analysis be carefully formulated and understood. We cannot here review all analyses that have been constructed. Rather, we shall develop a fairly general form to use as a framework for discussing particular results. To simplify the calculations at this point, we shall not display the details of the source terms $\mathcal{W}$, $\vec{\mathcal{F}}$ and $\mathcal{P}$ in equations (2.18)- (2.20).

Write all dependent variables as sums of mean and fluctuating parts, $p = \bar{p} + p'$, etc. We assume that average values do not vary in time. That assumption is occasionally violated in actual systems but we shall not elaborate here.To second order in the fluctuations, equations (2.19) and (2.20) are

$$\bar{\rho}\frac{\partial \vec{u}'}{\partial t} + \nabla p' = -\bar{\rho}(\bar{\vec{u}}_g \cdot \nabla \vec{u}'_g + \vec{u}'_g \cdot \nabla \bar{\vec{u}}_g) - \bar{\rho}(\vec{u}'_g \cdot \nabla\ \vec{u}'_g) - \rho' \frac{\partial \vec{u}'_g}{\partial t} + \vec{\mathcal{F}} \tag{2.27}$$

$$\frac{\partial p'}{\partial t} + \bar{\gamma}\bar{p}\nabla \cdot \vec{u}'_g = -\bar{\vec{u}}_g \cdot \nabla p' - \bar{\gamma} p' \nabla \cdot \bar{\vec{u}}_g - \vec{u}'_g \cdot \nabla p' - \bar{\gamma} p' \nabla \cdot \vec{u}'_g + \mathcal{P}' \tag{2.28}$$

The mean velocity varies in the chamber, but the average pressure is taken to be uniform. That amountsd to assuming that the average Mach number is relatively small, a restriction commonly violated in practice. Large Mach numbers cause quantitative corrections but do not introduce fundamental changes of behavior. The equations can easily be extended to cover flows at high Mach numbers.

Equations (2.27) and (2.28) contain the six dependent variables ρ', p', T' and three velocity components. A complete set is obtained by including the perturbed forms of the continuity equation (2.18) and the equation of state (2.16):

$$\frac{\partial \rho'}{\partial t} = -\bar{\vec{u}}_g \cdot \nabla \rho' - \vec{u}'_g \cdot \nabla \bar{\rho} + \vec{u}'_g + \mathcal{W}' \tag{2.29}$$

$$p' = \bar{R}(\rho'\bar{T} + \bar{\rho}T') + \bar{R}(\rho' T') \tag{2.30}$$

Many works have been based on solutions to the equations as written here. But the primary sources of information about combustion instabilities are oscillations of pressure. Thus much is gained by forming a wave equation for p'. Differentiate (2.28) with respect to time and substitute (2.27) for $\partial \vec{u}/\partial t$ to find

$$\nabla^2 p' - \frac{1}{\bar{a}^2}\frac{\partial^2 p'}{\partial t^2} = h \tag{2.31}$$

where $\bar{a}^2 = \bar{\gamma}\bar{R}\bar{T}$ is constant and

$$h = -\bar{\rho}\nabla \cdot (\bar{\vec{u}}_g \cdot \nabla \vec{u}'_g + \vec{u}'_g \cdot \nabla \bar{\vec{u}}_g) + \frac{1}{\bar{a}^2}\bar{\vec{u}}_g \cdot \nabla \frac{\partial p'}{\partial t} + \frac{\bar{\gamma}}{\bar{a}^2}\frac{\partial p'}{\partial t}\nabla \cdot \bar{\vec{u}}_g$$

$$- \nabla \cdot (\bar{\rho}\vec{u}'_g \cdot \nabla \vec{u}'_g + \rho' \frac{\partial \vec{u}'_g}{\partial t}) + \frac{1}{\bar{a}^2}\frac{\partial}{\partial t}(\vec{u}'_g \cdot \nabla p') + \frac{\bar{\gamma}}{\bar{a}^2}\frac{\partial}{\partial t}(p' \nabla \cdot \vec{u}'_g)$$

$$+\nabla \cdot \vec{\mathcal{F}}' - \frac{1}{\bar{a}^2}\frac{\partial \mathcal{P}'}{\partial t} \tag{2.32}$$

Boundary conditions set on the gradient of p' are found by taking the scalar product of the outward normal vector with equation (2.27)

$$\hat{n} \cdot \nabla p' = -f \tag{2.33}$$

with

$$\begin{aligned} f = \bar{\rho}\frac{\partial \vec{u}_g'}{\partial t} \cdot \hat{n} + \bar{\rho}(\vec{\bar{u}}_g \cdot \nabla \vec{u}_g' + \vec{u}_g' \nabla \vec{\bar{u}}_g) \cdot \hat{n} \\ + \bar{\rho}(\vec{u}_g' \cdot \nabla \vec{u}_g') \cdot \hat{n} + \rho' \frac{\partial \vec{u}_g'}{\partial t} \cdot \hat{n} - \bar{\mathcal{F}}' \cdot \hat{n} \end{aligned} \tag{2.34}$$

If all perturbations are absent, functions h and f vanish, and we recover the wave equation for the pressure in classical acoustics with the boundary condition for a rigid wall, $\hat{n} \cdot \nabla p' = 0$. We shall base our discussion on that case as the zeroth approximation. There may be circumstances when a different choice is more effective- e.g. if the average Mach number is high and the influence of the exhaust nozzle on the wave motions is substantial* - but it is good enough here to assume this limiting case. The general solution to the unperturbed problems can be written as a superposition of the normal modes $\psi_n(\vec{r})$ satisfying the equations.

$$\begin{aligned} \nabla^2 \psi_n + k_n^2 \psi_n = 0 \\ \hat{n} \cdot \nabla \psi_n = 0 \end{aligned} \tag{2.35)a, b$$

where k_n is the wavenumber, related to the frequency by

$$\omega_n = \bar{a} k_n \tag{2.36}$$

For three-dimensional problems, n stands for three indices.

The allowed values of the wavenumbers and the mode shapes ψ_n are determined entirely by the geometry of the chamber. These define the classical acoustic nodes which, when perturbed, become the most common kind of combustion instabilities. Purely longitudinal modes are represented by

$$\begin{aligned} \psi_l(z) = \cos(k_l z) \\ k_l = l\frac{\pi}{L} \end{aligned} \tag{2.37)a, b$$

and the cyclic frequencies are integral multiples of the fundamental, $f_1 = \omega_1/2\pi = \bar{a}/2h$. This is the result for a chamber closed at both ends, probably the most common case in propulsion systems.

For a cylindrical chamber of radius R, the mode shapes and natural frequencies are

$$\psi_{lmn} = \cos k_l z \; J_n(\kappa^{mn} r) \begin{Bmatrix} \cos m\theta \\ \sin m\theta \end{Bmatrix} \tag{2.38)a$$

$$k_n^2 \equiv k_{lmn}^2 = k_l^2 + \kappa_{mn}^2 \tag{2.38)b$$

and the κ_{mn} are the roots of

$$\left. \frac{dJ_n(\kappa_{mn} r)}{dr} \right|_{r=R} = 0 \tag{2.39}$$

Both (2.37)a and (2.38)a represent standing waves; representations of the corresponding traveling waves are easily constructed.

These results are the zeroth approximations to the majority of combustion instabilities observed in practical systems. Longitudinal nodes generally fall in the lower frequency range below 1-2kHz and are found in all three types of engines. The geometries of ramjet engines and augmentors cause distortions of the mode shapes from the simple result (2.37)a, but the essential idea remains. We discuss in Section 5.3.3 how mode shapes and frequencies are composed for those cases.

For a cylindrical chamber, equations (2.38)a,b are usually good approximations to the high frequency instabilities having frequencies as high as 10-20 kHz and higher. Although mixed modes with $l \neq 0$ (i.e. having axial distributions) occur, more commonly found are purely tangential or azimuthal modes ($m = 0$, $n \neq 0$); purely radial modes ($m \neq 0$, $n = 0$); or combinations when m, n are both non-zero, broadly called transverse modes. These modes are the basis for the instabilities called 'screeching'; the most common mode has been the 'first tangential' for which the classical mode shape is $J_1(\kappa_{11} r)\sin\theta$ or $J_1(\kappa_{11} r)\cos\theta$).

Equations (2.35)a,b do not contain the important case having the lowest frequencies commonly called 'chugging', a bulk or Helmholtz node. That corresponds to the solution $k_n = 0$, but exists only because there is either

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* Two examples are the analyses and experiments for a small liquid rocket by Crocco, Grey, and Harrje (1960) and for a small laboratory ramjet combustor by Laverdant, Poinsot and Candel (1986). The results are not interpreted in the fashion suggested here because the nozzle causes a large frequency shift. A more accurate choice must be made for the zeroth order approximation ψ_n.

a perturbation in the volume or, usually, at the boundary, so $\hat{n} \cdot \nabla\psi_n \neq 0$. When found in liquid rockets, those modes have often been referred to as low frequency instabilities [e.g. Harrje and Reardon (1972)]. It may happen also, as in ramjets, that a portion of the system may oscillate in a bulk node, while part exhibits a wave behavior - the frequency is of course the same throughout.

The main point here is that the classical unperturbed modes described by equations (2.35)a,b really are good approximations to a large proportion of observed combustion instabilities. The results (2.37)a,b and (2.38)a,b are the two most important special cases for actual systems but in general (2.35)a,b must be solved for the actual geometry. That is now a routine matter of numerical analysis and we may simply assume that for whatever system we wish to study, the natural node shapes $\psi_n(\vec{r})$ and frequencies ω_n are known.

The practical question is: how do the processes represented by the functions h and f, equations (2.32) and (2.34), affect the mode shapes and frequencies ? In fact, the details of the mode shapes are of less interest because the changes are relatively small. Central to the problem of linear stability are the frequencies, which become complex quantitites: the imaginary part is the growth or decay rate of the corresponding mode. For nonlinear behavior the main questions concern the conditions under which periodic limit cycles exist, what the amplitudes are, and how their characteristics are influenced by linear processes. The approximate analysis we now construct is a basis for examining both linear and nonlinear behavior. The method amounts to comparing the unperturbed problem, for which h and f vanish, with the actual problem to be analyzed ($h, f \neq 0$).

Multiply equation (2.31) by ψ_n, (2.35)a by p', subtract the results, and integrate over the chamber:

$$\int [\psi_n \nabla^2 p' - p' \nabla^2 \psi_n] dV - \frac{1}{\bar{a}^2} \int \psi_n \frac{\partial^2 p'}{\partial t^2} dV - k_n^2 \int p' \psi_n dV = \int h dV$$

Apply Green's theorem to the left hand side, substitute the boundary conditions (2.33) and (2.35)b and re-arrange the terms to give

$$-\frac{1}{\bar{a}^2} \int \psi_n \frac{\partial^2 p'}{\partial t^2} dV - k_n^2 \int \psi_n p' dV = \int \psi_n h dV + \oiint \psi_n f \, dS \tag{2.40}$$

We now use a form of the method of least residuals, essentially a form of Galerkin's method. This approach was first applied to combustion instabilities in liquid rockets by Zinn and Powell (1968, 1970). Independently, essentially the same idea was worked out for solid propellant rockets by Culick (1971, 1975, 1976), the basis for the discussion here. The unsteady pressure field is expressed as a synthesis of the normal modes $\psi_m(\vec{r})$ with time-varying amplitudes $\eta_m(t)$:

$$p'(\vec{r}, t) = \bar{p} \sum \eta_m(t) \psi_m(\vec{r}) \tag{2.41}$$

Correspondingly, the velocity field is written

$$\vec{u}'(\vec{r},\ t) = \sum \frac{\dot{\eta}_m}{\bar{\gamma} k_m^2} \nabla \psi_m(\vec{r}) \tag{2.42}$$

Term by term these series satisfy the perturbed problem, equations (2.31) and (2.33) with $h = f = 0$, providing the amplitudes satisfy

$$\ddot{\eta}_m + \omega_m^2 \eta_m = 0 \tag{2.43}$$

where $\omega_m^2 = \bar{a}^2 k_m^2$. Thus $\eta_m \sim e^{\pm i\omega_m t}$ and we recover the correct representation of natural modes.

Obviously (2.41) and (2.42) are not exact representations of the actual fields, for the correct boundary conditions are not satisfied. Equation (2.41) gives $\hat{n} \cdot \nabla p' = 0$ because all ψ_m satisfy (2.35)b. Consequently, (2.41) and (2.42) do not accurately reproduce the spatial structure of the unsteady motions near the boundary. The errors are small if h and f are small and because of the spatial averaging, the equations found for the amplitudes $\eta_m(t)$ will provide a satisfactory basis for studying real problems.

The set of normal modes can be constructed so the $\psi_n(\vec{r})$ are orthogonal:

$$\int \psi_m \psi_n dV = E_n^2 \delta_{mn} \qquad E_n^2 = \int \psi_n^2 dV \tag{2.44)a, b}$$

Substitute (2.41) in the left hand side of (2.40) and use the orthogonality property (2.44)a,b to find the system of equations for the amplitudes:

$$\frac{d^2 \eta_n}{dt^2} + \omega_n^2 \eta_n = F_n \tag{2.45}$$

with

$$F_n = -\frac{\bar{a}^2}{\bar{p} E_n^2} \{ \int h \psi_n dV + \oiint f \psi_n dS \} \tag{2.46}$$

The corresponding results for one-dimensional problems are

$$\frac{d^2 \eta_l}{dt^2} + \omega_l^2 \eta_l = F_l \tag{2.47}$$

$$F_l = -\frac{\bar{a}^2}{\bar{p}E_l^2}\{\int_0^L h_1\psi_l S_c dz + [f_1\psi_l S_c]_0^L\} \tag{2.45}$$

following the expansions for the acoustic field

$$\begin{aligned} p'(z,\ t) &= \bar{p}\sum_{j=1}^{\infty}\eta_j(t)\psi_j(z) \\ u'(z,\ t) &= \sum_{j=1}^{\infty}\frac{\dot{\eta}_j}{\bar{\gamma}k_j^2}\frac{d\psi_j(z)}{dz} \end{aligned} \tag{2.49)a, b}$$

Orthogonality is expressed as

$$\begin{aligned} \int_0^L \psi_j\psi_l S_c dz &= E_l^2\delta_{jl} \\ E_l^2 &= \int_0^L \psi_l^2 S_c dz \end{aligned} \tag{2.50)a, b}$$

Three points must be emphasized:

i.) Although the unsteady field has been synthesized of the mode shapes for standing waves, solutions to equations (2.45) and (2.47) may be used to represent standing waves with energy losses or gains, traveling waves, and discrete wave motions or pulses;

ii.) The forcing functions F_n and F_l are nonlinear functions of the pressure and velocity fluctuations, so (2.45) and (2.47) are sets of coupled nonlinear ordinary differential equations;

ii.) Many interesting one-dimensional problems involve piecewise representation of the acoustic field due to abruptly nonuniform distributions of cross-section area. The formulation (2.47)-(2.50) remains valid for those cases.

We should note also that the procedure beginning with spatial averaging and leading to (2.45) and (2.47) amounts to solving (2.31) and retaining the first term in a solution by iteration; that method is summarized in Section 4.3.

Solution to equations (2.45) or (2.47) requires first evaluation of the forces F_n or F_l. To do so the various sources must be represented, a subject discussed in the following section. The second order equations may then be solved numerically, a procedure followed by Zinn and Powell (1970, 1971). However, great advantage is gained in may problems by applying the method of time-averaging (or expansion in two time scales) to replace the second order system by an equivalent set of first order equations. This step greatly reduces the cost of routine calculations and also provides a more convenient basis for formal analysis of the general behavior. The following argument applies to both the three-dimensional and one- dimensional formulations.

Time averaging is an effective procedure for many practical problems, based on the observation that the oscillations commonly have amplitudes and phases varying slowly in time; their changes are small in one period of oscillation. Hence, the amplitudes $\eta_n(t)$ may be written in the form

$$\eta_n(t) = r_n(t)\sin(\omega_n t + \phi_n(t)) = A_n(t)\sin\omega_n t + B_n(t)\cos\omega_n t \tag{2.51}$$

The time varying phase $\phi_n(t)$ is observed as a frequency shift, the actual frequency for the perturbed node being $d/dt(\omega_n t + \phi_n) = \omega_n + \psi_n$.

We shall not cover the method for constructing the equations for $\eta_n(t)$ and $\phi_n(t)$ or the $A_n(t)$ and $B_n(t)$. The method was developed by Krylov and Bogoliubov (1947) in a form directly applicable here to the case of purely longitudinal modes when the frequencies are integral multiples of the fundamental, equation (2.37)b. When the frequencies are not so related, as for the common care of transverse modes in a cylindrical chamber, some difficulties arise which have been treated approximately by Culick (1976), and by Yang and Culick (1986) for problems of combustion instabilities in ramjet engines.

Here we quote only the results obtained for longitudinal nodes:

$$\begin{aligned} \frac{dA_n}{dt} &= \frac{1}{2\pi}\int_0^{\tau_n} F_n\cos\omega_n t' dt' \\ \frac{dB_n}{dt} &= -\frac{1}{2\pi}\int_0^{\tau_n} F_n\sin\omega_n t' dt' \end{aligned} \tag{2.52)a, b}$$

The interval of averaging has been taken equal to $\tau_n = 2\pi/\omega_n$: the equations for each mode are averaged over the period of that node. During this interval, *all* amplitudes are supposed not to change significantly. That is, the A_m, B_m appearing in F_n are taken to be constant when the integrals are performed.

If the nonlinear processes are due only to second order acoustics, then F_n has the form [Culick (1976)]

$$F_n = -\sum_{i=1}^{\infty}[D_{ni}\dot{\eta}_i + E_{\eta i}n_i] - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}[A_{nij}\dot{\eta}_i\dot{\eta}_j + B_{nij}\eta_i\eta_j] \tag{2.53}$$

The constants D_{ni}, E_{ni}, A_{nij}, B_{nij} depend on the unperturbed node shapes and frequencies. Linear processes *always* lead to the forms shown; the D_{ni}, E_{ni} are linear combinations of the various contributions, some of which are proportional to the Mach number of the mean flow. Others depend on other small parameters. For example, attenuation due to inert particles depends on the properties of the particles and on the mass fraction of condensed material. For the case of longitudinal modes, substitution of (2.53) in (2.52)a,b eventually leads to the equations

$$\begin{aligned} \frac{dA_n}{dt} &= \alpha_n A_n + \theta_n B_n + \frac{\beta_n}{2}\sum_{i=1}^{\infty}[A_i(A_{n-i} - A_{i-n} - A_{i+n}) \\ &\qquad - B_i(B_{n-i} + B_{i-n} - B_{i+n})] \\ \frac{dB_n}{dt} &= \alpha_n B_n - \theta_n A_n + \frac{\beta_n}{2}\sum_{i=1}^{\infty}[A_i(B_{n-i} + B_{i-n} - B_{i+n}) \\ &\qquad + B_i(A_{n-i} - A_{i-n} + A_{i+n})] \end{aligned} \tag{2.54)a, b$$

where

$$\begin{aligned} \alpha_n &= -\frac{1}{2}D_{nn} \\ \theta_n &= -\frac{1}{2}\frac{E_{nn}}{\omega_n} \end{aligned} \tag{2.55)a, b$$

and

$$\beta = \frac{\bar{\gamma}+1}{8\bar{\gamma}}\omega_1 \tag{2.56}$$

The modes will be linearly coupled only if very special influences are present; in general $D_{ni} = E_{ni} = 0$ if $i \neq n$. While equations (2.54)a,b are valid for any linear process, the nonlinear terms are special, representing only the gasdynamics to second order for longitudinal nodes.

With this formulation, the analytical problem for combustion instabilities has come down to constructing the functions h and f - i.e. modeling the contributions processes appearing in the sources $\mathcal{W}$, $\vec{\mathcal{F}}$ and $\mathcal{P}$, equations (2.21) - (2.23) and then carrying out the integrals defining F_n, equation (2.46). As a preliminary step, of course, the natural modes and frequencies must be computed. Because almost all of this paper will be concerned with linear behavior, a few general remarks are in order.

2.3 The Problem of Linear Stability

The nice feature of linear behavior is that the problem can be solved once for all cases. We have, of course, chosen to treat a rather special form of the general problem in order to reach such a conclusion. If, for example, the average Mach number is large, or if the action of the exhaust nozzle causes a substantial shift from the idealized condition for a rigid wall, then the functions h and f, equations (2.32) and (2.34) are not small perturbations of the classical problem. Nevertheless, the approach taken here is a simple first approximation and in any event offers what we really need - a convenient vehicle for comparing the behavior in different systems, and the possible consequences of different mechanisms.

We assume now that h and f are linear functions of the dependent variables p', $\vec{u}'$, ρ', and T'. Thus equation (2.45) is satisfied if all functions have exponential time dependence, so $p'/\bar{p} \sim e^{i\bar{a}kt}$, $\vec{u}' \sim e^{i\bar{a}kt}$, etc. and h, f, η_n have the same dependence:

$$\eta_n = \hat{\eta}_n e^{i\bar{a}kt}; \; h = \hat{h}e^{i\bar{a}kt}; \; f = \hat{f}e^{i\bar{a}kt} \tag{2.58)a, b, c$$

All amplitudes denoted by ( ) are complex functions - i.e. they are generally not in phase with one another. It is convenient to measure the phase relative to the pressure oscillation and its amplitude $\hat{\eta}_n$ is taken to be real; because coupling between modes is absent we are really treating one term in the series (2.41) and (2.42):

$$\frac{p'(\vec{r},\, t)}{\bar{p}} = \hat{\eta}_n e^{i\bar{a}kt}\psi_n(\vec{r}) \tag{2.59}$$

and

$$\vec{u}'(\vec{r},\, t) = \frac{\dot{\eta}_n}{\bar{\gamma}k_n^2}\nabla\psi_n(F) = \frac{i\bar{a}k}{\bar{\gamma}k_n^2}\hat{\eta}_n e^{i\bar{a}kt}\nabla\psi_n$$

Because the actual value of h differs from k_n by small quantitites, we have to first order

$$\frac{\vec{u}'(\vec{r},\, t)}{\bar{a}} = \frac{i}{\bar{\gamma}k_n}\hat{\eta}_n e^{i\bar{a}kt}\nabla\psi_n \tag{2.60}$$

The expressions (2.59) and (2.60) are to be substituted for p', $\vec{u}'$ when F_n is computed from its definition (2.46). Also to the order considered here, $p'/\bar{p}$ and $T'/\bar{T}$ can be approximated by their values for isentropic motions,

$$\frac{\rho'}{\bar{\rho}} \approx \frac{1}{\bar{\gamma}}\frac{p'}{\bar{p}}; \quad \frac{T'}{\bar{T}} \approx \frac{\bar{\gamma}-1}{\bar{\gamma}}\frac{p'}{\bar{p}} \tag{2.61)a,b$$

The wave number k is complex,

$$k = \frac{1}{\bar{a}}(\omega - i\alpha) \tag{2.62}$$

and with the definitions used here, $\alpha > 0$ if the mode is unstable, for then $p'/\bar{p} \sim e^{\alpha t}$. Substitution of (2.58) in (2.45) and (2.46) and cancellation of the common factor $e^{i\bar{a}kt}$ gives the formula for k^2:

$$k^2 = \frac{1}{\bar{a}^2}(\omega - i\alpha)^2 = \frac{\omega_n^2}{\bar{a}^2} + \frac{1}{\bar{p}E_n^2}\{\int \frac{\hat{h}}{\hat{\eta}_n}\psi_n \, dV + \oiint \frac{\hat{f}}{\hat{\eta}_n}\psi_n dS\} \tag{2.63}$$

Because we are treating only small perturbations, α/ω and $(\omega - \omega_n)/\omega_n$ are small. With this approximation, the real and imaginary parts of (2.63) give convenient formulas for the frequency and growth constant of the actual motions:

$$\begin{aligned} \omega &= \omega_n + \frac{\bar{a}^2}{2\omega_n \bar{p} E_n^2}\{\int \frac{\hat{h}^{(r)}}{\hat{\eta}_n}\psi_n dV + \oiint \frac{\hat{f}^{(r)}}{\hat{\eta}_n}\psi_n dS\} \\ \alpha &= \frac{-\bar{a}^2}{2\omega_n \bar{p} E_n^2}\{\int \frac{\hat{h}^{(i)}}{\hat{\eta}_n}\psi_n dV + \oiint \frac{\hat{f}^{(i)}}{\hat{\eta}_n}\psi_n dS\}\cdot \end{aligned} \tag{2.64)a,b$$

Now we relate these formulas to the results obtained in the preceding section with the method of averaging. First note that η_n, equation (2.5)a can be written:

$$\eta_n = \hat{\eta}_n e^{i\bar{a}kt} = \hat{\eta}_n e^{\omega - i\alpha}t = \hat{\eta}_n e^{\alpha t + i(\omega - \omega_n)t} e^{i\omega_n t}$$

or, if $\delta\omega_n = \omega - \omega_n$ denotes the frequency shift,

$$\eta_n = \hat{\eta}_n e^{\alpha t} e^{i(\omega_n + \delta\omega_n)t} \tag{2.65}$$

For linear behavior only, equations (2.54)a,b are

$$\begin{aligned} \frac{dA_n}{dt} &= \alpha_n A_n + \theta_n B_n \\ \frac{dB_n}{dt} &= \alpha_n B_n - \theta_n A_n \end{aligned} \tag{2.66)a,b$$

Direct substitution shows that these equations are satisfied by

$$\begin{aligned} A_n &= A_{no} e^{\alpha_n t} \cos\theta_n t \\ B_n &= -A_{no} e^{\alpha_n t} \sin\theta_n t \end{aligned} \tag{2.67)a,b$$

The assumed form (2.51) for the amplitude is therefore

$$\eta_n(t) = A_{no} e^{\alpha_n t} \sin(\omega_n + \theta_n)t \tag{2.68)a,b$$

This is exactly the imaginary part of (2.65). We conclude that the parameter $\alpha_n = -D_{nn}/2$ defined by (2.55)a is the growth constant of the nth perturbed mode equation (2.64)b; and $\theta_n = -E_{nn}/2\omega_n$ defined by (2.53)b is the frequency shift, calculated with equation (2.64)a, $\theta_n = \delta\omega_n = \omega - \omega_n$.

All of the above can be summarized in the following steps, a receipe for assessing the consequences of an analysis of linear behavior.

i.) Construct the contributions to the functions h and f by applying the definitions of the source functions $\mathcal{W}$, $\vec{\mathcal{F}}$ and $\mathcal{P}$ and extracting their linear forms;

ii.) Substitute the acoustic approximations $p' = \bar{p}\eta_n\psi_n$ and $\vec{u}' = \dot{\eta}_n \nabla\psi_n/\bar{\gamma}k_n^2$ in f and h; if required, the formulas (2.61)a,b are used for the density and temperature fluctuations;

ii.) Compute F_n according to its definition (2.46). If second derivatives of the amplitudes should arise, they are to be replaced by the zeroth order approximation, $\ddot{\eta}_n \approx -\omega_n^2 n_n$.

v.) Then F_n will have the form (2.53) with $D_{ni} = E_{ni} = 0$ for $i \neq n$. The values for the growth constant and the frequency shift for the nth mode can be found immediately from the coefficients of $\dot{\eta}_n$ and η_n:

$$\alpha_n = -\frac{1}{2}D_{nn}; \; \theta_n \equiv \omega - \omega_n = -\frac{1}{2}\frac{E_{nn}}{\omega_n} \tag{2.69}$$

In this way the primary information given by a linear analysis can be found in a straightforward manner. It's true that due to approximations made here, there may be quantitative inaccuracies greater than those accompanying a more careful computation of linear behavior. The great advantage of the procedure described above is that comparison of proposed mechanisms can readily be made.

2.4 Evaluating the Linear Contributions to the Frequency Shift and the Growth Constant

In this review we concentrate on the linear behavior. We shall find in Section 3 that proposed mechanisms for instabilities can be properly assessed only within the complete acoustical analysis. It is therefore essential to work out the details for the most general possible forms of the inhomogeneous forcing functions represented by h and f.

