AA103 Homework 5

Cantwell Spring 2020-21

Due May 4, 2021

Suggested Reading

AA283 Course reader Chapter 7.

Problem 1

Recently one of the popular toys being sold was called a stomp rocket. The launcher consists of a flexible plastic bladder connected to a 1.5 cm diameter rigid plastic tube. The rocket is a slightly larger diameter rigid plastic tube, closed at the top end, about $20cm$ long. The rocket weighs about $10gm$. The rocket slips over the tube as shown in figure 1.

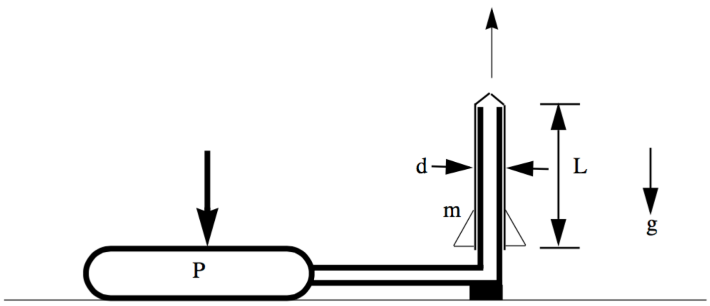

Figure 1: Stomp rocket toy.

Jumping on the bladder pressurizes the air inside and launches the rocket to a height which the manufacturer claims can exceed $50m$. The area of the bladder in contact with the ground is approximately $100cm^2$. Use basic principles of mechanics to roughly estimate how much a "child" would have to weigh to be able to achieve this height.

Problem 2

Consider a class of monopropellant thrusters based on the use of the noble gases including Helium $Mw = 4$, Neon $Mw = 20$, Argon $Mw = 40$, Krypton $Mw = 84$ and Xenon $Mw = 131$. Radon $Mw = 222$ is excluded because of its radioactivity. The thruster is comprised of a tank that exhausts through a simple convergent nozzle to the vacuum of space. Onboard heaters are used to maintain the gas in the tank at a constant stagnation temperature T_{t2} as it is exhausted.
1) The thrust is often expressed in terms of an effective exhaust velocity $T = \dot{m}C$. Show that the effective exhaust velocity of this system can be expressed as follows.

$$C = \left(\frac{2(\gamma + 1)}{\gamma} \left(\frac{R_u}{M_w} \right) T_{t2} \right)^{1/2} \tag{1}$$

2) The mass of propellant contained in the tank is

$$M_{propellant} = \frac{P_{t2_{initial}} V_{tank} M_w}{R_u T_{t2}} \tag{2}$$

The initial tank pressure is some rated value (a do-not-exceed pressure) independent of the type of gas used. The designer would like to choose the propellant gas so that the velocity increment produced by the propulsion system ΔV is as large as possible for fixed tank volume, initial pressure and gas temperature. The problem is to decide whether to choose a gas with low M_w, thus achieving a high value of C but low propellant mass, or a gas with high M_w reducing C but increasing propellant mass. By mixing two or more gases, any mean atomic mass between 4 and 131 can be selected by the designer. Note that γ is the same regardless of what gas or mixture of gases is used. Show that, once the tank volume pressure and temperature are determined and the vehicle empty mass $M_{structure}$ is known, the gas should be selected to have a mean atomic weight M_w such that $M_{propellant}/M_{structure} = 3.92$ for maximum ΔV.

Problem 3

One of the most common ways to demonstrate rocket propulsion is the "bottle rocket". A plastic drink bottle is partially filled with water and then attached to a fitting that seals the spout (nozzle). Air is pumped into the bottle to pressurize the space above the water. A release is triggered and the water is rapidly expelled while the bottle lifts off. The flight of the rocket is very short and so heat transfer between the bottle and surrounding air is small. Assume the expansion of gas during the expulsion of water is adiabatic and isentropic.

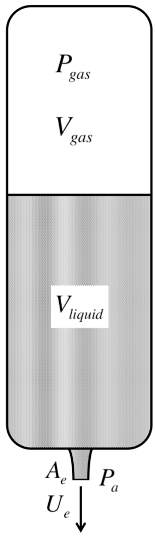

Figure 2: Bottle rocket sketch.

1) Use the Bernoulli equation to express the velocity, U_e , of water expelled from the bottle in terms of the gas pressure and ambient pressure. Ignore the hydrostatic pressure of the water column above the nozzle.
2) The initial gas pressure is generally determined by the maximum pressure that the plastic bottle can withstand independent of the initial relative amounts of water and gas. Let this pressure be $P = 3 \times 10^5 N/m^2$. Let the initial volume of water ($\rho_{water} = 10^3 kg/m^3$) be chosen so that when the last bit of water is expelled, the gas pressure in the bottle reaches ambient pressure, $P_{ambient} = 10^5 N/m^2$. The total volume of the bottle is $V = 4 \times 10^{-3} m^3$. Determine the initial water volume.
3) The diameter of the spout is $2cm$. Determine the initial thrust of the bottle rocket.
4) Derive the right hand side of the equation for $\frac{d}{dt}\left(V_{liquid}/V\right) =?$. Use this result to establish an expression for the characteristic time, τ_{liquid}, for the liquid to be expelled from the bottle. Evaluate it for this problem. Does the time roughly match your experience?