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AA103
Air and Space Propulsion

Topic 10 – Aircraft engine performance parameters

Suggested reading – AA283 Course reader Chapter 2

The problem of predicting/measuring thrust

Definition of thrust

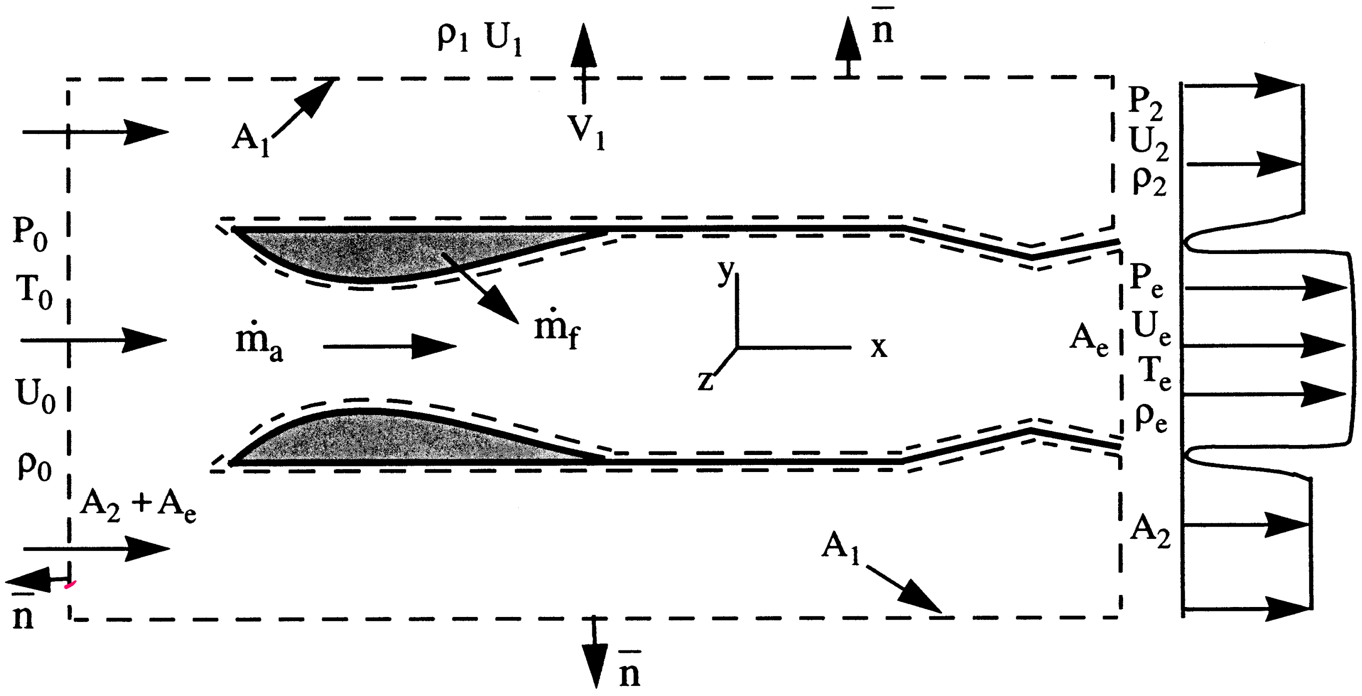

Figure 2.1 Ramjet control volume for developing a definition of thrust

$$\dot{m}_a = \text{Air mass flow rate}$$
$$\dot{m}_f = \text{fuel mass flow rate} \qquad (2.1)$$

Integral form of the equations of motion

$$\int_A \rho \bar{U} \bullet \bar{n} dA = 0$$

$$\int_A (\rho \bar{U}\bar{U} + P\bar{\bar{I}} - \bar{\bar{\tau}}) \bullet \bar{n} dA = 0 \qquad (2.2)$$

$$\int_A (\rho h_t \bar{U} - \bar{\bar{\tau}} \bullet \bar{U} + \bar{Q}) \bullet \bar{n} dA = 0$$

Stagnation enthalpy

$$h_t = e + pv + k \qquad (2.3)$$

Conservation of mass

$$\int_A \rho \bar{U} \bullet \bar{n} dA =$$

$$\int_{A_2} \rho_2 U_2 dA + \rho_e U_e A_e - \rho_0 U_0 (A_2 + A_e) - \dot{m}_f + \int_{A_1} \rho_1 V_1 dA = 0 \qquad (2.4)$$

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Conservation of momentum

$$\int_A (\rho\bar{U}\bar{U} + P\bar{\bar{I}} - \bar{\bar{\tau}}) \bullet \bar{n}\,dA \Big|_x =$$

$$\int_{A_2} (\rho_2 U_2^2 + P_2)dA + (\rho_e U_e^2 A_e + P_e A_e) - (\rho_0 U_0^2 + P_0)(A_2 + A_e) + \quad (2.5)$$

$$\int_{A_1} \rho_1 U_1 V_1 dA + \int_{A_w} (P\bar{\bar{I}} - \bar{\bar{\tau}})dA = 0$$