Some rearrangement leads to the following result for the linear parts of h and f:

$$\begin{aligned}\int \hat{h}\psi_n dV + \oiint \hat{f}\psi_n dS = \bar{\rho}k_n^2 \int (\vec{\bar{u}}_g \cdot \hat{\vec{u}}_g)\psi_n dV - \bar{\rho}\int (\vec{\bar{u}}_g \times \nabla \times \hat{\vec{u}}_g)\cdot \nabla\psi_n dV \\ + i\frac{k_n}{\bar{a}}\int \psi_n[\hat{\vec{u}}_g \cdot \nabla\hat{p} + \gamma\hat{p}\nabla\cdot\vec{\bar{u}}_g]dV \\ - i\frac{k_n}{\bar{a}}\int \psi_n \hat{\mathcal{P}} dV - \int \hat{\vec{\mathcal{F}}}\cdot\nabla\psi_n dV \\ + i\bar{\rho}\bar{a}k_n \oiint \psi_n \hat{\vec{u}}_g \cdot \hat{n} dS\end{aligned}$$

We assume throughout that all terms in h and f are small (e.g. many are of order of the mean flow Mach number) and since k differs from k_n by terms of that order, we replace k by k_n, thereby consistently neglecting terms of order square in small quantities. For the same reason, we replace $\hat{p}$ and $\hat{\vec{u}}$ by their unperturbed values,

$$\hat{p} = \bar{p}\hat{\eta}_n\psi_n;\ \hat{\vec{u}} = \frac{i\bar{a}}{\bar{\gamma}k_n}\hat{\eta}_n\nabla\psi_n \tag{2.70}$$

Taking the real and imaginary parts of the integrals and substituting in (2.64)a,b gives the basic formulas for studying linear stability:

$$\begin{aligned}\omega = \omega_n + \frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\{\frac{k_n}{\bar{a}}\int \psi_n\frac{\hat{\mathcal{P}}^{(i)}}{\hat{\eta}_n}dV - \int \frac{1}{\hat{\eta}_n}\hat{\vec{\mathcal{F}}}^{(r)}\cdot\nabla\psi_n dV \\ - \bar{\rho}\bar{a}k_n \oiint \frac{1}{\hat{\eta}_n}(\hat{\vec{u}}_g^{(i)}\cdot\hat{n})\psi_n dS\}\end{aligned} \tag{2.71}$$

$$\begin{aligned}\alpha = -\frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\{\frac{k_n}{\bar{a}}\int \psi_n\frac{\hat{\mathcal{P}}^{(r)}}{\hat{\eta}_n}dV + \int \hat{\vec{\mathcal{F}}}^{(i)}\cdot\nabla\psi_n dV \\ - (\bar{\gamma}-1)\frac{k_n}{\bar{a}}\bar{p}\int \psi_n^2(\nabla\cdot\vec{\bar{u}}_g)dV \\ + \bar{\rho}\bar{a}k_n \oiint [\frac{1}{\hat{\eta}_n}(\hat{\vec{u}}_g^{(r)}\cdot\hat{n})\psi_n + \frac{1}{\bar{\gamma}}\psi_n^2(\vec{\bar{u}}_g\cdot\hat{n})]dS\}\end{aligned} \tag{2.72}$$

The term $\hat{\vec{u}}\cdot\hat{n}$ in the surface integral arises from the contribution $\rho(\partial\vec{u}/\partial t)\cdot\hat{n}$ in f. Here $\vec{u}$ is not replaced by its unperturbed value ($\vec{u}\cdot\hat{n} = 0$) at the surface because in general the boundary is not rigid. It has long been a convention in classical acoustics to replace fluctuations of the velocity at a boundary by admittance functions. That has become common practice in analysis of combustion instabilities with account taken of the mean flow through the boundary, as at a burning surface is solid propellant rockets, and at the exhaust nozzle generally. The admittance function A_N at the nozzle entrance is defined as

$$A_N = \frac{1}{\bar{a}}\frac{\hat{\vec{u}}_g\cdot\hat{n}}{\hat{p}/\bar{\gamma}\bar{p}} = \frac{\bar{\gamma}}{\bar{a}}\frac{\hat{\vec{u}}_g\cdot\hat{n}}{\hat{\eta}_n\psi_n} \tag{2.73}$$

Thus the combination in the surface integral is

$$\frac{1}{\bar{a}}[\bar{\gamma}\frac{\hat{\vec{u}}_g\cdot\hat{n}}{\hat{\eta}_n\psi_n} + \vec{\bar{u}}_g\cdot\hat{n}] = A_N + \bar{M}_N \tag{2.74}$$

All types of systems treated in this paper use choked exhaust nozzles. Owing to the large gradients of mean flow properties in the convergent section, such a nozzle acts as an efficient reflector of acoustic disturbances under most conditions - but not always. Tsien (1952) first analyzed the unsteady behavior in nozzles, in a paper that set the essential basis for all subsequent calculations. Crocco (1953) and Crocco and Cheng (1956) elaborated on Tsien's treatment of one-dimensional (planar) wave motions. Some chief results of the theory were confirmed with tests performed by Crocco, Grey and Monti (1961). Culick (1961) reported limited results for three- dimensional motions but the most thorough treatment of the subject was given by Crocco and Sirignano (1967). The latter work is particularly useful because fluctuations of vorticity and entropy are accommodated. Some consequences of entropy disturbances incident in a nozzle were later investigated by Marble (1973) and Marble and Candel (1977); that is an issue which arises in connection with analysis of convective waves as a possible mechanism for instabilities. Some aspects of nonlinear behavior of a nozzle have been treated by Crocco and Sirignano (1966) and by Zinn and Crocco (1968a, 1968b); they will not be pursued here.

The exhaust nozzle provides a significant loss of acoustic energy particularly for longitudinal oscillations. Its effects are less clear for three-dimensional motions, particularly when the nozzle is submerged, a common feature in solid propellant rockets. There has therefore been considerable interest in measuring the nozzle admittance, beginning in the late 1960's [Buffum et al (1967); Culick and Dehority (1969)]. The most elaborate and effective experimental work has been done using a large impedance tube with flow, a method first used by Zinn et al (1973). For longitudinal modes, for which the wavelength is usually much greater than the nozzle length, the "short nozzle approximation" is quite accurate, $A_N = (\bar{\gamma}-1)\bar{M}_N/2$. There is no experimental data sufficiently accurate to prove or disprove the theoretical prediction that under some conditions transverse oscillations may be amplified by the exhaust nozzle: that is, a small amount of energy is transferred from the mean to the unsteady flow, not an unreasonable possibility (cf. whistles and sirens). For most purposes here the action of the nozzle may be viewed as causing attenuation of acoustic waves with a slight increase of axial wavelength.

Similarly, an admittance function can be introduced to represent the effects of fluctuations of the liquid fuel and oxidizer at the injector. There is no average flow of gas in that case, so we set $\vec{u}\cdot\hat{n}=0$ and define

$$A_I = -\frac{\bar{\gamma}}{\bar{a}}\frac{\hat{\vec{u}}_g\cdot\hat{n}}{\hat{\eta}_n\psi_n}$$

The minus sign is appended because the normal vector $\hat{n}$ is positive outward but the velocity is positive inward at injector ports. Equations (2.71) and (2.72) are now

$$\omega = \omega_n + \frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\{\frac{k_n}{\bar{a}}\int\psi_n\frac{\hat{\mathcal{P}}^{(r)}}{\hat{\eta}_n}dV - \int\frac{1}{\hat{\eta}_n}\hat{\vec{\mathcal{F}}}^{(r)}\cdot\nabla\psi_n dV + k_n\bar{p}\iint A_I^{(i)}\psi_n^2 dS - k_n\bar{p}\iint A_N^{(i)}\psi_n^2 dS\} \tag{2.75}$$

$$\alpha = \frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\{\frac{k_n}{\bar{a}}\int\psi_n\frac{\hat{\mathcal{P}}^{(i)}}{\hat{\eta}_n}dV - \int\frac{1}{\hat{\eta}_n}\hat{\vec{\mathcal{F}}}^{(r)}\cdot\nabla\psi_n dV - (\bar{\gamma}-1)\frac{k_n}{\bar{a}}\bar{p}\int\psi_n^2(\nabla\cdot\bar{\vec{u}}_g)dV + k_n\bar{p}\iint A_I^{(r)}\psi_n^2 dS - k_n\bar{p}\iint(A_N^{(r)}+\bar{M}_N)\psi_n^2 dS\} \tag{2.76}$$

The result (2.75) has not been particularly useful in predictions of linear stability because the frequency shifts $\omega-\omega_n$ are usually so small as not to be noticeable. As we noted earlier, predictions of the frequency are not a true test of a theory. However, 'prediction' means that all contributions on the right hand sides of (2.74) and (2.75) can be calculated. It has been common practice in studies of instabilities in liquid rockets to use both of these equations in the stability boundary ($\alpha=0$) to compute the real and imaginary parts of the function representing unsteady combustion. Hence the matter of prediction is not an issue. We discuss the procedure at greater length in Sections 4 and 5.

First we need to make explicit the contributions to the source functions $\vec{\mathcal{F}}$ and $\vec{\mathcal{P}}$, derived from the definitions (2.22) and (2.23). We make two assumptions to simplify the formulas:

i.) Viscous stresses and heat conduction are negligible within the volume of the chamber. This is true except for sharp fronted waves. Otherwise, viscous effects are important at boundaries, as for gas/particle interactions or at inert walls. Those losses are not included here but are easily taken into account.

ii.) In steady state, the liquid droplets are in equilibrium. Thus $\delta\bar{\vec{u}}_l=0$, an approximation that is valid only after the injected liquid has formed a spray moving with the chamber gases. Thus this approximation is not good over most of the region near the injector. As a result, losses are underestimated.

With these two assumptions, we find to first order in small quantities*

$$\vec{\mathcal{F}}' = \delta\vec{F}_l' + \delta\vec{u}_l'\bar{w}_l \tag{2.77}$$

$$\mathcal{P}' = \frac{\bar{R}}{\bar{C}_v}[Q' + \delta Q_l' + (\bar{h}_l - \bar{e})w_l' + (h_l' - e')\bar{w}_l] \tag{2.78}$$

Also, we replace $\nabla\cdot\vec{u}$ in the third term of (2.76) by using the averaged form of the continuity equation (2.1) with $\bar{\rho}_g$ approximately constant: then $\nabla\cdot\bar{\vec{u}}_g = \bar{w}_l/\bar{\rho}_g$. Equation (2.76) is now

$$\alpha = \frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\{\frac{k_n}{\bar{a}}\frac{\bar{R}}{\bar{C}_v}\int\frac{\psi_n}{\hat{\eta}_n}[(\hat{Q}+\delta\hat{Q}_l)+(\bar{h}_l-\bar{e})w_l'+(h_l'-e')\bar{w}_l]^{(r)}dV$$

unsteady energy addition

$$+\int\frac{1}{\hat{\eta}_n}[\delta\hat{\vec{F}}_l + \delta\hat{\vec{u}}_l]^{(i)}\cdot\nabla\psi_n dV$$

losses due to gas/liquid interactions

$$-(\bar{\gamma}-1)\frac{k_n}{\bar{a}}\frac{\bar{p}}{\bar{\rho}_g}\int\psi_n^2\bar{w}_l dV \tag{2.79}$$

loss associated with vaporization

* To simplify, we have also dropped arbitrarily a term $\bar{C}_v\bar{T}_g\nabla\cdot(\bar{\rho}_l\delta\vec{u}_l')$ in $\mathcal{P}$ representing a fluctuation of energy due to expansion (or contraction) of the droplet cloud.

$$+k_n\bar{p}\iint A_I^{(r)}\psi_n^2 dS - k_n\bar{p}\iint(A_N^{(r)}+\bar{M}_N)\psi_n^2 dS\}$$

injector exhaust nozzle

This result seems to contain all contributions considered in previous works and will therefore serve as the basis for discussion of mechanisms of instabilities in Section 3.

2.5 Rayleigh's Criterion

As a result of his studies of acoustic waves generated and sustained by heat addition, Lord Rayleigh (1878, 1945, Vol. II, p. 226) stated his famous criterion:

> "If heat be communicated to, and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged."

Probably no other principle has been so widely invoked in studies of combustion instabilities [e.g. Heidmann and Wieber (1966a,b); Putnam(1971); Harrje and Reardon(1972); Zinn(1980)]. Even some success has been achieved. But it seems that in all cases when Rayleigh's criterion has been applied to situations involving exchange of mass as well as heat, the conclusions reached have been incomplete and in some cases seriously misleading. The reason is that not all sources have been properly accounted for - that is, pieces of $\vec{\mathcal{F}}$ and $\mathcal{P}$, equations (2.22) and (2.23), have been omitted. Rayleigh never considered the possible influences of a condensed phase.

The approximate analysis worked out in Section 2.3 provides a convenient basis for deriving an explicit form of Rayleigh's criterion including all contributions [see Culick (1987) for the form accounting only for heat addition and applicable to nonlinear motions]. We begin with equation (2.45) for the time-dependent amplitude of the n^{th} mode,

$$\frac{d^2\eta_n}{dt^2}+\omega_n^2\eta_n = F_n$$

This is the equation for a driven oscillator whose "energy" is $\mathcal{E}_n = (\dot{\eta}_n^2+\omega_n^2\eta_n^2)/2$ which, within a constant multiplier, is the mechanical energy associated with the n^{th} acoustic mode. Energy flows to the mode at the rate $F_n\dot{\eta}_n$ and at time t the rate of change of energy in one period $\tau_n = 2\pi/\omega_n$ of the motion is

$$\Delta\mathcal{E}_n(t) = \int_t^{t+\tau_n} F_n\dot{\eta}_n dt' \tag{2.80}$$

Traditional use of Rayleigh's criterion has involved only heat addition, a strictly literal use of the statement quoted above. Correct application to liquid-fueled systems requires consideration of all contributions associated with heat addition and exchange of mass, momentum and energy between the gas and liquid phases. With only those terms retained, $\vec{\mathcal{F}}'$ and $\mathcal{P}'$ are given by (2.77) and (2.78) so h and f, equations (2.32) and (2.34), are*:

$$h = \nabla\cdot[\delta\vec{F}_l+\delta\vec{u}_l'\bar{w}_l] - \frac{\bar{R}/\bar{C}_v}{\bar{a}^2}\frac{\partial}{\partial t}[Q'+\delta Q_l'+(\bar{h}_l-\bar{e})w_l'+(h_l'-e'-\frac{p'}{\bar{\rho}_g})\bar{w}_l] \tag{2.81}$$

$$f = -[\delta\vec{F}_l'+\delta\vec{u}_l'\bar{w}_l]\cdot\hat{n} \tag{2.82}$$

Then F_n, equation (2.46), is

$$F_n = \frac{\bar{R}/\bar{C}_v}{\bar{p}E_n^2}\int\frac{\partial Q_R}{\partial t}\psi_n dV + \frac{\bar{a}^2}{\bar{p}E_n^2}\int\vec{\mathcal{F}}_R\cdot\nabla\psi_n dV \tag{2.83}$$

where

$$\begin{aligned} Q_R &= Q'+\delta Q_l'+(\bar{h}_l-\bar{e})w_l'+(h_l'-e'-\frac{p'}{\bar{\rho}_g})\bar{w}_l \\ \vec{\mathcal{F}}_R &= \delta\vec{F}_l'+\delta\vec{u}_l'\bar{w}_l \end{aligned} \tag{2.84a,b}$$

To find an explicit expression of Rayleigh's criterion, substitute (2.83) in (2.80) and integrate the first term by parts:

$$\Delta\mathcal{E}_n = \frac{\bar{R}/\bar{C}_v}{\bar{p}E_n^2}\int\{[Q_R\dot{\eta}_n]_t^{t+\tau_n} - \int_t^{t+\tau_n}\ddot{\eta}_n Q_R dt'\}\psi_n dV + \frac{\bar{a}^2}{\bar{p}E_n^2}\int\int_t^{t+\tau_n}\dot{\eta}_n\vec{\mathcal{F}}_R\cdot\nabla\psi_n dV$$

If the system is executing a steady oscillation, then $Q_R\,\dot{\eta}_n$ is strictly periodic and the term in square brackets vanishes. In the second term we may set $\ddot{\eta}_n = -\omega_n^2\eta_n$ as remarked earlier. With $\bar{R}/\bar{C}_v = \bar{\gamma}-1$, the energy added to the n^{th} mode during one period of oscillation is

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* This formula includes the terms that led to the part of equation (2.79) identified as a "loss associated with vaporization".

$$\Delta\mathcal{E}_n = \frac{\omega_n^2}{\bar{p}E_n^2}\int dV \int_t^{t+\tau_n} [(\bar{\gamma}-1)\frac{p_n'}{\bar{p}}Q_R + \bar{\gamma}\vec{\mathcal{F}}_R\cdot\vec{u}_n']dt \tag{2.85}$$

where we have used the definition of the acoustical pressure and velocity: $\eta_n\psi_n = p_n'/\bar{p}_n$, $\vec{u}_n' = \nabla\psi_n/\bar{\gamma}k_n^2$.

Equation (2.85) is an extended form of Rayleigh's criterion. The first term accounts for heat addition with and without charge of phase; the second term represents the effects of momentum transfer between the two phases. If $\Delta\mathcal{E}_n$ is positive, then the amplitude of the mode in questions grows providing the contributions accounted for here exceed the energy losses associated with the injector and the nozzle, the last two terms of equation (2.79).

The derivation of equation (2.85) has involved only some of the terms in h and f, a restriction imposed here only because previous works have emphasized the importance of heat and mass addition. It is evident from the derivation that all of the perturbations can be included. This will produce an extended form of Rayleigh's criterion applicable to all circumstances encompassed by the original conservation equations.

We need simply to use the entire definition (2.46) of F_n in equation (2.80) for $\Delta\mathcal{E}_n$. The contribution from the volume integral over $\psi_n^2\bar{w}_l$ in (2.79) arises from the term in F_n,

$$-\frac{\bar{R}/\bar{C}_v}{\bar{p}E_n^2}\int \frac{\partial p'}{\partial t}\frac{\bar{w}_l}{\bar{p}_g}\psi_l\, dV\ . \tag{2.86}$$

When this is placed in the definition of $\Delta\mathcal{E}_n$, the result can be re-written

$$-\frac{\bar{R}/\bar{C}_v}{\bar{p}E_n^2}\int_t^{t+\tau_n} dt' \int \frac{\bar{w}_l}{\bar{p}_g}\frac{\partial p'}{\partial t}(\dot{\eta}_n\psi_l)dV$$

The combustion $\dot{\eta}_n\psi_n = \partial(p'/\bar{p})/dt$ so this contribution to $\Delta\mathcal{E}_n$ is

$$-\frac{\bar{R}/\bar{C}_v}{E_n^2}\int_t^{t+\tau_n} dt' \int \frac{\bar{w}_l}{\bar{p}_g}\left[\frac{\partial}{\partial t}\left(\frac{p_n'}{\bar{p}}\right)\right]^2 dV \tag{2.87}$$

Similarly, the surface integral involving the real part* of the admittance in F_n is

$$\frac{\bar{a}^2}{\bar{p}E_n^2}\iint\left(\bar{\rho}\frac{\partial\vec{u}_g'}{\partial t}\cdot\hat{n}\right)^{(r)}\psi_n\, dS = \frac{\bar{a}^2}{\bar{p}E_n^2}\iint \bar{\rho}\bar{a}A^{(r)}\frac{\partial}{\partial t}\left(\frac{p'}{\bar{\gamma}\bar{p}}\right)\psi_n\, dS$$

where the definition (2.72) has been used. After substitution in the definition of $\Delta\mathcal{E}_n$, this term gives

$$\frac{\bar{a}^2}{\bar{p}E_n^2}\int_t^{t+\tau_n} dt' \iint \bar{\rho}\bar{a}A^{(r)}\frac{\partial}{\partial t}\left(\frac{p'}{\bar{\gamma}\bar{p}}\right)(\dot{\eta}_n\psi_n)\, dS\ .$$

Again $\dot{\eta}_n\psi_n$ is replaced by $\partial(p'/\bar{p}))\partial t$ and we have the contribution to $\Delta\mathcal{E}_n$

$$\frac{\bar{p}\bar{a}}{\rho_g E_n^2}\int dt' \iint A^{(r)}\left[\frac{\partial}{\partial t}\left(\frac{p_n'}{\bar{p}}\right)\right]^2 dS\ . \tag{2.88}$$

This result can be applied to both the injector and the exhaust nozzle, the two contributions appearing in equation (2.79).

With these additional contributions, the extended form of Rayleigh's criterion is now

$$\begin{aligned}\Delta\mathcal{E}_n = &\frac{\omega_n^2}{\bar{p}E_n^2}\int dV \int_t^{t+\tau_n} dt'\left[(\bar{\gamma}-1)Q_R\frac{p_n'}{\bar{p}} + \bar{\gamma}\vec{\mathcal{F}}_R'\cdot\vec{u}_n\right]\\ &-\frac{1}{E_n^2}\int dV\int_t^{t+\tau_n} dt'(\bar{\gamma}-1)\frac{\bar{w}_L}{\bar{p}_g}\left[\frac{\partial}{\partial t}\left(\frac{p_n'}{\bar{p}}\right)\right]^2\\ &+\frac{\bar{a}}{E_N^2}\iint dS\int_t^{t+\tau_n} dt'\frac{\bar{p}}{\bar{\rho}_g}A_I^{(r)}\left[\frac{\partial}{\partial t}\left(\frac{p_n'}{\bar{p}}\right)\right]^2\\ &-\frac{\bar{a}}{E_N^2}\iint dS\int_t^{t+\tau_n} dt'\frac{\bar{p}}{\bar{\rho}_g}(A_N^{(r)}+\bar{M}_N)\left[\frac{\partial}{\partial t}\left(\frac{p_n'}{\bar{p}}\right)\right]^2 .\end{aligned}$$

This formula for the energy charge* in one period of oscillation contains all the contributions included in the conservation equations (2.8)–(2.26). The statement of the crtierion is now: if $\Delta\mathcal{E}_n$ calculated with (2.89) is positive, then the oscillation is unstable. As the calculations in the following section confirm, this result is equivalent to the condition for stability based on the growth constant.

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* Why it is legimate to use the real part follows from the remarks in the next section.

* Actually $\Delta\mathcal{E}_n$ is only proportional to the energy change; because of its definition (2.80), $\Delta\mathcal{E}_n$ has units sec^{-2}

2.6 The Connection Between Rayleigh's Criterion and the Growth Constant

The growth constant, defined as the imaginary part of the frequency, is the exponential rate of growth of the pressure amplitude. It is therefore related to the rate of change of acoustic energy by a formula easily established in the following paragraphs. Further, because Rayleigh's criterion has to do with the time evolution of acoustic energy in the system, there must be a relation between that principle and the growth constant, a result we establish in this section.

Equation (2.45) for the amplitude of the n^{th} mode is the basis for computing both the growth constant α and the change of energy $\Delta\mathcal{E}_n$, and therefore provides the connection between these quantities.

We are concerned here only linear behavior; thus $\eta_n(t)$ is proportional to $e^{i\bar{a}kt}$ and with no loss of generality we can set the constant of proportionality equal to unity*, so $\eta_n = e^{i\bar{a}kt}$, a choice that simplifies the following calculations. Substitute in equation (2.45) to find the formula for k^2:

$$\begin{aligned} k^2 &= \frac{\omega_n^2}{\bar{a}^2} - \frac{F_n}{\bar{a}^2}e^{-i\bar{a}kt} \\ &= \frac{\omega_n^2}{\bar{a}^2} - \frac{\hat{F}_n}{\bar{a}^2} \end{aligned} \tag{2.90}$$

where as usual we write

$$F_n = \hat{F}_n e^{i\bar{a}kt} \ .$$

The real and imaginary parts of (2.90) give equation (2.64)a,b, here expressed in terms of F_n:

$$\begin{aligned} \omega^2 &= \omega_n^2 - \hat{F}_n^{(r)} \\ \alpha &= \frac{1}{2\omega_n}F_n^{(i)} \end{aligned} \tag{2.91)a, b$$

Now integrate the right hand side of the definition (2.80) of $\Delta\mathcal{E}_n$:

$$\begin{aligned} \Delta\mathcal{E}_n &= \int_t^{t+\tau_n} F_n\dot{\eta}_n dt' \\ &= \int_t^{t+\tau_n}\left[\frac{d}{dt}(F_n\eta_n) - \eta_n\frac{dF_n}{dt'}\right]dt' \\ &= [F_n\eta_n]_t^{t+\tau_n} - \int_t^{t+\tau_n}\eta_n\frac{dF_n}{dt'}dt' \ . \end{aligned}$$

The first term vanishes because we consider steady oscillations (at most $F_n\eta_n$ changes by a small amount in one period) and

$$\Delta\mathcal{E}_n = -\int_t^{t+\tau_n}\eta_n\frac{dF_n}{dt'}dt' \ . \tag{2.92}$$

Let ϕ be the phase of F_n (measured relative to the pressure oscillation) and

$$F_n = |F_n|e^{i(\bar{a}kt+\phi)} \ . \tag{2.93}$$

Real quantities must be used in the right hand side of (2.92); we choose the real parts of complex quantities, so $\eta_n = \cos(\bar{a}kt)$, $F_n = |F_n|\cos(\bar{a}kt+\phi)$. hence (2.92) is

$$\begin{aligned} \Delta\mathcal{E}_n &= -\int_t^{t+\tau_n}\cos(\bar{a}kt')\{-\bar{a}k|F_n|\sin(\bar{a}kt'+\phi)\}dt' \\ &= \bar{a}k\int_t^{t+\tau_n}|F_n|\cos(\bar{a}kt')\{\cos(\bar{a}kt')\sin\phi + \sin(\bar{a}kt')\cos\phi\}dt \ . \end{aligned}$$

Once again we apply the condition that F_n is a small perturbation so we can set $\bar{a}k \approx \omega_n$ and assume that $|F_n|$ and ϕ are nearly constant during one period of the motion. Hence the integrals can be done; only the first integral is non-zero and we find

$$\Delta\mathcal{E}_n = \omega_n|F_n|\sin\phi\frac{\tau_n}{2} \ .$$

With $\omega_n\tau_n = 2\pi$, we have the result

$$\Delta\mathcal{E}_n = \pi F_n^{(i)} \ . \tag{2.94}$$

Finally, comparison of (2.91)b and (2.94) gives the desired relation:

$$\Delta\mathcal{E}_n = 2\pi\omega_n\alpha \ . \tag{2.95}$$

This result establishes quite generally the connection suggested by the explicit forms (2.79) and (2.89). We should emphasize that as he originally formulated the statement known as his criterion, Rayleigh considered only the matter of heat addition to the acoustic field. It is perhaps stretching the point to account for all energy losses and gains to produce equation (2.95). We have done so here to clarify some abuses that have appeared in the literature; an example is discussed in Section 3.2

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* This is usually a matter of normalization; see also remarks in Section 4.3 after equation (4.10).

3. MECHANISMS OF COMBUSTION INSTABILITIES

Both Rayleigh's criterion and the growth constant provide means of analyzing the stability of unsteady motions. Properly interpreted, Rayleigh's criterion can be applied to nonlinear behavior, but the growth constant is strictly defined for linear instabilities. The results expressed as equations (2.79) and (2.90) are largely formal. Their physical content derives from the assumptions forming the basis for the conservation equations (2.18) - (2.26). To proceed further it is necessary to provide more explicit representations, or models, of the processes dominating the behavior in a combustion chamber.

Apart from differences in geometry, the primary distinctions between different propulsion systems are due to the internal physical processes. Some are independent of geometry, but others - such as flow separation - are not. In this section, we discuss the four main ideas that have been proposed for explaining combustion instabilities in liquid-fueled systems. Although all have been prompted by experimental results, they differ greatly in the extent to which they have been developed. We begin with the most widely used idea, the "time lag model".

3.1 Interpretation With a Time Lag

The basic idea is simple, and quite general, related to the familiar experience that a forced oscillating system will gain energy if the force has a component in phase with the velocity of the point of application. Stability of dynamical systems characterized in some sense by a phase or time lag had been studied prior to the concern with combustion instabilities [for example, see Callender et al (1936) and Minorsky (1942)]. In 1941, Summerfield (1951) had observed low frequency 'chugging" during firings of a liquid rocket. Discussion with von Karman led to the idea of a time lag as a possible explanation. Gunder and Friant (1950) independently introduced a time lag in their analysis of chugging, but it was Summerfield's paper and subsequent work at Princeton by Crocco that established the time lag theory in the firm widely used.

The essential idea in all applications of the time lag is that a finite interval - the lag - exists between the time when an element of propellent enters the chamber and the time when it burns and releases its chemical energy. Such a time lag must exist in steady operation, and, since combustion is distributed throughout the chamber, there is no unique value. Evidently a complete analysis of injection and subsequent processes could then be interpreted in terms of a time lag; results exist only for approximate analyses.

Now suppose that at time t the pressure in the chamber suddenly decreases, causing an increase in the flow of propellant through the injector. The increased mass burns at some later time $t + \tau$, where τ is the time lag. If the pressure is increasing when the added mass burns, the energy released will tend to encourage the pressure increase, a destabilizing tendency. This elementary process is easily interpreted with Rayleigh's criterion. Assume that the pressure varies sinusoidally,

$$p' = \hat{p} \sin \omega t \tag{3.1}$$

and that the energy occurs later with constant time lag τ,

$$Q' = \hat{Q} \sin \omega(t - \tau) \tag{3.2}$$

Integration of the product $p'Q'$ over one period $2\pi/\omega$ gives

$$\int_t^{t+2\pi/\omega} p'Q'dt' = \hat{p}\hat{Q} \int_t^{t+2\pi/\omega} \sin \omega t' \sin(\omega t' - \omega\tau)dt' = \hat{p}\hat{Q}\frac{\pi}{\omega} \cos \omega\tau \tag{3.3}$$

Thus, according to Rayleigh's criterion (2.7), we expect that net energy is added to the oscillation if $\cos \omega\tau$ is positive, so the time lag must lie in the ranges

$$0 < \tau < \frac{\pi}{2\omega}, \qquad \frac{3\pi}{2\omega} < \tau < \frac{5\pi}{2\omega}, \qquad ....\text{etc.} \tag{3.4}$$

Suppose that the system is unstable and the τ lies in the range $3\pi/2\omega < \tau < 5\pi/\omega$. Then the strategy for fixing the problem is based on modifying the system so that τ is either increased or decreased, placing its value outside the range for instability.

Because the processes subsequent to injection are surely dependent on the flow variables, pressure, temperature, velocity, ..., it is unrealistic to assume that the time-lag is constant. The most widely used form of the representation with a time lag are dominated by its dependence on pressure.

Figure 3.1, taken from Dipprey (1972), is a sketch illustrating the behavior for a sinusoidal pressure oscillation imposed on the system. The total time delay to burning is supposed in this case to be composed of two parts, due to the propellant feed system, and the combustion delay (injection, atomization, vaporization, mixing, and chemical kinetics). It is the second part that is sensitive to the flow conditions in the chamber.

Let $\dot{m}$ denote the mass flow (mass/sec.) of propellant. At this point we are not concerned with details and we need not distinguish between fuel and oxidizer. The arguments based on the idea of a time lag are directed mainly to constructing a representation of the mass source term $w_l(mass/vol. - sec.)$ in the continuity equation (2.1). Thus the result is intended to express the rate of conversion of liquid to gas in a volume element of the chamber. These is no consideration of combustion processes; the usual assumption is that combustion occurs instantaneously, a view that determines how the time lag model ought to be incorporated in the equations.

