Introduction to Symmetry Analysis - Edits and Errata June 27, 2024

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Removed text is indicated by parentheses (..)

Historical Preface

1) Page xxi, line 18 from the top - ...to our experience had a (an) direct or indirect...
2) Page xxv, line 5 from the top - "Crelle had become very wealthy from the railroad business...". This somewhat perfunctory description of the source of Crelle's wealth deserves some expansion given the fact that commercial railroads were in their infancy at the time. Although the opening of the Liverpool and Manchester railroad in 1830 is usually taken to be the beginning of modern rail transportation, the practical development of railroads began in Britain in the early 1800's. Railroads were extensively used in mining and for hauling coal in Britain and the Continent throughout that period. Horses often provided the power but early steam designs were being developed. Crelle worked as a self-taught civil engineer for the Prussian government designing roads and railroads until he left in 1828 at the age of 48 to put more attention to his journal. His wealth derived from the fact that most railroads in Germany were built to his designs. He didn't put the railroad business aside altogether and played a key role in the planning and construction of the first commercial line between Berlin and Potsdam that opened in 1838.
3) Page xxx - Shortly after publication, I sent a copy of the book to my good friend Grisha Barenblatt who was visiting the UC Berkeley math department at the time. In his November 23, 2002 thank you letter, he correctly pointed out that my discussion of **Gauss, Riemann and the New Geometry** failed to to refer to the central role of Nikolai Lobachevsky in the discovery of Non-Euclidean (hyperbolic) geometry, "one of the greatest discoveries in the history of mankind". He further noted, "In fact, namely Lobachevsky's publication in 1829 in the Bulletin of Kazan University, a regular scientific magazine, was the first publication of the great idea". The three people most associated with this advance are Gauss, who never published on the subject, Bolyai who only published once, and Lobachevsky who published widely over his career and is often referred to as the Copernicus of geometry.
4) Page xxxviii, line 3 - ...leading Danish (Norwegian) mathematician...

Chapter 1

1) Page 26, line 4 from the bottom - ...which is a locally analytic...

Chapter 2

1) Page 54, Reference [2.5] - ...at Mach numbers from 0.3 to 2.0 and Reynolds numbers approaching 10^7...

Chapter 3

1) Page 57, Equation (3.8) - The left side should read $d\psi$ not $d\Psi$.
2) Page 70, Equation (3.82) - The third term in the equation should read $W[x, y, z]\partial\Psi/\partial z$ not $W[x, y, z]\partial\Psi/\partial x$.
3) Page 74, Equation (3.102) - The third term after the equality should read $= -\nabla^2\mathbf{A}$ not $= \nabla^2\mathbf{A}$.
4) Page 76, Equation (3.116) - The last equality should read $\psi_4 = z + 2(x - q)(p/q) - (p/q)x^2$.

Chapter 4

1) Page 110, Equation (4.80) - The right-hand equation should read $\mathbf{r}_2 = -m_1\mathbf{r}/m_1 + m_2$.
2) Page 120, Equation (4.125) - The first term in the equation should read d^2x/dt^2.

Chapter 5

1) Page 144, lines 5 and 6 - The structure constants......are connected to the structure constants...
2) Page 144, line 9 - However, the definitions (5.91) and (5.92) do not....
3) Page 144, line 16 - These are called *Jordan algebras* after the German physicist Pascual Jordan (1902-1980) who developed the matrix theory of quantum mechanics with Heisenberg and Born. Remove "...great French group theorist Marie-Ennemond Camille Jordan.....to.....and Felix Klein". I made reference here to the wrong Jordan.
4) Page 146, Equation (5.103) - The equation should read $\tilde{x} = x\,cosh[a] + ct\,sinh[a]\,, \quad \tilde{t} = (x/c)\,sinh[a] + t\,cosh[a]$.
5) Page 147, line 1 - ...of the system (5.105). Use the...

Chapter 6

1) Page 149, Equation (6.2) - The first relation should read $\tilde{x} = a\,x + b\,y$.
2) Page 162, Equation (6.73) - The equation should read

$$\begin{aligned}&\left(\tfrac{xy}{f[y]}\tfrac{\partial}{\partial x} + \tfrac{y^2}{f[y]}\tfrac{\partial}{\partial y}\right)\left(\int_{y/x}\tfrac{1}{\alpha^2 g[\alpha] d\alpha} - \int_y \tfrac{f[\alpha]}{\alpha^2}d\alpha\right)\\ &= \tfrac{xy}{f[y]}\left(-\tfrac{1}{\alpha g[\alpha]}\tfrac{1}{x}\right)_{\alpha=y/x} + \tfrac{y^2}{f[y]}\left(-\tfrac{1}{\alpha^2 g[\alpha]}\tfrac{1}{x}\right)_{\alpha=y/x}\\ &+\tfrac{y^2}{f[y]}\left(\tfrac{f[\alpha]}{\alpha^2}\right)_{\alpha=y} = -1\end{aligned} \tag{1}$$