Surface forces

$$\int_{A_w} (P\bar{\bar{I}} - \bar{\bar{\tau}})dA = Thrust + Drag = 0 \quad (2.6)$$

Momentum flux over the outer surface of the control volume

$$\int_{A_1} \rho_1 U_1 V_1 dA \cong \int_{A_1} \rho_1 U_0 V_1 dA \quad (2.7)$$

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Subtract the mass equation multiplied by U_0

$$\rho_e U_e (U_e - U_0) A_e + (P_e - P_0) A_e + \dot{m}_f U_0 + \int_{A_2} \rho_2 U_2 (U_2 - U_0) + (P_2 - P_0)\, dA = 0 \tag{2.8}$$

$$Thrust = \rho_e U_e (U_e - U_0) A_e + (P_e - P_0) A_e + \dot{m}_f U_0 \tag{2.9}$$

$$Drag = \int_{A_2} \rho_2 U_2 (U_2 - U_0) + (P_2 - P_0)\, dA \tag{2.10}$$

Overall mass balance

$$\rho_e U_e A_e = \dot{m}_a + \dot{m}_f \qquad (2.11)$$

Definition of thrust

$$T = \dot{m}_a(U_e - U_0) + (P_e - P_0)A_e + \dot{m}_f U_e \qquad (2.12)$$

Turbojet control volume

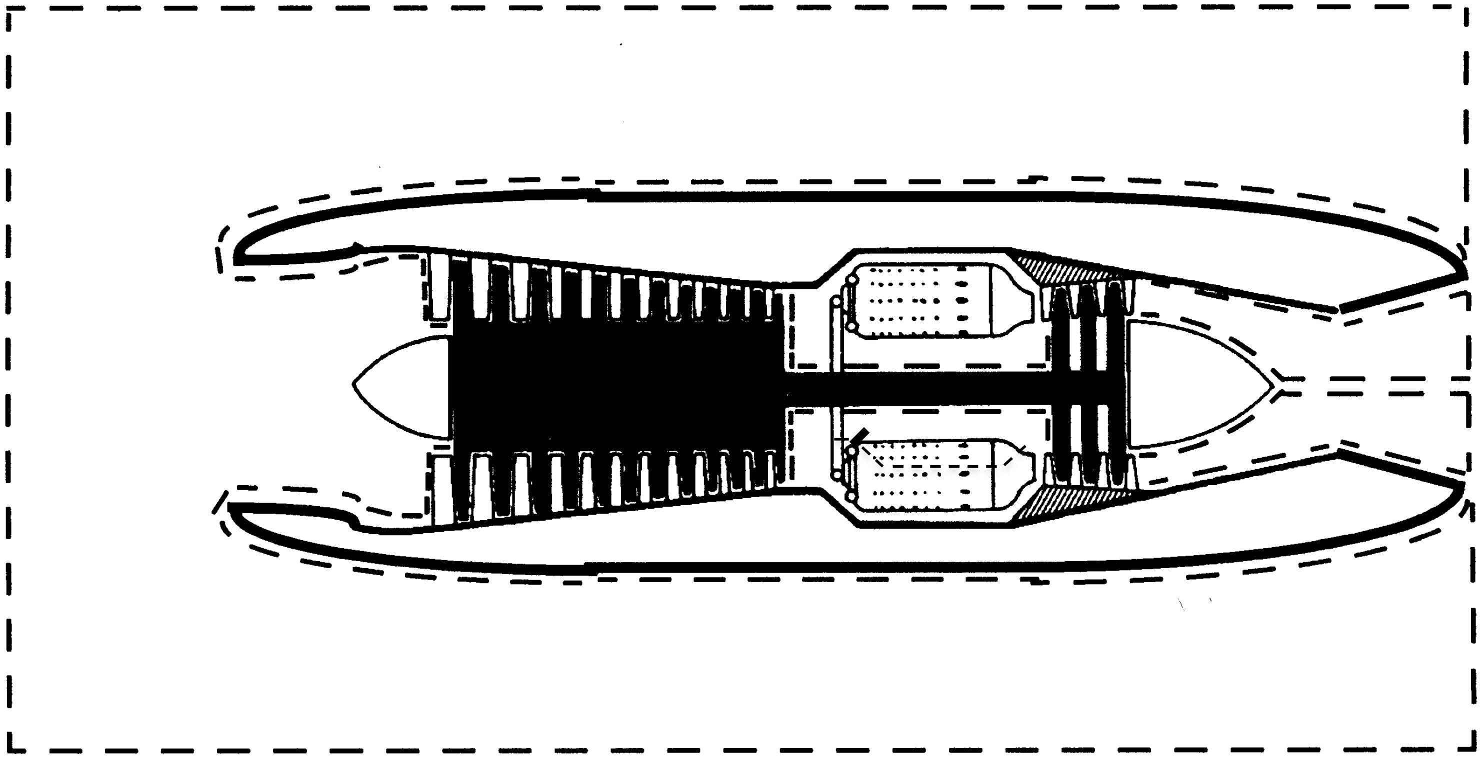

Figure 2.2 Turbojet control volume

Energy balance

$$\int_A (\rho h_t \overline{U} - \bar{\bar{\tau}} \bullet \overline{U} + \overline{Q}) \bullet \bar{n} dA =$$