Let ($\vec{r}$, dV) denote the volume element at position $\vec{r}$ in the chamber and let (t, dt) denote the small time interval dt at time dt. The idea is that the amount of liquid $w_l dVdt$ converted to gas in the element (r, dV)

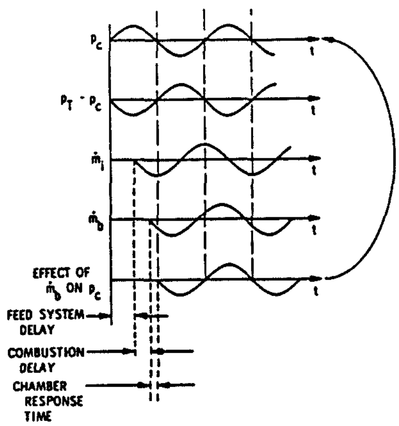


FIGURE 3.1

in the interval (t, dt) was injected as $\delta\dot{m}(t-\tau)d(t-\tau)$ at the time $t-\tau$ in the interval $d(t-\tau)$. Hence by conservation of mass,

$$w_l dV dt = \delta\dot{m}_i(t-\tau)d(t-\tau) \tag{3.5}$$

According to earlier remarks, the time lag is supposed to be variable, and can be written as the sum of average and fluctuating values, $\tau = \bar{\tau} + \tau'$. In steady-state operation, (3.5) is

$$\bar{w}_l dV dt = \delta\dot{m}_i(t-\bar{\tau})d(t-\bar{\tau}) = \delta\dot{m}_i(t-\bar{\tau})dt \tag{3.6}$$

Expanding $\delta\dot{m}(t-\tau)$ in Taylor series for use in (3.5) we have

$$\delta\dot{m}_i(t-\tau) = \delta\dot{m}_i(t-\bar{\tau}) + \tau'[\frac{d}{dt}\delta\dot{m}_i(t)]_{t-\tau} + \cdots \tag{3.7}$$

The second term is non-zero if the injected mass flow is not constant. There are many situations (notably for low frequency instabilities) for which variations are important. But for the instabilities at high frequencies, variations of the propellant flow are generally not important. Hence we ignore the second term in (3.7) and substitute (3.6) in (3.5) to find

$$w_l(\vec{r},\ t) = \bar{w}_l(1 - \frac{d\tau}{dt}) \tag{3.8}$$

The variations of the local converstion of liquid to gas depend in this simple fashion on the time - dependence of the time lag. Note that τ may in general depend on position: the reasoning here is quite widely applicable.

The difficult problem is of course to predict τ - in fact it has never been done. Crocco introduced the idea that the time lag is the period required for the processes leading to vaporization to be completed. He assumed that this integrated effect can be represented by an integral over the time lag of some function f of the variables affecting the processes

$$\int_{t-\tau}^{t} f\{p,\ T,\ \vec{u},\ \vec{u}_l,\ \ldots\}dt' = E \tag{3.9}$$

The constant E is supposed to be a measure of the level to which the integrated effects must reach in order for vaporization to occur. Almost all applications of the time lag model rest on the assumption that the time lag is sensitive only to the pressure. The function f may then be expanded about its value at the mean pressure,

$$f(p) = f(\bar{p}) + p'\left.\frac{df}{dp}\right|_{\bar{p}} = f(\bar{p})[1 + p'\frac{1}{f(\bar{p})}\left.\frac{df}{dp}\right|_{\bar{p}}]$$

If $f = cp^n$ then $df/dp = ncp^{n-1}$ and $(df/dp)/f(p) = n/p$. The *interaction index* n is defined as

$$n = \frac{\bar{p}}{f(\bar{p})}\left.\frac{df}{dp}\right|_{\bar{p}} \tag{3.10}$$

and $f(p)$ is approximated as

$$f(p) = f(\bar{p})[1 + n\frac{p'}{\bar{p}}] \tag{3.11}$$

This form is now used in approximate evaluation of (3.9).

First differentiate (3.9) with $f(p) = f\{p(t)\}$ to find

$$f\{p(t)\} - (1 - \frac{d\tau}{dt})f\{p(t-\tau)\} = 0$$

Substitution of (3.11) gives

$$1 - \frac{d\tau}{dt} = \frac{1 + n\frac{p'(t)}{\bar{p}}}{1 + n\frac{p'(t-\tau)}{\bar{p}}} \approx 1 + n\left[\frac{p'(t)}{\bar{p}} - \frac{p'(t-\tau)}{\bar{p}}\right] \tag{3.12}$$

Set $w_l = \bar{w}_l + w_l'$ in (3.8) and substitute (3.12) to find the basic result of the time lag theory:

$$w_l' = \bar{w}_l n\left[\frac{p'(t)}{\bar{p}} - \frac{p'(t-\tau)}{\bar{p}}\right] \tag{3.13}$$

For analyzing linear stability, $p' = \bar{p}e^{i\bar{a}kt}\psi(\vec{r})$ and $w_l' = \hat{w}_l e^{i\bar{a}kt}$, so

$$\hat{w}_l = \bar{w}_l n(1 - e^{-i\omega\tau}) \tag{3.14}$$

where the usual approximation has been made, $\alpha\tau << \omega\tau$ in the exponent.

Equation (3.17) is a two-parameter representation of the conversion of liquid to gas. The two parameters, the time lag τ and the interaction or pressure index n, are unknown *a priori*. All work with the time lag theory requires experimental measurements to determine their values. The general idea is simple. After substituting (3.14) in the linearized conservation equations, solution is found for the stability boundary ($\alpha = 0$) with n and τ as parameters. Experimental data for the stability boundary are used to determine n and τ. Crocco, Grey and Harrje (1960) were first to obtain sufficient data to confirm the value of this approach. Figure 3.2 reproduces some of their results for the time lag and interaction index inferred from tests with two injectors. The data were taken for the stability boundary of the fundamental longitudinal mode and show the strong dependence on fuel/oxidizer ratio.

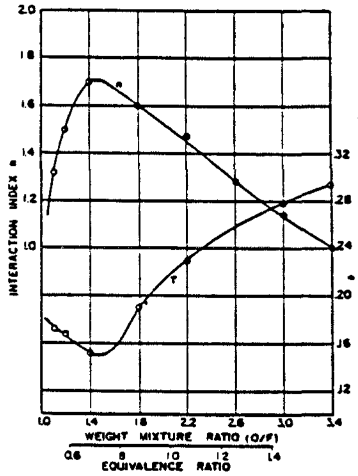


Fig. 11 Values of the sensitive time lag τ and interaction index n determined from the experimental *lower* stability boundary (Fig. 5) for the fundamental longitudinal mode. This figure shows results for the first injector (design O/F = 1.4) at a nominal chamber pressure of 300 psia

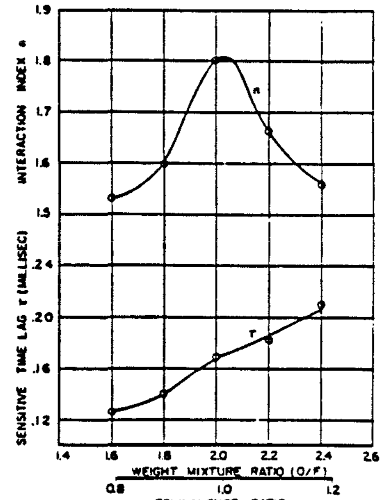


Fig. 12 Experimental values of n and τ for the second injector (design O/F = 2.0)

FIGURE 3.2

Obviously, there are many limitations. The analysis leading to (3.14) is entirely phenomenological; the final result containing two parameters only is an enormous simplification of the real situation, but there is no way to assess the imperfections. The formula (3.14) can be extended to include, for example, dependence on velocity fluctuations [Reardon, Crocco, and Harrje (1964)]. Because the values of all parameters must be found from experimental data, the difficulties become prohibitive.

The time lag model (it is, after all, not really a theory) is based on an appealing physical argument but no processes are treated explicity. Probably the most serious deficiency is that no detailed treatment is given of combustion, which is ultimately the source of the energy driving all combustion instabilities. Nevertheless, the model has been the basis for some success in treating instabilities in liquid rockets, primarily as the basis for correlating data. The two-parameter representation provides a convenient framework for detecting trends of stability with design changes. Its predictive value is very restricted indeed.

3.2 Atomization, Droplet Vaporization and Burning

Some years after the time lag model had been developed, work at the NASA Research Center [Priem and Guentert (1962) and Priem (1965)] showed that the stability of a liquid rocket motor could be controlled by varying the characteristics of the vaporization process. The conclusion followed from the results of numerical solutions to the equations for nonlinear unsteady motions in a chamber. The source terms were approximated with models of the atomization, vaporization and burning. Variations of characteristic parameters showed that atomization and vaporization were the dominant rate processes determining the stability limits. That conclusion led to a series of studies particularly emphasizing vaporization.

Because of the difficulty of extracting precise conclusions from numerical analyses, Heidmann and Wieber (1966a, 1966b) devised a method for assessing the vaporization process alone. A droplet is injected axially in a steady flow. An acoustic field is superimposed having the form of the lowest first tangential mode for a cylindrical chamber ($\sin\theta\ J_1(\kappa_{11}r)$). The motion and vaporization rate of the droplet is calculated throughout its history. By superimposing the results for an array of injected drops, assumed not to interact with one another, one may find the local fluctuation of vaporization rate throughout the chamber. That is the mass source term w_l' in the continuity equation (2.1) for the gas phase.

Heidmann and Wieber (1966a) defined a "response factor", N, to interpret their results:

$$N = \sum \frac{w_l'/\bar{w}_l}{p'/\bar{p}} \tag{3.15}$$

where $\sum$ here denotes the sum over all droplets in the volume considered. They gave results for N as a function of various parameters. Typically, N shows a peak of about .6-.9 in a frequency range .04-.1 Hertz. Results obtained for n-heptane over fairly wide flow conditions were correlated with a dimensionless parameter containing droplet size, chamber pressure, gas velocity and a dimensionless amplitude of the oscillation.

In a later work, Heidmann and Wieber (1966b) used a restricted form of Rayleigh's criterion and a simpler linear analysis to produce essentially the same conclusions. The new definition of the response factor was

$$N = \sum \frac{\int_0^{2\pi/w} \frac{\hat{w}_l^{(r)}}{\bar{w}_l}\frac{\hat{p}}{\bar{p}}dt}{\int_0^{2\pi/w} (\frac{\hat{p}}{\bar{p}})^2 dt} \tag{3.16}$$

These analyses amount to detailed examination of a particular process contributing to the time lag discussed above. Substitution of the real part of (3.14) in (3.16) gives

$$N = n(1 - \cos\omega\tau) \tag{3.17}$$

Heidmann and Wieber found that their numerical results could be approximated quite well in the range $\tau_v\omega < 1$ by the values

$$\begin{aligned} n &= 0.21 \\ \tau &= 4.5\tau_v \end{aligned} \tag{3.18}$$

where τ_v is the mean droplet lifetime. This comparison is shown in Figure 3.3 taken from Heidmann and Wieber (1966).

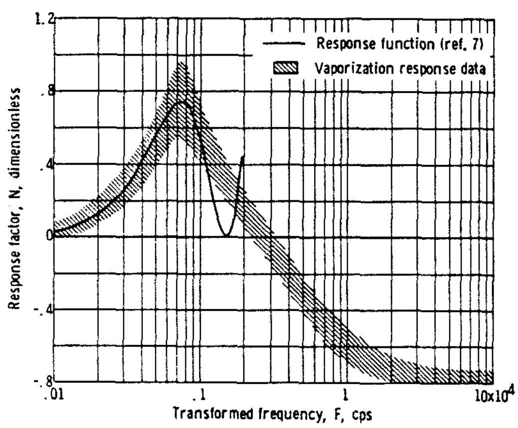


FIGURE 3.3

Note that the function (3.17) oscillates and therefore becomes a poor approximation for $\omega\tau_v > 1$, as show by the solid line in Figure 3.3. The vaporization rates seem physically reasonable for the conditions shown, so one must conclude that the time lag model fails at higher frequencies. Recently Tong and Sirignano (1986a, 1986b, 1987) have re-examined the problem of unsteady vaporization. With their more detailed model including the effects of unsteady heat transfer in the gas phase, they conclude that their vaporization rates are much higher than those found by Heidmann and Weiber.

More strongly, Tong and Sirignano propose that unsteady droplet vaporization is a potential mechanism for driving combustion instabilities. Heidmann and Wieber had earlier noted that the response factor they calculated for the vaporization process was less than that calculated for the nozzle losses. Thus, although vaporization itself did add energy to the acoustic field according to their analysis, the effect was too small to be a mechanism for instabilities. Tong and Sirignano conclude that their results show sufficient energy transfer from the vaporizing droplets to the acoustic field to qualify as a mechanism in actual systems*. Their conclusion is based solely on the $p - v$ work done by the process of vaporization and does not include any energy release due to combustion. The proposal is evidently wrong for the following reasons.

* Later application of this work to ramjet combustors is discussed briefly in Section 5.3.4.

It is significant that none of the preceding conclusions involved combustion: the assertion is that coupling between pure vaporization and the acoustic field produces net flow of energy to the oscillations in the gas. The contrary conclusion was reached by Marble and Wooten(1970) and Marble (1969), that both condensing and vaporizing droplets attenuate acoustical motions.

The reason for the opposite conclusion seems to be that not all interactions between the droplets and the acoustic field are accounted for in the calculations by Heidmann and Wieber and by Tong and Sirignano. Their conclusions were based on using Rayleigh's criterion, but only one term was considered. They argued that by analogy with Raleigh's original statement concerning fluctuations of heat addition, the same criterion should apply to mass addition. Therefore, as in equation (3.16) only the integral involving w_l' was computed; a positive value indicates the possibility for driving the acoustic field. However, the derivation in Section 2.5 has shown that the correct form of the criterion involves several contributions. Considering only those associated with the conversion of liquid to gas, we combine equations (2.84)a,b and (2.83) to find

$$\Delta\mathcal{E}_n = \frac{\omega_n^2}{\bar{p}E_n^2}\int dV \int_t^{t+\tau_n} [(\bar{\gamma}-1)\{\delta Q_l' + (\bar{h}_l - \bar{e})w_l' + (h_l' - e' - \frac{p'}{\bar{\rho}_g})\bar{w}_l\} + \bar{\gamma}(\delta\vec{F}_l' + \bar{w}_l\delta\vec{u}_l')\cdot\vec{u}_n']dt \tag{3.19}$$

There is indeed a term proportional to the integral of $w_l'p'$ but it is multiplied by $(\bar{h}_l - \bar{e})$ which contains the heat of vaporization. There are also significant amounts of energy transfer associated with the terms involving $\delta Q_l' + \delta F_l'$ which for non-vaporizing drops represent the attenuation of sound waves. Those effects are included in the work by Marble and Wooten: their results show that the accompanying energy losses dominate so that in fact if combustion is ignored, vaporizing droplets cause damping, not driving, of unsteady gas motions.

We must emphasize that the conflicting results, and the conclusion that vaporization is not a mechanism for driving combustion instabilities rests on proper computation of the energy transfer. In the earlier work, an incorrect or, rather, incomplete form of Rayleigh's criterion was used. It is certainly true that the process represented by $w_l'p'$ alone does cause driving if the fluctuation of mass release has a component in phase with the pressure fluctuation, but that is only part of the story.

Priem (1988) has recently used Heidmann and Wieber's model of vaporization, combined with the model worked out by Feiler and Heidmann (1967) for a gaseous fuel, to study combustion instabilities in the LOX/methane system. He bases his conclusions concerning stability boundaries on numerical results for the combustion responses, of which that for liquid oxygen is computed with equation (3.16) and the method described above; and on corresponding results found for the losses associated with the exhaust nozzle and baffles. His results seem to compare fairly well with recent experimental work. The reason that this could be so - even though vaporization causes net energy losses if all contributions are accounted for - is that the energy released by combustion, immediately following vaporization, is the dominant factor. That is, in equation (3.17) the terms involving energy transfer are larger than those representing losses. Comparison with experimental results seems always to involve multiplicative factors which are determined to provide best fit to data, or are absent in normalized forms. Then when good agreement is found, it seems that it is largely the qualitative behavior that is being checked.

Despite the heavy emphasis, in many works, on vaporization as the rate controlling process, it is generally recognized that other processes contribute and in some situations may be dominant. The injection process itself may be affected under unsteady conditions due to the varying streams, impact of jets, and atomization all are sensitive to unsteady flow fields. Those problems are extremely complicated, difficult to describe in a fashion suitable for use in a general analysis, and are very much dependent on details of the hardware. Thus the work has largely been experimental with some effort to correlate results in a form useful for design [e.g. Levine (1965); Sotter, Woodward and Clayton (1969); Webber (1972); Webber and Hoffman (1972)]. The time-lag model has been used essentially as a means of correlating all of those processes without concern for details [Reardon, Crocco and Harrje (1964); Reardon, McBride and Smith (1966)]. Summaries of experimental results obtained prior to 1971 may be found in the reference volume edited by Harrje and Reardon (1972).

Of recent work, the most fundamental and detailed is that carried out at ONERA as a result of problems due to combustion instabilities in the Viking motor. Special effort has been made to understand the unsteady behavior of the injectors used in that engine. The intentions of the research program were described by Souchier, Lemoine and Dorville (1982); and by Lourme and Schmitt (1983). Considerable effort has since been expended to characterize the steady and unsteady behavior as the basis for analyzing instabilities in the engine [Lourme, Schmitt and Brault (1984); Lecourd, Foucaud and Kuentzmann (1986); Lourme (1986); Lecourd and Foucaud (1987)]. The results range from detailed measurements of the spray (droplet size and velocity distributions) to the more global unsteady response of the injector, using a device adapted from a method developed for solid propellant rockets. Incorporation of the results of these works in analysis of the instabilities in engines is in progress.

3.3 Convective Waves

Following work by Kovasznay (1953), Chu and Kovasznay (1957) showed one way of decomposing general small disturbances of a viscous compressible fluid into three classes: acoustic, viscous, and entropy waves. Acoustic waves carry no entropy changes, while viscous and entropy waves have no accompanying pressure fluctuations. The direct effects of viscous stresses and heat conduction on combustion instabilities are generally negligible except in the vicinity of surfaces. That entropy fluctuations evidently have second order effects on the acoustic waves is implied by the formal analysis covered in Section 2; there was no need to introduce the entropy.

However, both viscous effects and nonuniform entropy may affect the acoustic field indirectly through processes at the boundaries. First we examine here the possible influences of entropy fluctuations. These fall within the general class of convective waves, that is, disturbances that are carried with the mean flow: their propagation speed is the average flow speed. Entropy fluctuations are associated with the portion of temperature fluctuations not related isentropically to the pressure fluctuation; such as non-uniformities of temperature due, for example, to combustion of a mixture having non-uniformities in the fuel/oxidizer ratio. In general, an entropy wave may be regarded as a nonuniformity of temperature carried with the mean flow.

As shown by Chu(1953) pressure waves incident upon a plane flame will cause generation of entropy waves carried downstream in the flow of combustion products. Thus one should expect that when combustion instabilities occur, there must be ample opportunity for the production of entropy fluctuations. That process has negligible effect directly on stability (the coupling between acoustic and entropy waves is second order within the volume) but there has long been interest in the possible consequences of entropy waves for the following reason.

When an entropy wave is incident upon the exhaust nozzle, it must pass through a region containing large gradients of mean flow properties. A fluid element must retain its value of entropy and for this condition to be satisfied, the pressure and density fluctuations cannot be related by the familiar isentropic relation, $\delta p \sim \gamma \delta \rho$. As a result, within the nozzle pressure changes are produced that will generate an acoustic wave that will propagate upstream. Thus, an entropy wave incident upon an exhaust nozzle can produce an acoustic wave in the chamber, augmenting the acoustic field due to other sources.

An artificial elementary example will illustrate the proposition. Consider a chamber admitting uniform constant mean flow at the head end, say through a choked porous plate; the flow exhausts through a choked nozzle (Figure 3.4). Suppose that

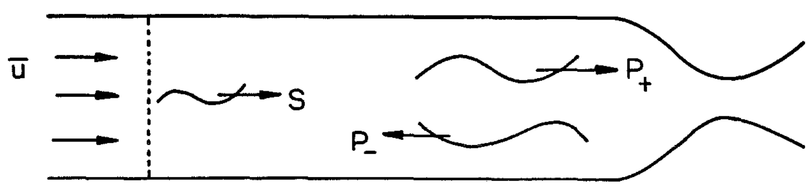


FIGURE 3.4

at the head end a heater is placed, arranged so that its temperature can be varied periodically, with frequency ω. This action produces a continuous temperature or entropy wave convected with the flow. An experimental realization of this situation has been described by Zukoski and Auerbach (1976). We assume no losses within the flow, so a fluid element retains its entropy; small perturbations s' of the entropy satisfy the equation

$$\frac{\partial s'}{\partial t} + \bar{u}\frac{\partial s'}{\partial z} = 0 \tag{3.18}$$

If S is the amplitude of the fluctuation at the heater ($z = 0$), the solution for s' is

$$s' = Se^{-i\omega(t - \frac{z}{\bar{u}})} \tag{3.19}$$

To simplify the calculations, assume that the flow speed is vanishingly small so that we may ignore its effect on acoustic waves (we relax this assumption in Section 5). Then the acoustic pressure and velocity fields can be expressed as sums of rightward and leftward traveling plane waves:

$$\begin{aligned} p' &= [\mathrm{P}_+ e^{ikz} + \mathrm{P}_- e^{-ikz}]e^{-i\omega t} \\ u' &= [U_+ e^{ikz} + U_- e^{-ihz}]e^{-i\omega t} \end{aligned} \tag{3.20a, b}$$

As usual, the complex wavenumber is $k = (\omega - i\alpha)/\bar{a}$. The acoustic pressure and velocity must in this problem satisfy the classical acoustic momentum equation, (2.27) with $\bar{u}_g = \mathcal{F}' = 0$:

$$\bar{\rho}\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial z} = 0 \tag{3.21}$$

Separate substitution of the forms for the rightward and leftward traveling waves shows that $U_\pm$, $P_\pm$ are related by

$$\bar{\rho}\bar{a}U_+ = \mathrm{P}_+ \quad ; \qquad \bar{\rho}\bar{a}U_- = -\mathrm{P}_- \tag{3.22}$$

Assume that the head end acts as a perfect reflector for the acoustic waves, so

$$u' = 0 \; ; \quad \frac{\partial p'}{\partial z} = 0 \qquad (z = 0) \tag{3.23a, b}$$

In a real case (e.g. if the heater were actually a flame) the pressure fluctuations would cause fluctuations of entropy at the head end. To represent this effect, set s' proportional to p' at $z = 0$:

$$s' = A_0 p' \qquad (z = 0) \tag{3.24}$$

Tsien (1952), Crocco (1953) and Crocco and Cheng (1956) have shown that the boundary condition at the nozzle entrance may be written in the form

$$p' + \bar{\rho}\bar{a}A_1 u' + A_2 s' = 0 \qquad (z = L) \tag{3.25}$$

We may now show that the problem formulated here admits solutions representing steady acoustic oscillations in the chamber, whose stability depends on the values of the coefficients A_0, A_1, A_2. We eliminate the unknown amplitudes S, P_+, P_- and obtain a characteristic equation for the complex wavenumber k, by satisfying the boundary conditions (3.22)-(3.25). Substitute equations (3.20) and (3.22) into (3.23) to find

$$\mathrm{P}_+ - \mathrm{P}_- = 0 \tag{3.26}$$

With (3.19) and (3.20)a, the condition (3.24) is satisfied if

$$S = A_0(\mathrm{P}_+ + \mathrm{P}_-) \tag{3.27}$$

Finally, substitution of (3.19), (3.20)a,b and (3.27) in (3.25) gives

$$[(1 + A_1)e^{ikL} + A_0 A_2 e^{i\frac{\omega}{\bar{u}}L}]P_+ + [(1 - A_1)e^{-ikL} + A_0 A_2 e^{i\frac{\omega}{\bar{u}}L}]P_- = 0 \tag{3.28}$$

With $P_- = P_+$ from (3.26) we have the characteristic equation

$$e^{i2kL} = \frac{-1}{(1 + A_1)}[1 - A_1 + 2A_0 A_2 e^{i(k+\frac{\omega}{\bar{u}})L}] \tag{3.29}$$

Generally A_0, A_1, A_2 are complex numbers. The real and imaginary parts of (3.29) provide transcendental equations for the real and imaginary parts ($\omega/\bar{a}$, $\bar{\alpha}/\bar{a}$) of k. The solutions are unstable if $\alpha > 0$, corresponding to self-excited waves. Note that in the limiting case of no entropy fluctuations ($A_0 = 0$) and a rigid wall ($A_1 \to \infty$) at $z = L$, (3.29) reduces to $e^{i2kL} = +1$ or $\cos 2kL = 1$ and $\sin 2kL = 0$. Then $k = n\pi/L$ and the allowable wavelengths are $\lambda = 2\pi/k = 2L/n$, the correct values for a tube closed at both ends.

This example suggests the possibility for producing instabilities *if* entropy waves are generated and *if* those waves interact with the boundary in such a way as to produce acoustic disturbances. It is in fact a genuine possibility that has been considered both in laboratory tests and as an explanation of instabilities observed in actual engines. The difficulties in applying this idea are largely associated with treating the processes responsible for causing the entropy waves.

In a combustion chamber, possible sources of entropy fluctuations may be distributed throughout the chamber. Burning of non-uniform regions of fuel/oxidizer ratio and interactions of pressure distributions with combustion zones are important causes, both producing non-isentropic temperature fluctuations. Thus in general the property that in inviscid flow free of sources an element of fluid has constant entropy, is inadequate. A proper description of entropy waves should be placed in the broader context accounting also for convective waves of vorticity as worked out first by Chu and Kovasznay (1957). We cannot provide a complete discussion here, but for later purposes in Section 5.3 it is helpful to have at hand the more general equation governing entropy fluctuations.

Combination of the first law of thermodynamics for a perfect gas and the definition $ds = dq/T$, valid if the heat transfer dq is not too abrupt, gives

$$ds = C_v \frac{dT}{T} - \frac{p}{\rho_g}\frac{d\rho_g}{\rho_g}$$

Now introduce the perfect gas law to eliminate the temperature change. Writing the result for motion following a fluid element we have

$$\frac{1}{\bar{C}_v}\frac{Ds}{Dt} = \frac{1}{\rho}\frac{Dp}{Dt} - \frac{\bar{\gamma}}{\rho_g}\frac{D\rho_g}{Dt} \tag{3.30}$$

where $D/Dt = \partial/\partial t + \vec{u}_g \cdot \nabla$ is the convective derivative. After substitution of (2.18) and (2.20) we find the equation for entropy,

$$\begin{aligned} \frac{1}{\bar{C}_v}\frac{Ds}{Dt} = \frac{1}{\rho}\frac{\bar{R}}{\bar{C}_v}[Q + \delta Q_l + \nabla\vec{q} + \Phi + \delta\vec{u}_l \cdot \vec{\mathcal{F}}_l + \frac{p}{\rho_g}\nabla \cdot (\rho_l \delta\vec{u}_l) \\ + \{(h_l - e) + \frac{1}{2}(\delta u_l)^2\}w_l] \end{aligned} \tag{3.31}$$

The right hand side contains all sources of entropy changes including viscous effects, combustion and conversion of liquid to gas.

Equation (3.31) completes the set of equations required for complete analysis of combustion instabilities including entropy waves. The equations governing vorticity waves are obtained by splitting the velocity field into two parts: the acoustic field which is irrotational, and the rotational vorticity field which, if treated in all generality, includes turbulence as well as large vortex structures and shear waves.

The subject of convective waves in the presence of acoustic motions has not been exhaustively treated. Some special examples are discussed in Section 5.3

3.4 Vortex Shedding and Combustion

The presence of swirling, spinning or vortex motions in propulsion systems has long been recognized as a serious problem. They fall broadly into two classes: those with angular momentum directed along the axis, usually (if the rocket itself isn't spinning) related to standing or spinning transverse acoustic modes of the chamber; and those having angular momentum mainly perpendicular to the axis, associated with vortex shedding from bluff bodies or rearward facing steps.

Motions identified as forms of transverse or tangential modes do not normally qualify as mechanisms: they are themselves the combustion instability. Male, Kerslake and Tischler (1954) gave an early summary of severe transverse oscillations ("screaming" at 10K Hertz) and noted what has always been a serious consequence: greatly increased surface heat transfer.

Here we are concerned with vortex motions growing in unstable shear layers. Those vortices, now commonly called "large coherent structures" [Brown and Roshko(1974)] are convected downstream at approximately the average speed of the two streams forming the shear layer. In propulsion systems, the shear layers in question are generally formed in flow past bluff body flame holders (in thrust augmentors) or past rearward facing steps (in ramjet engines).

Observations of vortex shedding from flameholders, and recognition of the importance of this process as a possible mechanism for combustion instabilities were first independently reported by Kaskan and Noreen (1955) and by Rogers (1954) and Rogers and Marble (1956). Both experiments used premixed gaseous fluel and air flowing past a flameholder in a rectangular channel. However, the particular mechanisms proposed were very different. Figures 3.5 and 3.6 taken respectively from Kaskan and Noreen (1955) and Rogers and Marble (1956) clearly show the vortex shedding.