3) Page 170, 6 lines from the bottom - "....using the relation $F[\phi,k] = K + F[\phi - \pi/2, k]$. When we consider...". Between these two sentences, insert the following sentence. "The quantity K refers to the complete elliptic integral which for $D > 0$ is $K = F[\pi/2, sin(5\pi/12)] = 2.76804$ and for $D < 0$, is $K = F[\pi/2, sin(\pi/12)] = 1.59814$".
4) Page 171, equation (6.129) - The second of the two relations should read as follows.

$$q^0_{r>0}[\tau] = -\left(\frac{1}{1-(1/\sqrt{3})\tau}\right)^2 \qquad (0 < \tau < \sqrt{3}) \tag{2}$$

5) Page 173, line 12 - ... construction attributed to (used by) the Greek mathematician Diophantus....
6) Page 176, equation (6.147) - in the right equation, the y to the left of the equals sign needs a tilde $\tilde{y}$.

Chapter 8

1) Page 202, line - ....associated....
2) Page 206, Equation (8.73) - This equation should read as follows.

$$\psi^2 = \int_x \left\{C\,exp\left[\int_{x'}(B + 2\,f\,C)\,dx''\right]\right\}dx' + h\,exp\left[\int_x (B + 2\,f\,C)\,dx'\right] \tag{3}$$

3) Page 207, Equation (8.82) - This equation should read as follows.

$$sinh^{-1}[y/\sqrt{\psi_1}] = tanh^{-1}[y_x] + tanh^{-1}[\psi^2] \tag{4}$$

4) Page 207, Equation (8.84) - The middle term in the equality should read $\Omega[\psi^1, \psi^2]$.
5) Page 211, Equation (8.99) - The equation as written has unphysical (imaginary) solutions. It should read as follows.

$$y_{xx} + \frac{1}{x} y_x - e^y = 0 \tag{5}$$

6) Pages 211 and 212, Equations (8.100), (8.101),(8.102),(8.103),(8.104), (8.105) and (8.106) - To be consistent with the change to (8.99), change e^y to $-e^y$. The symmetries do not change.
7) Page 217, last line - "...solvable Lie subalgebra where...". Remove the period.
8) Page 228,three lines from the bottom - "...since f, g, h and F are assumed to be...".
9) Page 230, equation (8.210) - This equation should read as follows.

$$\frac{dv}{du} + \left(\frac{\theta_u}{\theta} - \left(\frac{f[u]}{h[u]} \right) \frac{\theta}{\theta_u} \right) v - \frac{F[u]}{h[u]\,\theta_u} = 0 \tag{6}$$

10) Page 231, equation (8.214) - This equation should read as follows.

$$\begin{aligned} &\tfrac{d\tilde{v}}{d\tilde{u}} + \left(\tfrac{\tilde{\theta}_{\tilde{u}}}{\tilde{\theta}} - \left(\tfrac{f[\tilde{u}]}{h[\tilde{u}]} \right) \tfrac{\tilde{\theta}}{\tilde{\theta}_{\tilde{u}}} \right) \tilde{v} - \tfrac{F[\tilde{u}]}{h[\tilde{u}]\,\tilde{\theta}_{\tilde{u}}} \\ &= \tfrac{dv}{du} + \left(\tfrac{\theta_u}{\theta} - \left(\tfrac{f[u]}{h[u]} \right) \tfrac{\theta}{\theta_u} \right) v - \tfrac{F[u]}{h[u]\,\theta_u} = 0 \end{aligned} \tag{7}$$

11) Page 231, equations (8.216), (8.217), (8.218), (8.219) - Where f appears inside the integral replace it with F.