$$\int_{A_2} \rho_2 h_{t2} U_2 dA + \rho_e U_e h_{te} A_e - \rho_0 h_{t0} U_0 (A_2 + A_e) - \dot{m}_f h_f + \qquad (2.13)$$

$$\int_{A_1} \rho_1 h_{t1} V_1 dA = 0$$

Energy content in jet fuel

$$h_f \big|_{JP-4} = 4.28 \times 10^7 J/kg \qquad (2.14)$$

$$h \big|_{Air\ at\ 288.14°K} = C_p T_{SL} = 1005 \times 288.14 = 2.896 \times 10^5 J/kg$$

$$\frac{h_f \big|_{JP-4}}{h \big|_{Air\ at\ 288.14°K}} = 148 \qquad (2.16)$$

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Assume the flow about the engine is adiabatic

$$h_{t2} = h_{t1} = h_{t0}$$

Then

$$\int_{A_2} \rho_2 h_{t0} U_2 dA + \rho_e U_e h_{te} A_e - \rho_0 h_{t0} U_0 (A_2 + A_e) - \dot{m}_f h_f + \int_{A_1} \rho_1 h_{t0} V_1 dA = 0 \quad (2.17)$$

Subtract the mass equation multiplied by h_{t0}

$$\rho_e U_e h_{te} A_e - \rho_e U_e h_{t0} A_e - \dot{m}_f (h_f - h_{t0}) = 0 \quad (2.18)$$

Overall energy balance for an adiabatic engine

$$(\dot{m}_a + \dot{m}_f) h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f \quad (2.19)$$