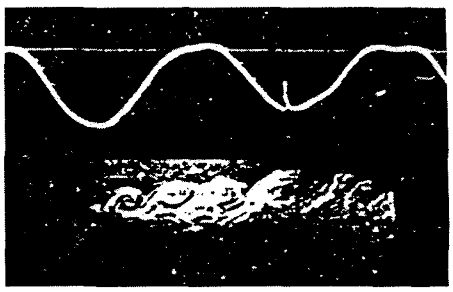


FIGURE 3.5

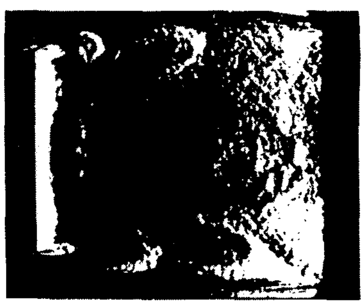


FIGURE 3.6

Motivated partly by earlier observations of Blackshear (1953) and Putnam and Dennis (1953), Kaskan and Noreen speculated that stretching of the flame front accompanying roll-up in the vortex causes a pressure distrubance. Periodic disturbances generated by periodic vortex shedding may then sustain either transverse or longitudinal acoustic fields. (They observed both in their tests.) As a quantitative basis for interpreting their results they modified a theoretical relation derived by Chu (1953) for plane flames. Although they had modest success comparing their reasoning with their data, Kaskan and Noreen did not provide a complete explanation of the closed-loop process required to generate self-excited oscillations. This mechanism has not subsequently received much notice as a cause for combustion instabilities, although the idea has recently been revived in connection with work on ramjet combustion; see remarks in Section 5.3.2.

Rogers and Marble gave detailed reasoning to support their idea that delayed periodic combustion in shed vortices generates periodic pressure pulses that serve as sources of the acoustic field (transverse in their tests).

The fluctuating velocity of the acoustic field itself initiates the vortex shedding, thereby closing the loop. Rogers and Marble drew on earlier data for the ignition delay in flow past bluff bodies [Zukoski and Marble (1955)] to demonstrate that vortex combustion could in fact occur in proper phase to support the acoustic vibrations.

During the past six years, the idea that vortex shedding is a dominant factor in mechanisms for many combustion instabilities has gained growing support. Practically all of the work has been motivated by problems of longitudinal oscillations in ramjet engines. Even though the frequencies are substantially lower than those of the oscillations treated by Rogers and Marble, the essentials of the idea seem to hold true.

The problem of longitudinal oscillations in small ramjet engines was apparently first recognized by Hall (1978). Rogers (1980a, 1980b) gave thorough summaries of the available experimental work. Those reports served as the basis for an early analysis of the problem by Culick and Rogers (1983); that work did not include a satisfactory mechanism. Concurrently, Byrne (1981, 1983) proposed that vortex shedding in a dump combustor appeared to be a likely cause of the observed instabilities. Apparently unaware of the earlier work by Rogers and Marble on transverse oscillations, he based his argument on known results for cold jet flows.

Since the early 1980's a great deal of attention has been given to the role of vortex shedding in dump combustors, both in cold flow and in laboratory combustion tests [e.g. Keller et al (1982); Smith and Zukoski (1985); Biron et al (1986); Schadow et al (1987); Sterling and Zukoski (1987); Poinsot et al (1987); Yu et al (1987)]. There is little doubt now that indeed the coupling between periodic energy released by combustion in shed vortices and the acoustic field is the dominant mechanism in dump combustors. The extent to which the same mechanism is active in contemporary thrust augmentors is less well-established but there is good reason to believe that it is often, if not usually, the main cause.

Extensive experimental work on vortex shedding in shear layers and jets at room temperature has provided a fairly complete picture of the formation of vortices; vortex pairing; and the general features of the flow without heat addition [see Schadow et al (1987) for a brief review of the literature relevant to problems in ramjet engines]. Tests in various configurations, including those appropriate to ramjets [e.g. Flandro et al (1972); Culick and Magiawala (1979); Dunlap and Brown (1981); Brown et al (1981, 1983); Schadow et al (1984)] established the ability of shed vortices to drive acoustic resonances over a broad range of flow conditions. The works cited above have extended that conclusion to flows with large heat addition accompanying combustion under circumstances simulating those found in actual ramjet engines. We shall discuss those results further in Section 5.3.2.

The obvious qualitative importance of combustion in large vortices has prompted several recent analytical investigations of the process. Broadly the idea is that the shear layer is formed at the edge of a bluff body, the high speed stream consisting of an unburnt mixture of reactants; the low speed stream is composed largely of hot combustion products forming the recirculation zone behind the body. As Smith and Zukoski (1985) and Sterling and Zukoski (1987) have shown, the shear layer exhibits widely varying degrees of stability depending on the operating conditions. We are concerned here with cases when the layer is highly unstable, a situation encouraged by the action of the acoustic velocity forcing oscillations of the layer at the lip. Large vortices may then rapidly form, entraining unburnt mixture on one side of an interface, with the combustion products on the other side. A flame is initiated at the interface and the question to be answered is: how does the rate of combustion, and therefore heat release, vary as the vortex rolls up and propagates downstream?

Marble (1984) treated an idealized case of a diffusion flame initiated along a horizontal plane when simultaneously the velocity field of a line vortex is imposed along an axis in the interface. Elements of flame initially in the interface are caused to execute circular motions and are stretched by the vortex field, causing an increase in the rate at which reactants are consumed. The expanding core contains combustion products but as the vortex roll-up continues, the rate of consumption always remains greater than that for flame in the flat interface having the same length as that in the rolled-up vortex. Karagozian and Marble (1986) carried out a similar analysis accounting for the influence of stretching along the axis of the vortex. They found that, following a transient period during which the core grows to its asymptotic form, the augmented consumption rate is unaffected by axial stretching. In those cases the rate of heat release reached a constant value monotonically: there is no distinguished period of pulsed combustion as required for the mechanism for instability described above.

More recently, Laverdant and Candel (1987, 1988) have treated both diffusion and premixed flames in the presence of vortex motion with finite chemical kinetics. Their analysis is entirely numerical giving good agreement with those of Karagozian and Marble and Karagozian and Manda (1986) for a vortex pair.

Perry (1983) also analysed the influence of finite chemical kinetics in the problem posed and solved by Marble (1984) who had assumed infinite reaction rates. Under some conditions, the heart release rate shows a modest peak in time. However, neither his results, nor those of Laverdant and Candel, suggest the sort of time delay to pulsed combustion one might like to see to complete the picture.

No work has been accomplished to determine whether or not the augmented reaction rates found in the analysis are sufficient to explain the mechanism of instabilities driven by vortex combustion. On the other hand, the experimental results reported by Smith and Zukoski (1985), Sterling and Zukoski (1987), and Yu et al (1987) show vividly and beyond doubt that unsteady combustion associated with vortex motions is a vigorous source indeed. Figure 3.7 is a sequence of photographs taken by Smith and Zukoski during one cycle of a high amplitude oscillation. They propose the following mechanism. A vortex is initiated at the edge of the step at a time determined partly by the local acoustic velocity. The vortex propagates downstream, releasing energy of a rate that seems to reach maximum when the vortex impinges on the wall. In order for impingement to occur at a favorable time during the acoustic oscillation, the propagation rate and hence strength of the vortex must increase with frequency. Because the vortex strength depends on the magnitude of velocity fluctuation initiating the motion at the lip, it is necessary that the steady amplitude of the acoustic field increase with frequency. That behavior is observed. Moreover, numerical calculations by Hendricks (1986) have shown quite similar behavior for the unsteady flow induced by an abrupt change of velocity past a rearward facing step. Figure 3.8 is a sketch taken from Hendricks' work showing the development of a vortex calculated for those conditions.

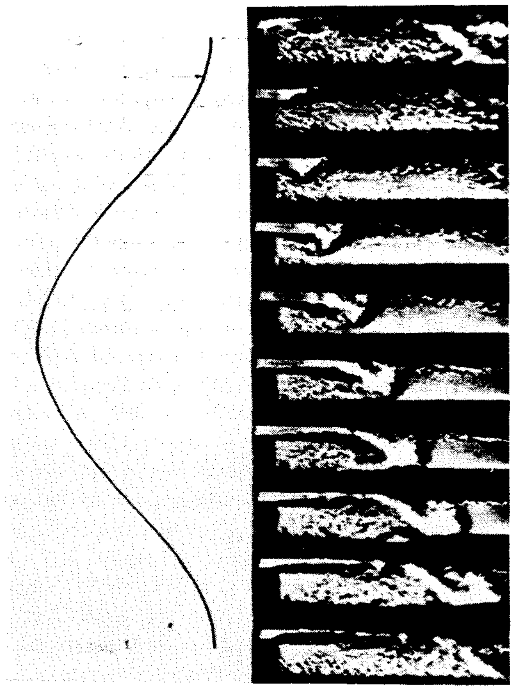


FIGURE 3.7

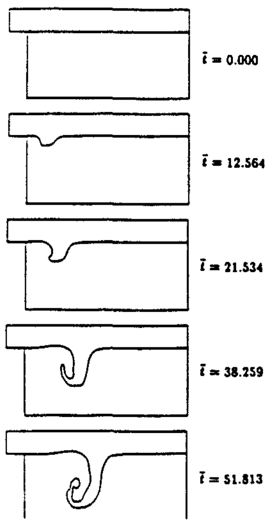


FIGURE 3.8

The essential ideas of vortex combustion as a mechanism for driving instabilities can be easily incorporated in the approximate analysis summarized in Sections 2.2 and 2.3. There is ample experimental evidence that large vortices in cold flow can sustain resonances in a duct; Flandro (1986) has shown one means of handling the process analytically based on direct fluid mechanical coupling between vortical and acoustic motions. See also Aaron and Culick (1985) for an elementary model of coupling associated in the impingement of a vortex on an obstacle. Tests with combustors have shown, however, that the amplitudes of oscillation are substantially greater when burning occurs. That result is most likely due to the unsteady energy release. We therefore assume that this is the main source of the driving.

Hence in the forcing function F_n, equations (2.45) and (2.46), we retain only the term Q' in P, equation (2.23); Equation (2.45) for the time-dependent amplitude of the n^{th} mode is

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = \frac{\bar{\gamma}-1}{\bar{p}E_n^2}\int \psi_n \frac{\partial Q'}{\partial t} dV \tag{3.32}$$

A formula for Q' must be constructed to account for the trajectory of the vortex and its associated rate of energy release along the trajectory. To illustrate with a simple example, we consider excitation of longitudinal modes and assume that the vortex travels parallel to the axis. Within the one-dimensional approximation, that implies averaging the presence of the vortex over planes transverse to the axis. The situation is sketched in Figure 3.9. The origin $z = 0$ is at the step, which is not the location of a pressure anti-node. In fact, we must allow the acoustic velocity to be non-zero at the beginning of the shear layer at $z = 0$, so the mode shape is

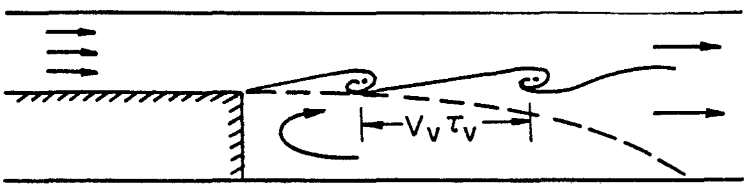


FIGURE 3.9

$$\psi_n(z) = \cos(k_n z + \phi) \tag{3.33}$$

The values of k_n and ϕ can be set by imposing a boundary condition at $z = l$ and choosing some location $z < 0$ for a pressure anti-node. For example, if pressure anti-nodes occur at $z = -\delta L_0$ and $z = L + \delta L_1$, the two conditions must be satisfied

$$\begin{aligned} &\sin(-k_n \delta L_0 + \phi) = 0 \\ &\sin[k_n(L + \delta L_1) + \phi] = 0 \end{aligned} \tag{3.32}$$

from which k_n and ϕ can be determined. For the purposes here, the particular values of k_n and ϕ are immaterial. With (3.33), the acoustic pressure and velocity are

$$\begin{aligned} p' &= \bar{p}\eta_n(t)\cos(k_n z + \phi) \\ u' &= -\frac{\dot{\eta}_n}{\bar{\gamma} k_n}\sin(k_n z + \phi) \end{aligned} \tag{3.33)a, b}$$

For simplicity, assume that the vortices propagate with constant speed v_v and are launched periodically with period τ_v, at the times $t = 0,\ \tau_v,\ 2\tau_v,\ \cdots$. Assume further that these are point vortices releasing energy at the rate $q(t)$ each. Hence the energy release associated with a train of shed vortices can be represented by δ-functions moving with speed v_v multiplying the energy release:

$$\begin{aligned} Q'(z_1 t) &= q_1(t)\delta[z - v_v t] + q_2(t)q[z - v_v(t - \tau_v)] + q_3(t)\delta[z - v_v(t - 2\tau_v)] + \cdots \\ &= \sum_{j=0}^{\infty} q_j(t)\delta[z - v_v(t - j\tau_v)] \end{aligned} \tag{3.34}$$

In accordance with the behavior reported by Smith and Zukoski we should relate the strength of each vortex and, therefore by assumption its energy release, to the velocity fluctuation causing its birth. For simplicity we ignore the influence of the mean flow speed and set q_j proportional to the acoustic velocity at the step and at the time when the vortex is launched. Hence, we assume

$$q_j(t) = \bar{q}_j(t)u'(0, jT_v) = -\bar{q}_j(t)\frac{\dot{\eta}_n(j\tau_v)}{\bar{\gamma}k_n}\sin\phi \tag{3.35}$$

where $q(t)$ is supposed to be common to all vortices. With (3.37) for $q_j(t)$, differentiate (3.36):

$$\frac{\partial Q'}{\partial t} = -\sum_{j=0}^{\infty}\frac{\dot{\eta}_n(j\tau_v)}{\bar{\gamma}k_n}\sin\phi\{\dot{\bar{q}}_j\delta f[z - v_v(t - j\tau_v)] - \bar{q}_j v_v\delta[z - v_v(t - j\tau_v)]\} \tag{3.38}$$

Now substitute (3.33) and (3.38) in the integral on the right hand side of (3.30), with $dV = S_c dz$ where S_c is the cross-section area of the chamber:

$$\begin{aligned} \int \psi_n \frac{\partial Q'}{\partial t} dV = S_c \int_0^L \cos(k_n z + \phi) \sum_{j=0}^{\infty} \frac{\dot{\eta}_n(j\tau_v)}{\bar{\gamma}k_n} \sin\phi\{\dot{\bar{q}}_j\delta[z - v_v(t - j\tau_v)] \\ - \bar{q}_j v_v \delta[z - v_v - q_i v_v \delta(t - j\tau_v)]\}dz \end{aligned}$$

Use the properties

$$\int \delta(x-a)f(x)dx = f(a); \qquad \int \delta'(x-a)f(x)dx = -f'(a)$$

to find:

$$\int \psi_n \frac{\partial Q'}{\partial t} dV = - S_c \sum_{j=0}^{\infty} \zeta_{nj} \{ \dot{\bar{q}}_j(t) \cos[k_n v_v (t - j\tau_v)] + \bar{q}_j(t) k_n v_v \sin[k_n v_v (t - j\tau_v)] \} \tag{3.39}$$

with

$$\zeta_{nj} = \frac{\dot{\eta}_n(j\tau_v)}{\bar{\gamma} k_n} \sin\phi \tag{3.40}$$

Thus we have an expression for the right hand side of (3.30) representing the forcing due to a train of burning vortices, launched at $t = 0$, τ_v, $2\tau_v$, $\cdots$ from the lip of the step at $z = 0$.

The results (2.52)a,b of time averaging may now be used to determine the functions $A_n(t)$, $B_n(t)$ in the form (2.51) for $\eta_n(t)$. To find explicit results, the time dependence $q(t)$ for the energy release rate of each vortex must be prescribed. Although contrary to the results for vortex combustion cited earlier, we assume for simplicity that negligible energy is released by each vortex until some time τ_c later, the time delay for this process. Hence we set all $q_j(t)$ equal except for the difference in the times of initiation:

$$\bar{q}_j(t) \doteq q_v \delta[t - j(\tau_v + \tau_c)] \tag{3.41}$$

With this form the integrals in (2.52)a,b can be carried out explicitly. In this step, consistent with the approximations required for the method of time averaging we set

$$\dot{\eta}_n(j\tau_v) = -\omega_n [A_n(t) \cos\omega_n j\tau_v - B_n(t) \sin\omega_n j\tau_v]$$

and take $A_n(t)$, $B_n(t)$ to be constant when the integrals are performed. As a result, the formulas (2.52)a,b lead to the results always found for linear behavior,

$$\frac{dA_n}{dt} = \alpha_n A_n + \theta_n B_n$$
$$\frac{dB_n}{dt} = \alpha_n B_n - \theta_n B_n \tag{3.42)a,b}$$

Hence the α_n, θ_n depend on the various parameters k_n, ϕ_n, τ_v, τ_c etc. introduced to define the mode shape and the stream of vortices. Their particular forms are unimportant here. The point is that the idea of producing combustion instabilities by unsteady combustion in vortex shedding can be translated to an approximate description allowing correlation and interpretation of data. The details of applying this analysis to experimental data are presently incomplete. It would be particularly interesting to determine whether this model of the process predicts frequency shifts as large as those sometimes observed in laboratory tests.

4. METHODS OF ANALYSIS

Ultimately the purpose of research on combustion instabilities is to provide the basis for understanding and curing the problem in actual propulsion systems. Experimental data taken with laboratory or full-scale devices, combined with analytical estimates, have suggested the most likely mechanisms for instabilities. Analysis based on the conservation equations incorporating one or more of those mechanisms provides the means for translating empirical results to a form useful in design and development.

Thus the analyses of unsteady motions that have been carried out have been conditioned to a considerable extent by the *a priori* view of the mechanism, or model, chosen to represent the unsteady combustion processes. (That is not entirely a necessary rule, but seems to be partly a social matter or a question of taste.) In this section we shall cover briefly four classes of analysis that seem to encompass almost all of the work that has been done on combustion instabilities in liquid rockets. Historically, the problems were first treated for liquid rockets, research in the subject was particularly active during the 1960's because of the needs of the Apollo program. Some of the ideas have since been adapted with suitable modifications, to analyze instabilities in augmentors and ramjets. We shall discuss those subjects in Section 5.

4.1 Numerical Analysis and Simulation of Combustion Instabilities

By 'numerical analysis' we mean works that are devoted to solving the differential conservation equations, usually in nonlinear forms. Thus, it is necessary to prescribe in complete detail the mechanism selected as the cause for the instabilities. In view of the discussions in Section 3, it is therefore not surprising that numerical analyses have uniformly been concerned with mechanisms emphasizing injection, atomization, vaporization, and combustion of liquid propellants.

Priem and Guentert (1962) were first to treat combustion instabilities in liquid rockets by solving numerically the conservation equations. Their approach was later adopted by others [notably by Hoffman, Wright and Breen (1968) based on work reported by Beltran, Wright, and Breen (1966)] with some important detailed charges, but the strategy remained unchanged. The work has been summarized in Harrje and Reardon (1972), pp. 194-207 and pp. 286-293.

Because of computational limitations, only one- or two- dimensional problems were treated. The two-dimensional problem was formulated for concentric annuli in the chamber (Figure 4.1) The nonlinear conservation equations were solved with source terms representing vaporization and combustion. Hoffman, Wright, and Breen distinguished the liquid fuel and oxidizer and were able to represent the energy release somewhat more realistically: for example, they were able to account for variable oxider/fuel ratio and conditions when either fuel or oxidizer may be excess. Moreover, they included the droplet drag and momentum thransfer between the liquid and gas phases. They assumed, with Priem, that the vaporization rate was quasi-steady, but they improved the representation by including a dependence on Reynolds number. Like Priem, they assumed also that combustion occurred immediately upon vaporization. As an improvement in detail, they used a lognormal distribution of sizes for both the fuel and oxidizer droplets. Break-up and atomization of the injected liquids were ignored.

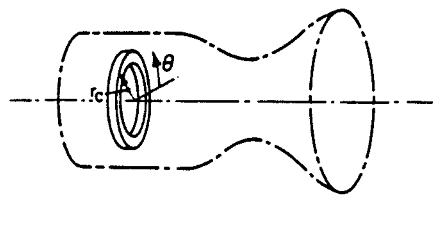


FIGURE 4.1

The same initial value problem was treated in all these works: a wave having sinusoidal amplitude distribution around the annulus is initiated with maximum amplitude $\Delta\bar{p}/\bar{p}$. It is time evolution is calculated as it propagatees around the annulus. The annulus can be placed anywhere along the axis of the chamber, but axial propagation of the wave is not treated. Axial locations are distinguished because the properties of the droplet sprays vary as the flow proceeds from the injector. The average properties of the spray are assumed uniform within an annulus.

Thus the wave is confined to an annulus and the chief results are the waveform as a function of time and space. From that can be determined the growth or decay of the wave and the dependence on the various parameters characterizing the system. It's a curious result - never satisfactorily explained - that all calculations apparently showed linear stability. The initial disturbance had to have finite amplitude in order to be amplified. When unstable, the wave steepened, as shown by Figure 4.2 taken from Hoffman, Wright, and Breen. Owing to the constraints placed in the problem (especially the required propagation in an annulus) it is not very surprising that the possibility of finite amplitude transverse waves without shocks, predicted by Maslen and Moore (1956) was not confirmed; their results always show substantial steeping after short times.

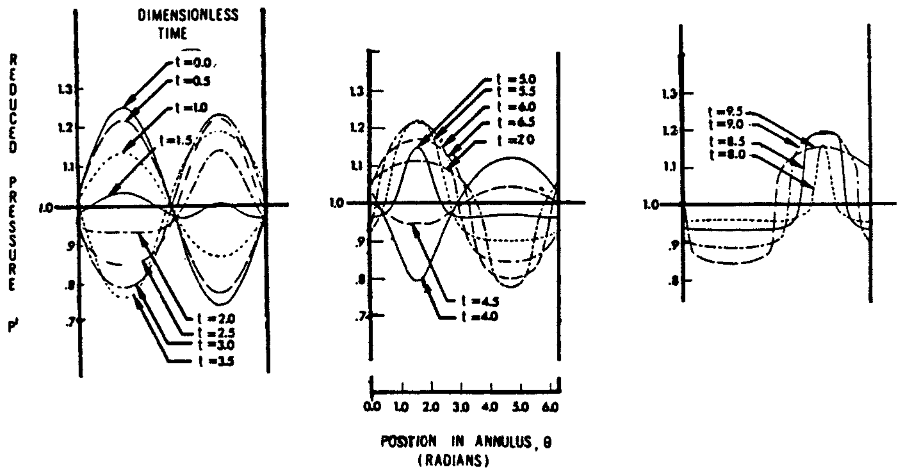


FIGURE 4.2

Figure 4.3 shows some of the main results found by Hoffman, Wright, and Breen. The initial maximum amplitude of the wave is plotted versus a burning rate parameter, essentially a measure of the average energy release rate in unit volume. The curves are stability boundaries: for values of amplitude below the curve, the wave decays. Several boundaries are shown for values for a 'drag parameter' equal to 1, 10 and 100. The drag parameter is a dimensionless measure of the momentum exchange per unit volume between the liquid and gas phases. Figure 4.3 illustrates the obvious result that the region of instability is decreased as the momentum exhange increases.

Transverse waves have long been troublesome and destructive instabilities in liquid rockets [Male, Kerslake and Tischler (1954); Reardon, Crosco and Harrje (1964); Levine (1965); Clayton, Rogers, and Sotter (1968)]. The problem had motivated the analyses described above, and for similar reasons Burstein, Chinitz and Schechter (1972) also carries out a numerical analysis. Like the earlier formulations, they treated wave propagation in an annulus, including the influences of a droplet cloud, but with the added feature that they could include axial

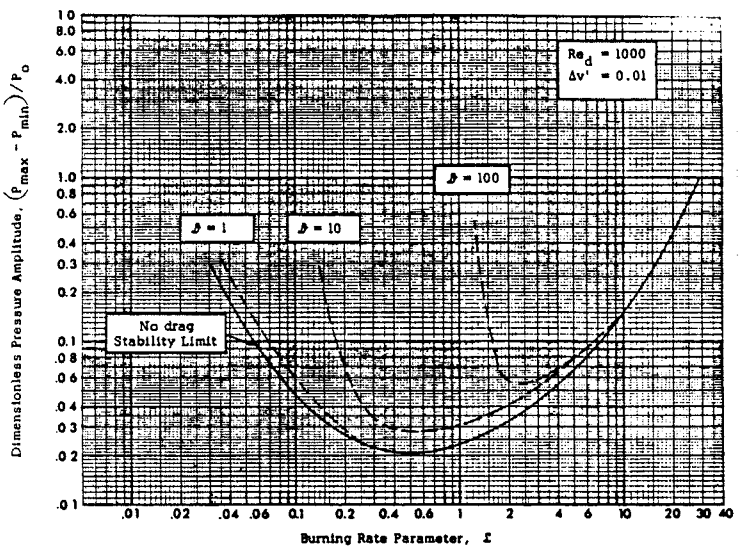


FIGURE 4 EFFECT OF DRAG PARAMETER ON STABILITY LIMITS

FIGURE 4.3

baffles. Thus their main contribution seems to have been their more extensive treatment of propagation in the axial direction. They too evidently found that small amplitude waves were stable.

The serious constraints placed on those earlier works, mainly due to the limitations of computational resources, cause them to be of limited value. It is not clear that the problems solved have been formulated in a physically consistent fashion, causing one to question the meaning of the results. In particular, the restriction to wave propagation in an annulus is no longer an acceptable approximation. Habiballah, Lourme and Pit (1988) have recently reported their most recent progress in a program devoted to modern numerical analysis - or numerical simulation - of instabilities. Although the calculations are being done specifically for the Viking engine, the ideas are more general and the essentials of the approach are broadly applicable.

This sequence of works has been previously described in several places [Schmitt and Lourme (1981); Habiballah and Monin (1984); Habiballah, Maraffa, and Monin (1985)] and we shall not cover the details here: the results are yet to be completed. The work amounts to be as thorough as possible analysis of both the liquid and gas phases in unsteady flow, including the influences of turbulence and combustion based on recent analysis and experiment. A series of two-dimensional and three-dimensional computer programs is being constructed. Ultimately, solutions (simulations) will be obtained for the conservation equations discussed in Section 2, but with careful account taken of chemical species. This seems clearly to be the proper direction for current and future numerical analysis of combustion instabilities. Only with full use of modern computational resources will it be possible to include the necessary details of the processes from injection to combustion.

4.2 Analyses Based on the Time Lag Model

By 'time lag model' we mean here the most common form, expressed by equation (3.14) for the unsteady conversion of liquid to gas. Crocco and Cheng (1956) examined various elaborations, including spatial variations of the sensitive time lag, but here we shall assume τ to be uniform everywhere and the same for all elements of injected propellant. Also we shall not distinguish between oxidizer and fuel. Both assumptions have been adopted in almost all applications, a notable exception being an analysis of chugging in which two time lags were introduced [Wenzel and Szuch (1965)].

Although some analysis has been done of nonlinear behavior with the time lag model [Sirignano and Crocco (1964); Mitchell, Crocco and Sirignano (1969); Mitchell and Crocco (1969)] by far most results, and all applications, have been worked out for linear behavior. To illustrate here we use the approximate analysis described in Sections 2.2 and 2.3. Although differences in detail will arise, the results will contain all the essential ideas discussed in previous works.

Broadly, the central idea is to use the formula (2.72) for the growth constant, α, *evaluated on the stability boundary*, so $\alpha = 0$. Those terms containing ω will of course depend on the interaction index, n, and the time lag, τ. If we assume that all other contributions to the formula are known, then the condition $\alpha = 0$ provides a relation between n and τ that must, within the approximations used, hold on the stability boundary.

There is no need to work out all details. Comparison of equations (2.90) and (2.96) shows that equation (2.72) will take the form

$$\alpha = C_1 \int \psi_n \hat{w}_l^{(r)} dV - C_2$$

where C_1, C_2 are constants. The constant C_2 contains the various effects of liquid/gas interactions, the nozzle, mean flow/acoustics interactions and damping devices. Now with $\hat{w}_l$ given by (3.13), its real part is $n(1-\cos\omega\tau)$, and for $\alpha = 0$, (4.1) gives

$$n(1-\cos\omega\tau) = \frac{C_1}{C_2 \int \bar{w}_l \psi_n^2 dV} = G_R \tag{4.1}$$

since E_n^2 is defined by (2.44)b and therefore becomes a common factor. The function G_R is supposed to be known, with value depending on the various parameters (geometrical, etc....) defining the system. Then equation (4.1) is the relation between n and τ referred to above.

Figure 4.4 shows the unstable regions defined by equation (4.1). This is a reproduction of Figure 4.2.2a, p.180, in an article prepared by Crocco [Harrje and Reardon (1972)]. The calculations carried out by Crocco were quite different from those summarized here, but the result has the same form, another illustration of the fact that, there is, in certain deep sense, only one 'linear stability problem'. Differences in detail among analyses arise only because representations of processes, and therefore characteristic parameters, may differ.

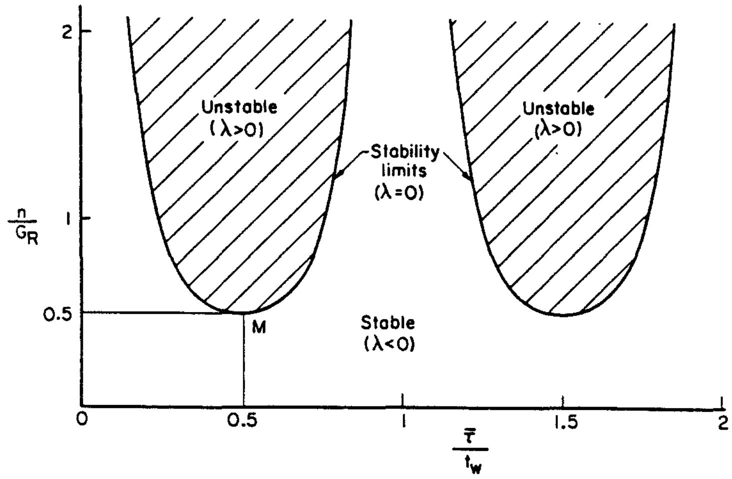


FIGURE 4.4

In this normalized form, Figure 4.4 is a kind of universal chart for the $n - \tau$ model. The multiple regions appear because of the factor $1 - \cos\omega\tau$ in (4.2) and correspond to the multiple peaks in the response, noted in respect to Figure 3.3. They are usually not physically realistic and are another reflection of limitations of the elementary time lag model. A formulation of the $n - \tau$ model showing only a single peak was reported by Crocco (1966) but need not be discussed here.

