AA103
Air and Space Propulsion

Chapter 8 - Solid Propellant Rockets

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10.1 Introduction

Section view of a typical solid propellant rocket

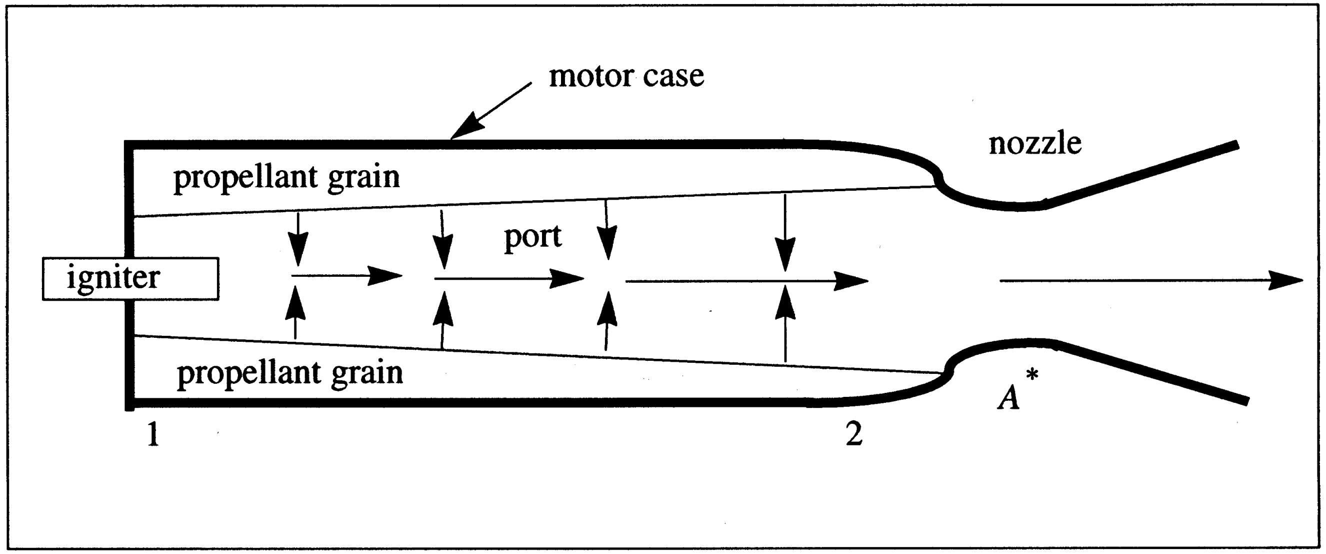


Figure 10.1 Solid rocket cross section

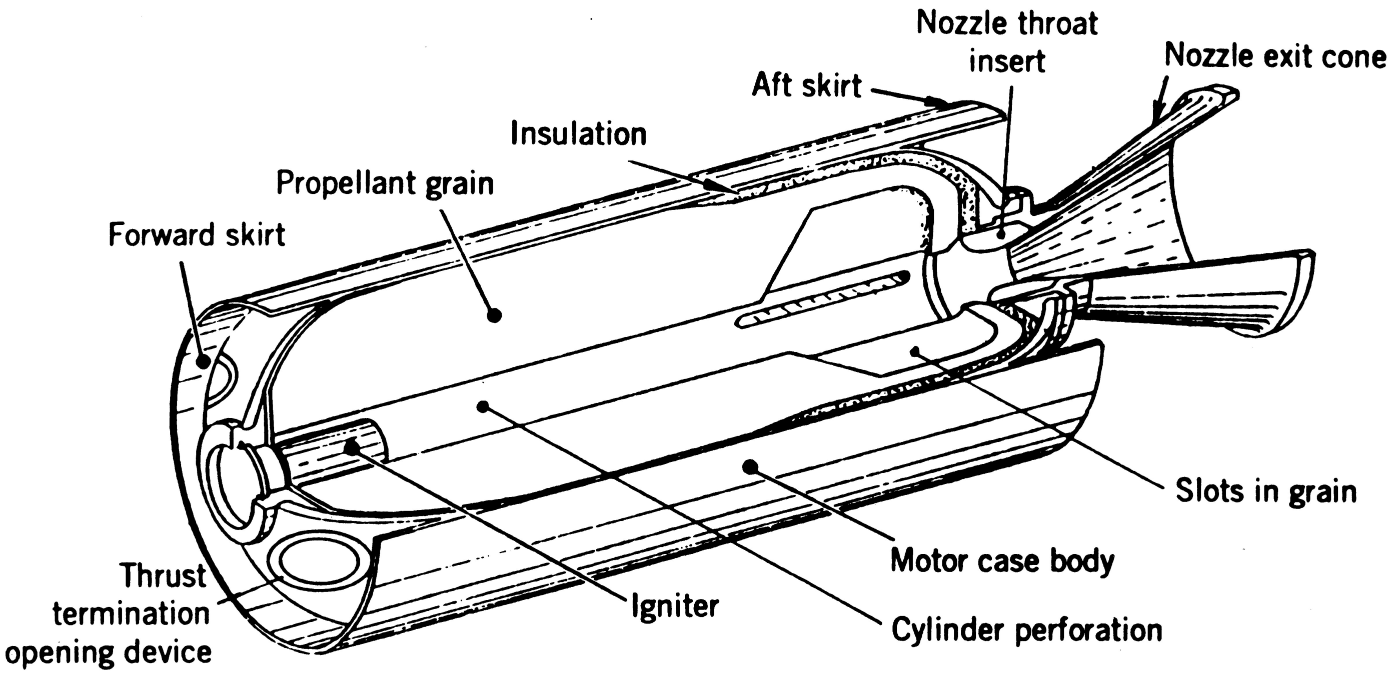
The are basically two types of propellant grains.

1) Homogeneous or double base propellants - Here fuel and oxidizer are contained within the same molecule. Typical examples are Nitroglycerine and Nitrocellulose

2) Composite propellants - heterogeneous mixtures of oxidizing crystals in an organic plastic-like fuel binder typically synthetic rubber.

Sometimes metal powders such as Aluminum are added to the propellant to increase the energy of the combustion process as well as fuel density. Typically these may be 12 to 22 % of propellant mass although in the space shuttle boosters Aluminum is the primary fuel.

Typical solid rocket motor design



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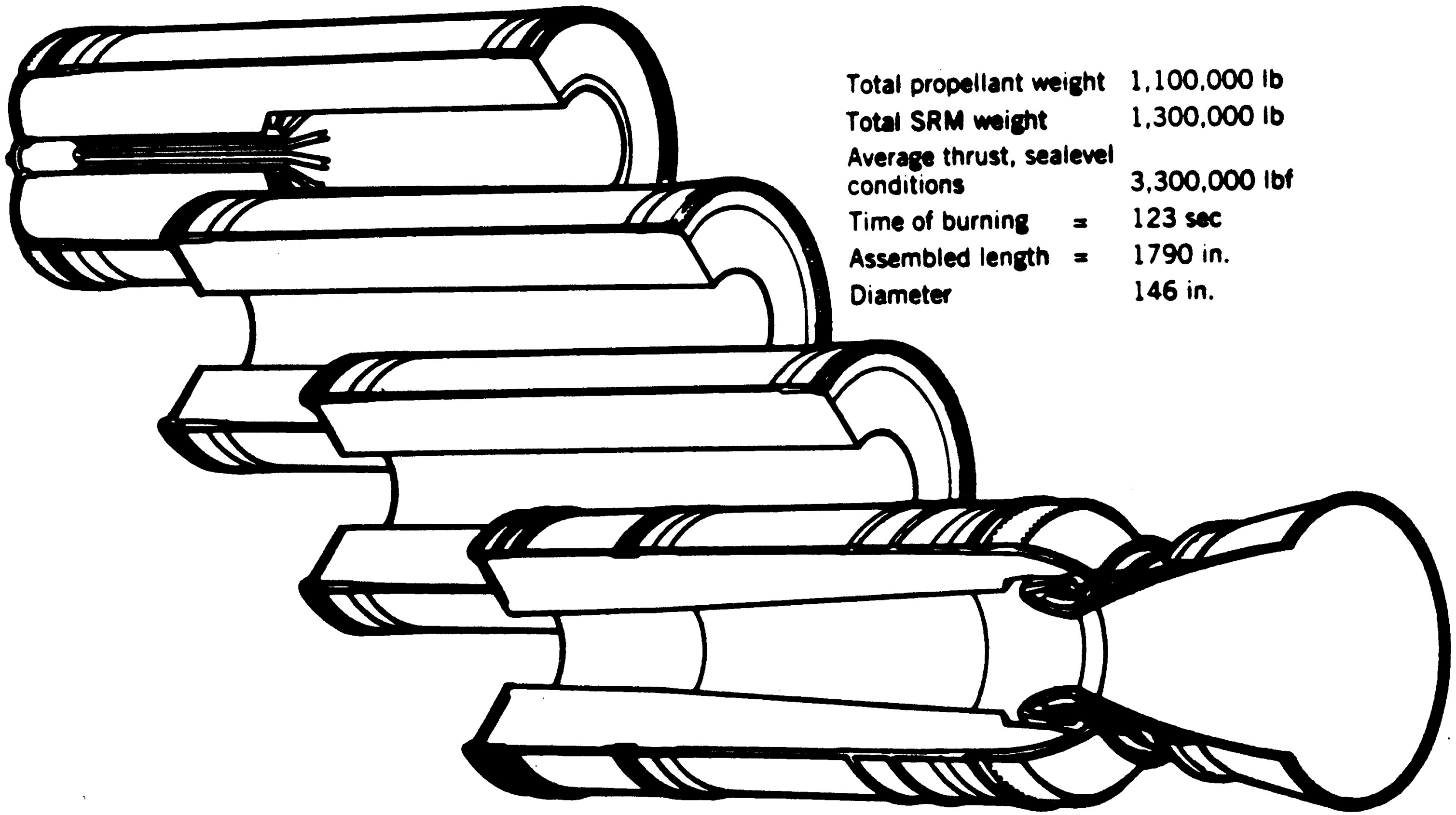
Space shuttle solid rocket booster - note segmented design



Total propellant weight	1,100,000 lb
Total SRM weight	1,300,000 lb
Average thrust, sealevel conditions	3,300,000 lbf
Time of burning =	123 sec
Assembled length =	1790 in.
Diameter	146 in.

