



# AA 103 2018–2019

## Project 2

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### 1 Problem

A very important situation to model in rocket propulsion (for cold/warm gas thrusters, pressurization systems, etc.) is the expansion of a gas from a tank into a lower pressure environment. There are two main assumptions that can be made for these models:

- The temperature inside the tank remains constant: Isothermal blowdown
- There is no heat transferred to the gas (adiabatic): Isentropic blowdown

For this project you will derive the equations for both assumptions and simulate the results using software of your choice (Matlab, python, etc...). You will then gather experimental data in the lab and compare to your model. You will be working in pairs and must submit an individual report using the AIAA JPC conference paper format. It is highly recommended that you solve parts 2 and 3 before going to the lab.

Throughout the problem you can assume that the gas is ideal ( $\gamma = 1.4$ ,  $R = 296.8$  J/kg/K). The tank can be modeled as a cylinder with a radius of 1 inch and a length of 6.75 inches (hardware is typically machined in imperial units!). The orifice diameter is 700 microns and you can assume a discharge coefficient  $C_d = 0.98$  (reducing the area of the orifice to an effective “area”). You can consider the ambient pressure to be 14.7 psi.

Please use the following notations:

- The subscript  $i$  refers to initial values
- $P$  is the pressure in the tank, with ( $P_i \approx 200$  psi the initial pressure before the blowdown)
- $T$  is the temperature in the tank, with  $T_i$  the initial temperature before the blowdown, around ambient)
- $m_{tank}$  is the mass of gas in the tank
- $\dot{m}$  is the mass flow rate of gas exiting the tank through the orifice
- $V$  is the tank volume
- $A$  is the area of the orifice

## 2 Isothermal blowdown

1. When the valve is initially opened, will the flow be choked at the orifice? Justify your answer by determining the condition that must be satisfied for it to be true.
2. In the regime defined in question 1, derive the equation for the mass flow rate of nitrogen leaving the tank:

$$\dot{m} = C_d A \sqrt{\gamma \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \sqrt{\frac{1}{RT}}} \times P \quad (1)$$

Hint: The mass flow rate through the orifice is  $\dot{m} = \rho_{or} \times C_d A \times U_{or}$  with  $\rho_{or}$  the density and  $U_{or}$  the velocity AT the orifice. To write these two parameters as functions of  $P$  and  $T$  (pressure and temperature inside the tank) you must assume that these values are equal to the stagnation values at the orifice ( $T_{or,t}$  and  $P_{or,t}$ ). Note that the speed of sound at the orifice is a  $a = \sqrt{\gamma RT_{or}}$ .

3. Relate the mass flow rate  $\dot{m}$  and the derivative of the mass in the tank  $\frac{dm_{tank}}{dt}$ .
4. Using the ideal gas law and assuming that the process is isothermal ( $T = T_i$ ), derive the first order differential equation governing  $m_{tank}$  as a function of  $C_d$ ,  $A$ ,  $V$ ,  $\gamma$ ,  $R$  and  $T_i$ .
5. Solve this differential equation to obtain an explicit expression for  $m_{tank}$  as a function of time.
6. Use a software of your choice to simulate the tank blowdown. Plot  $m_{tank}$  (in grams) and  $P$  (in psi) as functions of time. Remember to end your simulation when the condition derived in question 1 is no longer satisfied.

## 3 Isentropic blowdown

Now we will assume that the blowdown is isentropic. This assumes (among other things) that no heat is transferred from the outside to the gas. As a result, because the gas is expanding, its temperature will decrease. This assumption is very good when considering a rapid process. However, if the blowdown is very slow, there is enough time for heat to be transferred to the gas. In this case, deriving an explicit equation for  $m_{tank}$  is very difficult, so we will go about solving it numerically.