For applications, equation (4.1) and Figure 4.4 have always been unfolded to give plots of n versus τ; n and τ versus some characteristic parameter, such as the fuel/oxidizer ratio as in Figure 3.2 above; or in some cases the stability boundaries have been presented in terms of system variables, with n and τ parameters along the curves.

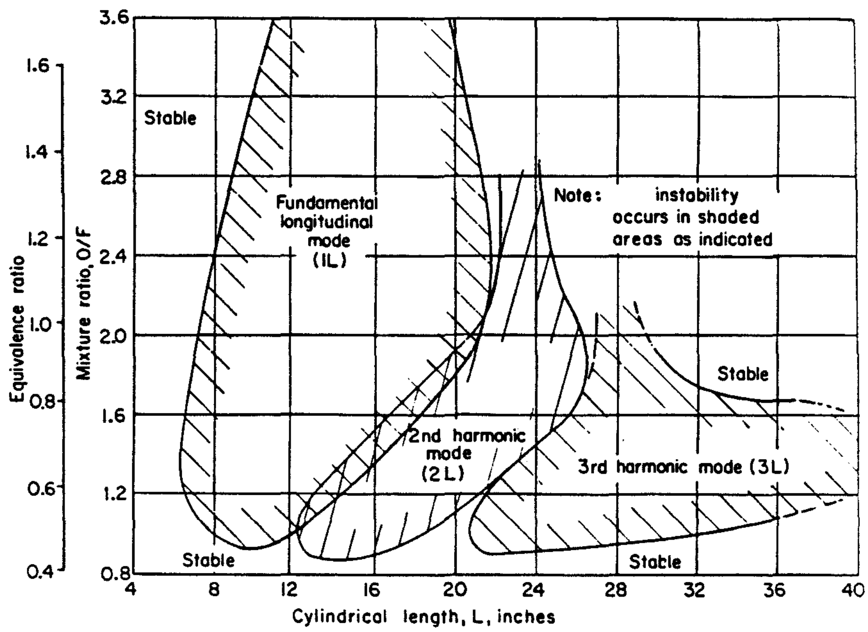


FIGURE 4.5

An example of the latter is reproduced in Figure 4.5 taken from Crocco, Grey and Harrje (1960). The preparation of this figure, and other quantitative results for n and τ, rests on extensive experimental work. In all cases the strategy is the same: the stability boundary, marking the transition between stable and unstable small amplitude waves, is located experimentally, as a function of the variables defining the instabilities. Then the theoretical relation (4.1) is used to compute the required values of n and τ along the boundary.

That procedure has been used successfully to interpret longitudinal modes [Crocco, Grey and Harrje (1960)] and transverse modes [Crocco, Harrje and Reardon (1962) and Reardon, Crocco and Harrje (1964)]. By applying the method to large numbers of tests, extensive correlations have been worked out for the interaction index and time lag as functions of geometric variables, injector design, propellant types and operating conditions. A brief summary has been given by Reardon in Harrje and Reardon (1972), pp.277-286. Figure 4.6 is an example, of results for n and τ determined from tests for storable hypergolic propellants, with various types of injectors.

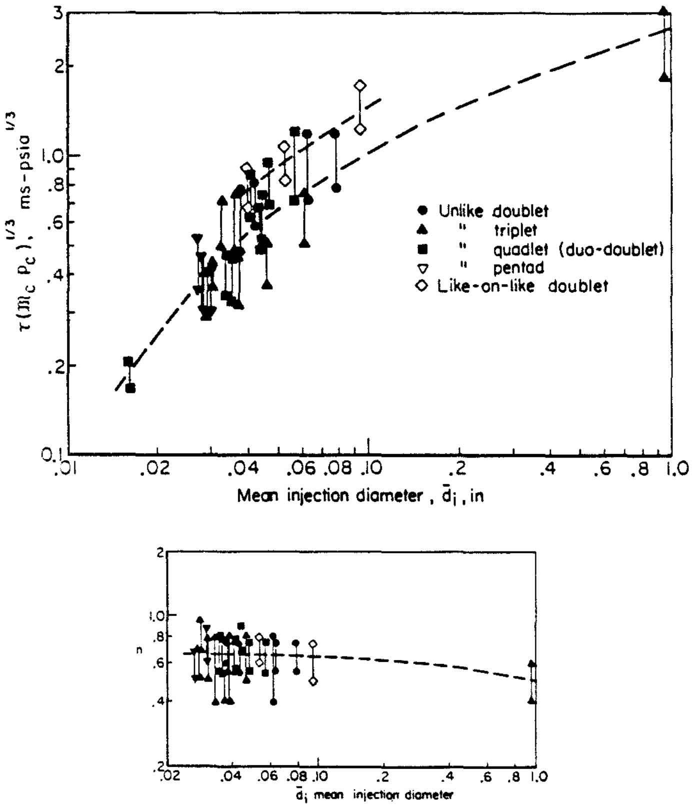


FIGURE 4.6

Having values of n and τ, one is now presumably in a position to return to the theoretical result for the growth constant and apply the results to designing new systems. An obvious shortcoming is that the data correlations can be assumed valid only for the systems actually tested. How far the results can be extrapolated cannot be known with great confidence. Nevertheless, this semi-empirical approach has been apparently used successfully both as a framework for correlating data and as an aid to design. It is essential in this procedure that the same theoretical result for the growth constant be used for correlating the data and for subsequent predictions. Otherwise, inconsistent and meaningless results will be obtained.

Although the ideas leading to the definitions of n and τ are appealing, the time lag model should be regarded truly as a framework for correlating data and not as a theory explaining fundamental mechanisms of combustion instabilities. With a different two-parameter represention of the unsteady process, the left hand side of (4.1) might have a different functional form, but the formula could be used in the same fashion to interpret stability boundaries. Only the forms of the correlations would be changed.

We must also note that because only the single formula for the growth constant (2.79) or (4.1) has been used, the method described above uses one equation to determine two unknowns (n, τ). Thus in practice, some difficulties may arise in obtaining consistent restuls. That trouble is avoided if, more correctly, both the real and imaginary parts, (2.75) and (2.76) of the complex wavenumber are used. In that event, measured values of the frequency are used and since (2.75) contains the imaginary part of the unsteady mass source (3.14), the two equations (2.75) and (2.76) have the form

$$\omega = \omega_n + C_3 \int \psi_n \hat{w}_l^{(i)} dV - C_4$$

$$\alpha = C_1 \int \psi_n \hat{w}_l^{(r)} dV - C_2$$

Hence with (3.14)

$$
\begin{aligned}
n\sin\omega\tau &= \frac{\omega-\omega_n+C_4}{C_3\int\psi_n^2\bar{w}_l dV}\\
n(1-\cos\omega\tau) &= \frac{C_2}{C_3\int\psi_n^2\bar{w}_l dV}
\end{aligned}
\qquad (4.2)a,b
$$

The left hand sides could equally be regarded, within a multiplier, as the real and imaginary parts of the mass source,

$$
\begin{aligned}
\hat{w}_l^{(r)} &= \bar{w}_l n(1-\cos\omega\tau)\\
\hat{w}_l^{(i)} &= \bar{w}_l n\sin\omega\tau
\end{aligned}
\qquad (4.4)a,b
$$

and correlations could be done with $\hat{w}_r = n(1-\cos\omega\tau)$ and $\hat{w}_i = n\sin\omega\tau$ instead of $(n,\ \tau)$. Thus, even though the heuristic argument leading to $\hat{w}_l$ in the form (4.4) is based on a time lag associated with motions of the propellant (a Lagrangian view), the end result is equivalent to a purely Eulerian representation of local combustion process. The time lag associated with motions in space can be reinterpreted as a phase lag in time at a fixed location in space.

The formulas (4.3)a,b have been deduced from the approximate analysis discussed in Section 2 and therefore have a particularly simple form. Although it is true that a linear analysis will always produce two formulas, for the real and imaginary parts of complex wavenumber, the forms may be wildly different in detail, depending in the method of solution. Crocco, Grey and Harrje (1960) solved their differential equations directly, a method used later by Crocco, Harrje and Reardon (1962) and Reardon, Crocco and Harrje (1964) to study transverse modes.

The time lag models of the combustion process has been used also in analysis of nonlinear behavior, both for longitudinal oscillations [Sirignano and Crocco (1964); Mitchell, Crocco and Sirignano (1969); Crocco and Mitchell (1969)] and for transverse oscillations [Zinn (1966); Zinn and Savell (1968)]. In those and other works discussed in the following two sections, either n and τ are assigned values; or the unsteady behavior is studied as a function of n and τ. Thus, sufficient experimental data had been gained to support the time lag model that it could be used in a general fashion for theoretical work. However, remarks above emphasize that this practice really amounts to using any combustion response having real and imaginary parts related to n and τ by equations (4.4)a,b. Expressing results and interpreting behavior in terms of n and τ carries no uniqueness.

4.3 Use of Green's Function to Compute Linear Stability

Beginning in the late 1960's, Mitchell and co-workers (1972, 1975, 1979, 1984, 1985, 1986, 1987) introduced use of a Green's function to produce an integral equation solved by interaction. Good summaries of this approach have been given by Mitchell and Eckert (1979) and by Mitchell (1984). The time lag model was assumed to represent the combustion response, the reason for mentioning those works here. Apart from that matter, the method is attractive for analying complications such as baffles and resonators that are quite awkward to handle when differential equations are used.

The approximate results (2.63) and (2.64)a,b are in fact the first order terms in an iterative procedure like that used by Mitchell. They are also, as the discussion in Section 2 showed, found by applying Galerkin's method to the problem of linear stability of sinusoidal disturbances. We therefore end this section with brief resume of those two methods.

With the introduction of Green's function, a differential equation is converted to an integral equation that may be conveniently solved by iteration. The general theory, with applications, is thoroughly described by Morse and Feshbach (1953). Culick (1963) first applied the method to problems of combustion instabilities; it was later adopted by Oberg and Kulava (1969, 1971) to study acoustic liners.

Mitchell and his co-workers (e.g., 1969, 1979, 1984, 1985) have chosen to express the acoustic field terms of a velocity potential, an unneccessary limitation to irrotational flows, as the developments in Section 2 demonstrate. We can illustrate the essentials of his method with the formulation derived here. The problem is to solve the inhomogeneous equation (2.31) for the pressure, subject to the boundary condition (2.33). For steady waves, all dependent variable are proportional to $e^{i\bar{a}kt}$: $p' = \hat{p}e^{i\bar{a}kt}$ and equation (2.31) is

$$\nabla^2\hat{p} + k^2\hat{p} = \hat{h} \qquad (4.5)$$

Define the Green's function satisfying the same equation as $\hat{p}$ but with a unit source at position $\vec{r}_0$ and homogeneous boundary conditions

$$
\begin{aligned}
\nabla^2 G(\vec{r}\mid\vec{r}_0) + k^2 G(\vec{r}\mid\vec{r}_0) &= \delta(\vec{r}-\vec{r}_0)\\
\hat{n}\cdot\nabla G(\vec{r}\mid\vec{r}_0) &= 0
\end{aligned}
\qquad (4.6)a,b
$$

Now multiply (4.5) by $G(\vec{r}\mid\vec{r}_0)$, (4.6) by $\hat{p}$, integrate over the chamber, use Green's theorem and insert the boundary conditions (2.33) and (4.6)b to find* the 'solution' for $\hat{p}$:

$$\hat{p} = \int G(\vec{r}\mid\vec{r}_0)\hat{h}(\vec{r}_0)dV_0 + \oiint G(\vec{r}\mid\vec{r}_0)\hat{f}(\vec{r}_0)dS_0 \qquad (4.7)$$

* In these manipulations, the exchange of variables is made, $\vec{r}\leftrightarrow\vec{r}_0$, and the reciprocity property of G is used: $G(\vec{r}\mid\vec{r}_0) = G(\vec{r}_0\mid\vec{r})$.

It is convenient to express $G(\vec{r} \mid \vec{r}_0)$ as an expansion in the normal modes ψ_n of the system, satisfying (2.35)a,b. That is, assume the form

$$G(\vec{r} \mid \vec{r}_0) = \sum A_n(\vec{r}_0)\psi_n(\vec{r}_0)$$

Substitution in (4.6)a, multiplication by ψ_n, and integration over the chamber gives

$$A_n = \frac{\psi_n(\vec{r}_0)}{k^2 - k_n^2}$$

so the Green's function is

$$G(\vec{r} \mid \vec{r}_0) = \sum \frac{\psi_m(\vec{r}_0)}{k^2 - k_m^2}\psi_m(\vec{r}) \tag{4.8}$$

Now insert (4.8) in (4.7) and assume that we are examining that mode which, when the perturbations vanish, reduces to the nth classical acoustic mode. Split that term from the remainder of the expansion and apply the normalization $\hat{p} \to \psi_n$ in the limit $\hat{h} = \hat{f} = 0$. Those operations give the formulas for $\hat{p}$ and k^2.

$$\hat{p}(\vec{r}) = \psi_n(\vec{r}) + \sum_{m \neq n} \frac{\psi_m(\vec{r})}{k^2 - k_n^2}\{\int \psi_n(\vec{r}_0)\hat{h}(\vec{r}_0)dV_0 + \oiint \psi_n(\vec{r}_0)\hat{f}(\vec{r}_0)dS_0\} \tag{4.9}$$

$$k^2 = k_n^2 + \frac{1}{E_n^2}\{\int \psi_n(\vec{r}_0)\hat{h}(\vec{r}_0)dV_0 + \oiint \psi_n(\vec{r}_0)\hat{f}(\vec{r}_0)dS_0\} \tag{4.10}$$

Equation (4.10) is exactly equation (2.63), except for the factor $\bar{p}\hat{\eta}_n$ which appeared because to obtain (2.63), equation (2.59) was used, $p' = \bar{p}\hat{\eta}_n e^{akt}\psi_n$. (i.e. here $\bar{p}\hat{\eta}_n = 1$, allowed because there is an arbitrary factor in the normalization; thus $\hat{p} \to \psi_n$ in (4.9) while in (2.59), $\hat{p} = \bar{p}\hat{\eta}_n\psi_n$).

Equation (4.9) shows explicitly the perturbations of the classical mode shape, ψ_n, giving the actual mode shape $\hat{p}$. Because $\hat{h}$ and $\hat{f}$ depend on p' and $\vec{u}'$ (i.e. $\hat{p}$ and $\hat{\vec{u}}$)., equation (4.9) is an integral equation for $\hat{p}$ and of course k^2 cannot be calculated with (4.9) until $\hat{p}$ and $\hat{(\vec{r})}$ are known. In the approximate method discussed in Section 2, we simply set $\hat{p} \approx \psi_n$, $\hat{\vec{u}} \approx i(\bar{a}/\bar{\gamma}k_n)\nabla\psi_n$ and use (4.10) directly. To proceed further, the integral equation (4.9) must be solved. That is where most of the labor in Mitchell's work is expended. He solved the equation numerically using an iteration procedure. We shall not discuss the details further.

We must note, however, that when an iteration procedure is used, care must be exercised that *all* terms of consistent order are retained. The small parameter here is a Mach number, $\bar{M}_r$, characterizing the average flow. Corrections to $\hat{p} = \psi_n$ are thus of order $\bar{M}_r$; that is, if the successive steps in the iteration are labeled $\hat{p}^{(i)}$, the procedure gives:

$$\begin{aligned}
\hat{p}^{(0)} &= \psi_n \\
\hat{p}^{(1)} &= \psi_n + \bar{M}_r\phi^{(1)} \\
\hat{p}^{(2)} &= \psi_n + \bar{M}_r\phi^{(1)} + \bar{M}_r^2\phi^{(2)} \qquad \text{....etc}
\end{aligned}$$

The functions h and f are constructed by expansion of the primitive conservation equations according to remarks in Section 2.2. If they are not carried to order higher than $\bar{M}_r$, then it is *not* correct to proceed beyond the zeroth approximation $\hat{p}^{(0)} = \psi_n$ of the mode shape to compute k^2 with (4.10). That is why the approximate method discussed in Sections 2.2-2.4 was not carried to higher order. To carry $\hat{p}$ and $\hat{\vec{u}}$ to higher order than allowed by the construction of $\hat{h}$ and $\hat{f}$ may yield misleading and incorrect results.

Properly used, the formulation based in Green's function is a powerful method to obtain results for complicated problems. For practical purposes it is much superior to solution of the differential equations owing to the ease with which arbitrary boundary conditions and volumetric sources can be accommodated. In that respect, this method has the same advantages as the approximate method discussed earlier, except that use of Green's function as described here and in Mitchell's work is strictly limited to linear problems. On the other hand, the approximate method given earlier is a form of Galerkin's method and can be used to analyze nonlinear behavior.

4.4 Application of Galerkin's Method

Zinn and Powell (1968, 1970) first published work applying a form of Galerkin's method to combustion instabilities. The chief difference from the classical Galerkin's method was addition of a recipe for handling the inhomogeneous boundary conditions. In a subsequent series of papers, the method was used to investigate both longitudinal and transverse instabilities, with main emphasis on special aspects of nonlinear behavior [Powell and Zinn (1971, 1974); Lores and Zinn (1972, 1973)].

Because the purpose of this review is mainly to cover characteristics of linear stability, space does not permit a survey of nonlinear problems. That is not intended to imply that nonlinear behavior is unimportant. On the contrary, there is much to be learned both theoretically and for applications to design. As implied by the discussion of the approximate method in Section 2, this author believes that the most promising method on both counts is some form of Galerkin's method.

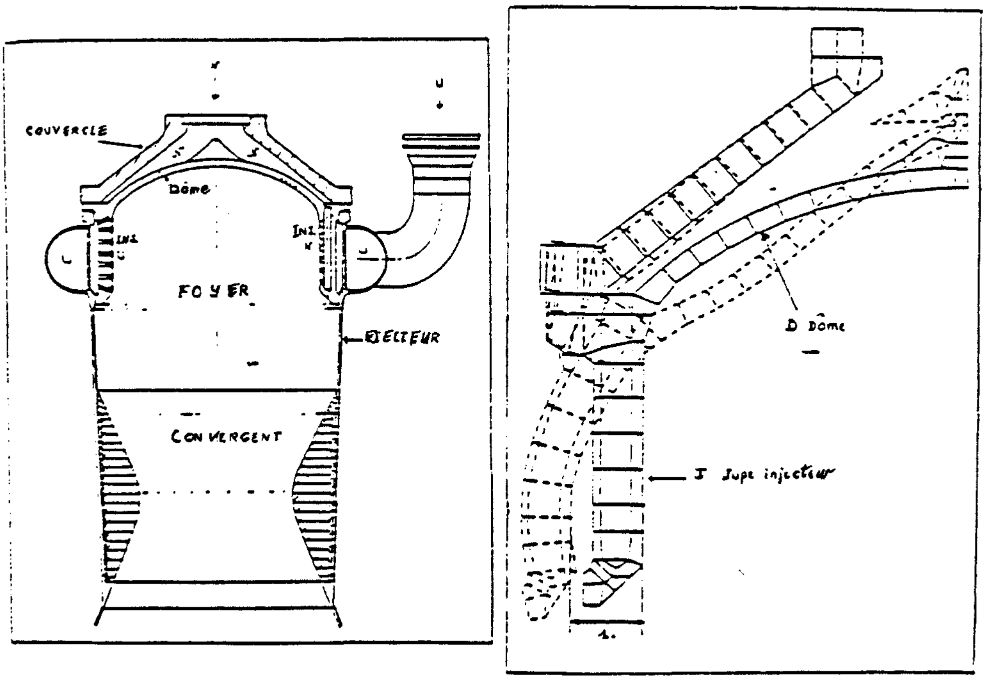
5. REMARKS ON INSTABILITIES IN THE THREE TYPES OF SYSTEMS

Most of the preceding discussion has been concerned with matters common to all three types of systems. Much of the work was in fact carried out originally for liquid-fueled rockets, the strongest motivation being applications to engines intended for the Apollo vehicle. Some of the ideas and methods developed for liquid rockets have been modified or extended for analysis of combustion instabilities in augmentors and ramjets. Moreover, there are special problems peculiar to the different systems themselves. We therefore examine now those particular matters.

5.1 Combustion Instabilities in Liquid Rockets

We have covered almost all the basic material related to liquid rockets. Little work was done in the problem in the later 1970's. With the flight failure of an Ariane vehicle due to combustion instability in a first stage Viking motor, a comprehensive research program was initiated in France in 1981. Most of the available reports of that work have already been referred to and little more needs to be added here.

Within the present context, the most important parts of the French work are the experimental and analytical efforts to characterize the liquid spray; and the extensive numerical simulations of unsteady motions, incorporating the results obtained for the propellant sprays. The problem causing the failure involved coupling between the pressure oscillations in the chamber and structural vibrations of the injector which is placed in the lateral boundary [Figure 5.1 taken from Souchier, Lemoine and Dorville (1982)] Figure 5.1(b) shows the computed distortion of the injector plane. As a result, the fuel and oxidizer jets were shaken, causing (apparently) perturbations of the distribution and phase of the energy release, therby closing the loop and making possible self-excited motions.



(a) (b)

FIGURE 5.1

Such effects on the injection processes have long been known to be a potential cause of instabilities [Levine (1965); Harrje and Reardon (1972)] but they have yet to be well-characterized. They are likely to be particularly important in cases when the amplitudes of motion are large. It is quite possible that the *forms* of the representation of the unsteady sources of mass and energy are strongly dependent on the amplitudes of motion as well as on the hardware design. Such behavior is far outside any successes of the time lag model and is likely to remain so. Careful experimental work is essential to clarify the situation.

During the past several years, serious interest in developing new liquid-fueled rockets has grown in the U.S. primarily for use in proposed heavy lift launch vehicles. Because of their high densities and good performance, liquid oxygen and hydrocarbon fuels are being considered as propellants. In particular, methane has been selected by the NASA Lewis Research Center is the favored fuel. As a result, studies of combustion instabilities are in progress at the Aerojet TechSystems Company and at the Rocketdyne Division of Rockwell Internation.

Rocketdyne has designed and fabricated two engines, for the Lewis Research Center (LeRC) and for the Marshall Space Flight Center(MSFC). Both use LOX/methane and have identical thrust chambers but different injectors. The MFSC engine has an acoustic resonator; the LeRC engine has no damping device. A small number of firings directed to determining stability characteristics have been completed [Jensen, Dodson and Trueblood (1988); Philippart and Moser (1988)].

Development of computer programs for analysis of instabilities is in progress and only incomplete reports have been used. [Fang (1984, 1987); Fang and Jones (1987); Mitchell, Howell and Fang (1987); Nguyen (1988)]. The program IFAR (Injector Face Acoustic Resonator) has been in existence for some years; the time lag model was used to represent the combustion process. That program has been revised and modified for application to both retangular and axisymmetric chambers to become HIFI (High Frequency Intrinsic Stability) [Nguyen (1988)].

These computer programs are being used in the manner described in Section 4.2. With all other variables and parameters specified, the values of n and τ are calculated on the stability boundary. Then to predict whether the engine is stable or not, the values of n and τ must be determined. Traditionally this has been done with correlations exists for injectors using hydrocarbon fuels, so as part of their work the group at Aerojet has been performing sub-scale tests and carrying out analysis of the injector response [Muss and Pieper (1987); Nguyen and Muss (1987)]. The analysis and tests is intended to provide correlations of n and τ for the injector with those on the stability boundary calculated with the analyses cited above.

Aerojet is pursuing a program combining analysis, sub-scale tests using both retangular and axisymmetric chambers prior to full- scale firings. The chief purpose is to provide as certain as possible basis for confidently predicting the stability of the large engines, thereby reducing development costs. This program has been recently described by Muss and Pieper (1988).

Philippart (1987) and Philippart and Moser (1988) have reported comparisons of predictions of the sort mentioned above, with firings of the two Rocketdyne engines. One operating condition was examined for which the LeRC engine was stable and the MSFC engine was unstable. Three calculations of the stability boundary in the $n-\tau$ plane were done, using the programs IFAR, HIFI and a modified from (NDORC) of Mitchell and Eckert's (1979) MODULE. Figure 5.2, taken from Philippart and Moser, shows the results obtained with HIFI for the two engines. Results obtained with the other two programs

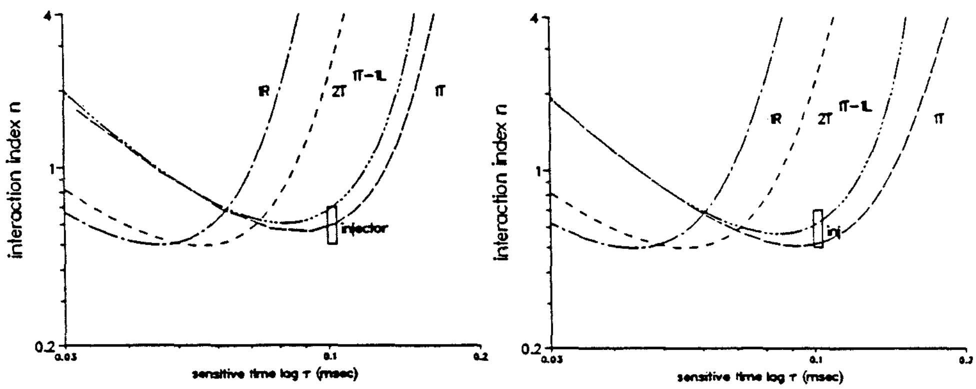


Fig. 9: HIFI MSFC Analysis (1/4 wave axial cavity)

Fig. 5: HIFI Analysis LeRC Engine (no cavity)

FIGURE 5.2

differ in details that are unimportant here. Also shown as filled boxes are estimates of the 'combustor response' (i.e. the values of $n-\tau$) based on correlations for LOX/hydrogen injectors. Apparently the predictions of the three codes agreed fairly well. However, there are uncertainities owing to differences between the codes; a significant distinction is that IFAR and HIFI assume that combustion is concentrated in a transverse plane, while MODULE is written for distributed combustion. Comparison with the test data is ambiguous and must be viewed as preliminary because the true characteristics of the injectors are unknown.

Jensen, Dodson and Trueblood (1988) have given an early progress report in their tests with the LeRDCF engine. They have measured growth rates (α) and, using the MODULE program, have inferred the necessary values of n and τ. Two examples are shown in Figure 5.3. The striking result is that the values of the interaction index are found to be considerably greater than those computed by Philippart and Moser and those provided previous correlations of data. It is impossible at this point to determine the cause for these differences.

Also at Rocketdyne some interesting work is in progress to analyze the characteristics of sprays vaporizing and burning under steady conditions [Liang et al (1986, 1987)]. The calculations are being done for various injector types placed in chambers, with provision for computing the internal flow field. When extended to cover transient motions, this work is potentially an important contribution to analysis of combustion instabilities. Indeed, it appears that one of the most important outstanding problems in the subject is the production of the liquid drops; unsteady spray combustion; and incorporation of the results in a complete formation allowing realistic numerical simulations.

5.1.1 Pogo Instabilities

The problem of low frequency POGO instabilities is well- documented and understood. Due to the POGO instability in the Apollo vehicle, it is also probably the best known among people otherwise not familiar with combustion instabilities.

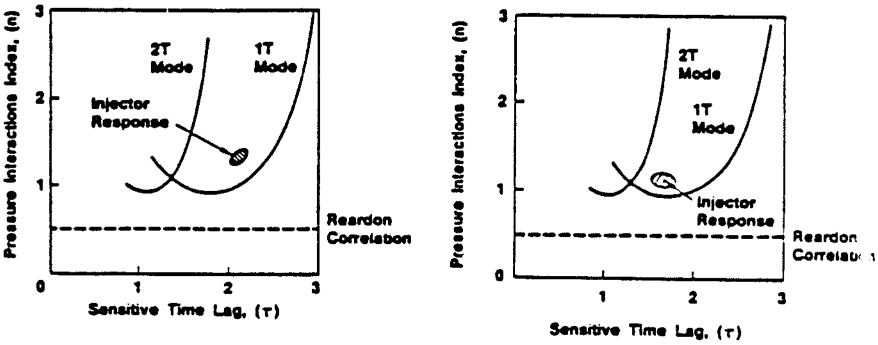


FIGURE 5.3

Low frequency instabilities ('chugging') arise due to coupling between the fluid dynamics in the combustion chamber, and the propellant supply system. They are perhaps the first sort of combustion instability definitely identified and analyzed for liquid rockets (see the remarks at the beginning of Section 3.1). POGO instabilities involve the further complication of coupling between the propulsion system and the structure of the vehicle. The low frequency structural vibrations are the origin of the name, by analogy with the motions of a POGO stick.

During the 1960's, the POGO instability received much attention as a serious problem in several vehicles including the Thor, Atlas, and Titan vehicles. Rubin (1966) has given a clear brief summary, including particular emphasis on pump cavitation and wave propagation in the propellant feed lines. Those are matters often overlooked by those concerned with motions in the combustion chamber. Yet they provide significant contributions to time lags in the system.

More recent work in France has been reported by Dordain, Lourme and Estoureig (1974) for the Europa II and Diamant B vehicles; and by Ordonneau (1986) for the Ariane.

5.2 Combustion Instabilities in Thrust Augmentors

It has long been standard practice that acoustic liners are integral parts of thrust augmentors. Since high frequency or 'screech' instabilities were first encountered as a serious problem in the late 1940's and early 1950's, liners have been developed largely by trial and error to act as passive control devices designed to suppress the oscillations. The staff of the Lewis Laboratory (1954) compiled most of the existing data and performed some tests to provide a basis for general guidelines for design; Harp et al. (1954) reported the results of extensive tests, also at Lewis Laboratory. Of the methods investigated to solve the problem, including baffles and vanes as well as adjusting the distribution of injectors, perforated liners worked best. Groups at Pratt and Whitney Aircraft and the United Aircraft Research Laboratory had already tried Helmholtz resonators and in 1953 demonstrated the first successful use of perforated liners is a full-scale afterburner on a J57. The physical basis for the success of liners is explained in Section 6.1.