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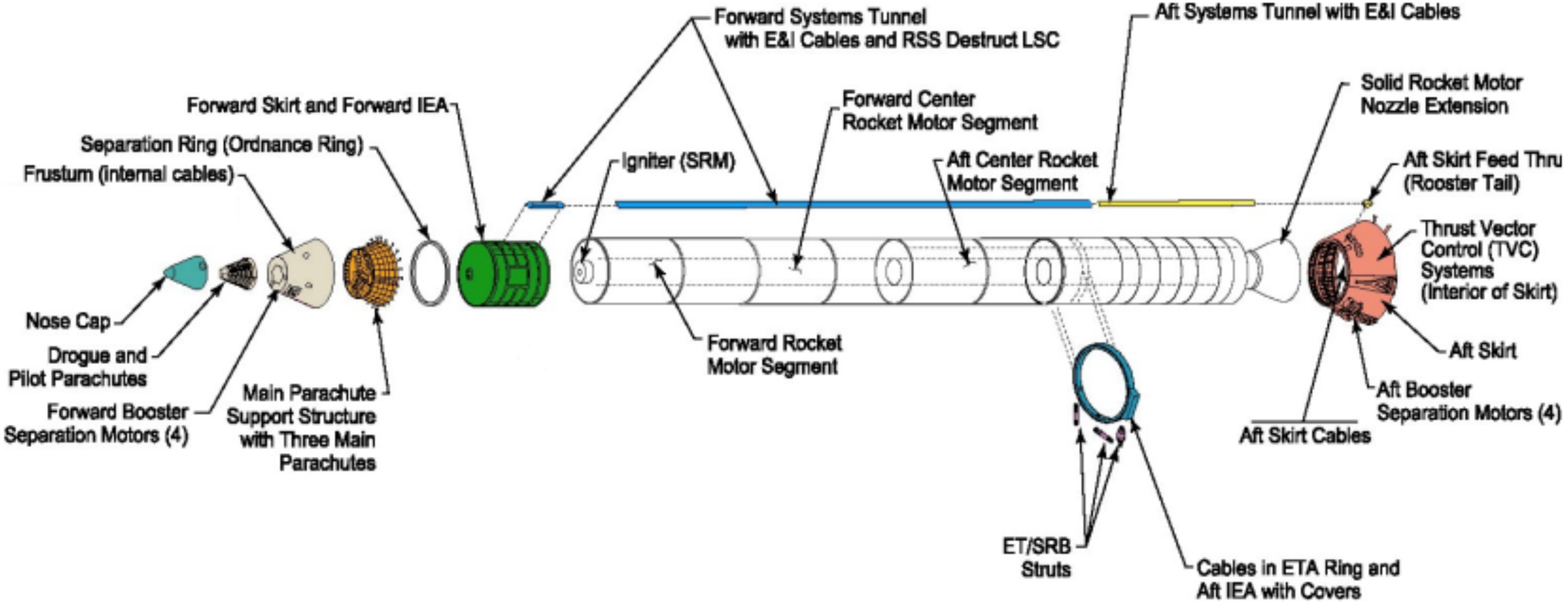
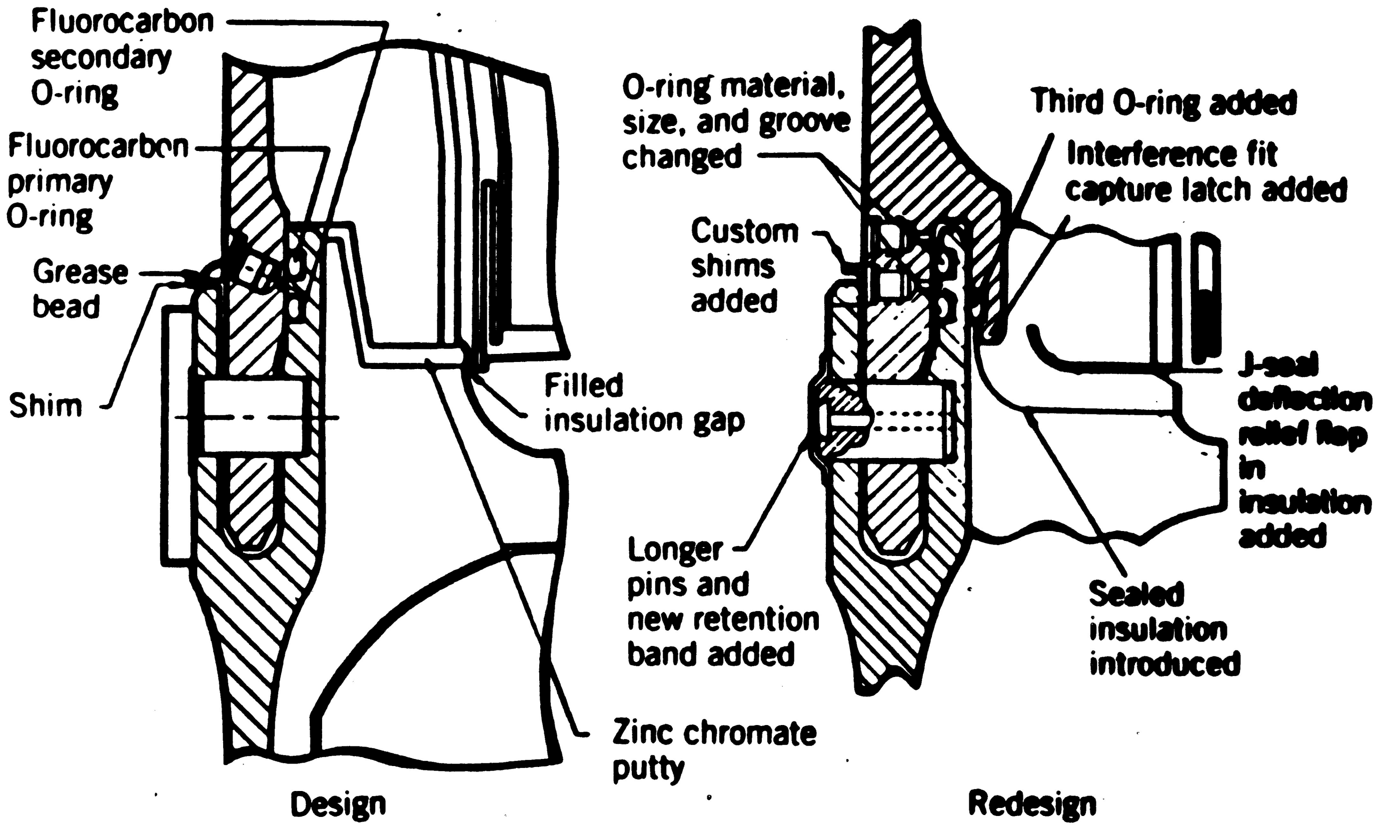
Space shuttle solid rocket booster - exploded view

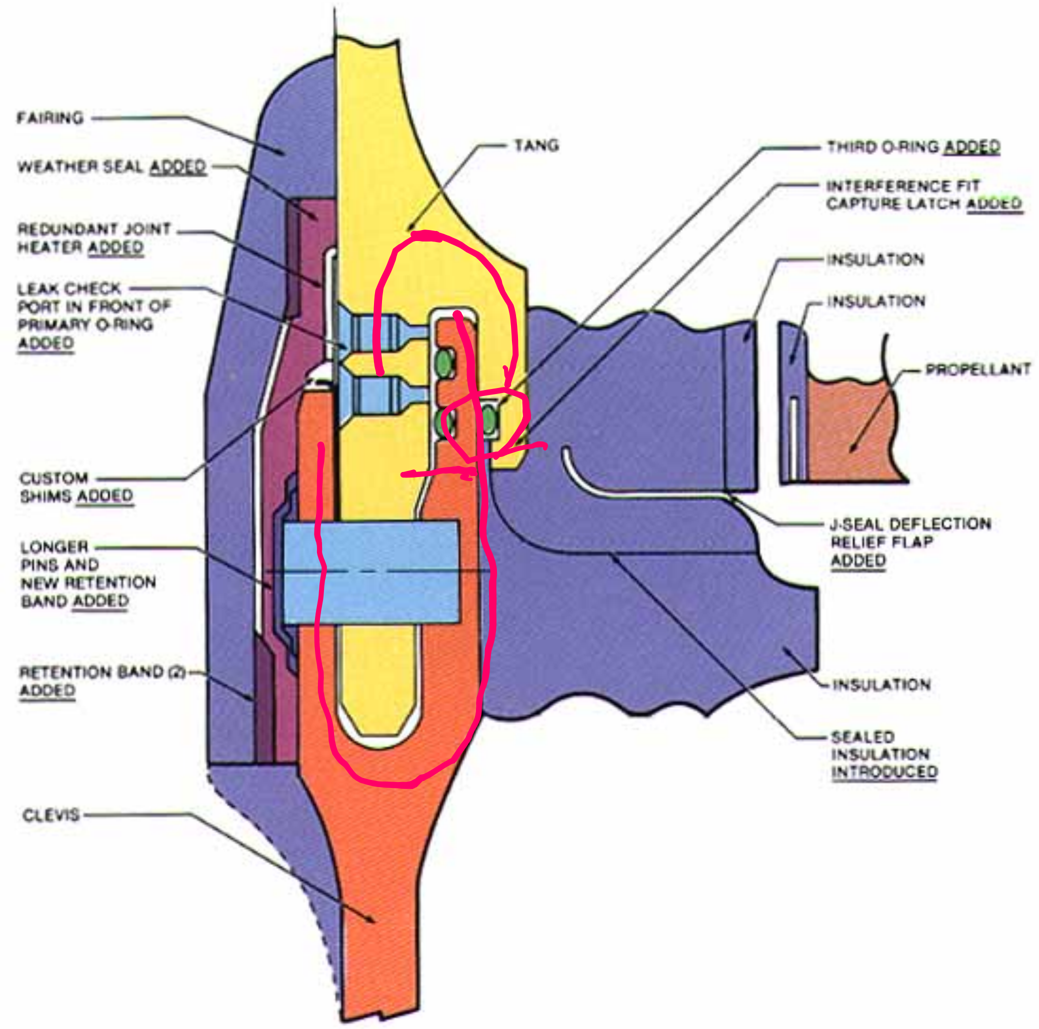


SRM field joint redesign after Challenger disaster



https://www.youtube.com/watch?v=01CfiyP0_7A

New SRM field joint



https://www.youtube.com/watch?v=01CfiyP0_7A

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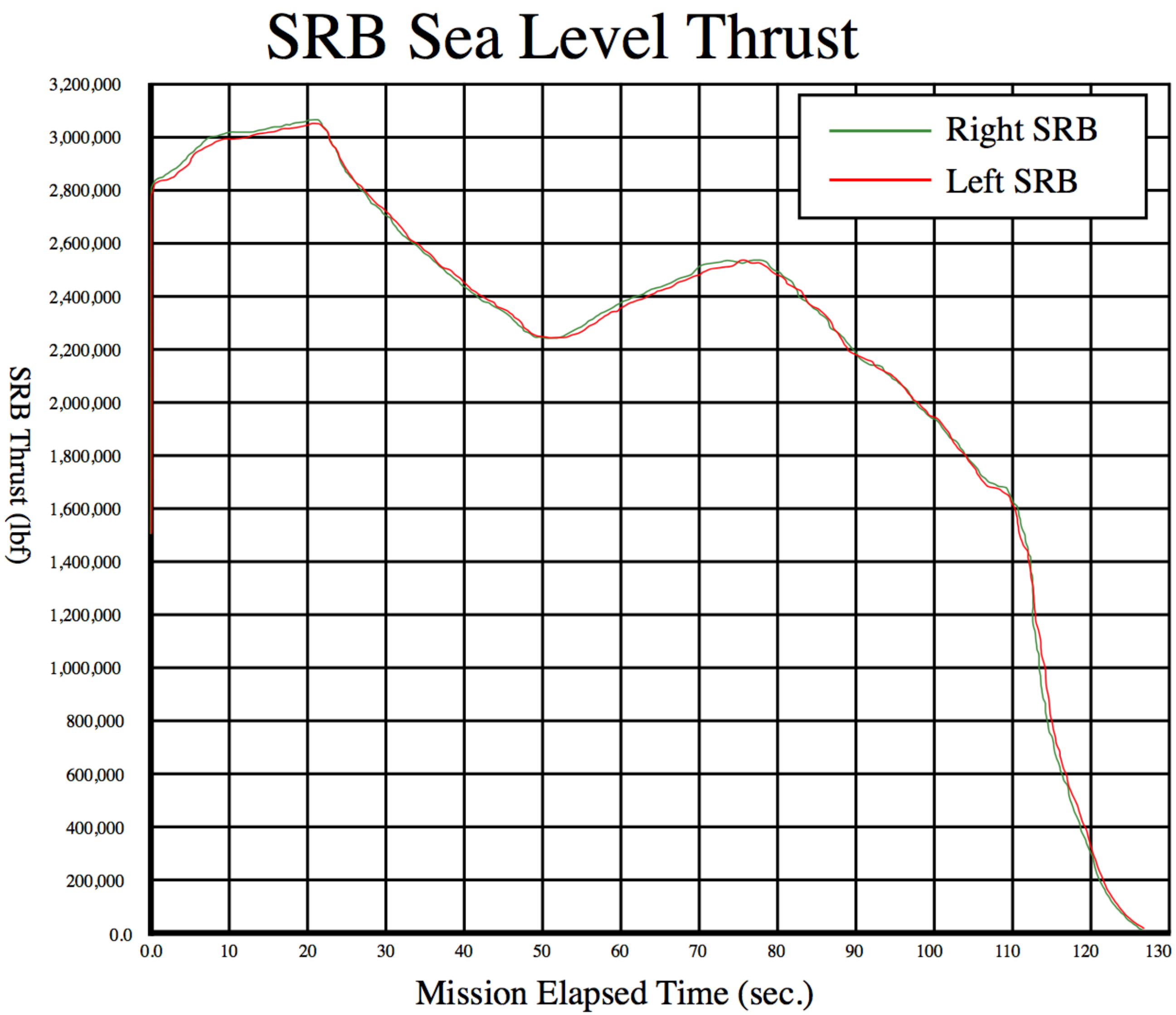
Environmental concerns over AP

There are Increasing concerns about groundwater contamination by perchlorates produced in the manufacture of solid rocket propellants. Even very low levels of contamination are correlated with reduced iodine intake in women.