Capture area

$$\dot{m}_a = \rho_0 U_0 A_0 \qquad (2.20)$$

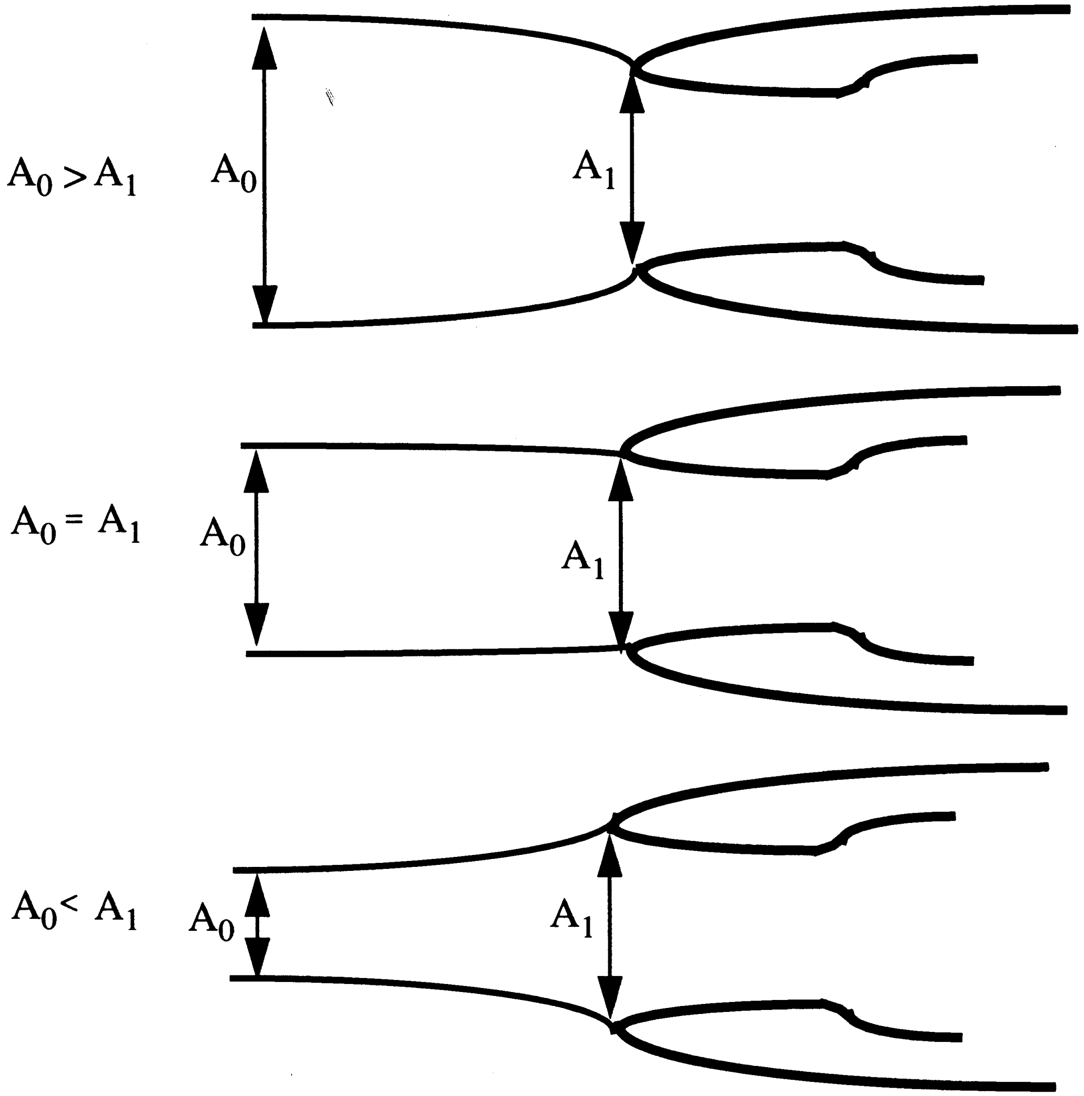

Figure 2.3 Variation of inlet capture area with engine operating point.

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Overall efficiency

$$\eta_{ov} = \frac{\textit{The power delivered to the vehicle}}{\textit{The total energy released per second through combustion}} \qquad (2.21)$$

$$\eta_{ov} = \frac{TU_0}{\dot{m}_f h_f}$$

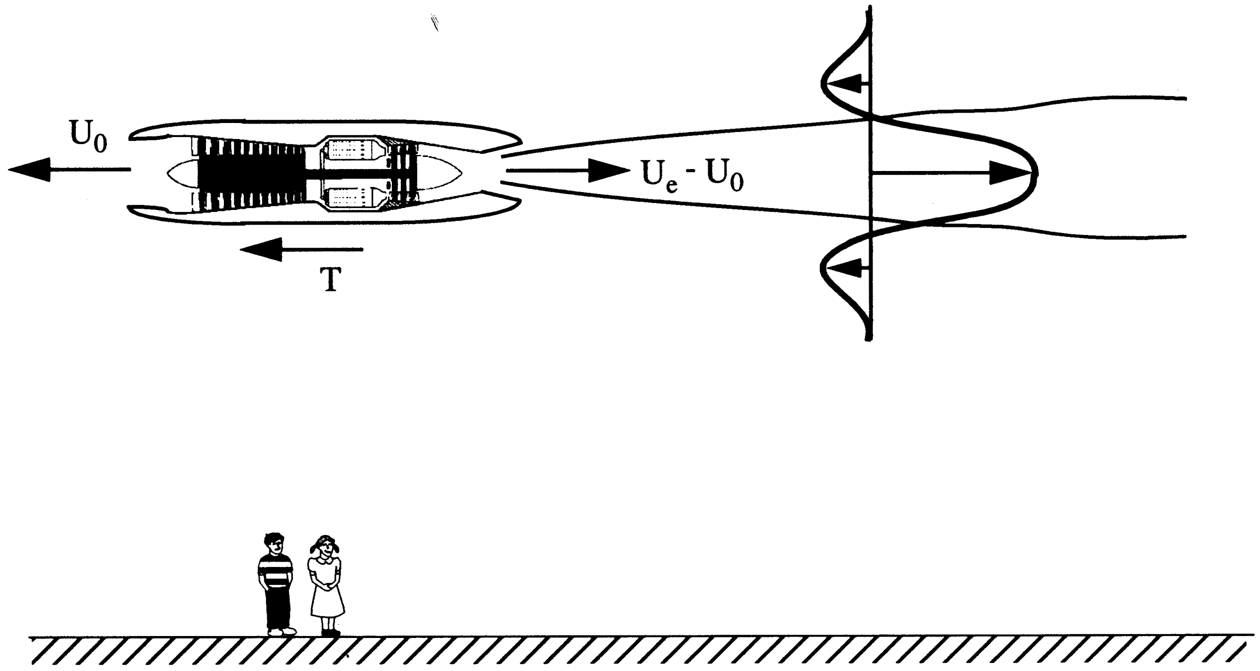

Figure 2.4 Frame of reference used to define efficiencies.