1. Assuming the blowdown is isentropic, write the condition relating  $T(t)$  with  $P(t)$ ,  $P_i$ ,  $T_i$ , and  $\gamma$ .
2. Replace the expression for  $T(t)$  in the ideal gas law to obtain an expression for  $P(t)$  as a function of  $m_{tank}(t)$ ,  $V$ ,  $T_i$ ,  $P_i$ ,  $R$  and  $\gamma$ .
3. Using equation 1 (from part 1), determine a system of equations to model the flow in the following form (where  $f$ ,  $g$ ,  $h$  denote functions that you have already derived).

$$\begin{cases} \dot{m} = C_d A \sqrt{\gamma \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \frac{1}{RT}} \times P \\ \frac{dm_{tank}}{dt}(t) = f(m_{tank}(t)) \\ P(t) = g(m_{tank}(t)) \\ T(t) = h(P(t)) \end{cases} \quad (2)$$

4. Solve the previous system of equations using the most simple numerical scheme possible. Choose a reasonably small time step to obtain smooth curves. Plot  $m_{tank}$  (in grams),  $P$  (in psi), and  $T$  (in °C) as functions of time.

## 4 Experiment

In this lab you will be manipulating a pressurized gas system. As a result, you are **REQUIRED** to wear safety glasses at all times inside the test room. Please read all of the following instructions before starting.

The setup you will be working with consists of:

- A vessel filled with nitrogen gas ( $\approx 200$  psi)
- A thermocouple inside the tank
- A pressure transducer
- An orifice (700 micron diameter)
- A manual ball valve and two electronically operated solenoid valves

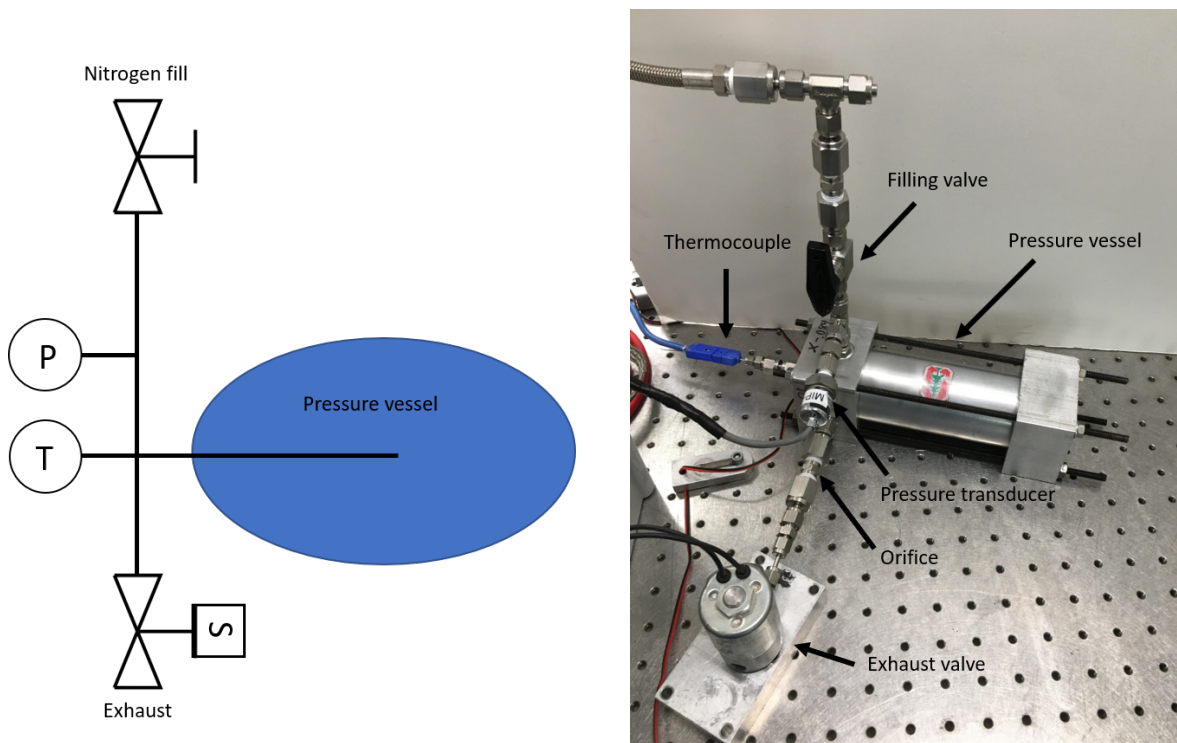


Figure 1: Schematic and picture of the experimental setup.