Reference: CDC Report doi:10.1289/ehp.9466 October 5, 2006. Available at http://dx.doi.org/

Space shuttle solid rocket booster - thrust vs time



<u>Propellant</u>

Ammonium Perchlorate - 69.8%
Aluminum - 16%
PBAN binder - 12%
Epoxy curing agent - 2%
Iron oxide catalyst - 0.2%

Ammonium perchlorate
NH_4ClO_4

PBAN
Polybutadiene acrylonitrile

<u>Specific impulse</u>

Sea level 242 sec
Vacuum 268 sec

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Propellant densities

PROPERTIES OF ROCKET PROPELLANTS

Compound	Chemical Formula	Molecular Weight	Density	Melting Point	Boiling Point
Liquid Oxygen	O_2	32.00	1.14 g/ml	-218.8°C	-183.0°C
Liquid Fluorine	F_2	38.00	1.50 g/ml	-219.6°C	-188.1°C
Nitrogen Tetroxide	N_2O_4	92.01	1.45 g/ml	-9.3°C	21.15°C
Nitric Acid	HNO_3	63.01	1.55 g/ml	-41.6°C	83°C
Hydrogen Peroxide	H_2O_2	34.02	1.44 g/ml	-0.4°C	150.2°C
Nitrous Oxide	N_2O	44.01	1.22 g/ml	-90.8°C	-88.5°C
Chlorine Pentafluoride	ClF_5	130.45	1.9 g/ml	-103°C	-13.1°C
Ammonium Perchlorate	ClH_4NO_4	117.49	1.95 g/ml	240°C	N/A
Liquid Hydrogen	H_2	2.016	0.071 g/ml	-259.3°C	-252.9°C
Liquid Methane	CH_4	16.04	0.423 g/ml	-182.5°C	-161.6°C
Ethyl Alcohol	C_2H_5OH	46.07	0.789 g/ml	-114.1°C	78.2°C
n-Dodecane (Kerosene)	$C_{12}H_{26}$	170.34	0.749 g/ml	-9.6°C	216.3°C
RP-1	$C_nH_{1.953n}$	≈175	0.820 g/ml	N/A	177-274°C
Hydrazine	N_2H_4	32.05	1.004 g/ml	1.4°C	113.5°C
Methyl Hydrazine	CH_3NHNH_2	46.07	0.866 g/ml	-52.4°C	87.5°C
Dimethyl Hydrazine	$(CH_3)_2NNH_2$	60.10	0.791 g/ml	-58°C	63.9°C
Aluminum	Al	26.98	2.70 g/ml	660.4°C	2467°C
Polybutadiene	$(C_4H_6)_n$	≈3000	≈0.9 g/ml	N/A	N/A

NOTES:

- Chemically, kerosene is a mixture of hydrocarbons; the chemical composition depends on its source, but it usually consists of about ten different hydrocarbons, each containing from 10 to 16 carbon atoms per molecule; the constituents include n-dodecane, alkyl benzenes, and naphthalene and its derivatives. Kerosene is usually represented by the single compound n-dodecane.
- RP-1 is a special type of kerosene covered by Military Specification MIL-R-25576. In Russia, similar specifications were developed under specifications T-1 and RG-1.
- Nitrogen tetroxide and nitric acid are hypergolic with hydrazine, MMH and UDMH. Oxygen is not hypergolic with any commonly used fuel.
- Ammonium perchlorate decomposes, rather than melts, at a temperature of about 240 °C.

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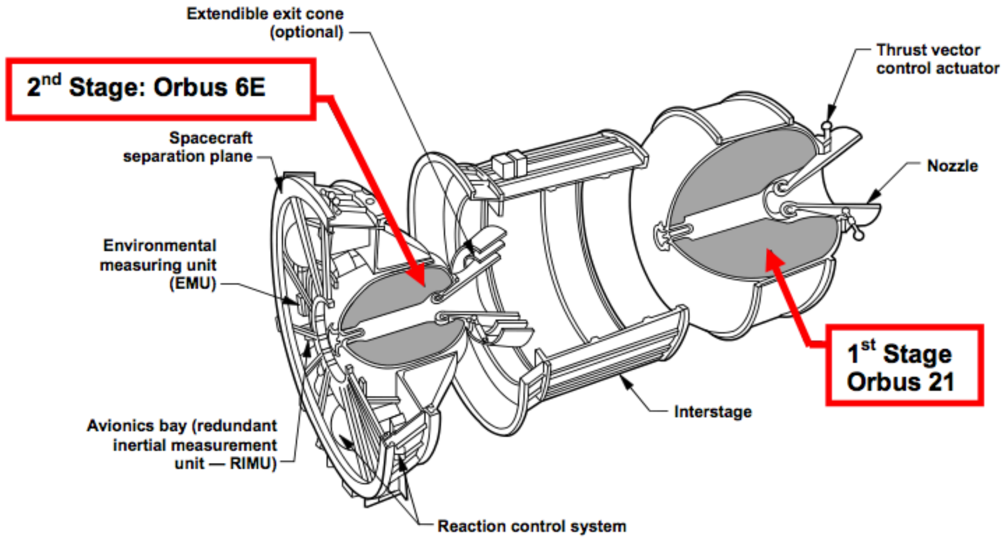
Propellant performance

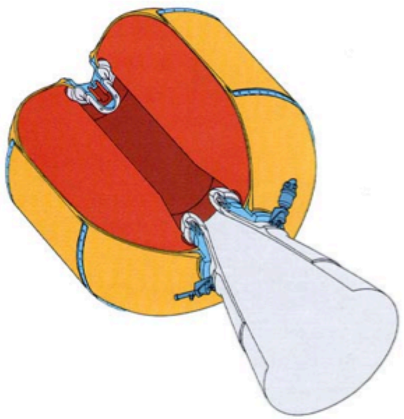
ROCKET PROPELLANT PERFORMANCE					
Combustion chamber pressure, P_c = 68 atm (1000 PSI) ... Nozzle exit pressure, P_e = 1 atm					
Oxidizer	**Fuel**	**Hypergolic**	**Mixture Ratio**	**Specific Impulse (s, sea level)**	**Density Impulse (kg-s/l, S.L.)**
Liquid Oxygen	Liquid Hydrogen	No	5.00	381	124
	Liquid Methane	No	2.77	299	235
	Ethanol + 25% water	No	1.29	269	264
	Kerosene	No	2.29	289	294
	Hydrazine	No	0.74	303	321
	MMH	No	1.15	300	298
	UDMH	No	1.38	297	286
	50-50	No	1.06	300	300
Liquid Fluorine	Liquid Hydrogen	Yes	6.00	400	155
	Hydrazine	Yes	1.82	338	432
FLOX-70	Kerosene	Yes	3.80	320	385
Nitrogen Tetroxide	Kerosene	No	3.53	267	330
	Hydrazine	Yes	1.08	286	342
	MMH	Yes	1.73	280	325
	UDMH	Yes	2.10	277	316
	50-50	Yes	1.59	280	326
Red-Fuming Nitric Acid (14% N_2O_4)	Kerosene	No	4.42	256	335
	Hydrazine	Yes	1.28	276	341
	MMH	Yes	2.13	269	328
	UDMH	Yes	2.60	266	321
	50-50	Yes	1.94	270	329
Hydrogen Peroxide (85% concentration)	Kerosene	No	7.84	258	324
	Hydrazine	Yes	2.15	269	328
Nitrous Oxide	HTPB (solid)	No	6.48	248	290
Chlorine Pentafluoride	Hydrazine	Yes	2.12	297	439
Ammonium Perchlorate (solid)	Aluminum + HTPB (a)	No	2.12	266	469
	Aluminum + PBAN (b)	No	2.33	267	472

NOTES:
- Specific impulses are theoretical maximum assuming 100% efficiency; actual performance will be less.
- All mixture ratios are optimum for the operating pressures indicated, unless otherwise noted.
- LO_2/LH_2 and LF_2/LH_2 mixture ratios are higher than optimum to improve density impulse.
- FLOX-70 is a mixture of 70% liquid fluorine and 30% liquid oxygen.
- Where kerosene is indicated, the calculations are based on n-dodecane.
- Solid propellant formulation (a): 68% AP + 18% Al + 14% HTPB.
- Solid propellant formulation (b): 70% AP + 16% Al + 12% PBAN + 2% epoxy curing agent.

Boeing – CSD Inertial Upper Stage

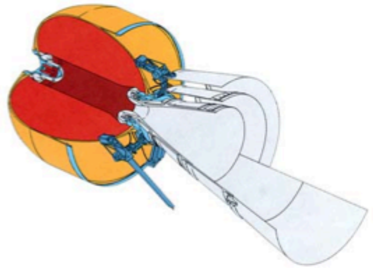


Air Force/NASA IUS, built by Boeing, a 2-Stage Space Vehicle using CSD's Orbus 21 and Orbus 6E Solid Propellant Rockets. It was Configured to Fly off both the Shuttle and Titan Launch Vehicles



Orbus 21: IUS 1st Stage

Diameter = 92-in
Wp = 21,400-lb



Orbus 6E: IUS 2nd Stage

Diameter = 63-in
Wp = 6,000-lb

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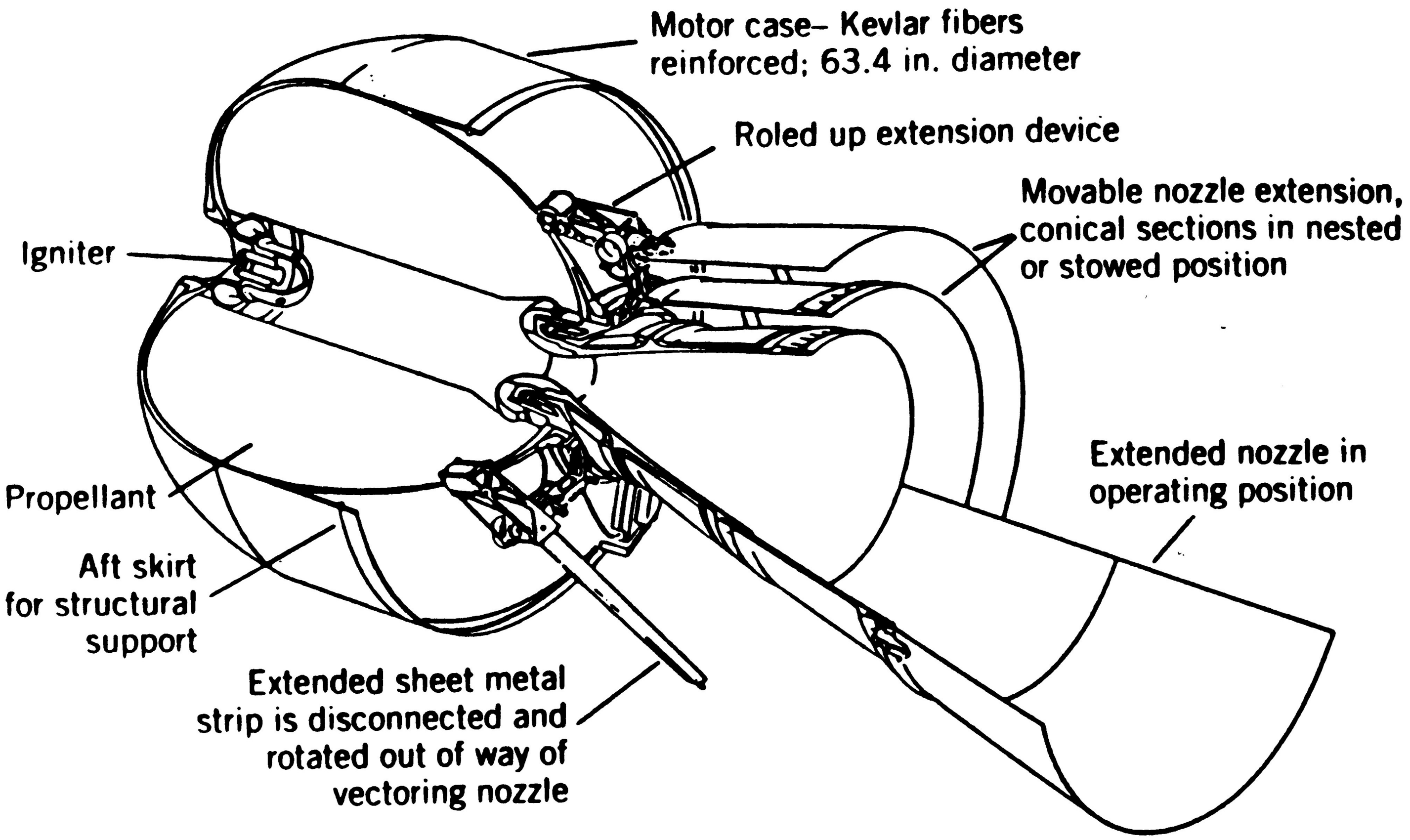
Boeing inertial upper stage (IUS) with extensible vectored nozzle.
Nozzle area ratio can change from 49.3 to 181 increasing specific impulse by 14 seconds.