Breguet aircraft range equation

$$R = \int U_0 dt = \int \frac{\dot{m}_f h_f \eta_{ov}}{T} dt .$$

Fuel mass flow

$$\dot{m}_f = -\frac{1}{g}\frac{dw}{dt} \qquad (2.24)$$

$$T = D = \left(\frac{D}{L}\right)L = \left(\frac{D}{L}\right)w \quad . \qquad (2.25)$$

$$R = -\eta_{ov}\frac{h_f}{g}\left(\frac{L}{D}\right)\int_{w_{initial}}^{w_{final}} \frac{dw}{w} . \qquad (2.26)$$

$$R = \eta_{ov}\frac{h_f}{g}\left(\frac{L}{D}\right)Log\left(\frac{w_{initial}}{w_{final}}\right) \quad . \qquad (2.27)$$

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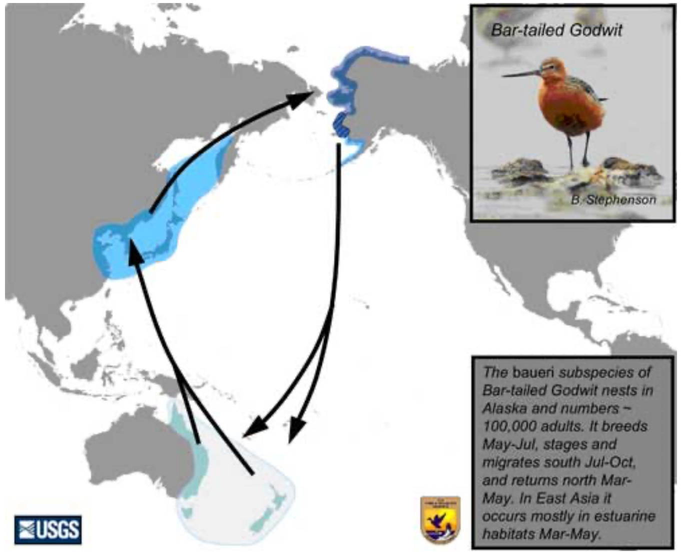
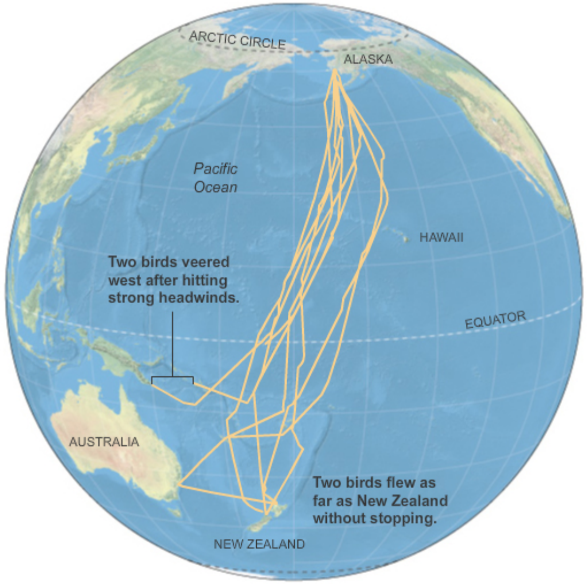
Epic bird flight - The Bar-tailed Godwit - 11,000km non-stop





Propulsive efficiency

$$\eta_{ov} = \eta_{pr} \times \eta_{th} \tag{2.28}$$

$$\eta_{pr} = \frac{\textit{Power delivered to the vehicle}}{\textit{Power to the vehicle} + \frac{\Delta\ \textit{kinetic energy of air}}{\textit{second}} + \frac{\Delta\ \textit{kinetic energy of fuel}}{\textit{second}}} \tag{2.29}$$

$$\eta_{pr} = \frac{TU_0}{TU_0 + \left[\frac{\dot{m}_a(U_e - U_0)^2}{2} - \frac{\dot{m}_a(0)^2}{2}\right] + \left[\frac{\dot{m}_f(U_e - U_0)^2}{2} - \frac{\dot{m}_f(U_0)^2}{2}\right]} \tag{2.30}$$

For a fully expanded exhaust and neglecting fuel flow

$$\eta_{pr} = \frac{2U_0}{U_e + U_0}. \tag{2.31}$$

Thermal efficiency

$$\eta_{th} = \frac{\textit{Power to the vehicle} + \dfrac{\Delta\ \textit{kinetic energy of air}}{\textit{second}} + \dfrac{\Delta\ \textit{kinetic energy of fuel}}{\textit{second}}}{\dot{m}_f h_f} \quad (2.32)$$

$$\eta_{th} = \frac{TU_0 + \left[\dfrac{\dot{m}_a(U_e - U_0)^2}{2} - \dfrac{\dot{m}_a(0)^2}{2}\right] + \left[\dfrac{\dot{m}_f(U_e - U_0)^2}{2} - \dfrac{\dot{m}_f(U_0)^2}{2}\right]}{\dot{m}_f h_f} \quad (2.33)$$

$$\eta_{th} = \frac{(\dot{m}_a + \dot{m}_f)\dfrac{U_e^2}{2} - \dot{m}_a\dfrac{U_0^2}{2}}{\dot{m}_f h_f} \quad (2.34)$$