Chapter 9

1) Page 238, line 1 - ....of many important equations. This is especially....
2) Page 249, equation (9.59) - The third relation should read.

$$\eta_{\{xx\}} = D_x \eta_{\{x\}} - u_{xx} D_x \xi - u_{xt} D_x \tau \tag{8}$$

3) Page 250, line 20 - ....first term on the right of (9.60), η_x, stands for the....
4) Page 252, line 22 - ....is called **SolveDeterminingEquations**, and the numbering....
5) Page 255, line 25 - ....solutions of the same invariance condition used to identify point....
6) Page 256, six lines from the bottom - ....discussed in Chapter 3 and, when collected together....Remove the reference to sections 3.1 and 3.8.
7) Page 259, equation (9.89) - The group parameter in this set of relations is a not s.
8) Page 260, line 1 - The two integrals of (9.90) are
9) Page 270, paragraph 2 - Line 2 change (9.86) to (9.78), line 3 change (9.86) to (9.78) in two positions.
10) Pages 275 and 276 - Somehow the publisher managed to drop the first letter of each name in the references. They are [9.1] Ibragimov, [9.2] Abramowitz, [9.3] Barenblatt, [9.4] Baikov, [9.5] Baikov, [9.6] Mahalingham, [9.7] Aksenov, [9.8] Hydon, [9.9] Baumann, [9.10] Bluman, [9.11] Clarkson.

Chapter 10

1) Page 281, Equation (10.10) - In the third relation change $\psi = e^c \psi$ to $\tilde{\psi} = e^c \psi$.
2) Page 282, first line below equation (10.18) - ...is only satisfied if we require....
3) Page 284, Equation (10.26) - In the right hand relation change $(F - \alpha F_\alpha)$ to $(\alpha F_\alpha - F)$.
4) Page 285, Equation (10.32) - The last term in the series of equalities should be $\frac{F_{\alpha\alpha\alpha}}{0}$.
5) Page 294, Paragraph 3 line 2 - ...there appears to be an infinite number....
6) Page 297, Line 4 - ...the diverging separatrix shown as a....
7) Page 305, Line 8 - ...by applying the rule $\partial U_e / \partial y = 0$ to the invariance condition....
8) Page 313, second paragraph line 1 - ...one of the few known exact solutions of....

Chapter 11

1) Page 318, line 9 - ...and the distinction between streamlines and particle paths in unsteady flow...
2) Page 329, equation (11.49) - Remove the second relation in this equation with $M/12\nu$ on the left hand side. The differential in the first relation should be $d\hat{F}$.
3) Page 337, second paragraph line 2 - ...governing equations in these variables...
4) Page 354, equation (11.139) - The second factor on the right hand side is $(1+Q_A+R_A)^2$. The I should be a 1.
5) Page 357, equations (11.146) and (11.147) - The exponent in the first relation in each of these equations is $(x+x_0)^{(1-\beta)/2}$ and $\theta^{(1-\beta)/2}$ respectively.

Chapter 12

1) Page 372, equation (12.35) - The second term in the second relation in this equation is $-\frac{x^i\,t}{({t_0}^2+t^2)^{1/2}}$ respectively.
2) Page 377, equation (12.48) - Change $\hat{\rho}$ to $\hat{\rho}_\infty$.
3) Page 385, equation (12.87) - The equation should read as follows.

$$\frac{\partial^2\phi_1}{\partial{x_1}^2}+\frac{\partial^2\phi_1}{\partial{y_1}^2}=0 \tag{9}$$

4) Page 386, equation (12.94) - The second relation should read as follows.

$$\left.\frac{\partial\phi_2'}{\partial x_2}\right|_{x_2\rightarrow-\infty}=0 \tag{10}$$

5) Page 392, exercise 12.4, second sentence - ...the package....

Chapter 13

1) Page 404, third paragraph - ...into the Reynolds equations (13.6), the result....
2) Page 405, ten lines from the bottom - Change "See Figures 11.11 and 11.12". to "See Figure 11.11".
3) Page 407, last paragraph - ...where the constant varies from flow to flow and tends to fall between 10 and 50 (Reference [13.12]).
4) Page 417, equation (13.61) - On the right hand side, G should have a superscript G^i.
5) Page 422, line 3 - ...where the jet Reynolds number is $u_{jet}d/\nu$. Change v to ν.
6) Page 431, line 10 - ...near the origin and the only.... Remove "however,".
7) Page 434, line 20 - ...indicated by the label ② in Figure 13.9, although vortex... Drop ...Figure 13.6, Figure 13.7, and...
8) Page 434, equation (13.102) - This equation should read as follows.

$$\left(\frac{2^{4/3}}{3}K_k^i+\frac{1}{3}\delta_k^i\right)\left(\frac{2^{4/3}}{3}K_j^k+\frac{1}{3}\delta_j^k\right)=\delta_j^i \tag{11}$$

9) Page 442, exercise 13.11 item (5) - Choose the appropriate value of f.