10.2 Combustion chamber pressure

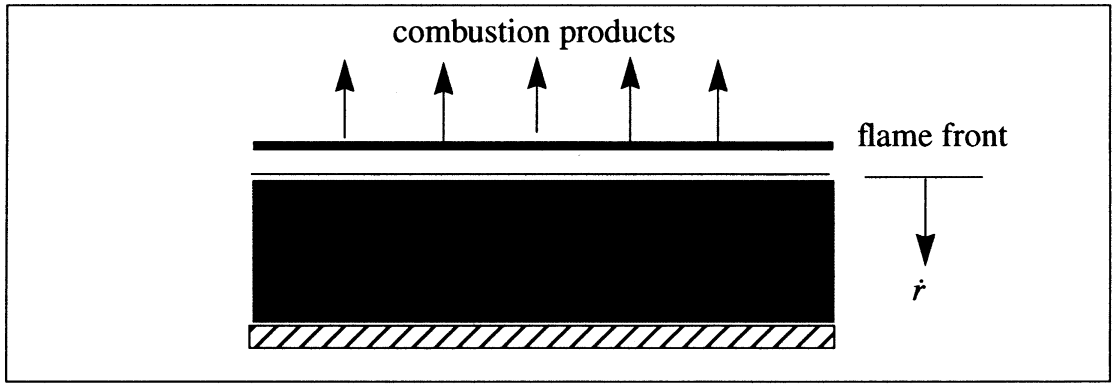


Figure 10.2 Surface regression and gas generation

The gas generation rate integrated over the port surface area is

$$\dot{m}_g = \rho_p A_b \dot{r} \tag{10.1}$$

$$\begin{aligned} \rho_p &= \textit{solid propellant density} \\ A_b &= \textit{area of the burning surface} \\ \dot{r} &= \textit{surface regression speed} \\ \dot{m}_g &= \textit{rate of gas generation at the propellant surface} \end{aligned} \tag{10.2}$$

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In general the regression rate of the propellant surface depends on chamber pressure and propellant temperature

$$\dot{r} = \frac{K}{T_1 - T_p}(P_{t2})^n \qquad (10.3)$$

Propellant temperature

P_{t2} = *combustion chamber pressure*

K = *impirical constant for a given propellant*

T_1 = *impirical detonation temperature* (10.4)

n = *impirical exponent, approximately independent of temperature*

The exponent n is usually between 0.4 and 0.7 and the detonation temperature is substantially larger than the propellant temperature.

A very good comprehensive paper on solid propellants

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Combustion of Solid Propellants

G. Lengellé, J. Duterque, J.F. Trubert
Research Scientists, Energetics Department
Office national d'études et de recherches aérospatiales (ONERA)
29 avenue de la Division Leclerc
BP 72 – 92322 Châtillon Cedex
FRANCE

2.0 Energetics of the AP Combustion

The model of Ref. [19] is subscribed to in order to describe the combustion of AP alone. The AP undergoes a phase transition at 513 K, melts around 830 K and, in the thin (a few microns) superficial liquid layer thus created, an exothermic reaction, affecting 70 % of the AP, takes place and creates the final combustion gases, O_2 in particular. The remaining 30 % of the AP sublime into NH_3 and $HClO_4$ which react exothermically in a premixed flame very close to the surface (a few microns), Fig. 16.

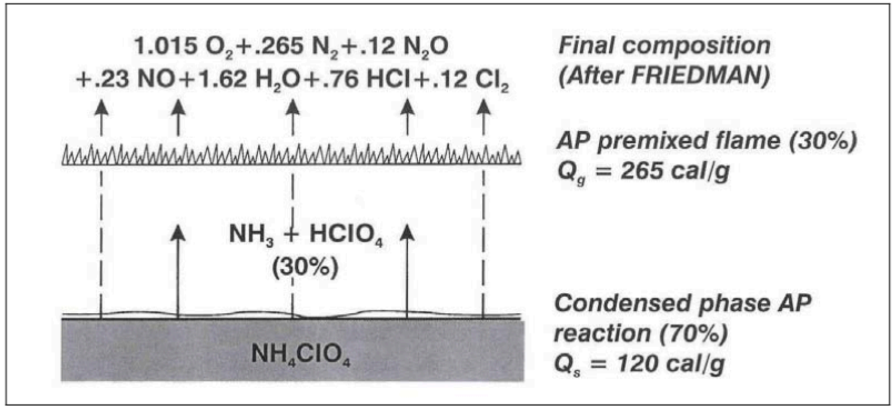
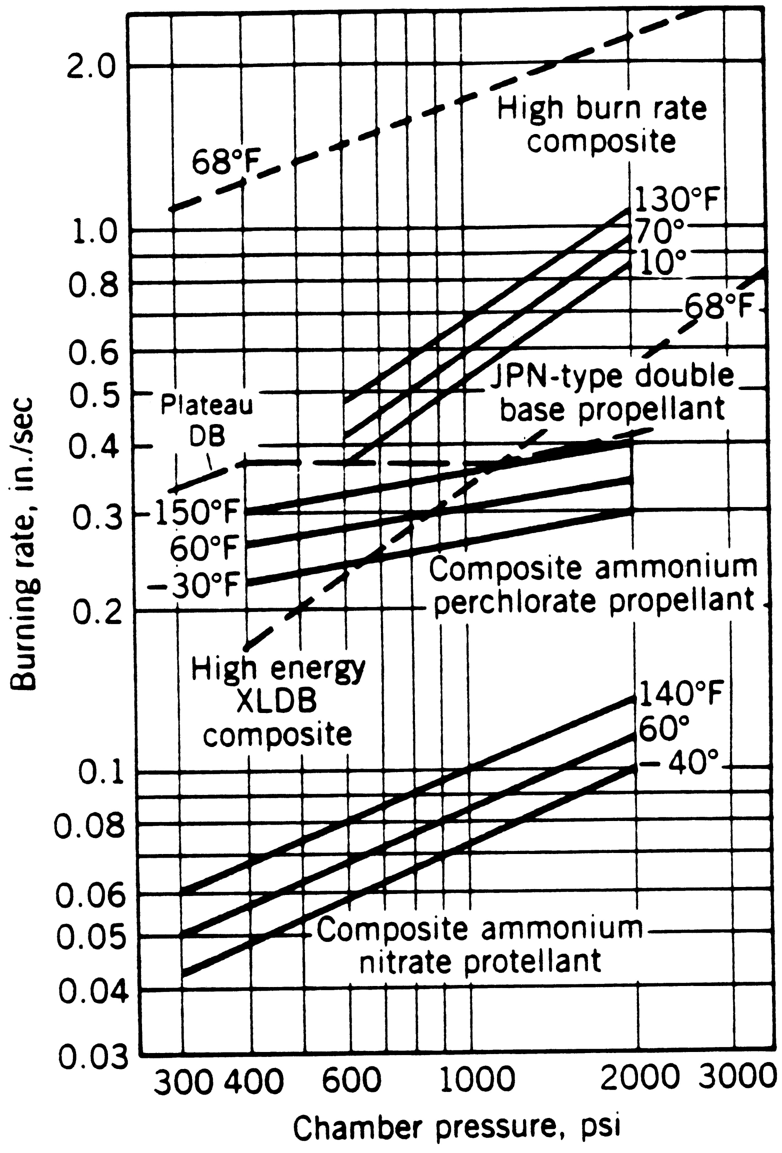
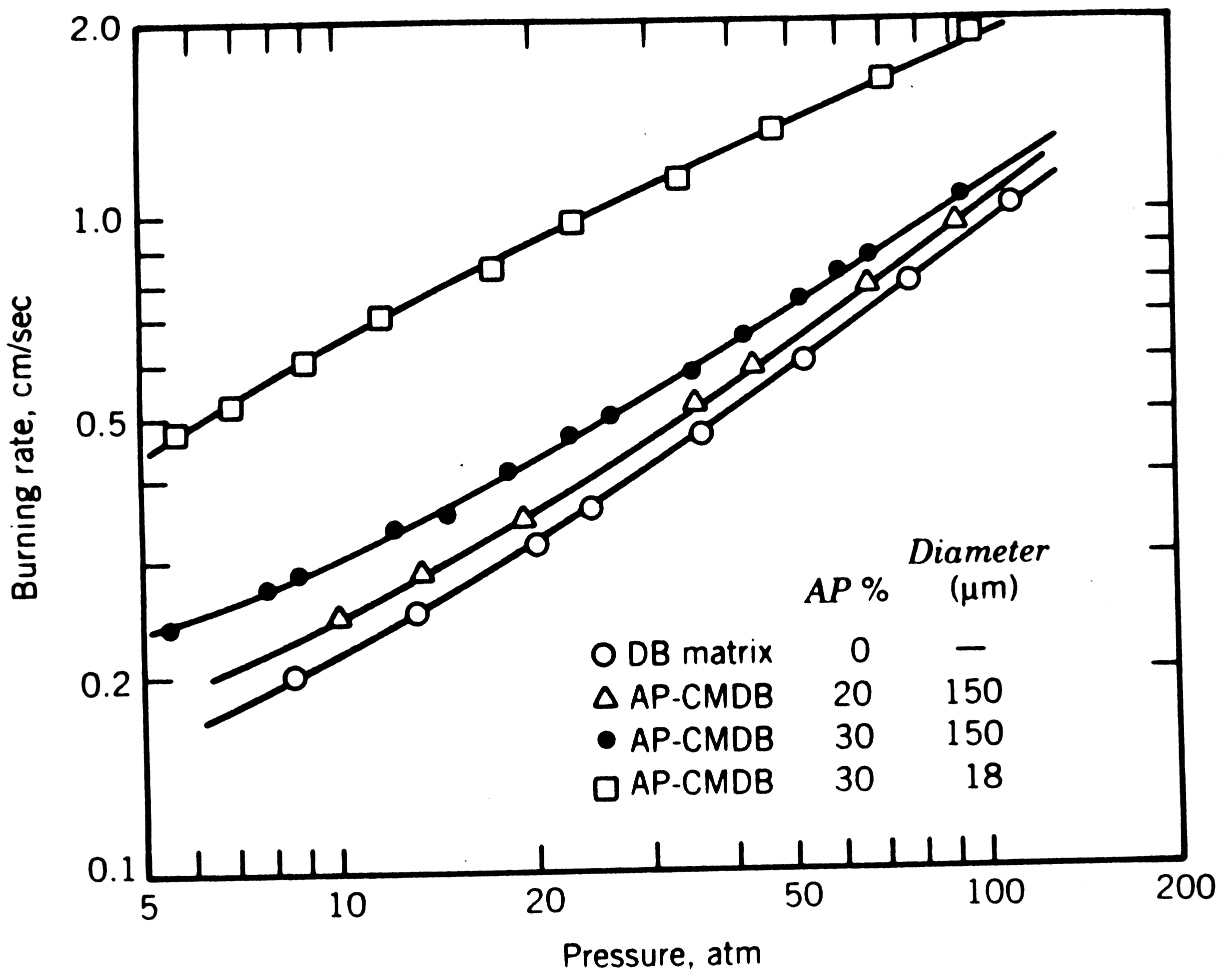


Figure 16: Autonomous Combustion of Ammonium Perchlorate.