Despite several attempts to develope analytical methods and a more quantitative basis for design, treatment of combustion instabilities in thrust augmentors has remained almost entirely an empirical matter. Kenworthy, Woltmann and Corley (1974) reported the results of an experimental program devoted to studying screech instabilities in 3 different designs of augmentors. The report also contains analysis used to correlate data and to provide some quidance for design of acoustic liners. This seems to be the last extended work on high frequency instabilities in full-scale augmentors; the mechanisms remain obscure. Chamberlain (1983) has given the most recent status report: little has changed in the past decade, it seems.

Perforated liners effectively attenuate the high frequency oscillations related to radial and tangential acoustic nodes. Low frequency instabilities, often called 'rumble', tend to be more troublesome. Liners are ineffective at low frequencies and the problem of rumble is solved or reduced in practice by careful control and coordination of the distribution of injected fuel and the nozzle opening. It's a costly process to develop the system, inevitably requiring several designs of the injection system and flameholders, and expensive full-scale tests in altitude simulation test facilities.

The problem of combustion instabilities in thrust augmentors is arguably more difficult then that in liquids rockets for at least two reasons: the processes involved in flame stabilizations are sensitive to pressure and velocity fluctuations; and the device is usually required to perform over a wider range of operating conditions. The first explains the importance of injector and flameholder design. As a result of the second, the high and low frequency instabilities are typically found in different regions of the flight envelope. Figure 5.4, reproduced from the excellent summary (as of 1971) by Bonnell, Marshall and Riecke (1971) illustrates the point.

Instabilities in the lower frequency range became increasingly troublesome with the development of turbofan engines, a consequence of the geometry (see Figure 5.5 taken from Bonnell, Marshall and Rieche (1971) and Figure 5.6 taken from Zukoski (1985). In the pure turbojet, the fluctuations may propagate upstream past the turbine disk but the turbine generally seems to act as a good reflector. In fan engines, it is common that the entire length of the fan duct participates in the oscillations, reducing the frequencies sometimes as low as 50 Hz. See Nicholson and Radcliffe (1953) for an early report of very low frequency oscillations; observations in turbofans have been discussed by Bonnell, Marshall and Riecke (1971); Mach (1971); Ernst (1976); Underwood et al (1977); and Cullom and Johnsen (1979). Figure 5.7 reproduces power spectral densities taken from turbofan augmentors [Bonnell, Marshall and Riecke (1971)]. Because of the rotating parts, spectra of the acoustic field in gas turbine

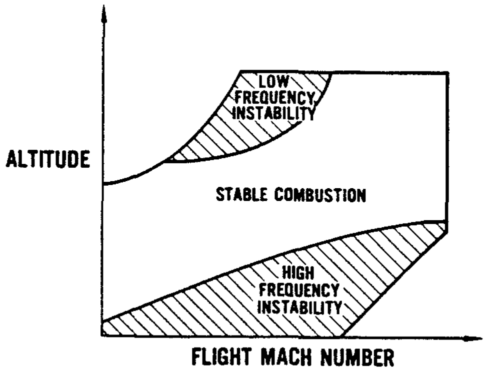


FIGURE 5.4

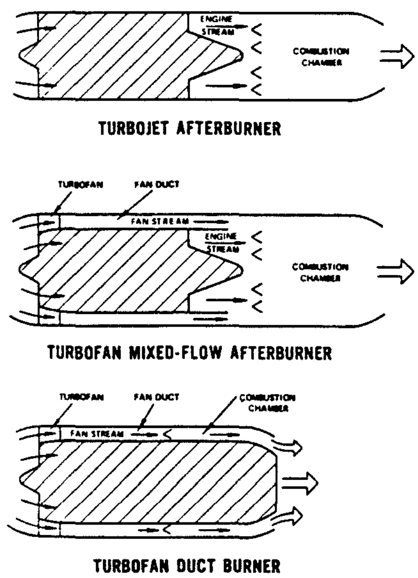


FIGURE 5.5

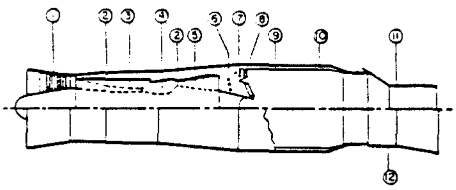
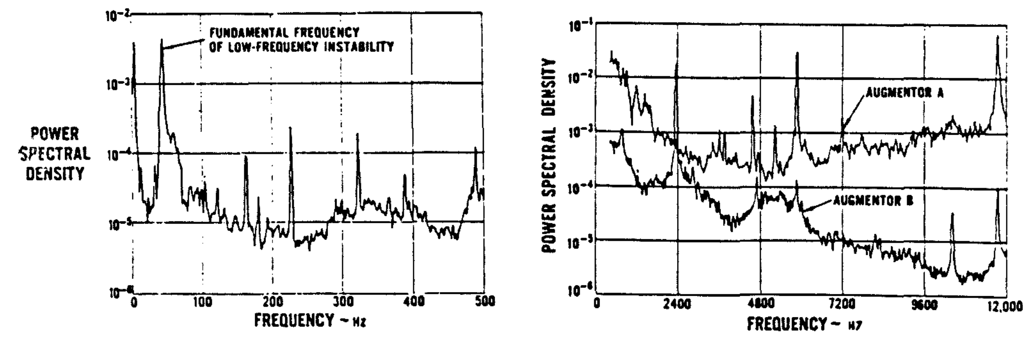


Figure 21.0.2 Pratt and Whitney F100-PW-100 augmented turbofan engine. (1) Three stage fan; (2) bypass air duct; core engine compressor (3), burner (4), and turbine (5); (6) fuel injectors for core engine gas stream; (7) fuel injectors for bypass air stream; (8) flame stabilizer for afterburner; (9) perforated afterburner liner; (10) afterburner case; nozzle closed to minimum area (11) and opened to maximum area (12).

FIGURE 5.6

engines tend to exhibit a greater variety of discrete oscillations than do those for liquid rockets. The peaks at the higher frequencies in Figure 5.7(a) are 'screech' modes.

The combustion processes in an augmentor differ in several fundamental respects from those in a liquid rocket. Only fuel is injected as liquid; the oxidizer is unburnt oxygen in the fuel-lean flows from the bypass and the core engine. There are no impinging fuel and oxidizer liquid streams, but the formation of drops and



(a) (b)

FIGURE 5.7

vaporization of the fuel must obviously occur. Normally, it is intended that the fuel drops should be entirely vaporized prior to ignition in the core flow so burning occurs in the fuel/air gaseous mixture. Because the flame propagation speed is less than the flow speed, a continuous source of ignition is required, normally supplied by the wake of a bluff body, the flame holder. Clearly, the performance of such a system depends not only on the flow conditions and physical properties of the fuel but also very strongly on the geometry of the injectors and flame holders. In the cooler bypass flow, vaporization is not completed upstream and liquid impinges on the flameholders; the liquid layer then vaporizes. Zukoski (1985) has provided a thorough and readable discussion of the combustion processes in afterburners. Figure 5.8 taken from his article, illustrates the general features of the flow in the vicinity of various flame holders.

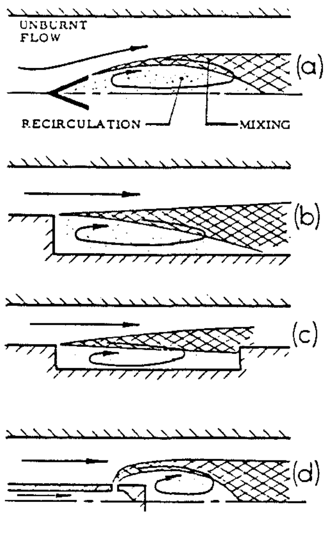


FIGURE 5.8

According to the preceding remarks, it appears unlikely that vaporization of the fuel droplets is a dominant mechanism for combustion instabilities in augmentors. Nevertheless, it is certainly quite possible that interaction of the acoustic field with the injection system could produce fluctuations of the fuel flow and hence subsequent fluctuations of fuel/oxidizer ratio and heat release in combustion. The process might be modeled in terms of a time lag but there seems to be no treatment of this sort in the published literature.

One would suspect that processes associated with the flame holder may dominate. That view is generally supported by practical experience with the strong effects of flame holder design on instabilities. We have discussed in Section 3.4 the mechanism based on vortex shedding and combustion suggested by Rogers and Marble (1956). Their argument is persuasive and there has never been evidence disproving that process as a possible mechanism. Theoretical developments and the necessary laboratory tests have not been carried far enough to incorporate the proposal in an analysis suitable for general design work with arbitrary geometries.

Russell, Brant and Ernst (1978), have worked out a one-dimensional analysis of instabilities in augmentors; the work is also discussed by Underhill et al (1977). Broadly the analysis represents the acoustic field as a synthesis of up and downstream traveling acoustic waves, and entropy waves, as in the example discussed earlier here in Section 3.3. The unsteady heat sources are derived as models of mixing and combustion in the wakes of the flame holders. Bypass and core flows are treated separately and superposed in parallel. It's a linear analysis; the equations for the time-dependent variables are solved by applying the Laplace transform. Conditions for stability are determined by applying the Nyguist criterion. It is difficult to understand all details of the analysis from the available (abbreviated) description. Although some success was evidently achieved with this work, it seems not to have been widely applied. Moreover, the results are mainly in a computer program which has not furthered general understanding of the problem although it may have been useful in treating particular cases.

Over a period of several years Dix and Smith and co-workers developed an analysis based on the formulation published by Culick (1963) for liquid rockets. See Dix and Smith (1971) and references cited there for a description of the work. Although that sort of approach should be useful in treating augmentors, that analysis has also not be widely applied. It is important to note that while their linear analysis is correct, Dix and Smith committed some basic errors in trying to extend their calculations to nonlinear behavior. The results they have reported for the influences of the amplitudes of oscillations are wrong.

The most recent work in instabilities in augmentors seems to be that reported by Dowling and Bloxsidge (1984); Langhorne (1988) and Bloxsidge, Dowling, and Langhorne (1988) at Cambridge University. Laboratory experiments were done in a configuration intended, roughly, to represent a longitudinal segment of an augmentor (Figure 5.9). A flame stabilized on a single vee gutter in a duct supplied with premixed gaseous reactants entering through a choked nozzle. With modifications

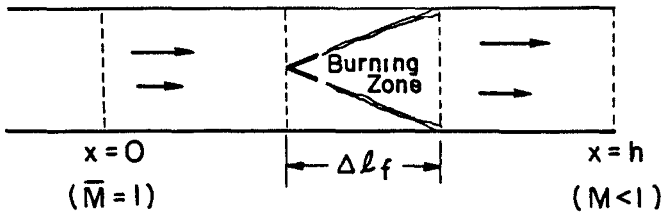


FIGURE 5.9

that may have significant influences on the unsteady behavior, this is a common configuration [Kaskan and Noreen (1954); Hegde et al (1986, 1987, 1988); Reuter et al (1988)]. The work by Kaskan and Noreen has already been described briefly (see Figure 3.5). They worked with a flame stabilized on a vee gutter whereas Hegde, Reuter, and their co-workers at Georgia Tech have been using one or two wires to stabilize the burning, although their work has presumably been directed to applications in ramjet engines and is mentioned further in Section 5.3.2.

All of these works are concerned in some broad sense with flames and flame instabilities the instabilities are ultimately manifested as vortices, so the mechanism for the instabilities discussed here could be classified as vortex shedding and combustion, as discussed in Section 3.4. Another similarity among these works is the use of electromagnetic radiation to identify the heat released by combustion products.

Langhorne (1988) concludes that for the device shown in Figure 5.9, two types of coupling exist between the burning processes and pressure oscillations. The transition between the two occurred in a narrow range of stoichiometric ratio around 0.65. For $\phi < 0.65$ a convective wave of entropy or spots of high temperature appeared to propagate well downstream of the flame holder. With increasing ϕ, that convective aspect seemed to have been confined to a short length and in the remainder of the duct the heat release (as measured by radiation from C_2 and $C(t)$ seemed to be in phase with the pressure oscillation. No results of flow visualization are available to confirm the behavior directly, but vortex shedding apparently may be involved.

At least partly as a result of the two kinds of coupling, two frequencies of instability were observed with larger amplitudes produced at higher stoichiometric ratios. Bloxsidge, Dowling, and Langhorne (1988) have worked out an interesting and useful one-dimensional analysis to interpret their observations.

Certain aspects of the Cambridge results are similar to those reported by Heitor, Taylor, and Whitelaw (1984). Sivesegaram and Whitelaw (1987) and by the Georgia Tech groups. The reasons for the similarities and reconciliation of differences are not known; a sufficiently general analysis has not been constructed to accommodate all the results on a common basis. There is little doubt that more than one mechanism may act, one or another dominant under different conditions. Because this is a relatively well-defined situation, (a premixed flame in a duct) the problem merits further attention both experimentally and theoretically to bring clearer understanding of the behavior.

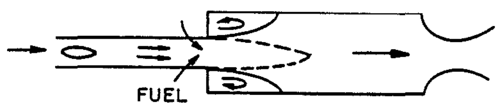
5.3 Combustion Instabilities in Ramjet Engines

During the past decade, substantially more attention has been paid to combustion instabilities in ramjet engines then can be discussed in this paper. Much progress has been made but several essential problems remain unsolved, mainly associated with the conversion of liquid fuel to gaseous reactants; coupling between combustion processes and the unsteady motions; and the inlet/diffuser.

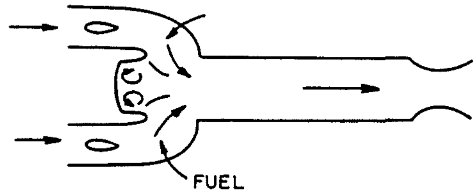
Sketches of two typical configurations are shown in Figure 5.10. Most contemporary liquid-fueled ramjets are "integral ramjet engines". The combustion chamber is initially filled with solid propellant that is burnt to boost the vehicle to supersonic speed. Liquid propellant is injected upstream of the region where the flow area abruptly increases at the "dump plane". Flame stabilization is achieved through continuous ignition by the hot combustion products in the recirculation zone. In some designs additional bluff body flame holders may also be used; and occasionally continuous burning of a pilot light may be required.

Zukoski (1985) has given a thorough discussion of steady flame stabilitization in thrust augmentors. Much of that material applies with virtually no change to the corresponding problems in ramjet engines. The presence of the rearward facing step and the sensitivity of shear layers and recirculation zones to fluctuations in the flow are major factors in the problem of combustion instabilities in ramjet engines.

Much of the material we have covered for liquid-fueled rockets and thrust augmentors is relevant as well to ramjet engines. There are, however, several distinguishing features. First, unlike the case for liquid rockets



COAXIAL DUMP COMBUSTOR



SIDE DUMP COMBUSTOR

FIGURE 5.10

but similar to that for afterburners, spray combustion seems a lesser issue. Although the published evidence is perhaps not wholly conclusive, [see,e.g, Edelman (1981); Edelman et al (1981); Harsha et al (1981)], it appears that in operating engines, the liquid droplets are largely vaporized before the flow reaches the zones of flame stabilization and combustion. Hence the processes in those regions involve mostly gaseous reactants, a great simplification for carrying out research on combustion instabilities; very little experimental work has been done recently in the coupling between spray combustion and unsteady motions. Laboratory tests have for the most part used gaseous fuels.

That is not to say that transient processes of droplet heating and vaporization are unimportant, for they are surely influential in arranging the distribution of fuel over the plane at the entrance to the combustor. But there is no operational or experimental evidence to support the proposal by Tong and Sirignano (1986a, 1986b, 1987) that the unsteady conversion of liquid to vapor is a potential mechanism for instabilities. This matter has already been discussed in Section 3.2 with the chief conclusion that if all processes *except combustion* are accounted for, the presence of evaporating liquid drops is a stabilizing influence on unsteady motions. We shall not consider further problems associated with injection, atomization and vaporization. However, it is true that insufficient attention has been paid to the distribution of fuel/oxidizer ratio in the flow. Little is known of the details, either theoretically or experimentally; yet laboratory tests [e.g. Schadow et al (1987)] have shown that the distribution of fuel can have a substantial effect on instabilities , a fact that has long been known qualitatively from experience gained in engine development. [Rogers (1980a, 1980b); Grenleski et al (1977)]. There seems to be no evidence of coupling between oscillations in the flow and the fuel supply system. Thus no oscillations have been observed in ramjets corresponding to 'chugging' or POGO instabilities in liquid rockets.

5.3.1 Unsteady Behavior of the Inlet/Diffuser

So far as combustion instabilities are concerned, the principal feature distinguishing ramjet engines from liquid-fuel rockets and afterburners is the inlet/diffuser. Within the inlet a system of shock waves exists to provide the mass flow and stagnation conditions demanded by the conditions set in the combustion chamber and exhaust nozzle. Under normal operating conditions the shocks are located downstream of the geometric throat in the expanding supersonic flow. The position of the shock depends chiefly in the stagnation pressure in the combustion chamber; increasing the stagnation pressure causes the shocks to move upstream where the Mach number and therefore loss of stagnation pressure are less. It is this sensitivity of the flow in the inlet to pressure changes downstream that has caused longitudinal oscillations to be such a serious concern in ramjet engines. In the late 1970's [Hall (1978, 1980); Rogers (1980a, 1980b)] first qualitative and later limited quantitative relations were established between the amplitudes of pressure oscillations and the loss of dynamic pressure margin.

Since those early works, extensive tests by Sajben and co-workers [Chen, Sajben and Kroutil (1979); Sajben, Bogan and Kroutil (1984); Bogan, Sajben and Kroutil (1983a, 1983b)] have shown that the unsteady behavior is greatly more complicated due to flow separation and instability of shear layers. High speed schlieren pictures [see also Schadow et al (1981)] have shown large shock oscillations as well as the formation of vortex structures. Although computations based in the one-dimensional approximation to flow in the diffuser [Culick and Rogers (1983); Yang (1984); Yang and Culick (1984, 1985, 1986)] are useful and seem to capture some of the dominant features of the behavior, it is quite clear that the true motions can be simulated well only by numerical analysis based on the Navier-Stokes equations for two-or three-dimensional flows [Hsieh, Wardlaw and Coakley (1984); Hsieh and Coakley (1987); and references cited there].

There is evidence that under some conditions inlets exhibit self- excited or 'natural' oscillations. Energy is transferred from the mean flow to the fluctuations associated at least partly with separated flow. Although a one-dimensional calculation [Culick and Rogers (1983)] and an approximation to some of Sajben's data by Waugh et al (1983, Appendix D) suggest the possibility that the inlet may drive combustion instabilities, there is no firm evidence from tests with combustors that those conclusions hold. Most experimental results strongly suggest that the major source of driving unstable motions is likely associated with processes in the combustion chamber.

Nevertheless, because the flow from the inlet is the initial state for flow in the chamber, it is fundamentally important that processes in the inlet be well- understood. In that respect, as we remarked above, perhaps the greatest deficiency is knowledge of the history of the injected fuel and the distribution of liquid droplets and gaseous fuel at the inlet phase.

5.3.2 Vortex Shedding and Combustion Instabilities

By far most attention has been directed to vortex shedding as the most likely mechanism for combustion instabilities in ramjet engines. In addition to extensive experimental work related to those ideas, much has been done, both with laboratory tests and analysis, to clarify the acoustical characteristics of the modes of oscillation. Much more is known, and understood, about vortex shedding and its role as a mechanism for causing combustion instabilities chiefly because that phenomenon is easily identified in experiments and is commonly encountered. Although vortex shedding is arguably the dominant feature causing instabilities in dump combustors - and might therefore be termed the most important mechanism - it cannot be separated completely from convective waves. Furthermore, neither mechanism can be understood apart from the acoustics of the chamber in which they occur; the type of mode that is unstable always provides some clues about the mechanism. For convenience here we nevertheless treat the phenomena separately and defer discussion of convective waves to Section 5.3.5. One distinction between the two mechanisms that seems to be true is that if direct coupling between large vortices and the acoustics field dominates, the frequencies of oscillations tend to be close to those of classical resonances. If convective waves are involved, the frequencies may be quite different, as shown with the elementary example in Section 3.3.

In Section 3.4 we discussed vortex shedding as a mechanism for combustion instabilities. The earliest ideas were developed in the 1950's to explain the occurrence of high frequency transverse or tangential waves in afterburners. Periodic combustion of reactants entrained in large vortex structures served as sources of acoustic energy. If properly phased, the sources may supply energy to an acoustic mode of the chamber. The fluctuations of velocity associated with the mode initiate vortex shedding, completing the cycle.

Roughly two decades later vortex shedding was again proposed as a possible mechanism for instabilities, but now periodic combustion was not part of the argument [Flandro and Jacobs (1975); Culick and Magiawala (1979)]. Laboratory tests in cold flow established the result that if vortices shed from a step or corner impinge on an obstacle downstream, there is sufficient coupling with unsteady motions to excite the sustain standing acoustic modes in a duct [Culick and Magiawala (1979); Dunlap and Brown (1981); Dunlap et al (1981); Nomoto and Culick (1982); Aaron and Culick (1985)]. In all those cases, longitudinal modes were drive. Large "vortex-like" structures were observed in some flow visualization work on dump combustors at AFWAL sometime in the late 1970's [Private communication, F.D. Stull].

It was therefore logical that vortex shedding should be proposed as a possible mechanism for causing the longitudinal modes in a ramjet engine. The idea seems to have been discussed first in this connection at a JANNAF workshop [Culick (1980)] in 1979. Byrne (1981, 1983) gave the first detailed discussion of the mechanism. He used known results for the stability of shear layers and jets, vortex shedding and vortex merging to argue that the frequencies of those processes taking place under the conditions occurring in ramjet engines are in the range of frequencies of the oscillation actually observed. He supported his conclusions by good comparisons of his estimated frequencies with data taken by others for both coaxial and side-dump configurations. Waugh et al (1983, Appendix B) showed modest success correlating amplitudes of instabilities with Strouhal number.

Since 1980, a large number of experimental works have established both by visualization and quantitative measurements that vortex shedding is a distinctive feature of dump combustors. [Schadow et al (1985, 1987); Smith and Zukoski (1985); Brown et al (1985); Biron et al (1986); Sterling and Zukoski (1987); Poinsot et al (1987); Yu et al (1987); Davis and Strahle (1987)]. All of those tests were performed either in cold flow or with premixed gaseous reactants. The most extensive summary of the subject has been given by Schadow et al (1987) who included also references to related work not discussed here.

The work by Schadow and co-workers at NWC is particularly noteworthy for its systematic progression from tests in cold flow to experiments in dump combustors with burning, as well as studies of vortex combustion in diffusion flames. Their program has used four different experimental facilities and has involved both forced and self-excited oscillations. They have also done limited tests in a water tunnel to show the formation of large vortices in their configuration. Overall, the work at NWC has established the existence of vortex shedding at the frequencies of instabilities in realistic coaxial configurations. Moreover, they have shown that combustion processes drive oscillations to much higher amplitudes than found in the cold flow tests. We should note that for the cases cited earlier, of oscillations driven by votex shedding in solid rocket motors, the vortices were formed in essentially non-reacting combustion products. The amplitudes of such instabilities have always been relatively small (<5% of mean pressure). Thus it seems true, as found also in the work by others cited above that truly large amplitude oscillations require the presence of combustion processes and the conversion of heat released to mechanical energy.

Hegde et al (1986, 1987) and Reuter et al (1988) have studied oscillations in a duct driven by a flame, in a situation similar to that devised by Kaskan and Noreen (1955) and by Dowling and co-workers at Cambridge for afterburners (Figure 5.9). In the Georgia Techo tests, the flame (or flames) is stabilized on one or two wires spanning a duct. Under broad conditions, the flame is unstable and vortices grow in the sheet. Interactions with the flow field are sufficiently strong to excite acoustic waves in the duct. The authors have proposed that fluctuations of the flame surface area - and hence of the reaction rate are responsible. They have given data based on emitted radiation, showing that the oscillations of surface area are in phase with the pressure variations. By Rayleigh's criterion for heat addition, it follows that the heat addition encourages growth of acoustic waves, a result established also by Sterling and Zukoski (1987) for a dump combustor.

Although most experimental work related to vortex shedding in ramjets has been done with coaxial configurations, the phenomenon has also been found in side-dump combustors. Stull et al (1983) have reported early work with that geometry and Nosseir and Behar (1986) have examined similar cases in a small scale. More extensive results with full-scale hardware were discussed by Zetterstöm and Sjöblum (1986) who investigated configuration having two or four inlets. Visualization in a water tunnel revealed the presence of vortex shedding. Instabilities in the operating engines were avoided by modifying the fuel injection systems in such a fashion as to minimize combustion within the vortices. That's an important practical result clearly supporting the general picture of vortex shedding as a dominant mechanism.

5.3.3 Mode Shapes: Experimental and Calculated

In practice, the first indications of combustion instabilities are almost always fluctuations in recordings of the pressure. If there is only one pressure transducer, one can infer only the amplitude and frequency - best displayed as a power spectral density. While the frequency alone may suggest what modes are involved, the configurations used for ramjet combustors are sufficiently complicated that the modes are not always easily identified. Moreover, in laboratory tests there may be an upstream plenum chamber and other parts of the apparatus that participate in the oscillations. As a general rule, it is essential that measurements of the pressure be taken at several locations in order to provide unambiguous identification of the modes. Sufficient care should be taken that distributions of both the amplitude and relative phase can be determined. This information has also proven extremely useful for confirming the results of analyses.

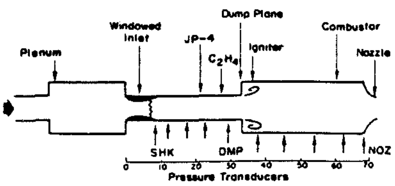
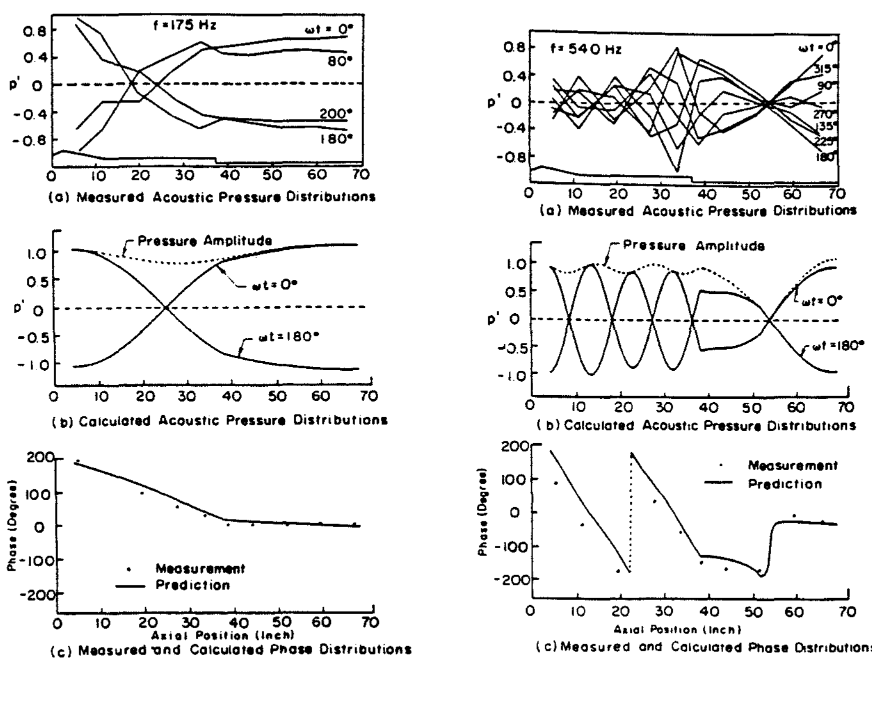


FIGURE 5.11

The most extensive measurements of mode shapes in dump combustors were made at the Naval Weapons Center by Schadow and co-workers. A summary of the results, with references to the previous work, was published by Crump et al (1986). Figure 5.11 shows the geometry of the sub-scale laboratory device; some results of measurements and analysis are reproduced in Figure 5.12. A case in which a bulk mode is excited in the combustion chamber (175 Hz) is shown in Figure 5.12(a); the fundamental wave mode was excited in the chamber excited for the case shown in Figure 5.12(b) (540 Hz). The calculated



(a) (b)

FIGURE 5.12

results were based on a one-dimensional analysis [Yang (1984)] in which combustion was ignored and the mean flow was accounted for only in the inlet. The good agreement is further evidence of the point emphasized in Sections 1 and 2, that the mode shapes and frequencies for combustion instabilities are often well-approximated by results based on classical acoustics. Here we also find that the one-dimensional approximation works well. For those calculations, the inlet shock was represented with the admittance function computed by Culick and Rogers (1983). It is apparently a good approximation that for these cases, the shock system is highly absorbing: the reflected wave has much smaller amplitude than the upstream-traveling incident wave. That fact, and the presence of the high speed average flow, explains why the relative phase varies linearly in the inlet.

Clark and Humphrey (1986) have also reported fairly good results obtained with a one-dimensional analysis applied to a side-dump configuration. The engine was supplied from a large plenum through inlets that were not always choked. Although the frequencies of oscillation, phase distributions throughout the device, and amplitude distributions within the combustor were predicted well, the amplitude distributions within the inlets diffuser considerably from the measured results. The reasons for the differences are not known. Yang and Culick (1985) later carried out a numerical analysis including vaporization of the liquid fuel and were able to predict quite well both the distribution and level of the pressure field.

A series of tests in a coaxial combustor have been reported by Sivasegaram and Whitelaw (1987), intended to examine the consequences of changing geometric parameters and fuel/air ratio. Data are given for frequencies and sound intensity at one location. Mode shapes were evidently not measured and no results of analysis are cited. It would appear that these data offer an opportunity for a straightforward application of a simple one-dimensional analysis.