Recall the efficiency of a thermodynamic cycle

$$\eta = \frac{W}{Q_{input\ during\ the\ cycle}} = \frac{Q_{input\ during\ the\ cycle} - Q_{rejected\ during\ the\ cycle}}{Q_{input\ during\ the\ cycle}} = 1 - \frac{Q_{rejected\ during\ the\ cycle}}{Q_{input\ during\ the\ cycle}} \quad (2.35)$$

This can be compared to the, just defined, thermal efficiency of an aircraft engine.
Add and subtract 1 to equation 2.34

$$\eta_{th} = 1 - \left\{ \frac{\dot{m}_f h_f + \dot{m}_a \frac{U_0^2}{2} - (\dot{m}_a + \dot{m}_f) \frac{U_e^2}{2}}{\dot{m}_f h_f} \right\} \quad (2.36)$$

Recall the energy balance for an adiabatic engine

$$(\dot{m}_a + \dot{m}_f) h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f$$

$$h_{te} = h_e + \frac{U_e^2}{2} \qquad h_{t0} = h_0 + \frac{U_0^2}{2} \quad (2.37)$$

$$\eta_{th} = 1 - \left\{ \frac{\dot{m}_f h_f + \dot{m}_a (h_{t0} - h_0) - (\dot{m}_a + \dot{m}_f)(h_{te} - h_e)}{\dot{m}_f h_f} \right\} \quad (2.38)$$

$$\eta_{th} = 1 - \frac{(\dot{m}_a + \dot{m}_f)(h_e - h_0) + \dot{m}_f h_0}{\dot{m}_f h_f} = 1 - \frac{Q_{rejected\ during\ the\ cycle}}{Q_{input\ during\ the\ cycle}} \quad (2.39)$$

Specific impulse, SFC

$$Isp = \frac{Thrust\ force}{Weight\ flow\ of\ fuel\ burned} = \frac{T}{\dot{m}_f g} \qquad (2.41)$$

$$SFC = \frac{pounds\ of\ fuel\ burned\ per\ hour}{pound\ of\ thrust} = \frac{3600}{Isp} \qquad (2.42)$$

$$\begin{aligned} SFC|_{JT9D-takeoff} &\cong 0.35 \\ SFC|_{JT9D-cruise} &\cong 0.6 \\ SFC|_{military\ engine} &\cong 0.9\ to\ 1.2 \\ SFC|_{military\ engine\ with\ afterburning} &\cong 2 \end{aligned} \qquad (2.43)$$

Dimensionless forms

Thrust

$$\frac{T}{P_0 A_0} = \gamma M_0^2\left((1+f)\frac{U_e}{U_0} - 1\right) + \frac{A_e}{A_0}\left(\frac{P_e}{P_0} - 1\right)$$

$$\frac{T}{\dot{m}_a a_0} = \left(\frac{1}{\gamma M_0}\right)\frac{T}{P_0 A_0} \tag{2.44}$$

Specific impulse

$$\frac{I_{sp} g}{a_0} = \left(\frac{1}{f}\right)\frac{T}{\dot{m}_a a_0} \tag{2.45}$$