Propellant regression rate versus chamber pressure for a variety of propellant types and propellant temperatures



Propellant regression rate versus chamber pressure effect of AP particle size



$$\frac{dM_g}{dt} = \frac{d}{dt}(\rho_g V) = \rho_g \frac{dV}{dt} + V \frac{d\rho_g}{dt} \tag{10.5}$$

The combustion chamber volume changes as the propellant is converted from solid to gas.

$$\frac{dV}{dt} = \dot{r} A_b \tag{10.6}$$

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To a good approximation the combustion chamber stagnation temperature is determined by the propellant energy density and tends to be approximately independent of the combustion chamber pressure. From the ideal gas law

$$\frac{d\rho_g}{dt} = \frac{1}{RT_{t2}}\frac{dP_{t2}}{dt} \tag{10.7}$$

The mass flow out of the nozzle is

$$\dot{m}_n = \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \frac{\gamma P_{t2} A^*}{\sqrt{\gamma R T_{t2}}} \tag{10.8}$$

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Propellants	$P_{chamber}$ bar	$T_{chamber}$ K	C^* M/Sec	$C_e\|_{A_e/A_t = 100}$ M/Sec	$C_e\|_{A_e/A_t \to \infty}$ M/Sec
$H_2 + \frac{1}{2}O_2$	50	3626	2186	4541	5285
	100	3730	2203	4562	5287
$N_2H_4 + \frac{1}{2}N_2O_4$	50	3379	1818	3637	4030
	100	3451	1829	3643	4032
$(1.0)RP-1 + (3.4)O_2$ by mass	50	3676	1733	3631	4467
	100	3787	1749	3654	4469
$(0.1)Al + (0.835)NH_4ClO_4 + (0.065)C_6H_6$ by mass	50	3434	1511	3160	3726
	100	3514	1520	3171	3728

The mass generated at the propellant surface is divided between the mass flow exiting the nozzle and the time dependent mass accumulation in the combustion chamber volume.

$$\dot{m}_g = \frac{dM_g}{dt} + \dot{m}_n \qquad (10.9)$$

Substitute for the terms in (10.9).

$$\rho_p A_b \dot{r} = \rho_g \dot{r} A_b + V\frac{d\rho_g}{dt} + \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \frac{\gamma P_{t2} A^*}{\sqrt{\gamma R T_{t2}}} \qquad (10.10)$$

Substitute the regression rate law (10.3) and the rate of change of density derived from the ideal gas law.

$$\frac{K(\rho_p - \rho_g)A_b}{T_1 - T_p}(P_{t2})^n = \frac{V}{RT_{t2}}\frac{dP_{t2}}{dt} + \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}\frac{\gamma P_{t2}A^*}{\sqrt{\gamma RT_{t2}}} \qquad (10.11)$$

Rearrange (10.11)

$$\frac{V}{RT_{t2}}\frac{dP_{t2}}{dt} + \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}\frac{\gamma P_{t2}A^*}{\sqrt{\gamma RT_{t2}}} - \frac{K(\rho_p - \rho_g)A_b}{T_1 - T_p}(P_{t2})^n = 0 \qquad (10.12)$$

This first order ODE governs the unsteady filling and emptying of the rocket chamber volume.

After a rapid start-up transient the combustion chamber pressure reaches a quasi-steady state where changes occur very slowly and to a good approximation

$$\left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}\frac{\gamma P_{t2}A^{*}}{\sqrt{\gamma R T_{t2}}} = \frac{K(\rho_p-\rho_g)A_b}{T_1-T_p}(P_{t2})^{n} \qquad (10.13)$$

Solve for the pressure.

$$P_{t2} = \left(\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}\frac{K(\rho_p-\rho_g)}{\gamma(T_1-T_p)}\left(\frac{A_b}{A^{*}}\right)\sqrt{\gamma R T_{t2}}\right)^{\frac{1}{1-n}} \qquad (10.14)$$

This formula is valid as long as the burning area is a slow function of time. Note that there is a tendency for the chamber pressure to increase as the burning area increases.

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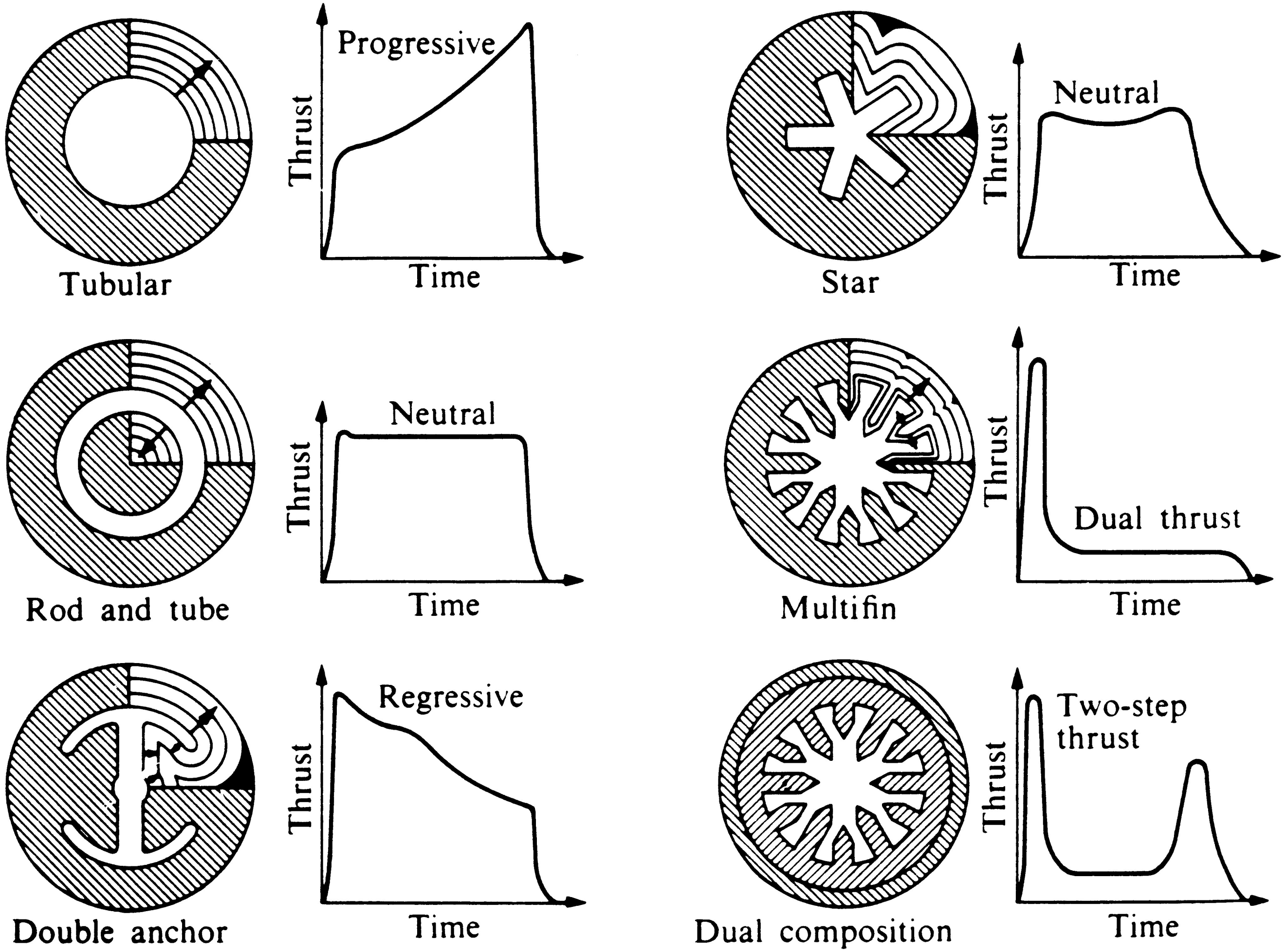
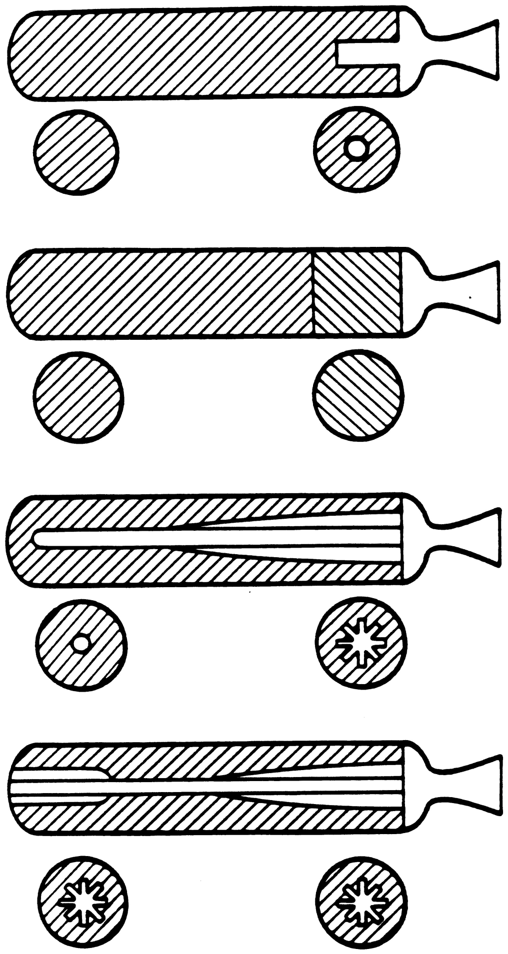
Propellant grain port design determines thrust-time behavior



Propellant grain design may vary along the port



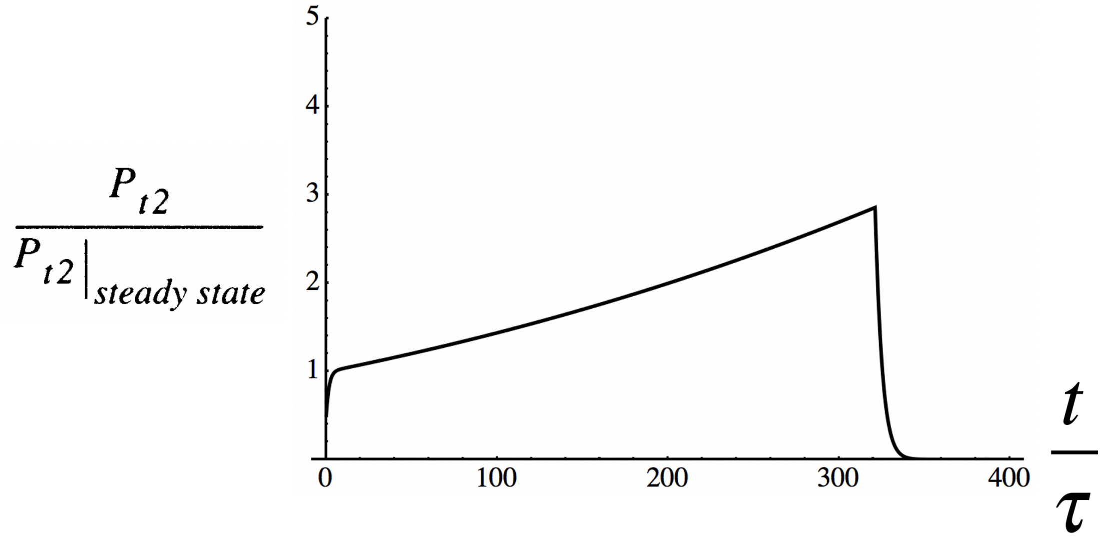
Single grain. Boost with radial burning, sustain with end burning

Duel end burning grains with two burning rates

Single grain. Boost with large burning area, sustain with smaller burning area (both radial)

Single grain. Boost-sustain-boost, with different burning areas (all radial burning)

10.3 Dynamic analysis



Rearrange (10.12)

$$\frac{dP_{t2}}{dt} + \left(\frac{(\gamma R T_{t2})^{1/2} A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} V}\right) P_{t2} - \left(\frac{K(\rho_p - \rho_g) A_b}{T_1 - T_p}\left(\frac{R T_{t2}}{V}\right)\right)(P_{t2})^n = 0 \qquad (10.15)$$

This is a nonlinear first order ODE of the form.

$$\frac{dP_{t2}}{dt} + \left(\frac{1}{\tau}\right) P_{t2} - \beta (P_{t2})^n = 0 \qquad (10.16)$$

Where the characteristic time is

This is the characteristic time for filling or emptying a volume containing a gas.

$$\tau = \frac{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}}{(\gamma R T_{t2})^{1/2}}\left(\frac{V}{A^*}\right) \qquad (10.17)$$

Note that this time is proportional to the time it would take for an acoustic wave to travel the length of the chamber multiplied by the internal nozzle area ratio.