The one-dimensional approximation works suprisingly well for rapid estimates of mode shapes and frequencies. It is worthwhile remarking on its application. Equation (2.25) and (2.26) with $\mathcal{F}_1 = \mathcal{P}_1 = 0$ determine the classical mode shapes. Few exact solutions exist for arbitrary variations of cross-section area $S_c(z)$, but in the case of ramjet configurations it is generally required to obtain results for piecewise variations. The problem comes down to solving the wave equation

$$\frac{d^2\hat{p}}{dz^2} + k_l^2\hat{p} = -\frac{d\hat{p}}{dz}\frac{1}{S_c}\frac{dS_c}{dz}$$

where dS_c/dz vanishes everywhere except at discontinuities of area where it is infinite.

Hence the general procedure is straightforward to find normal modes of the chamber. In uniform sections, the pressure field is represented by the usual forms, $A_i \cos(k_l z + \phi_i)$ or its equivalents, where A_i, ϕ_i are associated with segment i, and k_l is the wavenumber for mode l. These solutions are matched at the discontinuities by requiring continuity of the acoustic pressure and mass flow. Eventually the amplitudes A_i can be found to within a multiplicative constant, and the values of h_l are determined as roots of the characteristic equation.

This sort of analysis has long been known to give satisfactory results if the changes of area are not too large [Culick, Derr, Price (1972); Derr and Mathes (1974)]. Simple resonance tests at room temperature have confirmed the calculations, a method that is still useful for investigating the acoustic modes of combustion chambers. For application to actual systems, significant differences between these approximate results and observed values may arise due to uncertainties in the boundary conditions at the inlet and exhaust.

5.3.4 Numerical Analysis of Flows in Ramjet Combustors

It is the nature of the sort of approximate analysis discussed in Section 2 that reasonable results for the frequencies of oscillations can be obtained with rather crude approximations to the actual mode shapes. Furthermore, the stability of small amplitude notions can be assessed with some confidence if all the important processes are modeled reasonably well. The approximate analysis of course provides no information about the details of the situation in an actual combustor. Indeed, qualitative knowledge of the real state of affairs is required to make this sort of approach productive. Independently of experimental results, the only other source of information is numerical analysis based on the complete conservation equations. More importantly, results of accurate numerical analysis provide the only basis for judging the accuracy of an approximate analysis.

Thus, thorough numerical analysis of both the steady and unsteady flows in a combustion chamber is potentially extraordinarily important for investigating combustion instabilities. Even with recent developments in high speed computers, capabilities and resources fall considerably short of those required to handle "real" problems. For example, it is still not realistic to treat three-dimensional problems, even without combustion; and of course proper accounting of turbulence and combustion processes already taxes available resources beyond practical limits even for flows that are two-dimensional in the mean.

Numerical analysis, or simulation - really the application of computational fluid dynamics to investigate internal flows - has never been seriously pursued either for liquid rockets or for thrust augmentors. Because the surge of interest in treating combustion instabilities in ramjet engines has been quite recent, application of CFD has become a sensible endeavor. Although results obtained to date do not approach closely the goals cited above, some progress has been made.

The most advanced works have been reported by Kailasanath et al (1985, 1986, 1987) and by Jou and Menon (1986, 1988). Although eventually combustion processes will be accounted for, results published to date are only for cold flow, in the configuration tested by Schadow et al, Figure 5.11. The two chief differences between the two works are that Kailasanath et al solve the inviscid (Euler) equations, arguing that the effects of nonlinear damping of the numerical scheme gives a high (but not infinite) "effective Reynolds number", while Jou and Minon solve the full Navier-Stokes equations; and while Khailasanath et al assume multiple exhaust nozzles, Jou and Minon treat a single nozzle smoothly joined to the chamber as in the experimental apparatus. Both analsyses are carried out for axisymmetric flows using similar grid sizes. Figure 5.13, reproduced from Kailasanath et al

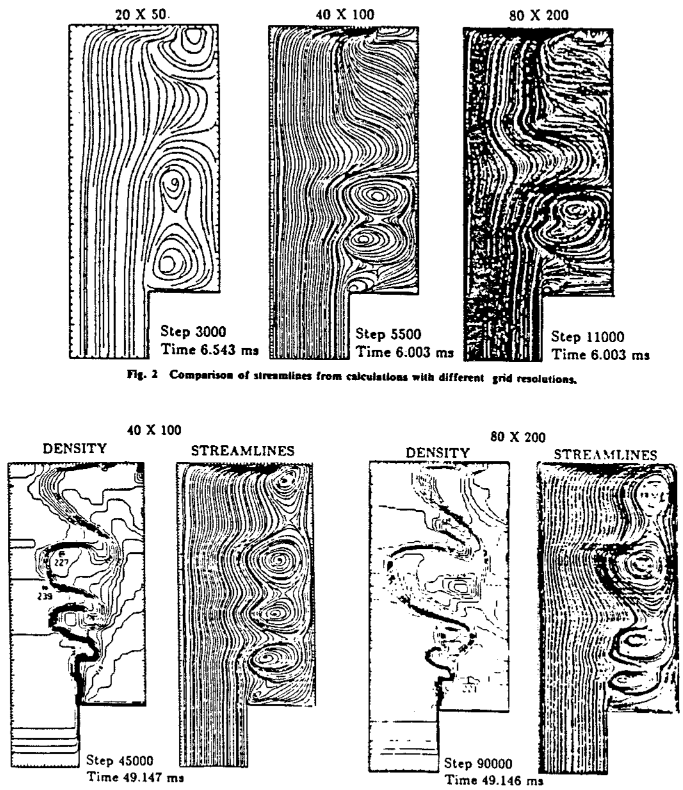


FIGURE 5.13

(1987b) shows density contours and streamlines for two different meshes. Figure 5.14 shows a time-sequence of vorticity contours computed by Jou and Menon (1988a). Obviously both results show the generation of large vortex structures.

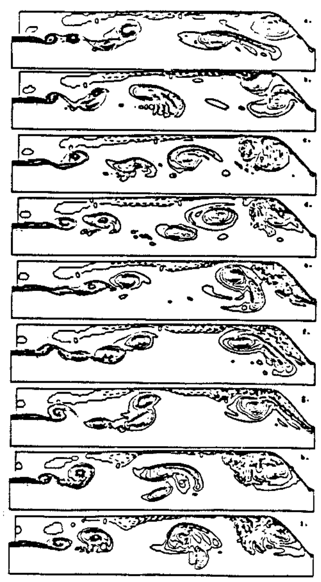


FIGURE 5.14

Those numerical results have not been subject to any objective detailed comparison, and it is impossible to do so here. Both works have concentrated in the vortex/acoustics interactions. much of the discussion of the results tends to descriptive. There are of course both similarities and differences between the two sets of works. At this time, one might be inclined to favor calculations based on the Navier-Stokes equations for which all boundary conditions can be correctly satisfied.

For some purposes, it can be an awkward feature that numerical analysis produces results for the total values - mean plus fluctuating - of the flow variables. Perhaps the easiest way to display the properties of the unsteady flow is with power spectral densities and various correlations. Power spectral densities are computed in the works cited, showing peaks not all of which can be related to classical acoustic modes. Jou and Menon (1988b) have attempted the more difficult task of extracting the acoustic field itself by computing the solenoidal and potential parts of the velocity field. Their results show that the large vortices are approximately quadrupole sources of the acoustic field; and that impingement of a vortex on a choked nozzle appears as a dipole source. They have used the latter conclusion, and an approximation to the influence of the acoustic field on separation of the shear layer, to construct a model for coupled acoustic/vorticity modes. This is a form of an instability based on a mechanism involving convected waves (in this case vorticity waves). Evidence of such coupled modes appears in the numerical results: their frequencies are not particularly close to those of classical acoustic modes.

More recently, Molavi and Sirignano (1988) have published their initial results for numerical analysis of the unsteady field in a two-dimensional dump configuration. The $k - \epsilon$ model is used to describe the gas flow. Vaporization of liquid fuel drops is accounted for, followed by combustion with finite reaction rate. Calculations of the gas motions were done using codes written by others (a "Teach-based algorithm with the Simple method for solving the pressure field").

Unlike the analyses discussed above, there is no evidence of vortex shedding and no discussion of that striking difference from previous calculations and experimental results. The emphasis in the work is on the behavior of the liquid phase. No oscillations appear spontaneously, and forced oscillations (i.e. unsteady motions numerically superimposed on the steady solutions) decay rapidly.

Two recent analyses have been done to demonstrate the existence of recirculation zones (confined or trapped vortex motions) in the head end of a side-dump combustor [Liou et al (1988) and Hong et al (1988)]. Those works use the same or similar computer codes as those used by Molavi and Sirignano; the results show steady vortex motions apparently in good agreement with observations of flow visualization tests [Stull et al (1983); Vanka et al (1983, 1985); Liou and Wu (1985)]. The calculations do not include combustion. Like another recent work by Vanka et al (1988), these analyses have to do with the steady flow in a ramjet combustor and make no attempt to treat combustion instabilities. In that respect they are nevertheless relevant because knowledge of the steady flow field is prerequisite to understanding unsteady motions.

It is not possible to reconcile the preceding numerical analyses. While all must contain some realism, the results show only limited qualitative agreement with experimental results and among themselves. The calculations are difficult, time consuming and as always are subject to peculiar - sometimes unpredictive - influences of numerical schemes. Others not involved in performing the calculations can only assume that the published results are in fact numerical solutions to the formulated physical problems and do not reflect in any significant fashion the computational procedures. Success with computational fluid dynamics applied to internal flows can be an enormously important contribution to treating problems of combustion instabilities.

5.3.5 Convective Waves of Entropy and Vorticity

We gave in Section 3.3 an elementary example showing the possibility for exciting instabilities by coupling entropy and acoustic disturbance at the exhaust nozzle. The calculations carried out there are equally valid for vorticity: just replace the entropy fluctuation by the vorticity fluctuation. That is essentially the gist of the model discussed by Jou and Menon (1988) cited above in Section 5.3.4.

Those computations have produced two main results: they confirm the view that convective waves constitute a possible mechanism for instabilities; and they show that the frequencies of coupled acoustic/convective wave modes can be significantly different from those of perturbed classical acoustic modes. Some of the numerical results cited in Section 5.3.4, and some experimental tests as well, have shown peaks in power spectral densities that apparently are not related to excitation of classical modes. Those observations strongly suggest that convective waves participate in some combustion instabilities, although incontrovertible proof has not been given.

Waugh treated two models of instabilities associated with entropy waves [Waugh et al (1983); Waugh and Brown (1984)]. In one model, the source of entropy fluctuations was concentrated at a single axial location, and in the second, several concentrated sources were used. The calculations required are modest extensions of the example given in Section 3.3. According to those results, distributed combustion tends to more stable than concentrated combustion when the chief mechanism for instability is the convected entropy wave.

In a work intended to investigate the stability of unsteady motions with combustion in a dump combustor, Humphrey and Culick (1986, 1987) used the results worked out by Chu (1953) for the unsteady behavior of a plane flame. The upstream boundary condition at the inlet was set with the one-dimensional analysis of the shock response [Culick and Rogers (1983)]. Those works once again established the existence of coupled acoustic/entropy modes that do not reduce to classical modes when the entropy fluctuations vanish: they arise in addition to the classical modes which themselves are of course slightly modified when entropy fluctuations are present.

Prompted by high speed films of the unsteady flow in a dump combustor [Davis (1981)], Abouseif, Keklak and Toong (1984) postulated that the instabilities were due to coupling between entropy waves and acoustic waaves. The basic model was essentially that described in Section 3.3. Periodic shedding of hot spots from the recirculation zone near the dump plane was interperted as a consequence of periodic heat release causing oscillations of temperature. Predictions of the frequency were about 10 per cent below the observed values. The authors speculated that the difference may be due to their assumption that the combustion zone - and hence the source of entropy waves - was concentrated at the dump plane. Apparently no effort was made to model a distributed combustion zone and no comparison was made between those coupled modes and classical acoustic modes that could be excited directly by interaction with shed vortices. The stability of the modes was calculated (i.e. values of the growth constant) but data was not available for comparison.

Waugh and Brown (1984) also applied their analysis of acoustics with convective waves to Davis's data. They noted that Abousseif et al had used an incorrect boundary condition at the nozzle. The corrected calculations produced frequencies quite close to those observed, and the mode shapes as well showed better agreement with test results.

5.3.6 The Time Lag Model Applied to Combustion Instabilities in Ramjet Engines

During the past seven years, Reardon (1981, 1983, 1984, 1985, 1988) has used the time lag model to correlate and interpret the extensive data taken by Davis (1981). Because the work is summarized in a paper given at this conference [Reardon (1988)], there is little to add here apart from noting the general approach and how it fits with other works.

The time lag model is unwieldy (at best) to use if combustion is allowed to be distributed and the time lag is variable. Hence as in many previous applications to liquid rockets, Reardon assumes that the energy release is concentrated in a transverse plane, that the parameters $(n,\ \tau)$ are constant, and that the flow field is one-dimensional. Then the combustion response is given by the part of equation (3.14) depending on frequency; to represent concentrated combustion, the average distribution $\bar{w}_l$ is replaced by δ-function. A modest change in the argument allows one to use this form for the unsteady conversion of liquid to vapor, or for unsteady energy release.

Reardon assumes that the oscillations observed by Davis are bulk modes in the combustor: the pressure is essentially uniform in space and pulsates in time. Hence the mode shape $\psi(\vec{r})$ is approximately constant and one may assume that the total unsteady energy release due to combustion processes in the chamber, E_c, is given by

$$\dot{E}_c = \dot{E}_0 n(1 - e^{-i\omega\tau})\frac{p'}{\bar{p}}$$

The rate of change of energy in the chamber is the net result of energy released by combustion and the rates at which energy is convected in and out of combustor:

$$\frac{dE}{dt} = \dot{E}_c + \dot{E}_{in} - \dot{E}_{out}$$

This relation is the basis for Reardon's treatment of the experimental results.

As we discussed in Section 4, in applications of the time lag model to instabilities in liquid rockets, both parameters $(n,\ \tau)$ were determined by matching a theoretical result to experimental results for the stability boundary. The idea then is that those values of $(n,\ \tau)$ can be used to predict the stability characteristics for new (but in some sense similar) designs. Here, Reardon has chosen to use values of n calculated by Crocco and Cheng (1956) and to compute the time lag independently, using previous results obtained by others. In short, Reardon essentially assumes that the combustion model is known (defined by the two parameters $(n,\ \tau)$ with concentrated combustion) and then uses the relation for the balance of energy in the chamber to correlate data.

Stability of oscillations may be determined by application of the Nyquist criterion after the unsteady energy balance is re-written by using the Laplace transform. This possibility arises because, as we have briefly described earlier, the problem of self-excited combustion instabilities can be interpreted as a linear system with a negative feedback loop. The stability criterion, expressed with the growth constant α, depends on other processes included in the energy balance. The formal result may therefore be used to test the importance of those processes by comparison with data.

Reardon has used this procedure to study the effects of several processes and geometrical parameters, with mixed results. It seems that this sort of approach suffers from the intrinsic limitation noted earlier: it is really only a method for correlating data and therefore in the first instance has little predictive value without assurance that the models used are accurate. Confidence in the results comes only from good correlations with data over broad ranges of parameters. The results to date do not seem to provide that confidence.

6. PASSIVE AND ACTIVE CONTROL OF COMBUSTION INSTABILITIES

There are two general strategies to follow in treating a combustion instability: change the design of the system to reduce the amplitudes of the self-excited oscillations; or introduce some form of control. When instabilities are encountered in a development program, significant design changes usually cannot be accepted. Moreover, even after nearly fifty years' experience with the general problem, it is often difficult to recommend effective modifications with great confidence. Since the earliest instances of serious instabilities, much effort has therefore been devoted to controlling the oscillations, generally by passive means.

Small solid propellant rockets produced during World War II posed serious problems of high frequency transverse modes. They were usually eliminated by incorporating 'resonance rods' extending along the axis of the chamber, or baffles, extending radially from the outer case. Perforated liners were evidently first used in about 1950 to reduce the amplitudes of screech oscillations in amplitudes of screech oscillations in afterburners [Lewis Laboratory Staff (1954)]. At about the same time, individual resonating cavities were first used in a small supersonic inlet to reduce shock oscillations ('buzz') [Fox (1951)]. During the 1960's, much was accomplished on the use of liners and damping cavities in liquid rocket engines, motivated chiefly by applications to the F-1 and lunar ascent engines of the Apollo program [Oberg (1971); Oberg and Kukula (1971)].

It is now standard practice to incorporate acoustic liners in the design of afterburners to attenuate screech oscillations. Baffles and resonating cavities, or liners are used in many liquid rockets, particularly larger engines. The way in which these devices work is quite well understood. Reasonably good designs can be produced if the mode shapes and frequencies of the troublesome oscillations are known, but in practice, successful applications always requires testing and modifications for reasons we shall explain below.

The two greatest disadvantages of passive devices are the space required to fit them in the combustion chamber, a constraint that limits their use in solid propellant rockets; and that a given design is effective only over a fairly narrow frequency range. It is possible to design baffles for operation at low frequencies, but acoustic liners and resonators can be used only to treat high frequency oscillations because their linear dimensions increase as the frequency is reduced. Generally, the necessary sizes are too large for application at frequencies below, say, 500-1000 Hz in actual propulsion systems.

Greater difficulties with low-frequency combustion instabilities during the past few years has generated serious interest in active control. The idea is not new, dating at least back to a proposal by Tsien (1953) for servo-stabilization of low- frequency chugging modes in sensors, instrumentation and microprocessors make active control an attractive possibility for treating combustion instabilities over broad ranges of frequency and operating conditions. Recent research on this subject has just begun and only a few results have been obtained.

6.1 Passive Control Devices

The design and operation of baffles, resonators and acoustic liners for liquid-fueled rockets have been thoroughly covered in two lengthy summaries: Chapter 8 of Harrje and Reardon (1972), and a report prepared largely by the group at Rocketdyne with the collaboration of other organizations [NASA Design Criteria Office (1974)]. In liquid rocket engines, baffles are mounted on the injector face along radii and extending some distance downstream. Figure 6.1 taken from Harrje and Reardon (1972, p. 428) shows the installation in the Lunar Module Ascent engine manufactured by Rocketdyne. The design also included slots around the periphery that acted as acoustic resonators. It has often been the practice to use both baffles and resonators or acoustic liners in liquid rocket engines. The liners may extend along the chamber wall to the nozzle entrance, an example of which is shown in Figure 6.2 [Mitchell (1965)]

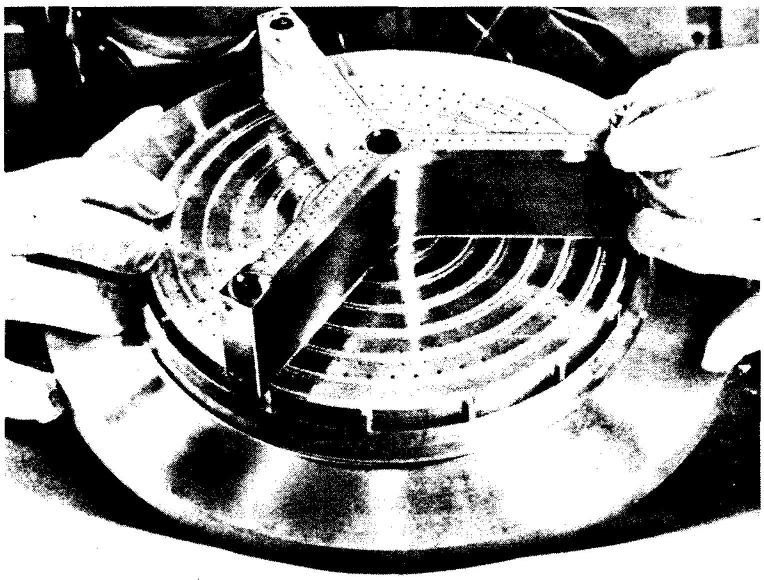


FIGURE 6.1

The number and orientation of baffles depends on the modes to be treated. Figure 6.3 taken from NASA Design Criteria Office (1974) shows the three most common cases encountered in practice. The idea is that the baffles are essentially rigid walls, changing the geometry locally and therefore allowing fewer modes than possible in the unobstructed chamber. For example, a single baffle extending over a diameter prevents traveling or spinning modes, Figure 6.3(a). If more baffles are added, as in Figure 6.3(b), a limited number of the lower modes are eliminated. Thus, if there are three radial baffles, the first and second tangential modes cannot exist, but the boundary conditions do allow the third tangential mode.

Because of performance losses and the practical problem of maintaining structure integrity, baffles can extend only part way along the axis of the chamber, so they cannot be totally effective. A measure of their operation is the rate at which a mode decays after excitation by injecting a pulse. Figure 6.4 [Oberg et al (1969)]. The time required for a mode to damp to $1/e$ of its initial value must obviously decrease as the length of baffle is increased in the axial direction.

In addition to discouraging the presence of some resonances - purely a geometrical effect - baffles also are sources of energy losses due to viscous effects. While skin friction may have some contribution, probably the largest effects are due to oscillating flow past edges and the associated formation of shear layers and vortex shedding.

Preventing certain modes is beneficial if the coupling between combustion process and the unsteady motions is primarily in the frequency range of those modes. Because the baffles are mounted on the injector, they may also affect the coupling directly by shielding the sensitive regions near the injector face from oscillating disturbances. Thus baffles are evidently effective both because they provide damping of unsteady motions and because they influence the processes tending to excite the motions.

In contrast to baffles, resonant acoustic cavities and acoustic liners have essentially no affects on energy transfer from combustion processes to the acoustic field. Their intended function is entirely to provide energy

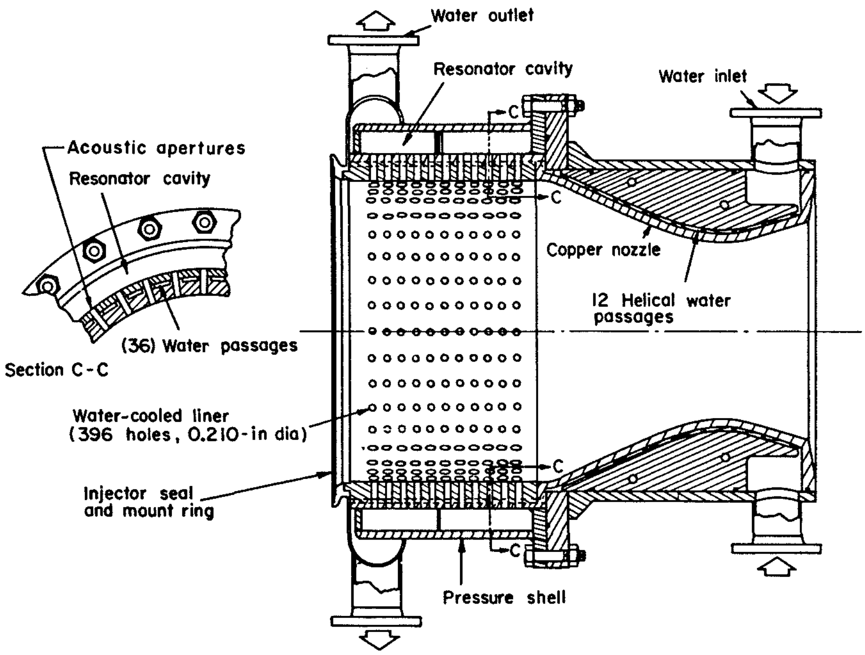


FIGURE 6.2

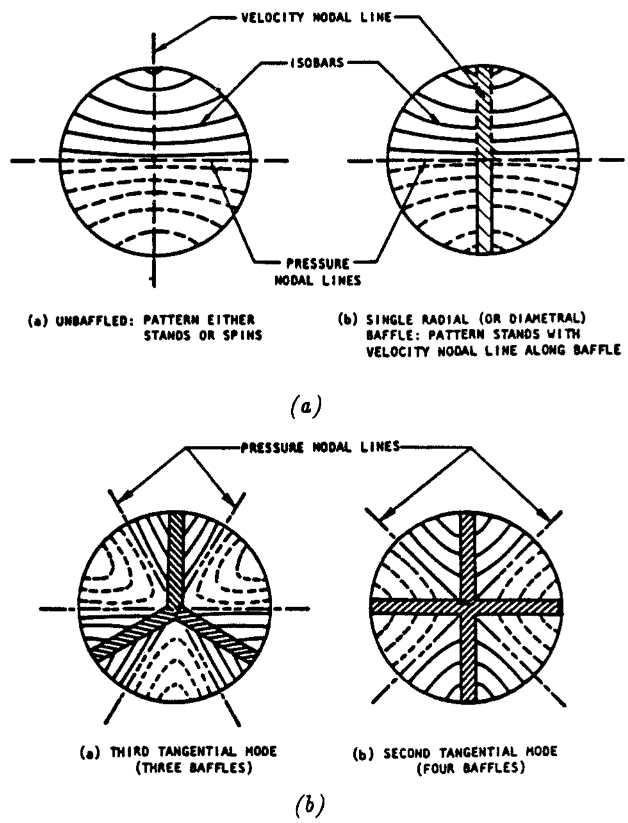


FIGURE 6.3

losses in the frequency range of instabilities. Their effectiveness depends on the local flow field induced by the unsteady field in the chamber, and in the consequent action of viscous forces to dampen the motions. The elementary cavity is the Helmholtz resonator, sketched in Figure 6.5. Its action can be visualized most simply as a mass/spring/dashpot system. The mass is the plug of gas in the orifice and the spring is provided by the compressibility of the gas in the volume of the chamber. Hence, one should expect that the natural frequency should decrease as the volume of the cavity increases(because a given displacement of the plug is a smaller fraction of the volume); and should also decrease as the mass of the plug increasees. [frequency $\sim$ (spring constant/mass)$^{\frac{1}{2}}$]. The formula for the resonant frequency is

$$\omega_r = \frac{\bar{a}}{l_0}\sqrt{\frac{V_0}{V_c}} \tag{6.1}$$

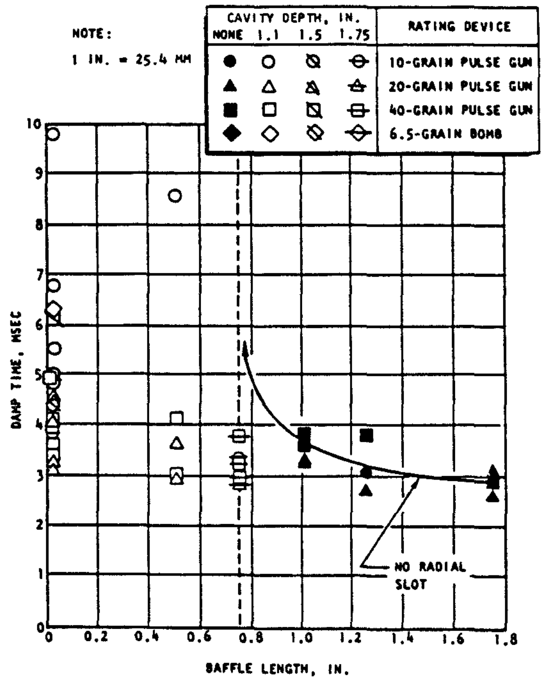


FIGURE 6.4

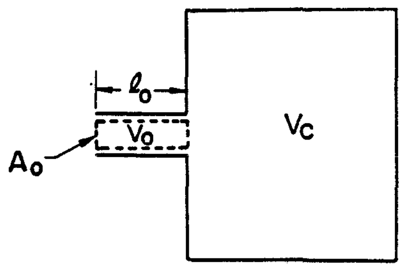


FIGURE 6.5

where l_0 is the length of the orifice, V_0 is its volume and V_c is the volume of the cavity. This formula is quite a good approximation to observed values providing the average temperature is everywhere the same, and if l_0 is assigned an approximate value. The length is really an effective length of the plug of gas, so in a sense, (6.1) really defines l_0. Theoretical estimates for l_0 may be found in several of the references cited, e.g. Harrje and Reardon (1972, p. 410).

Damping of the motions occurs primarily because of the motion of the plug of gas in the orifice. It is strongly a function of the amplitude of motions because of flow separation in the orifice [Ingaard and Labate (1950); Ingaard (1953); Sirignano et al (1967); Zinn (1969)].

Acoustic liners are essentially arrays of small acoustic resonators. Figure 6.6 [Harrje and Reardon (1972, p. 410) is a sketch of a typical configuration. Each
backing cavity has several orifices. Equation (6.1) is the basic formula for designing resonators and liners but clearly many compromises must be made in specific applications.

The design issues for liners are thoroughly discussed in the references. In addition to the obvious geometrical variables, we should mention that special consideration must be given to the effects of the amplitude of motion, mean flow past or into- the orifices, and temperature variations. The influence of temperature is important because it determines the speed of sound and hence the resonant frequency. Tuning cavities and liners is therefore seriously affected by the temperatures of the gases in the orifices and cavities. That is a major reason why acoustic resonators have not been successfully used in solid propellant rockets. Because of the short firing times, resonators are almost always operating under changing conditions.

A practical limitation of resonators is the finite bandwidth: a given geometrical configuration effectively attenuates only over a relatively narrow band of frequencies. Figure 6.7 [Nestlerode and Oberg (1969)] shows the influence of changing geometry of a liner used in the Lance Booster engine.

Increasing the amplitude of motions tends to reduce the peak of the resonance curve for a liner, but broadens the bandwidth. Figure 6.8 [Blackman (1960)] shows data taken at room temperature for a small section of liner in an impedance tube. Agreement with calculations is fairly good (θ is essentially the real part of the admittance function for the liner.)