Chapter 14

1) Page 447, second paragraph, line 8 - ...in agreement with Olver point out that neither....
2) Page 448, first line after equation (14.6) - ...with this property can be used to transform....
3) Page 455, equation (14.29) - The group parameter is s. In the leftmost relation replace t with s.
4) Page 455, equation (14.68) - This equation should read as follows. The superscripts 2 and p on the D operator are redundant.

$$\mu^i_{\{j_1\}} = D_{j_1}\mu^i$$

$$\mu^i_{\{j_1 j_2\}} = D_{j_2}\mu^i_{\{j_1\}} = D_{j_1 j_2}\mu^i \,,$$

$$\vdots \tag{12}$$

$$\mu^i_{\{j_1 \ldots j_p\}} = D_{j_1 \ldots j_p}\mu^i \,,$$

$$\vdots$$

5) Page 463, equation (14.71) - The superscripts 2 and 3 on the D operator are redundant and should be removed.
6) Page 464, equation (14.79) - Again, the superscripts 2 and 3 on the D operator are redundant and should be removed.
7) Page 468, the line following equation (14.102) - ...to the invariance condition (14.97), and the remaining....
8) Page 480, first line of Theorem 14.3 - Replace y with **y**.
9) Page 486. After the third line below equation (14.192) add the sentence: "The software package identifies the symmetries (14.192) correctly".
10) Page 487, equation (14.198) - The first group operator on the second line of the equation should read as follows.

$$X^4 = xt\frac{\partial}{\partial x} + t^2\frac{\partial}{\partial t} + (x - tu)\frac{\partial}{\partial u} \tag{13}$$

11) Page 492, first line after equation (14.227) - ...and we finally have....

Chapter 15

1) Page 500, equation (15.14) - Change $+D(s^2) + \ldots$ to $+O(s^2) + \ldots$.
2) Page 508, equation (15.52) - The equation is written on five lines. On line 2, change $-$ to $+$. On line 4, change $-\frac{3}{2}$ to $+\frac{3}{2}$.
3) Page 508, equation (15.54) - On the right hand side, change $\frac{1}{2}mv^2$ to $\frac{1}{4}mv^2 + \frac{1}{2}\gamma/r$.
4) Page 510, equation (15.67) - Remove the 2.
5) Page 510, equation (15.68) - The equation should read as follows.

$$L = \frac{1}{2}\psi_t^2 - \frac{1}{2}(\psi_{xx} + \psi_{yy})^2 \tag{14}$$

6) Page 510, equations (15.72), (15.73) and (15.74) - Remove the factors of 2 in each equation.
7) Page 513, equation (15.79) - Remove the 2.
8) Page 513, equation (15.80) - The equation should read as follows.

$$L = \frac{1}{2}\psi_t^2 - \frac{1}{2}(\psi_{xx} + \psi_{yy})^2 \tag{15}$$

Chapter 16

1) Page 524, equation (16.45) - The first term on the right should read $(f_\tau - f_{\chi\chi})e^{\phi/2}$.
2) Page 525, equation (16.51) - The second relation should read $\frac{dg}{da} = 1 + a + a^2 \ldots$.
3) Page 526, equation (16.55) - Change D_x to D_χ.
4) Page 542, equations (16.140) and (16.141) - Remove the exponent 2 on the right bracket of each equation.
5) Page 543, line 6 - ...and singular cases of the solution $\tilde{w}^-$ are plotted...
6) Page 544, equation (16.147) - Remove the exponent 2 on the right bracket of the equation.
7) Page 546, equation (16.163) - Change $+2$ to -2 ahead of the bracket enclosing the second term

8) Page 547, figure (16.7) - The relevant parameter values for this figure are $\kappa_0 = 1$, $\kappa_1 = 0.75$, $x_{a0} = 0$, $x_{a1} = -10$, $t_{a1} = t_{a0} = 0$ and $c_1 = c_0 = 0$.

Appendix A1

1) Page 556, figure A1.1 - In the caption change S to ψ.

Appendix A3

1) Page 577, line 12 - .....where α has been replaced by j_1.
2) Page 577, equation (A3.31) - This equation should read as follows.

$$D_\alpha(\eta^i)|_{p-1} = \frac{\partial \eta^i}{\partial x^\alpha} + y^\beta_\alpha \frac{\partial \eta^i}{\partial y^\beta} + y^\beta_{\gamma_1 \alpha} \frac{\partial \eta^i}{\partial y^\beta_{\gamma_1}} + \ldots + y^\beta_{\gamma_1 \ldots \gamma_{p-1} \alpha} \frac{\partial \eta^i}{\partial y^\beta_{\gamma_1 \ldots \gamma_{p-1}}} \tag{16}$$

4) Page 580, theorem (A.3.1), line 3 - ...point transformations for $m \geq 1$ and extensions of....