The constant multiplying the nonlinear forcing term is

$$\beta = \left(\frac{K(\rho_p - \rho_g)A_b}{T_1 - T_p}\left(\frac{RT_{t2}}{V}\right)\right) \qquad (10.18)$$

10.3.1 Exact solution

The nonlinear first order ODE governing the chamber pressure can be solved exactly.

$$\frac{dP_{t2}}{dt} + \left(\frac{1}{\tau}\right)P_{t2} - \beta(P_{t2})^{n} = 0. \qquad (10.26)$$

The steady state solution is

$$P_{t2}\Big|_{steady\ state} = (\tau\beta)^{\frac{1}{1-n}} \qquad (10.27)$$

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Let

$$H = \frac{P_{t2}}{P_{t2}\big|_{steady\ state}} \qquad \eta = \frac{t - t_0}{\tau} \qquad (10.28)$$

The governing equation becomes

$$\frac{dH}{d\eta} = H^n - H \qquad (10.29)$$

Which can be rearranged to read

$$\frac{dH}{H^n - H} = d\eta \qquad (10.30)$$

Integrate

$$-(1-n)\eta = Log\left[\frac{1-H^{1-n}}{1-H_0^{1-n}}\right] \qquad (10.31)$$

H_0 is the initial value of $P_{t2}/P_{t2}\big|_{steady\ state}$

Solve for H

$$H = \left(1-\left(1-H_0^{1-n}\right)e^{-(1-n)\eta}\right)^{\frac{1}{(1-n)}} \qquad (10.32)$$

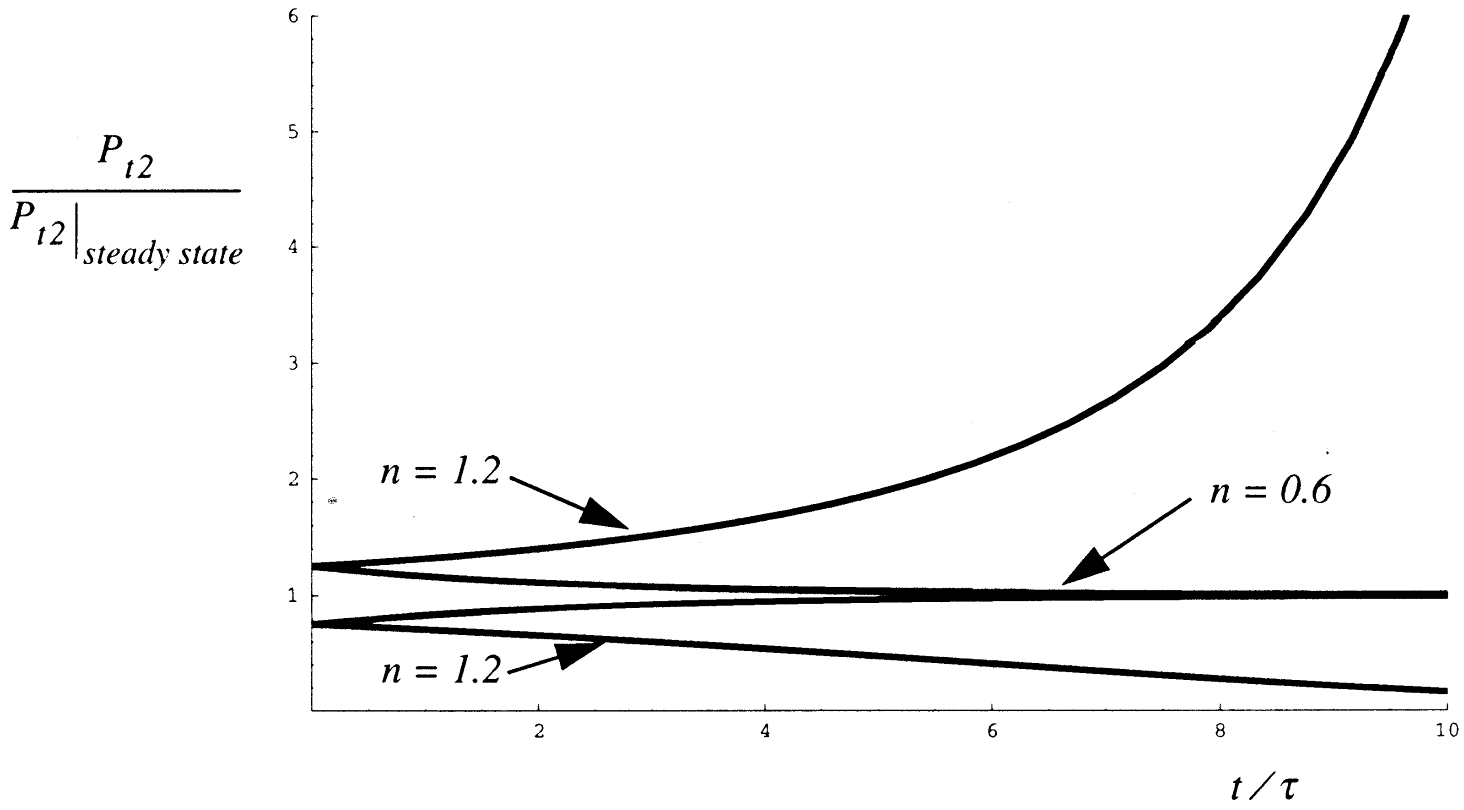

Figure 10.3 Chamber pressure response of a solid rocket.

10.3.2 Chamber pressure history – circular port

Quasi-steady chamber pressure

$$P_{t2} = \left(\alpha\left(\frac{r}{r_i}\right)\right)^{\frac{1}{1-n}}$$

where

$$\alpha = \left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \frac{K(\rho_p - \rho_g)}{\gamma(T_1 - T_p)}\left(\frac{2\pi r_i L}{A^*}\right)\sqrt{\gamma R T_{t2}}$$

Regression rate law

$$\frac{dr}{dt} = \frac{K}{(T_1 - T_p)}\left(\alpha\left(\frac{r}{r_i}\right)\right)^{\frac{n}{1-n}}$$

$$\frac{d(r/r_i)}{(r/r_i)^{\frac{n}{1-n}}} = \left(\frac{K}{(T_1 - T_p)r_i}(\alpha)^{\frac{n}{1-n}}\right)dt$$

Integrate

$$\frac{r}{r_i} = \left(1 + \left(\frac{1-2n}{1-n}\right)\left(\frac{K}{(T_1 - T_p)r_i}(\alpha)^{\frac{n}{1-n}}\right)t\right)^{\frac{1-n}{1-2n}} \qquad n \neq 0.5$$

$$\frac{r}{r_i} = Exp\left[\left(\frac{K}{(T_1 - T_p)r_i}(\alpha)^{\frac{n}{1-n}}\right)t\right] \qquad n = 0.5$$

Characteristic burn time

$$\tau_{burn} = \left(\frac{(T_1 - T_p)r_i}{K(\alpha)^{\frac{n}{1-n}}}\right)$$

Burnout time

$$t_{burnout} = \left(\left(\frac{r_f}{r_i}\right)^{\frac{1-2n}{1-n}} - 1\right)\left(\frac{1-n}{1-2n}\right)\tau_{burn} \qquad n \neq 0.5$$

$$t_{burnout} = Log\left[\frac{r_f}{r_i}\right]\tau_{burn} \qquad n = 0.5$$

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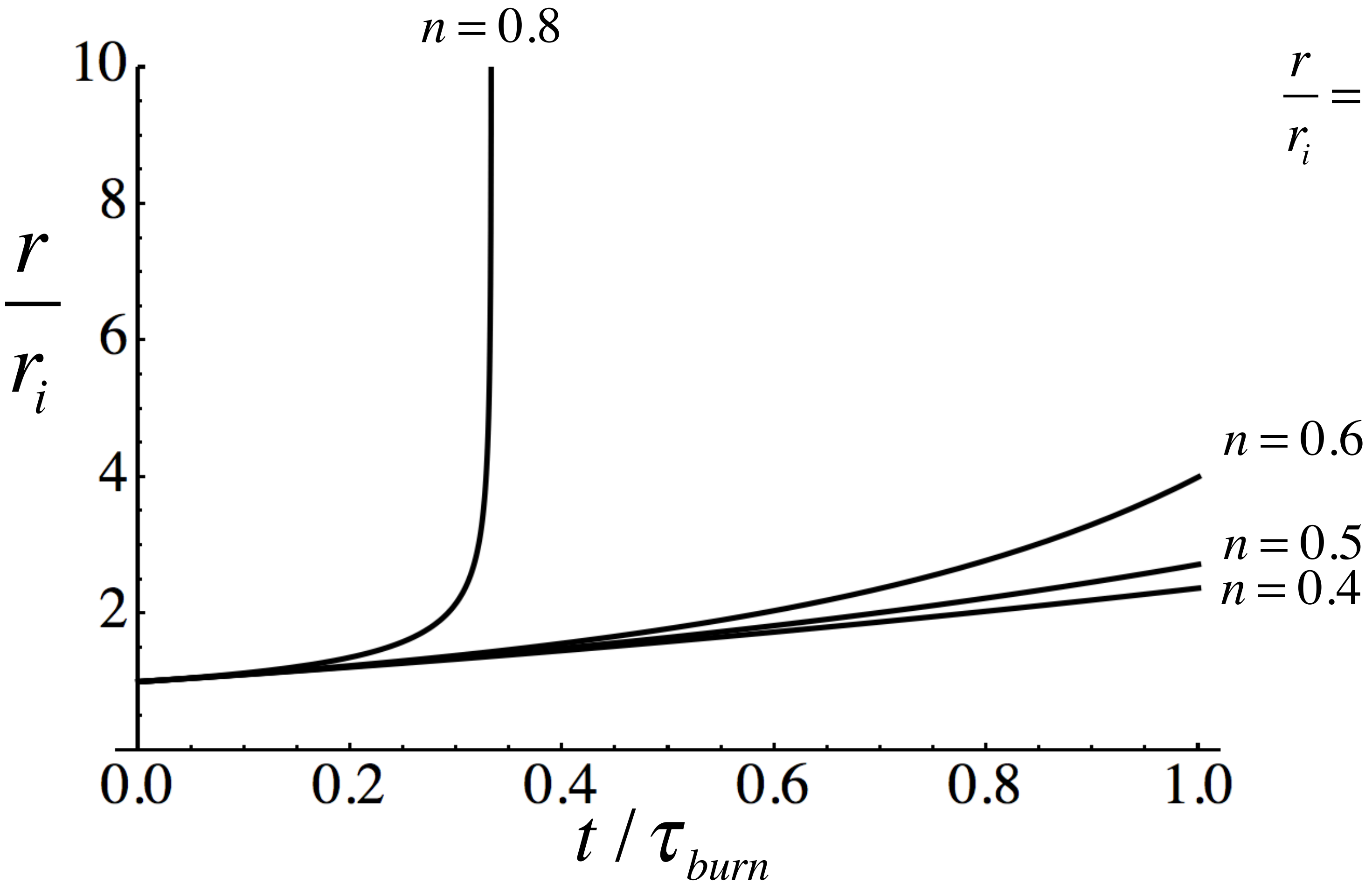
$$\frac{r}{r_i} = \left(1 + \left(\frac{1-2n}{1-n}\right)\left(\frac{K}{(T_1 - T_p) r_i}(\alpha)^{\frac{n}{1-n}}\right)t\right)^{\frac{1-n}{1-2n}} \qquad n \neq 0.5$$

$$\frac{r}{r_i} = Exp\left[\left(\frac{K}{(T_1 - T_p) r_i}(\alpha)^{\frac{n}{1-n}}\right)t\right] \qquad n = 0.5$$

$n = 0.8$

$$\frac{r}{r_i} = \frac{1}{\left(1 - 3\frac{t}{\tau_{burn}}\right)^{1/3}}$$



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Fully coupled chamber-pressure-port-radius history circular port

Define constant values of the characteristic time, the coefficient multiplying the nonlinear forcing term and a normalizing chamber pressure using the initial radius of the port.