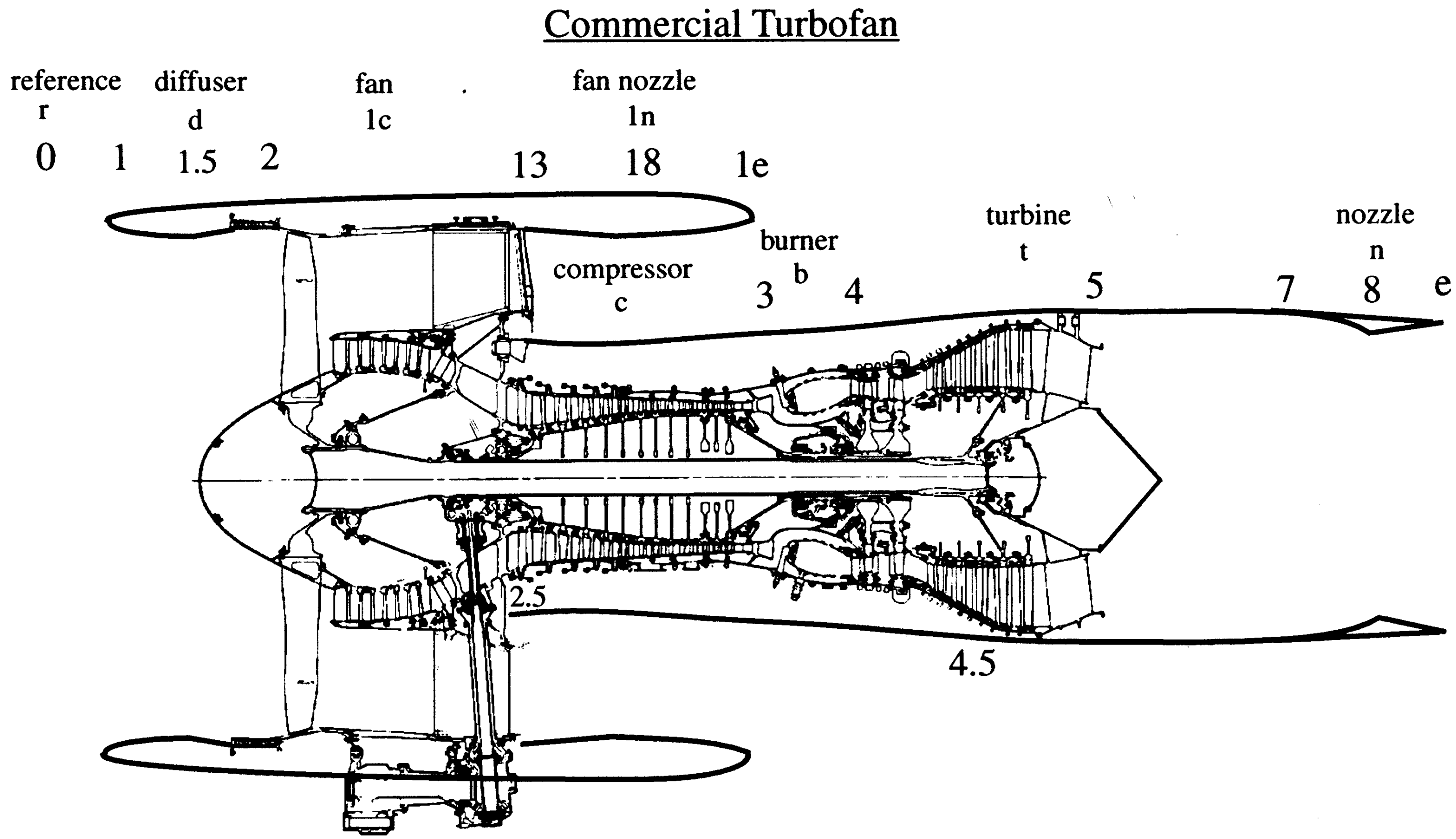
$$f = \frac{\dot{m}_f}{\dot{m}_a} \tag{2.46}$$

Overall efficiency

$$\eta_{ov} = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{1}{f\tau_f}\right)\left(\frac{T}{P_0 A_0}\right) \qquad (2.47)$$

$$\tau_f = \frac{h_f}{C_p T_0} \qquad (2.48)$$

2.10 Engine notation



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Military Turbofan

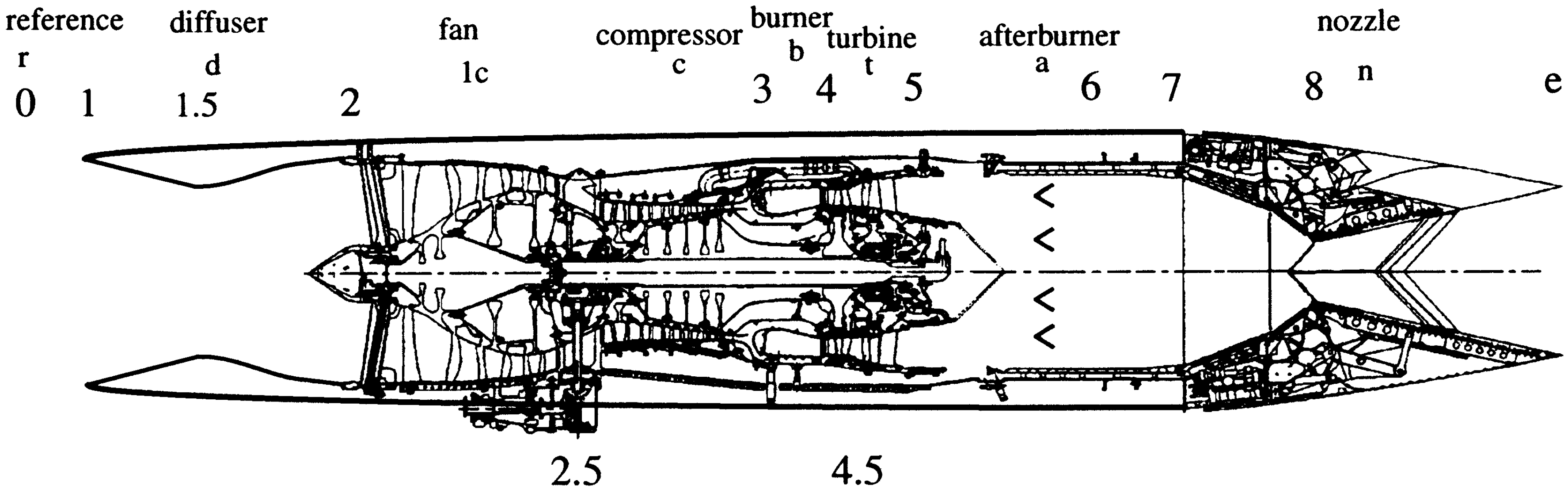

Figure 2.5 Engine numbering and component notation

Component temperature and pressure parameters

$$\tau = \frac{\textit{The stagnation temperature leaving the component}}{\textit{The stagnation temperature entering the component}}$$

$$\pi = \frac{\textit{The stagnation pressure leaving the component}}{\textit{The stagnation pressure entering the component}} \quad . \qquad (2.49)$$

