# AA103 Homework 6

Cantwell Spring 2020-21

Due May 11, 2021

## Suggested Reading

AA283 Course reader Chapters 8 and 9.

### Problem 1

A group of investors join together to develop a very low-cost, four stage rocket to launch small cubesat payloads to low Earth orbit (8,000 m/sec) from a barge off the island of Singapore. The concept proposed for the system utilizes propellants that are safe and cheap but provide a specific impulse of only 220 sec. All four stages are identical. What structural efficiency is required to reach orbit with a finite payload?

### Problem 2

A two stage rocket is to be used to put a payload of 1000 kg into low earth orbit. The vehicle will be launched from Kennedy Space Center where the speed of rotation of the Earth is 427 m/ sec. Assume gravitational velocity losses of about 1200 m/ sec and aerodynamic velocity losses of 500 m/sec. The first stage burns kerosene and oxygen producing a mean specific impulse of 320 sec averaged over the flight, while the upper stage burns hydrogen and oxygen with an average specific impulse of 450 sec. The structural coefficient of the first stage is 0.05 and that of the second is 0.07. Determine the payload ratios and the total mass of the vehicle. Suppose the same vehicle is to be used to launch a satellite into a north-south orbit from a launch complex on Kodiak island in Alaska. How does the mass of the payload change?

### Problem 3

I would like you to consider the Space Shuttle Main Engine (SSME). Use the thermochemical calculator CEA or an equivalent application to help solve the problem. The chamber pressure and nozzle area ratio of the SSME are

$$\begin{aligned} P_{t2} &= 3000psia, \\ A_e/A_t &= 77.5. \end{aligned} \tag{1}$$

The fuel and oxidizer mass flow rates are

$$\begin{aligned} \dot{m}_{H2} &= 69kg/sec, \\ \dot{m}_{O2} &= 400kg/sec. \end{aligned} \tag{2}$$

1) Determine the adiabatic flame temperature and equilibrium mass fractions of the chamber pressure gas mixture including $H, H_2, HO, H_2O, H_2O_2, O, O_2$ at the given chamber pressure.
2) Determine the exhaust velocity and sea level thrust assuming that the mixture remains at equilibrium during the expansion process.
3) Suppose the mass flow rates are changed to the stoichiometric ratio

$$\begin{aligned} \dot{m}_{H2} &= 52.5kg/sec, \\ \dot{m}_{O2} &= 416.5kg/sec. \end{aligned} \tag{3}$$

How much does the exhaust velocity change?

## Problem 4

The highest performing liquid propellant combination of fuel and oxidizer is hydrogen, $H_2(M_w = 2)$, reacting with fluorine, $F_2(M_w = 38)$, to produce $HF(M_w = 20)$. One of the problems with these propellants

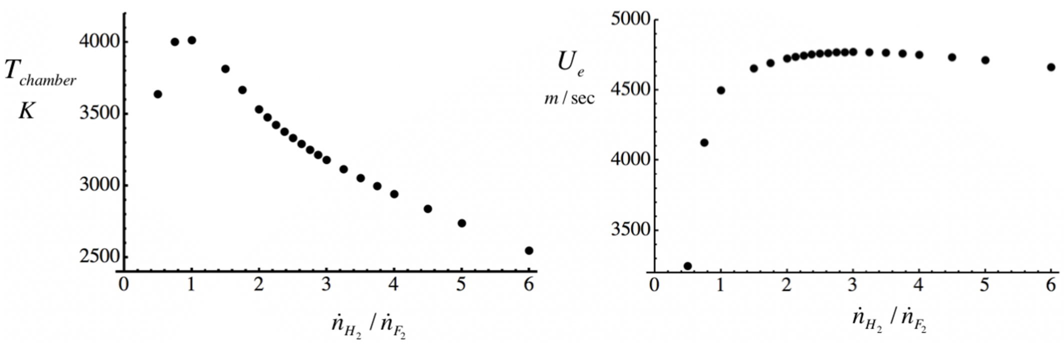


Figure 1: Effect of mixture ratio on $T_{t2}$ and $U_e$ of a fluorine hydrogen rocket.

is that if the ratio of moles of $H_2$ and $F_2$ fed to the combustion chamber is $1:1$, the chamber temperature is so high that any available alloy will soften and melt. Consider a thruster designed to operate in the vacuum of space. Both propellants are stored in gaseous form at high pressure at $298.15K$. The chamber pressure is $10^5 Pascals(1bar)$. The designer of the motor uses CEA to make the calculations shown in figure 1. The horizontal axis represents the ratio: moles per sec of hydrogen/moles per sec of fluorine injected into the combustion chamber. The nozzle area ratio is $100$.

1) Explain the trends in the data. What did the designer learn by doing this calculation?

2) The bond enthalpies for the reactants at $298.15$ and $1bar$ are: $H_2 = 4.36 \times 10^8 J/kgmole, F_2 = 1.58 \times 10^8 J/kgmole, HF = 5.65 \times 10^8 J/kgmole$. Suppose the gas exits through a very large area ratio nozzle. For $\dot{n}_{H_2}/\dot{n}_{F_2} = 1$, estimate the maximum possible value of $U_e$ and compare with the results shown above.

3) Use CEA to check the two points at $\dot{n}_{H_2}/\dot{n}_{F_2} = 1$ and $\dot{n}_{H_2}/\dot{n}_{F_2} = 5$.