$$\tau = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{\left(\gamma R T_{t2}\right)^{1/2}} \left(\frac{L_{port}\pi r_{initial}^{\ 2}}{A^*}\right)$$

$$\beta = \frac{K\left(\rho_p - \rho_{gi}\right) R T_{t2}}{T_1 - T_p} \left(\frac{L_{port} 2\pi r_{initial}}{L_{port}\pi r_{initial}^{\ 2}}\right) = \frac{2K\left(\rho_p - \rho_{gi}\right) R T_{t2}}{T_1 - T_p} \left(\frac{1}{r_{initial}}\right)$$

$$P_{t2\,quasi\text{-}steady\ state_{initial}} = \left(\tau\beta\right)^{\frac{1}{1-n}}$$

$$Note: \quad P_{t2} = \rho_g R T_{t2} \ \text{ and } \ P_{t2_{quasi-steady\ state_{initial}}} = \rho_{g_i} R T_{t2}$$

$$A_{bi} = 2L_{port}\pi r_i \quad \text{and} \quad V_i = L_{port}\pi r_i^2$$

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Dimensionless chamber pressure equation

$$\frac{dP_{t2}}{dt}+\frac{P_{t2}}{\tau\left(\frac{r}{r_{initial}}\right)^2}-\frac{\beta P_{t2}{}^n}{\left(\frac{r}{r_{initial}}\right)}\left(\frac{\rho_p-\rho_g}{\rho_p-\rho_{g_i}}\right)=\frac{dP_{t2}}{dt}+\frac{P_{t2}}{\tau\left(\frac{r}{r_{initial}}\right)^2}-\frac{\beta P_{t2}{}^n}{\left(\frac{r}{r_{initial}}\right)}\left(\frac{\frac{\rho_p}{\rho_{g_i}}-\frac{\rho_g}{\rho_{g_i}}}{\frac{\rho_p}{\rho_{g_i}}-1}\right)=0$$

$$H=\frac{P_{t2}}{P_{t2_{\text{quasi-steady state}_{initial}}}}\qquad R=\frac{r}{r_{initial}}\qquad \eta=\frac{t}{\tau}$$

$$\frac{\rho_g}{\rho_{g_i}}=\frac{P_{t2}}{P_{t2_{\text{quasi-steady state}_{initial}}}}=H$$

$$\tau\beta=\left(P_{t2_{\text{quasi-steady state}_{initial}}}\right)^{1-n}$$

$$\frac{dH}{d\eta}+\frac{H}{R^2}-\frac{H^n}{R}\left(\frac{\frac{\rho_p}{\rho_{g_i}}-H}{\frac{\rho_p}{\rho_{g_i}}-1}\right)=0$$

Dimensionless port radius equation

$$\frac{dr}{dt} = \frac{K}{T_1 - T_p} P_{t2}{}^n$$

$$\frac{dR}{d\eta} = \frac{K}{T_1 - T_p}\left(\frac{\tau}{r_{initial}}\right)\left(P_{t2_{quasi-steady\ state_{initial}}}\right)^n H^n$$

$$\tau\left(P_{t2_{quasi-steady\ state_{initial}}}\right)^n = \frac{P_{t2_{quasi-steady\ state_{initial}}}}{\beta}$$

$$\frac{dR}{d\eta} = \left(\frac{P_{t2_{quasi-steady\ state_{initial}}}}{r_{initial}}\right)\frac{K}{T_1 - T_p}\left(\frac{1}{\beta}\right)H^n = \left(\frac{P_{t2_{quasi-steady\ state_{initial}}}}{r_{initial}}\right)\frac{K}{T_1 - T_p}\frac{\left(T_1 - T_p\right)r_{initial}}{2K\left(\rho_p - \rho_{gi}\right)RT_{t2}}H^n$$

$$\frac{dR}{d\eta} = \left(\frac{P_{t2}}{2\left(\frac{\rho_p}{\rho_{g_i}} - 1\right)\left(\frac{\rho_{gi}}{\rho_g}\right)\rho_g RT_{t2}}\right)\left(\frac{P_{t2_{quasi-steady\ state_{initial}}}}{P_{t2}}\right)H^n = \left(\frac{P_{t2}}{2\left(\frac{\rho_p}{\rho_{g_i}} - 1\right)\left(\frac{P_{t2_{quasi-steady\ state_{initial}}}}{P_{t2}}\right)\rho_g RT_{t2}}\right)\left(\frac{P_{t2_{quasi-steady\ state_{initial}}}}{P_{t2}}\right)H^n$$

$Note:\quad P_{t2} = \rho_g RT_{t2}$ and $P_{t2_{quasi-steady\ state_{initial}}} = \rho_{g_i} RT_{t2}$

$$\frac{dR}{d\eta} = \frac{H^n}{2\left(\frac{\rho_p}{\rho_{g_i}} - 1\right)}$$

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Coupled system

$$\frac{dR}{d\eta} = \frac{H^n}{2\left(\frac{\rho_p}{\rho_{g_i}} - 1\right)}$$

$$\frac{dH}{d\eta} + \frac{H}{R^2} - \frac{H^n}{R}\left(\frac{\frac{\rho_p}{\rho_{g_i}} - H}{\frac{\rho_p}{\rho_{g_i}} - 1}\right) = 0$$

$$R(0) = 1$$

$$H(0) = Choose\ some\ initial\ value$$

$$Note:\ P_{t2\,quasi\text{-}steady\ state_{initial}} = (\tau\beta)^{\frac{1}{1-n}}\ and\ \rho_{g_i} = \frac{P_{t2_{quasi-steady\ state_{initial}}}}{RT_{t2}}$$

Adiabatic expansion after burnout

$$H = \frac{P_{t2}}{P_{t2_{quasi\text{-}steady\ state_{initial}}}} \qquad R = \frac{r}{r_{initial}} \qquad \eta = \frac{t}{\tau}$$

$$m = \frac{V_f}{R}\frac{P}{T}$$

$$m_0 = \frac{V_f}{R}\frac{P_0}{T_0}$$

$$\frac{m}{m_0} = \left(\frac{P}{P_0}\right)^{\frac{1}{\gamma}}$$

$$\frac{dm}{dt} = -\frac{\gamma A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R)^{1/2}}\frac{P}{T^{1/2}} = -\frac{\gamma P_0 A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R T_0)^{1/2}}\left(\frac{P}{P_0}\right)^{1-\left(\frac{\gamma-1}{2\gamma}\right)}$$

$$\frac{dm}{dt} = -\frac{\gamma P_0 A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R T_0)^{1/2}}\left(\frac{P}{P_0}\right)^{\left(\frac{\gamma+1}{2\gamma}\right)}$$

$$\frac{d}{dt}\left(\frac{P}{P_0}\right)^{\frac{1}{\gamma}} = -\frac{\gamma P_0 A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R T_0)^{1/2}\left(\frac{V_f}{R}\frac{P_0}{T_0}\right)}\left(\frac{P}{P_0}\right)^{\left(\frac{\gamma+1}{2\gamma}\right)}$$

$$\frac{d}{dt}\left(\frac{P}{P_0}\right) = -\frac{\gamma^2 P_0 A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R T_0)^{1/2}\left(\frac{V_f}{R}\frac{P_0}{T_0}\right)}\left(\frac{P}{P_0}\right)^{\left(\frac{\gamma+1}{2\gamma}\right)-\left(\frac{2}{2\gamma}-\frac{2\gamma}{2\gamma}\right)}$$

$$\frac{d}{dt}\left(\frac{P}{P_0}\right) = -\frac{\gamma P_0 A^*}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}(\gamma R T_0)^{1/2}\left(\frac{V_f P_0}{\gamma R T_0}\right)}\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)} = -\frac{\gamma A^*(\gamma R T_0)^{1/2}}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}V_f}\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)}$$

$$\frac{d}{dt}\left(\frac{P}{P_0}\right) = -\frac{\gamma A^*(\gamma R T_0)^{1/2}}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}}\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)}\frac{1}{V_f}\frac{V_i}{V_i} - \frac{\gamma A^*(\gamma R T_0)^{1/2}}{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}V_i}\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)}\frac{V_i}{V_f}$$

$$\frac{d}{dt}\left(\frac{P}{P_0}\right) = -\frac{\gamma}{\tau}\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)}\frac{1}{R_{final}{}^2}$$

$$\frac{d}{d\eta}\left(\frac{P}{P_0}\right) = -\gamma\left(\frac{P}{P_0}\right)^{\left(\frac{3\gamma-1}{2\gamma}\right)}\frac{1}{R_{final}{}^2}$$

$$\boxed{\frac{dH}{d\eta} = -\gamma H^{\left(\frac{3\gamma-1}{2\gamma}\right)}\frac{1}{R_{final}{}^2}}$$

$$\frac{dH}{H^{\left(\frac{3\gamma-1}{2\gamma}\right)}} = -\gamma d\eta\frac{1}{R_{final}{}^2}$$

$$\left.\frac{1}{-\left(\frac{3\gamma-1}{2\gamma}\right)+1}H^{-\left(\frac{3\gamma-1}{2\gamma}\right)+1}\right|_{H_0}^{H} = -\gamma(\eta-\eta_0)\frac{1}{R_{final}{}^2}$$

$$-\frac{1}{\left(\frac{\gamma-1}{2\gamma}\right)}\frac{1}{H^{\left(\frac{\gamma-1}{2\gamma}\right)}} + \frac{1}{\left(\frac{\gamma-1}{2\gamma}\right)}\frac{1}{H_0^{\left(\frac{\gamma-1}{2\gamma}\right)}} = \gamma(\eta-\eta_0)\frac{1}{R_{final}{}^2}$$

Include the final expansion after all propellant is expended

Assume that after all the propellant is consumed the final expansion to the vacuum of space is <u>isentropic</u>. In the equations the unit step function is used to turn off the isothermal term and turn on an isentropic term. The chamber stagnation temperature is constant until the propellant is expended and the isentropic expansion begins.

$$\frac{dR}{d\eta} = \left(1 - u_{step}\left(R - \frac{r_{final}}{r_{initial}}\right)\right)\frac{H^n}{2\left(\frac{\rho_p}{\rho_{g_i}} - 1\right)}$$

$$\frac{dH}{d\eta} + \left(1 - u_{step}\left(R - \frac{r_{final}}{r_{initial}}\right)\right)\frac{H}{R^2} + u_{step}\left(R - \frac{r_{final}}{r_{initial}}\right)\frac{H^{\frac{3\gamma-1}{2\gamma}}}{R_{final}^{\ 2}} - \left(1 - u_{step}\left(R - \frac{r_{final}}{r_{initial}}\right)\right)\frac{H^n}{R}\left(\frac{\frac{\rho_p}{\rho_{g_i}} - H}{\frac{\rho_p}{\rho_{g_i}} - 1}\right) = 0$$

$$where\ u_{step}(x) = 0\ if\ \ x < 0\ and\ u_{step}(x) = 1\ if\ x \ge 0$$

$$R(0) = 1$$

$$H(0) = Choose\ some\ initial\ value$$

$$Choose\ r_{final}/r_{initial}$$

$$Note:\ P_{t2quasi\text{-}steady\ state_{initial}} = (\tau\beta)^{\frac{1}{1-n}}\ and\ \rho_{g_i} = \frac{P_{t2_{quasi-steady\ state_{initial}}}}{RT_{t2}}$$