To open the program and run the experiment:

1. Open the folder entitled AA103 on the desktop and open the AA103 LabVIEW file.
2. Click the white arrow in the top left corner (under the Edit button) to run the program.
3. Check that the “Arduino communication” and the “Data Working Indicator” lights are both on. If one or both are off, please come see me in the office.

The first thing to do is to fill the tank to about 200 psi. To fill the tank using the LabVIEW program:

1. Enter the test room and open the manual ball valve (“Filling valve” in figure 1). Remember to wear safety glasses!

2. Exit the test room and click “Fill tank”. This will let nitrogen gas flow into the tank for 6 seconds. You can monitor the pressure increase on the schematic to the left of the screen. (Note the large increase in temperature!)
3. Once the “Fill tank” button is off, enter the test room to close the manual ball valve on the experiment.

Now that your tank is filled, you can start taking measurements as you vent the vessel. To vent the gas:

1. Make sure no one is in the test room (but you may keep the door open).
2. Make sure the manual ball valve is closed (so that you only vent the gas from the tank).
3. Select an exhaust time (in milliseconds) between 0 and 20,000 ms and input it in the “Set time” box in the program.
4. Click on “Exhaust”. The real time for which the exhaust valve was opened will be displayed in the “Real time” box.

You have the option of taking manual measurements between exhaust events or record the time-dependent data. The output data file will be in an lvm. format. You can open this file using Excel or any equivalent program. To record data:

1. Make sure the folder in the “Select directory” box is C:\Users\Nozzle\Desktop\AA103\RecordedData.
2. Enter a test name of your choice and click “Set path”. Your selected file name should appear in the box below.
3. Click “Record data”. To stop recording, click on the button again.

**Important:** The program will overwrite your data if you do not select a new file path. If you wish to record multiple sets of data, make sure to select a new test name and click on “Set path” every time. Please do not leave any data on the computer and cut everything onto your own USB stick when you are finished (use the USB hub on the desk). **Do not insert or remove a USB stick while the LabVIEW program is running, this messes with the data acquisition.**

**Final considerations:** To simulate an isothermal blowdown, you must try to maintain the temperature in the tank (indicated on the schematic to the left of the screen) as close to ambient as possible. Keep your exhaust time low and wait between events to let the gas heat up. Gather  $P$  and  $T$  manually as well as the real exhaust time to reconstruct the pressure-time profile. To simulate an isentropic blowdown, select an exhaust time that will allow the tank to vent all the way to ambient pressure in one event based on your simulation results (part 3). Record the time dependent data during the process.

You can refill the tank a reasonable amount of times to gather all of your experimental data. But if you are experiencing difficulties please come see me in the office.

**Important:** When you are finished, click on the “STOP PROGRAM button in the top left corner. Do not close the LabVIEW program directly.

## 5 Write-up

In your write-up, answer questions in Parts 2 and 3 as well as the following questions:

1. Explain your experimental process to simulate an isothermal blowdown.
2. Plot the experimental pressure on top of your simulation results. To do so, you might need to readjust the initial pressure and temperature in your simulation.

3. Explain your experimental process to simulate an isentropic blowdown.
4. Plot the experimental data (Pressure and Temperature) on top of your simulation results.
5. Explain any discrepancies between your simulation and experimental results. What do you think are the main sources of error in this experiment?