AA103 Homework 4, 2020 - 2021

Cantwell Spring 2020-21

Due April 27, 2021

Suggested Reading

Read AA210 Course reader Chapter 10 and AA283 Course reader Chapter 7.

Problem 1

A monopropellant thruster using helium gas at a pressure and temperature of $10^7 N/m^2$ and $300K$ exhausts through a simple convergent nozzle to the vacuum of space. Assume helium is continually added to the plenum from an onboard storage tank to maintain the pressure and temperature. The nozzle diameter is $0.01m$. The universal gas constant is $R_u = 8314J/(kmole - K)$, the atomic weight of helium is 4 and $\gamma = 5/3$.

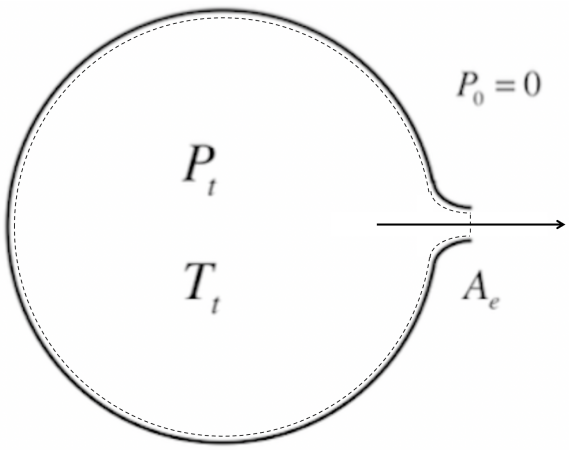

Figure 1: Steady flow from a plenum through a simple convergent nozzle to the vacuum of space.

a) What is the temperature and pressure of the gas at the nozzle exit ?
b) What is the velocity of the gas at the nozzle exit?
c) Integrate the momentum equation over the surface of the dashed line control volume shown in the figure. The control volume covers the inside surface of the plenum, A_s, as well as the nozzle exit plane, A_e. Integrate

$$\int_V \nabla \cdot \left(\rho \bar{U}\bar{U} + P\bar{\bar{I}} - \bar{\bar{\tau}} \right) dV = \int_{A_s + A_e} \left(\rho \bar{U}\bar{U} + P\bar{\bar{I}} - \bar{\bar{\tau}} \right) dA. \tag{1}$$

Assume that flow variables can be area averaged over the nozzle exit plane. Show that the thrust is given by

$$T = \rho_e {U_e}^2 A_e + P_e A_e. \tag{2}$$

d) Determine the thrust.

Problem 2

The thruster in problem 1 is modified to have a divergent nozzle with a nozzle exit Mach number of 4.
a) What is the diameter of the nozzle exit?
b) What is the temperature and pressure of the gas at the nozzle exit?
c) What is the velocity of the gas at the nozzle exit?
d) What is the thrust? Why is it higher than in Problem 1?

Problem 3

The thruster in problem 2 exhausts, not to the vacuum of space, but to an ambient atmosphere with a finite pressure. As a result there is a $M = 2$ shock between the nozzle throat and the exit. In this case the thrust formula is

$$T = \rho_e {U_e}^2 A_e + (P_e - P_0) A_e. \tag{3}$$

a) What is the ambient pressure?
b) What is the temperature and pressure of the gas at the nozzle exit?
c) What is the velocity of the gas at the nozzle exit?
d) What is the thrust?