Example n=0.35, Isentropic final expansion

$$\frac{dR}{d\eta}=u_{step}\left(\frac{r_{final}}{r_{initial}}-R\right)\frac{H^n}{2\left(\frac{\rho_p}{\rho_{g_i}}-1\right)}$$

$$\frac{dH}{d\eta}+u_{step}\left(\frac{r_{final}}{r_{initial}}-R\right)\frac{H}{R^2}+u_{step}\left(R-\frac{r_{final}}{r_{initial}}\right)\frac{H^{\frac{3\gamma-1}{2\gamma}}}{R_{final}^{\ 2}}-u_{step}\left(\frac{r_{final}}{r_{initial}}-R\right)\frac{H^n}{R}\left(\frac{\frac{\rho_p}{\rho_{g_i}}-H}{\frac{\rho_p}{\rho_{g_i}}-1}\right)=0$$

$$R(0)=1$$

$$H(0)=0.5$$

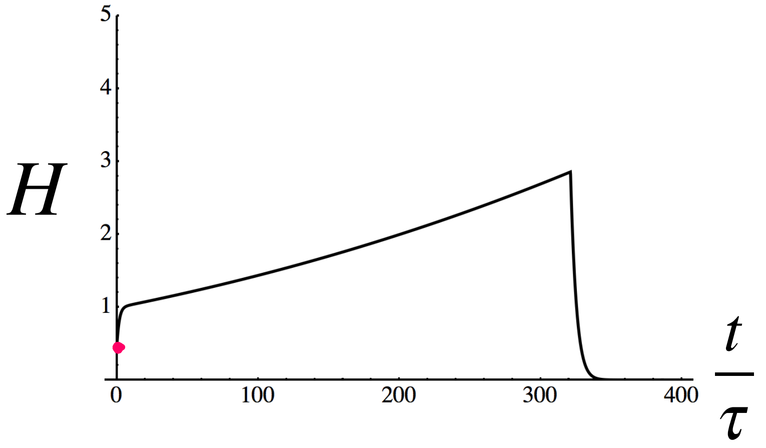
$$n=0.35$$

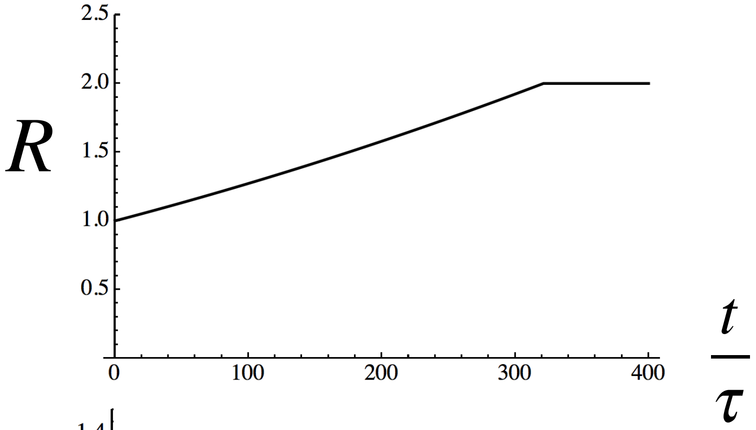
$$\frac{\rho_p}{\rho_{gi}}=196.66$$

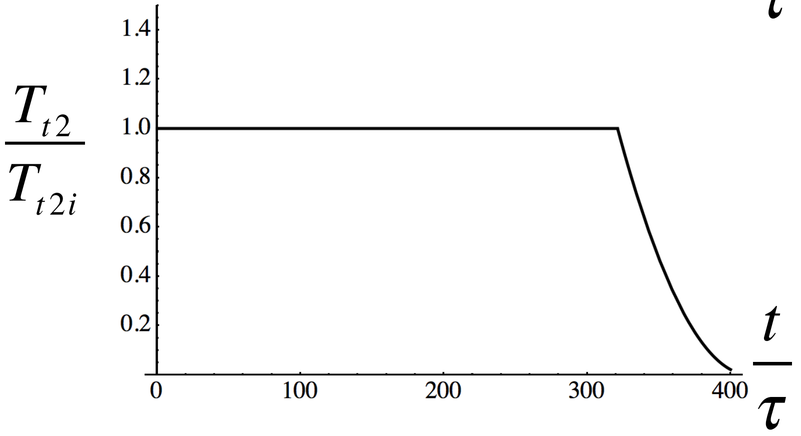
$$\frac{r_{final}}{r_{initial}}=2$$

Note that for an isentropic expansion $\frac{T_{t2}}{T_{t2i}}=\left(\frac{P_{t2}}{P_{t2_{endofburn}}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{P_{t2}}{P_{t2i}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{P_{t2i}}{P_{t2_{endofburn}}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{H}{H_{endofburn}}\right)^{\frac{\gamma-1}{\gamma}}$

where $T_{t2_i}=2500$







Shuttle SRB performance using RPA/CEA - Jonah Zimmermann

Pc = 622 psia (average, max = 910) - I ran it at the average pressure
Ae/At = 7.22
delivered Isp at altitude = 268.2 s

Propellant (from wikipedia, confirmed on a nasa site):
69.6% AP
16% Al
0.4% Fe2O3
12.04% PBAN
1.96% curative

I ran it with just 70% AP, 14% PBAN, and 16% Al

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Shuttle SRB performance using RPA

Theoretical (ideal) performance (O/F=2.333)

Parameter	Sea level	Optimum expansion	Vacuum	Unit
Characteristic velocity		1630.16		m/s
Specific impulse	2462.96	2501.88	2760.25	m/s
Specific impulse	251.15	255.12	281.47	s
Thrust coefficient	1.5109	1.5347	1.6932	

Estimated delivered performance (O/F=2.333)

Reaction efficiency: 0.9750

Nozzle efficiency: 0.9021

Overall efficiency: 0.8796

Parameter	Sea level	Optimum expansion	Vacuum	Unit
Characteristic velocity		1589.39		m/s
Specific impulse	2166.32	2200.55	2427.81	m/s
Specific impulse	220.90	224.39	247.57	s
Thrust coefficient	1.3630	1.3845	1.5275	

Ambient condition for optimum expansion: H=1.17 km, p=0.869 atm

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Shuttle SRB performance using CEA - mole fractions

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7337	42.665
P, BAR	42.885	24.737	1.0052
T, K	3395.44	3205.78	2288.17
RHO, KG/CU M	4.2328 0	2.6074 0	1.5206-1
H, KJ/KG	-1722.34	-2261.96	-4766.22
U, KJ/KG	-2735.50	-3210.67	-5427.27
G, KJ/KG	-34606.4	-33309.2	-26926.6
S, KJ/(KG)(K)	9.6848	9.6848	9.6848
M, (1/n)	27.865	28.096	28.780
MW, MOL WT	25.900	26.033	26.519
(dLV/dLP)t	-1.01880	-1.01438	-1.00190
(dLV/dLT)p	1.3370	1.2682	1.0459
Cp, KJ/(KG)(K)	3.8953	3.5170	2.1225
GAMMAs	1.1340	1.1376	1.1723
SON VEL,M/SEC	1071.9	1038.9	880.3
MACH NUMBER	0.000	1.000	2.803

PERFORMANCE PARAMETERS

Ae/At	1.0000	7.2200
CSTAR, M/SEC	1583.2	1583.2
CF	0.6562	1.5584
Ivac, M/SEC	1952.1	2735.3
Isp, M/SEC	1038.9	2467.3

MOLE FRACTIONS

*AL	0.00010	0.00005	0.00000
ALCL	0.00475	0.00302	0.00004
ALCL2	0.00038	0.00023	0.00000
ALCL3	0.00013	0.00010	0.00001
ALH	0.00002	0.00001	0.00000
ALHCL	0.00002	0.00001	0.00000
ALHCL2	0.00003	0.00002	0.00000
*ALO	0.00020	0.00008	0.00000
ALOCL	0.00044	0.00026	0.00000
ALOH	0.00436	0.00253	0.00002
ALOHCL	0.00063	0.00034	0.00000
ALOHCL2	0.00075	0.00052	0.00003
AL(OH)2	0.00021	0.00010	0.00000
AL(OH)2CL	0.00028	0.00018	0.00001
AL(OH)3	0.00009	0.00005	0.00000
AL2O	0.00005	0.00002	0.00000
AL2O2	0.00002	0.00001	0.00000
*CO	0.23587	0.23695	0.23739 ←
*CO2	0.01459	0.01482	0.01910
*CL	0.01175	0.00993	0.00182
CLO	0.00001	0.00000	0.00000
CL2	0.00002	0.00001	0.00000
*H	0.03501	0.02873	0.00469
HALO2	0.00002	0.00001	0.00000
HCN	0.00001	0.00000	0.00000
HCO	0.00002	0.00001	0.00000
HCL	0.13365	0.13947	0.15603
HOCL	0.00001	0.00000	0.00000
*H2	0.26340	0.26960	0.28931 ←
H2O	0.13231	0.13153	0.12949
*N	0.00001	0.00000	0.00000
*NH	0.00001	0.00000	0.00000
NH2	0.00001	0.00000	0.00000
NH3	0.00001	0.00001	0.00000
*NO	0.00055	0.00034	0.00001
*N2	0.08087	0.08140	0.08310
*O	0.00059	0.00034	0.00000
*OH	0.00818	0.00582	0.00037
*O2	0.00012	0.00007	0.00000
AL2O3(a)	0.00000	0.00000	0.07856 ←
AL2O3(L)	0.07051	0.07340	0.00000 ←

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Shuttle SRB performance using RPA - mass fractions

Table 2. Mass fractions of the combustion products

Species	Injector	Nozzle inl	Nozzle thr	Nozzle exi
AL	0.0001092	0.0001092	0.0000333	0.0000000
AL(OH)2	0.0004926	0.0004926	0.0001738	0.0000008
AL(OH)2CL	0.0010543	0.0010543	0.0005407	0.0000202
AL(OH)3	0.0002802	0.0002802	0.0001306	0.0000039
AL+	0.0000037	0.0000037	0.0000013	0.0000000
AL2O	0.0001272	0.0001272	0.0000278	0.0000000
AL2O2	0.0000635	0.0000635	0.0000128	0.0000000
AL2O3(L)	0.2775806	0.2775806	0.2904540	0.0000000
AL2O3(a)	0.0000000	0.0000000	0.0000000	0.3021025
ALCL	0.0114423	0.0114423	0.0058831	0.0000881
ALCL2	0.0014497	0.0014497	0.0006990	0.0000094
ALCL3	0.0006774	0.0006774	0.0004638	0.0000441
ALH	0.0000270	0.0000270	0.0000073	0.0000000
ALH2	0.0000002	0.0000002	0.0000000	0.0000000
ALH2CL	0.0000018	0.0000018	0.0000005	0.0000000
ALHCL	0.0000432	0.0000432	0.0000128	0.0000000
ALHCL2	0.0001165	0.0001165	0.0000519	0.0000007
ALN	0.0000002	0.0000002	0.0000000	0.0000000
ALO	0.0003271	0.0003271	0.0000965	0.0000000
ALO2	0.0000072	0.0000072	0.0000013	0.0000000
ALOCL	0.0013381	0.0013381	0.0006335	0.0000074
ALOCL2	0.0000109	0.0000109	0.0000037	0.0000000
ALOH	0.0074020	0.0074020	0.0033556	0.0000339
ALOHCL	0.0019387	0.0019387	0.0008005	0.0000064
ALOHCL2	0.0033213	0.0033213	0.0019896	0.0001250
CL	0.0160882	0.0160882	0.0124602	0.0023183
CL-	0.0000047	0.0000047	0.0000016	0.0000000
CL2	0.0000576	0.0000576	0.0000349	0.0000024
CLO	0.0000136	0.0000136	0.0000052	0.0000000
CN	0.0000001	0.0000001	0.0000000	0.0000000
CO	0.2550643	0.2550643	0.2547988	0.2505962
CO2	0.0247990	0.0247990	0.0252484	0.0318767
COCL	0.0000040	0.0000040	0.0000016	0.0000000
COOH	0.0000032	0.0000032	0.0000013	0.0000000
H	0.0013624	0.0013624	0.0010134	0.0001701
H2	0.0204997	0.0204997	0.0210040	0.0220031
H2O	0.0920388	0.0920388	0.0907104	0.0879142
H2O2	0.0000004	0.0000004	0.0000000	0.0000000
HALO	0.0000060	0.0000060	0.0000015	0.0000000
HALO2	0.0000373	0.0000373	0.0000124	0.0000000
HCHO,formaldehy	0.0000014	0.0000014	0.0000007	0.0000000
HCL	0.1881485	0.1881485	0.1978594	0.2146591
HCN	0.0000064	0.0000064	0.0000031	0.0000002
HCO	0.0000227	0.0000227	0.0000100	0.0000002
HCOOH	0.0000010	0.0000010	0.0000004	0.0000000
HNC	0.0000015	0.0000015	0.0000006	0.0000000
HNCO	0.0000014	0.0000014	0.0000007	0.0000000
HNO	0.0000012	0.0000012	0.0000004	0.0000000
HO2	0.0000011	0.0000011	0.0000003	0.0000000
HOCL	0.0000132	0.0000132	0.0000061	0.0000000
N	0.0000030	0.0000030	0.0000010	0.0000000
N2	0.0874711	0.0874711	0.0876281	0.0877818
NCO	0.0000002	0.0000002	0.0000000	0.0000000
NH	0.0000031	0.0000031	0.0000011	0.0000000
NH2	0.0000036	0.0000036	0.0000014	0.0000000
NH3	0.0000063	0.0000063	0.0000036	0.0000004
NO	0.0006341	0.0006341	0.0003212	0.0000094
O	0.0003666	0.0003666	0.0001649	0.0000020
O2	0.0001424	0.0001424	0.0000625	0.0000007
OH	0.0053758	0.0053758	0.0032666	0.0002222

Solid/liquid particles in the motor lead to:

1) reduced nozzle efficiency - two phase losses.

2) Improved motor stability through absorption of high frequency noise.