Calculation of the influences of resonators and acoustic liners have appeared in many publications [e.g. Oberg et al (1969, 1971, 1972); Baer and Mitchell (1974, 1977); Harrje and Rearden (1972, Chapter 8); and NASA Design Criteria Office (1974)]. Whatever the details of the computations, the results eventually come

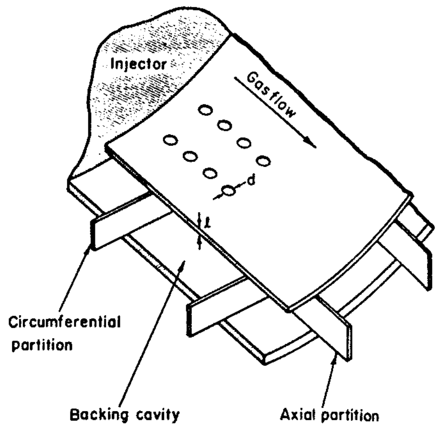


FIGURE 6.6

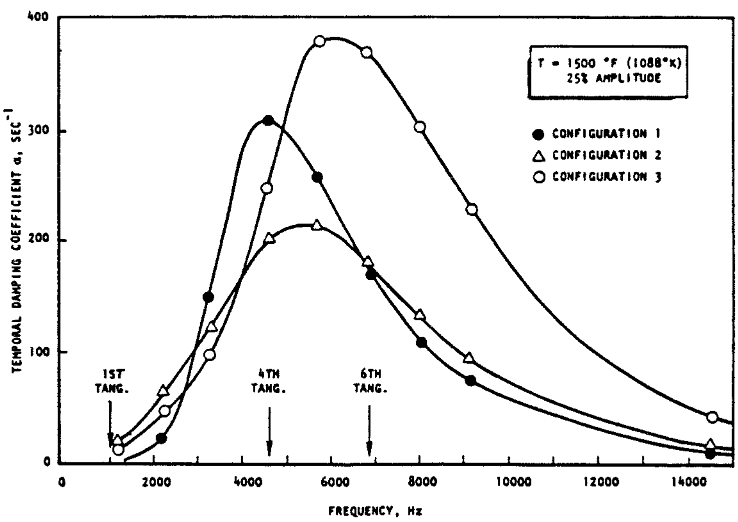
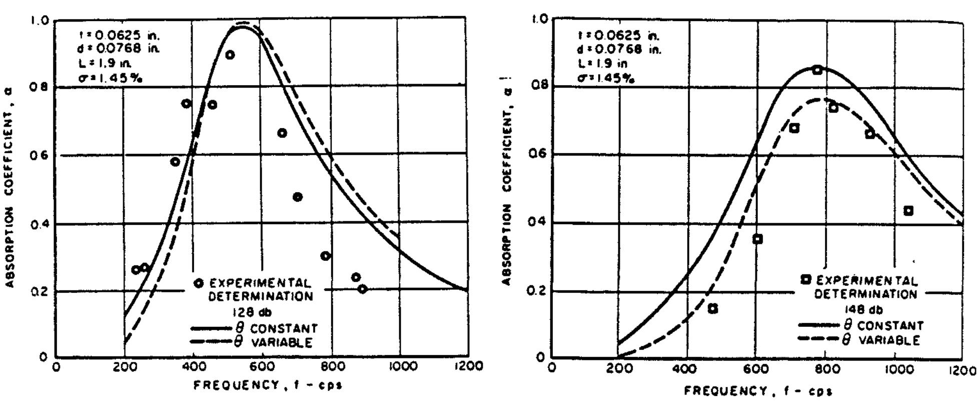


FIGURE 6.7



(a) (b)

FIGURE 6.8

down to a form of the last term in equation (2.79). Although it might appear that there are formal difficulties when the basic acoustic velocity is parallel to the surface, it can be shown that the same formula is valid [Culick

(1973)]. Thus, assuming no average flow normal to the plane of the liner, the attenuation constant is

$$\alpha_{\text{liner}} = -\frac{\bar{a}}{2E_n^2} \iint A_l^{(r)} \psi_n^2 dS \tag{6.2}$$

where $A_l^{(r)}$ is the real part of the admittance function defined for a unit area of liner; that is, it is an average of the admittance function for the orifices and the solid surface. If the impermeable surface is rigid, $A_l = 0$, and (6.2) can be written

$$\alpha_{\text{liner}} = -\frac{\bar{a}}{2E_n^2} \left(\frac{S_0}{S_l} \right) \iint A_l^{(r)} \psi_n^2 dS \tag{6.3}$$

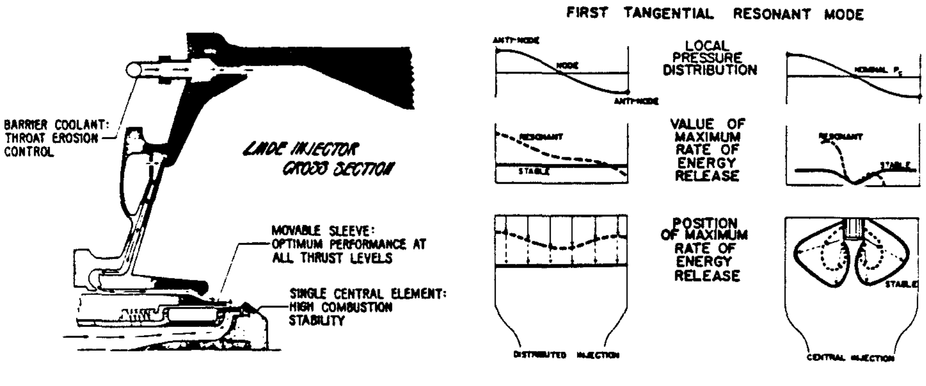
where S_0 is the total open area and S_l is the total area covered by the liner. Obviously the liner is more effective if placed where the mode shapes (i.e. pressure fluctuation) is largest. We emphasize that this result is strictly valid only for linear behavior; nonlinear behavior is accomodated by the approximate analysis given in Section 2, but no results are available.

In view of the widespread use of baffles and liners, perhaps the most remarkable liquid rocket engine design is that used for the Lunar Module Descent engine [Cherne (1967); Elverum et al (1967)]. Neither baffles nor liners nor any other damping device was required: the engine possessed very robust intrinsic stability over its entire range of throttling, from 1000 pounds to 10,000 pounds thrust. Moreover, the design has been scaled to 50,000 pounds and still exhibited complete stability in static tests (the large engine has never been flown).

The chief reason for the stability seems to be the feature that the propellants are injected through a single (large) coaxial element located in the axis of the combustion chamber. As a result, the distribution of energy release tends to be concentrated near the axis of the chamber where the tangential modes have pressure nodes and even the radial modes have smaller pressure anti-nodes than at the periphery. Refer to the first term representing the main source of excitation in equation (2.79) and consider only the unsteady energy addition,

$$\alpha_c \sim \int \psi_n \hat{Q}^{(r)} dV \tag{6.4}$$

The integral is reduced if the energy release $\hat{Q}$ is small where ψ_n is large, and vice-versa. Figure 6.9(a) taken from Elverum et al is a sketch of the situation for the first radial mode. As shown in Figure 6.9(b), the



(a) (b)

FIGURE 6.9

energy release has this distribution because fuel is injected axially along the periphery of the injector. Oxidizer is injected radially through 36 holes. Thus, the liquids impinge as the intersection of a sheet with 36 jets. As suggest in Figure 6.9(a), the axial momentum of the fuel sheet tends to bias the energy release in the axial direction, the oxidizer jets cause spreading also towards the chamber walls. Although the injector projecting into the chamber acts as a baffle against the radial modes, it can have little influence on the tangential modes. Hence one must conclude that the high level of stability in the absence of baffles and liners derives from the low value of the driving, equation (6.4). Yodzis (1968) gave an interesting brief comparison of the stability characteristics of three engines in the Apollo vehicle.

6.2 Active Control of Combustion Instabilities

It is a fundamental property that combustion instabilities are self-excited oscillations. Their excitation occurs because the energy transferred from the combustion processes and mean flow to the unsteady motions depends on the unsteady motions themselves. That is why a small amplitude disturbance grows exponentially in time until a nonlinear process limits its amplitude, commonly producing a periodic limit cycle.

Prediction of instabilities in combustion chambers is difficult and is always accompanied by large uncertainties. Consequently, any strategy of design or correction must in practice rely heavily on experimental work; and on analysis providing a framework for basic understanding, interpretation of data and designing experiments.

The very nature of the instabilities, that they are self-excited, causes considerable trouble both for treating problems in propulsion systems and for laboratory experiments. Experimental work is time consuming and expensive, due to the difficulty of performing controlled and reproducible tests.

Passive measures, as described in the preceding section, were the earliest and continue to be the only practical means of treating combustion instabilities in operational propulsion systems. While often effective, they have the disadvantages noted above: there is no widely applicable theory for passive control; development is always a costly trial and error process; and the effectiveness of a particular design is inevitably limited to a relatively narrow range of frequency and operating conditions.

Active control is an attractive strategy for use in both practical and research problems. It is not a new idea and has appeared in several limited forms in the past 35 years. Tsien (1953) showed how the chugging instability in a liquid rocket motor could be stabilized by controlling the supply of liquid propellant. His proposal was based on the time lag model of the supply and combustion dynamics. As Figure 3.1 shows, the overall time lag in the system is composed of several contributions. Consequently, it is not necessary to affect only the combustion processes or the pump characteristics to alter the stability. Tsien's idea was to manipulate the propellant flow in the supply lines by controlling the capacitance, Figure 6.10.

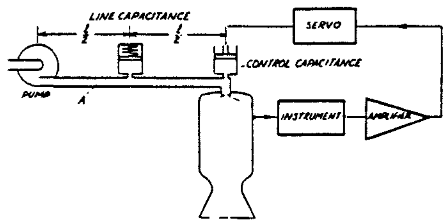


FIGURE 6.10

The theoretical basis is readily established by writing the equation for the chamber pressure, a result that can be obtained either by considering conservation of mass or energy, or by specializing the approximate analysis described in Section 2. Then we expect that pressure fluctuations will satisfy the equation for a damped oscillator, and if we include an excitation due to combustion with a time lag, the behavior is governed by

$$\ddot{p} + 2\alpha\dot{p} + \omega_0^2 p = \beta p(t-\tau) + u(t) \tag{6.5}$$

where $u(t)$ is an unspecified input control function. The damping coefficient is $\alpha > 0, \omega_0$ is the resonant frequency of the chamber and β is a constant, something like the interaction index. Now take the Laplace transform, with s the transform variable and $P(s), V(s)$ denoting transforms of $p(t), u(t)$, to find

$$P(s)\left[s^2 + 2\alpha s + \omega_0^2\right] = \beta e^{-s\tau}P(s) + U(s) \tag{6.6}$$

This equation can be interpreted with the block diagram drawn in Figure 6.11.

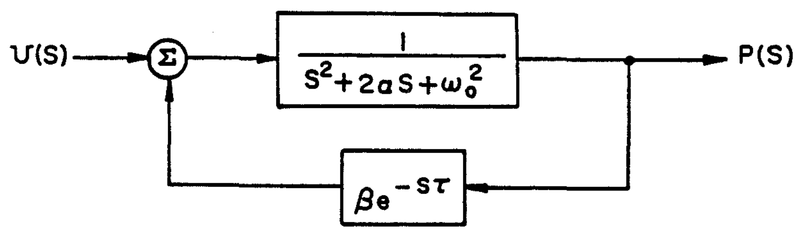


FIGURE 6.11

Equation (6.6) and the block diagram can be solved to give

$$P(s) = \frac{\beta e^{-s\tau}G}{1 + \beta e^{-s\tau}G}U(s) \tag{6.7}$$

where G denotes the transform function of the chamber (the "plant" in the terminology of control theory):

$$G(s) = \frac{1}{s^2 + 2\alpha s + \omega_0^2} \tag{6.8}$$

The solution (6.7) can be re-written as

$$\frac{P(s)}{U(s)} = \frac{\beta e^{-s\tau}}{s^2 + 2\alpha s + (\omega_0^2 + \beta e^{-s\tau})} \tag{6.9}$$

Stability of the system requires that the roots of the denominator have negative real parts.

For the case of a purely bulk mode or chugging, the term s^2 is missing from the denominator of $G(s)$. Tsien treated the problem of stability by applying the Nyquist criterion to the denominator of (6.9). For a system with time delay, the form due to Satche (1949) is required; see Tsien (1953, 1956). This method was applied by Marble and Cox (1953) and Marble (1955) to more complicated forms of the low frequency stability problem.

This example, progressing from formulation of wave motions in a chamber, as expressed by equation (6.5), to the block diagram in Figure 6.11 and the transfer function (6.9) for the closed loop system is the simplest illustration connecting combustion instabilities to control theory. There is no point pursuing details here, but this brief discussion should make appealing the application of modern control theory suggested below.

Application of "servo-stabilization" as proposed by Tsien may have been attempted in a laboratory somewhere, but it appears that no results have been published, and certainly this method has never been adopted in practice. The primary reasons seem to be inadequate sensors and instrumentation. That situation is very different now, with the recent developments in solid state devices and microprocessors. While practical applications may still be some time from realization, there is ample reason to pursue research on the problem, and indeed some interesting results have already been reported.

Short of the sort of intelligent control systems envisioned here, there have been several efforts in research programs to gain control over self-excited instabilities in order to obtain better data. A device invented at ONERA [Kuentzmann and Nadaut (1975)] used a rotating exhaust valve to modulate the flow and impose pressure oscillations on a burning solid propellant. The purpose was to provide a controllable means of measuring the frequency response of a burning surface. Subsequently the method was modified and used by several groups in the U.S. and England. Another technique for switching oscillations on and off involves a moveable baffle [described in the reference manual edited by Culick (1974)]. This technique has been used to produce several growth and decay periods of oscillations during a firing lasting less than one minute. Similar results have been obtained with a resonator, but with much greater difficulty because the temperature in the orifice and cavity changes rapidly during a test, causing great problems with tuning. All such methods are motivated by the need to gain some measure of control over unstable oscillations in laboratory tests. Here we are more concerned with techniques that have promise for application to full-scale propulsion systems.

Ffowcs-Williams (1984, 1986) has described his own successes and those of others in the use of active control to manipulate acoustic fields at normal temperatures and pressures by using acoustical interference. The essential idea — simply stated but not always easily realized in practice — is to determine the characteristics of the given acoustic field and use that information to control secondary sources of sound so as to produce desired results by interference. Most obviously a "desired result" is to reduce unwanted noise to silence—all parents' Holy Grail. Ffowcs-Williams calls "anti-sound" the acoustic field injected as the input to control the subject field.

In principle, the idea should be applicable to combustion systems, and some apparent examples have already been reported. The principle of interference well cover many situations, but not clearly in those just cited. When the primary sources of the acoustic waves are themselves sensitive to pressure and velocity disturbances - as the case is for all combustion instabilities - then interactions between the injected "control" field and the sources become a central issue. Consequently, although some authors may (possibly with justification) cite their results as examples of the application of "anti-sound," it seems that one might reasonably be skeptical.

Sreenivasan, Raghu and Chu (1985) demonstrated control of oscillations generated in a Rijke table by introducing a secondary heater. The idea has been at least discussed for many years and likely tried informally; some earlier results were reported by Collyer and Ayres (1972) but not in the context of control. If the fluctuation $\phi\prime$ has the proper phase and spacial distribution, then the contribution $\int \hat{\phi}^{(i)}\psi_n dV$ in α, equation (2.79), and the first term in (2.85) can be made negative, so disturbances are attenuated. In the Rijke tube, the control heater need not be oscillated by external means. If the source is placed in the upper half of the tube, the fluctuating heat addition arises from interactions with the velocity and automatically has the phase necessary to attenuate the waves. That is, the heat source in fact extracts energy from the field, on the average. Similar results were obtained by Sreenivasan it et al. with secondary heaters installed in an organ pipe and a "whistler-nozzle." The experiments are interesting and useful demonstrations but, if only because true external control is not exercised, application to propulsion systems seems a doubtful enterprise.

True active control of instabilities has been demonstrated in a series of works carried out at Cambridge University. Dines (1983) and Heckl (1985, 1986) injected acoustic waves with a loudspeaker placed near the end of the tube. The speaker was placed in a feedback loop allowing controllable gain and phase. Dines used a sensor to monitor the light emission as a measure of heat release. That information was processed to adjust the loop gain and phase. Heckl used the output of a microphone, sensing pressure fluctuations, as the signal in the feedback loop and showed that the instabilities could be suppressed over a broad range of phase. That result demonstrated that control of that combustion instability is not explained by the principle of "anti-sound," which requires a well-defined phase relation. Evidently the injected field had significant effects on the heat transferred to the oscillations.

More recently, Bloxsidge et al (1987) have controlled instabilities in the laboratory burner described in Section 5.2 (Figure 5.9). Control was exerted with an oscillating plug inserted in the choked inlet nozzle. Technically this amounts to controlling the inlet mass flow: its average value is always zero but the amplitude and phase of the oscillations can be varied. Those oscillations generate pressure waves that serve the same purpose as those produced by a loudspeaker (which itself actually appears as an oscillatory source of mass). The system was only partially successful in suppressing the instabilities, reducing the amplitude of the fundamental mode by somewhat more than half.

Lang et al (1987) and Poinsot et al (1988) have reported control of instabilities in a subsonic laboratory burner supplied with gaseous reactants. Two different flameholders have been used: a plate having 80 orifices, set in a stream of premixed gases; and a more realistic configuration comprising an array of three rearward facing steps through which fuel is injected into an air stream. In each case a microphone was used as sensor to excite loudspeakers placed upstream of the flames. Not only was suppression of the instabilities demonstrated, but in the most recent work, the authors have shown the great usefulness of control to study the transient behavior of the motions. Active control will no doubt be a useful tool in laboratory research.

Langhorne, Dowling, and Hooper (1988) have given an initial report of their results for a method which may well prove to be the most effective approach to controlling combustion instabilities in full-scale systems. Using the apparatus described in Sectin 52 [see Figure 5.9] they successfully reduced the amplitudes of an instability by introducing a controlled secondary supply of fuel upstream of the flameholder. That this is an attractive method follows from the discussion in the Introduction. Instabilities are encouraged in combustion chambers because of the high densities of energy release. The power involved cannot be matched by mechanical systems, such as loudspeakers, and response times as well may be inadequate. Evidentally the most direct method of control should be based on manipulating the source of energy. The work by Langhorne *et al.* is thoroughly discussed in a paper at this meeting.

In their work intended primarily for study the noise produced by convection of entropy fluctuations through a supersonic nozzle, Zuloski and Auerback (1976) demonstrated a form of control in a laboratory device. They produced entropy fluctuations by oscillating the temperature of a nichrome wire heater. The heat addition caused fluctuations of both temperature and pressure, the latter accompanying the unavoidable density fluctuations. With an oscillatory bleed valve, they were able to compensate the pressure fluctuations, leaving nearly pure temperature or entropy waves. Thus they demonstrated simultaneous control of mass and energy sources.

The approximate analysis developed in Section 2 has a form that is naturally adapted to applying the theory of control of distributed systems [Murray-Lasso (1966); Gould and Murray-Lasso (1966); Balas (1977; 1982)]. State-feedback control can be applied to distributed systems after decomposition of the general motion into modes, as accomplished in Section 2. The state of the system is then specified by the matrix of amplitudes η_n and amplitude velocities $\{\eta_1, \eta_2, \ldots; \dot{\eta}_1, \dot{\eta}_2, \ldots\}^T$. Yang, Sinha and Fung (1988) have first discussed the application of modal control to the problem of combustion instabilities and have given some preliminary results of a simulation. It appears that neither this nor any similar method has been tested on an actual combustion instability.

The basic ideas are easily explained, but successful application will require considerable further research. Simply incorporating some sort of feedback control is by no means a guarantee that the system can be stabilized — for example, a stable oscillator can be converted to an amplifier with the addition of feedback. In order to achieve success with active control of combustion instabilities, it is essential to have a thorough understanding of the system in question, particularly of the responsible mechanisms. There is no point here in speculating on possible control laws, or on the potential problems that may arise with use of modal control, but we can at least indicate why this appears to be a sensible strategy to pursue.

To control combustion instabilities means to exert external influence on the unsteady mass, momentum or energy in the chamber. Whatever physical means may be devised, the control inputs must theoretically appear as sources in the conservation equations (2.1) - (2.4) and in subsequent forms. Thus, in equations (2.18) - (2.20), we may add control inputs $\mathcal{W}_c, \vec{\mathcal{F}}_c$ and $\mathcal{P}_c$ on the right hand sides, their particular forms depending on the kind of control used.

In the examples treated above, Sreenivasan et al used control (limited) of an energy source; all others worked with a mass source, except in their most recent work, Langhorne *et al.* controle the energy source by modulating a secondary fuel supply. In order to give a conplete analysis of their results, it would be necessary also to analyze the basic mechanisms of the instabilities, for they were clearly affected by the motions induced by the control inputs.

The control source terms add contributions to h and f, and the wave equation (2.31) with its boundary condition (2.33) become

$$\begin{aligned} \nabla^2 p' - \frac{1}{a^2}\frac{\partial^2 p'}{\partial t^2} &= h + h_c \\ \hat{n}\cdot\nabla p' &= -f - f_c \end{aligned} \tag{6.10a,b}$$

Application of the expansion in the modes $\eta_n(t)\psi_n(\vec{r})$ proceeds as in Section 2.2, leading now, instead of (2.45), to the system of equations for the amplitudes:

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = F_n + u_n(t) \tag{6.11}$$

where $n_n(t)$ is the control input to the nth mode:

$$u_n(t) = -\frac{\bar{a}^2}{\bar{p}E_n^2}\left\{\int h_c\psi_n dV + \oiint f_c\psi_n dS\right\} \tag{6.12}$$

If there is no linear or coupling between modes, the equations (2.53) and (2.55) a, b give the linear part of F_n as $-D_{nn}\dot{\eta}_n - E_{nn}\eta_n = 2\alpha_n\eta_n + 2\omega_n\theta_n\eta_n$ and (6.11) is

$$\frac{d^2\eta_n}{dt^2} - 2\alpha_n\omega_n\frac{d\eta_n}{dt} + \left(\omega_n^2 - 2\omega_n\theta_n\right)\eta_n = F_n^{NL} + u_n(t) \tag{6.13}$$

where F_n^{NL} stands for the nonlinear part of the forcing function. If nonlinear behavior is ignored, each of the oscillator equations has the same form as (6.5) – it is trivial to incorporate a time lag model, and of course α_n here contains both energy sources and damping, for which the contributions are negative.

To incorporate feedback control, the state of the system must be sensed: that is, the unsteady pressure and its rate of change must be measured, say at S points, and the output signal is the matrix $\{y_1, y_2, \ldots, y_S\}$, where the s^{th} element is the signal measured at the position $\vec{r}_m$ in the chamber:

$$y_s^{(t)} = c_s \frac{p'(\vec{r}_s, t)}{\bar{p}} + d_s \frac{1}{\bar{p}} \frac{\partial p'(\vec{r}_s, t)}{\partial t} \tag{6.15}$$

Because the pressure field is represented by an expansion in modes, and in practice only a finite number, N, can be treated, (6.13)

$$y_s(t) = c_s \sum_{n=1}^{N} \eta_n(t)\psi_n(\vec{r}_s) + d_s \sum_{n=1}^{N} \dot{\eta}(t)\psi_n(\vec{r}_s) \tag{6.16}$$

In matrix form, the sensor output is

$$\{y\} = [C] \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix} \tag{6.16}$$

where C is an $S \times 2N$ matrix.

The control input (6.12) is due to a finite number of actuators; in the earlier examples, heaters or loudspeakers were used. Their presence can be represented in h_c and f_c, so $U_n(t)$ can be made explicit, except that the amplitudes for the actuator motions depend on the control law chosen. In practical systems, it is hardly likely that simple heaters or loudspeakers will be effective in combating combustion instabilities. The fast response necessary, and the energy requirements, probably will dictate either active control of the fuel supply, or some other method of directly affecting the mechanism of the instability.

Whatever the form of physical control chosen, the control matrix $\{u\}$ can be computed according to equation (6.12). In general, all actuators will affect all modes, and all $u_n(t)$ are non-zero. Yang *et al.* outlined the use of this formulism as the basis for a digital control system using a zero-order-hold technique. Their analysis is entirely formal, with a simulation to confirm their proposal. Neither they nor anyone else have attacked the far more difficult problems of applying these ideas to suppress combustion instabilities in a full-scale system. It's an area of research that holds much promise for productive results in the near future, a possible solution to the problem of combustion instabilities whose time has arrived.

7. Concluding Remarks

The operating conditions in high performance combustion chambers are such that there will always be high probabilities for disturbances to be unstable. Experience during the past four decades has clearly shown that one must expect a new design always exhibit instabilities. While research has established the principles for constructing stable combustion chambers, in practice the requirements may be poorly understood or violated in efforts to improve steady-state performance.

It is therefore essential that continuing work on these problems be directed to deeper understanding of the mechanisms of combustion instabilities; constructing more powerful and widely applicable analysis; and development of new methods for suppressing instabiliites when they arise in full-scale systems. Recent progress in sensors, instrumentation and computing resources offer significant new opportunities.

Understanding mechanisms requires extensive and careful laboratory experiments as well as thorough analysis of instabilities in full-scale systems. Three dominant mechanisms have been emphasized here: liquid droplet formation, vaporization and combustion; vortex shedding and combustion; and convective waves. For many years, mechanisms have been interpreted with a time lag model. This representation of the coupling between gasdynamics and unsteady mass and energy release has been successfully used in both research and design. However, as emphasized in Section 3.1, the use of the two parameters n, the interaction index, and τ, the time lag, is not to be confused with understanding what is really happening. Global correlations of data without firm basis on fundamental processes have limited predictive value and ranges of application.

With the development of higher performance propulsion systems, the most effective strategy (and cheapest in the long run) to treat combustion instabilities must be founded on research devoted to basic problems. It seems clear that for all three types of liquid-fueled systems, it is essential first to understand thoroughly the steady and unsteady processes leading from injected liquid to combustion of gaseous reactants. Already much has been learned in recent research on these subjects that has not been incorporated in studies of combustion instabilities. Much remains to be done, but the main point is that it should no longer be necessary to rely so heavily on vague applications of the $n - \tau$ model. Modern experimental methods, including high speed non-intrusive measurements and flow visualizaiton, provide wide opportunities for obtaining closer definition of the processes.

The recent work on vortex shedding and combustion in dump combustors is an indication of the possibilities. In addition to tests on realistic configurations, related laboratory tests and detailed numerical analysis have provided the beginnings for understanding the basic processes involved. The research is certainly not complete, and satisfactory connections with design have yet to be made, but the direction of progress is correct. Work on this subject and on the processes of spray formation and combustion should be directly applicable to thrust augmentors as well.

Analysis of the complete problem in combustors serves as a global framework important to both research and design. It is essential that proposed mechanisms be checked thoroughly with all relevant processes accounted for; failure to do so will quite likely produce incorrect and misleading results as discussed in Section 3.2. Modern computing resources are now capable of handling full three-dimensional problems and the time has arrived for serious application of computational fluid dynamics to problems of combustion instabilties.

Such calculations are expensive and will not in the forseeable future be useful for routine design work. Moreover, it will likely always be true that significant uncertainties will accompany some of the necessary input information, especially for the unsteady combustion processes. And of course there is always the feature that each numerical calculation provides results only for one case.

Nevertheless, numerical analysis is important for several purposes. As for the external aerodynamics of a missile or aircraft, CFD can be used in the design process. However, the corresponding application to combustion instabilities must await further progress in understanding the fundamental processes in a combustion chamber.

Recent work on the flow fields in dump combustors, and similar calculations for solid fueled rockets, have demonstrated that much can be learned even at this stage. Equally important, although incomplete and limited, the analyses serve as necessary steps in continued development of analytical tools.

For most applications and theoretical work, approximate analysis in one form or other will probably always be the primary method of doing calculations. One framework has been described here in Section 2; its usefulness has been illustrated in several places. It affords a rapid and simple way of assessing mechanisms and predicting trends of behavior; and in combination with test results, it provides guidelines for design.

An important step that has yet to be taken is close coordination of approximate and numerical analyses. Because of the assumptions required to simplify the equations, any approximate analysis is always accompanied by uncertainties in the results that are not completely known. The only way to assess accuracy is to compare approximate results with those obtained by an accurate numerical analysis. Limited comparisons of this sort have been done for instabilities in solid rockets, showing that the approximate analysis is accurate under broad useful conditions and can also be helpful in understanding unexpected numerical results.

Nonlinear behavior of combustion instabilities is an important topic not covered in this survey. Because combustion instabilities are self-excited oscillations, they reach limiting amplitudes only because one or more nonlinear processes are active. There are two classes of nonlinear problems to be considered theoretically: 1) what are the conditions for existence and stability of limit cycles?; and 2) under what conditions is a linearly stable system unstable to a sufficiently large initial disturbance? These problems have received some attention, chiefly with approximate analysis, but much remains to be learned. The subject of nonlinear behavior is fascinating theoretically, and already some results useful in practical situations have been obtained.

It appears that the confluence of modern experimental and analytical research will provide the basis for applying methods of active control to problems of combustion instabilities. The possibility is attractive, and may seem almost obvious, but practical realization is far off. Initial results obtained in laboratory tests with and without combustion have illustrated the promise. However, the differences between the conditions in those devices and in full-scale propulsion systems must be recognized. Successful applications will surely require thorough understanding of the mechanisms causing instabilities in the actual systems; results will be required of all the subjects covered in this review.

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DISCUSSION

A. P. Dowling, UK

I am surprised by your statement that the design of a successful active controller requires a detailed understanding of local combustion-flow interactions. Usually the implementation of 'anti-sound' or active control requires only global properties, and the transfer functions are more frequently measured than predicted. Would you care to elaborate?

Author's Reply:

I base my comment on the notion that the most effective control of a complicated physical system generally requires understanding the system itself. That forms part of the basis for designing the controller. While it's true that knowing only the transfer functions may on occasion be adequate, that information already implies at least partial knowledge of the system. Especially if, as the case is for combustion chambers, nonlinear processes may be important, I suspect that success will rest substantially on knowledge of the physical behavior.