Station 0 - This is the reference state of the gas well upstream of the engine entrance. The temperature/pressure parameters are

$$\tau_r = \frac{T_{t0}}{T_0} = 1 + \left(\frac{\gamma - 1}{2}\right) M_0^2$$
$$\pi_r = \frac{P_{t0}}{P_0} = \left(1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2.50}$$

Note that these definitions are exceptional in that the denominator is the *static* temperature and pressure of the free stream.

Station 1 - Entrance to the engine inlet. The purpose of the inlet is to reduce the Mach number of the incoming flow to a low subsonic value with as small a stagnation pressure loss as possible. From the entrance to the end of the inlet there is generally an increase in area and so the component is appropriately called a diffuser.

Station 1.5 - The inlet throat.

Station 2 - The fan or compressor face. The temperature/pressure parameters across the diffuser are

$$\tau_d = \frac{T_{t2}}{T_{t1}} \qquad \pi_d = \frac{P_{t2}}{P_{t1}} \tag{2.45}$$

Station 2.5 - All turbofan engines comprise at least two spools. There is a low pressure compressor (including the fan) driven by a low pressure turbine through a shaft along the centerline of the engine. A concentric shaft connects the high pressure turbine and compressor. Station 2.5 is generally taken at the interface between the low and high pressure compressor. Roll Royce turbofans commonly employ three spools with the high pressure compressor broken into two spools.

Station 13 - This is a station in the bypass stream corresponding to the fan exit and the entrance to the fan nozzle. The temperature/pressure parameters across the fan are

$$\tau_{1c} = \frac{T_{t13}}{T_{t2}} \qquad \pi_{1c} = \frac{P_{t13}}{P_{t2}} \tag{2.53}$$

Station 18 - The fan nozzle throat.

Station 1e - The fan nozzle exit. The temperature/pressure parameters across the fan nozzle are

$$\tau_{1n} = \frac{T_{t1e}}{T_{t13}} \qquad \pi_{1n} = \frac{P_{t1e}}{P_{t13}} \qquad (2.54)$$

Station 3 - The exit of the high pressure compressor. The temperature/pressure parameters across the compressor are

$$\tau_c = \frac{T_{t3}}{T_{t2}} \qquad \pi_c = \frac{P_{t3}}{P_{t2}} \qquad (2.55)$$

Note that the compression includes that due to the fan. The goal of the designer is to produce a compression system that is as near to isentropic as possible.

Station 4 - The exit of the burner. The temperature/pressure parameters across the burner are

$$\tau_b = \frac{T_{t4}}{T_{t3}} \qquad \pi_b = \frac{P_{t4}}{P_{t3}} \tag{2.56}$$

The temperature at the exit of the burner is regarded as the highest temperature in the Brayton cycle although generally higher temperatures do occur at the upstream end of the burner where combustion takes place. The burner is designed to allow an influx of cooler compressor air to mix with the combustion gases bringing the temperature down to a level that the high pressure turbine can tolerate. Modern engines operate at values of T_{t4} that approach 3700°R (2050°K).

Station 4.5 - This station is at the interface of the high and low pressure turbines.

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Station 5 - The exit of the turbine. The temperature/pressure parameters across the turbine are

$$\tau_t = \frac{T_{t5}}{T_{t4}} \qquad \pi_t = \frac{P_{t5}}{P_{t4}} \tag{2.57}$$

Station 6 - The exit of the afterburner if there is one. The temperature/pressure parameters across the afterburner are

$$\tau_a = \frac{T_{t6}}{T_{t5}} \qquad \pi_a = \frac{P_{t6}}{P_{t5}} \tag{2.58}$$

Station 7 - The entrance to the nozzle.

Station 8 - The nozzle throat.

Station e - The nozzle exit. The temperature/pressure parameters across the nozzle are

$$\tau_n = \frac{T_{te}}{T_{t7}} \qquad \pi_n = \frac{P_{te}}{P_{t7}} \tag{2.59}$$

In the absence of an afterburner

$$\tau_n = \frac{T_{te}}{T_{t5}} \qquad \pi_n = \frac{P_{te}}{P_{t5}} \tag{2.60}$$

Two additional parameters

$$\tau_f = \frac{h_f}{C_p T_0} \tag{2.61}$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \tag{2.62}$$