Design of Robust Global Power and Ground Networks

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Global power & ground network design

**Problem:** size wires (choose topology)

- minimize wire area subject to node voltage, current density constraints
- don’t consider fast dynamics (C,L)
- do consider (slow) variation in block currents
(Quasi-)static model

- segment conductance \( g_k = w_k/(\rho l_k) \); current density \( j_k = i_k/w_k \)
- conductance matrix \( G(w) = \sum_k w_k a_k a_k^T \); node voltages \( V = G(w)^{-1}I \)
- statistical model for block currents: \( \mathbf{E} \mathbf{I} \mathbf{I}^T = \Gamma \)
  - \( \Gamma \) is block current correlation matrix
  - \( \Gamma^{-1/2} = \text{RMS}(I_j) \); \( \Gamma_{ij} \) gives correlation between \( I_i, I_j \)
Sizing problem

minimize \[ A = \sum_k l_k w_k \] (area)
subject to \[ V_j \leq V_{\text{max}} \] (node voltage limit)
\[ \mathbf{E} j_k^2 \leq j_{\text{max}}^2 \] (RMS current density limit)
\[ w_k \geq 0 \] (nonneg. wire widths)

can't solve, except special case \( I \) constant

- (Erhard & Johannes) can improve any mesh design by pruning to a tree
- (Chowdhury & Breuer) can size P&G trees via geometric programming
**Meshes, trees and current variation**

- \( I_1, I_2 \) constant (or highly correlated): set \( w_2 = 0 \) (yields tree)
- \( I_1, I_2 \) anti-correlated: better to use \( w_2 > 0 \) (yields mesh)
Average power formulation

- power dissipated in wires: \( P = V^T I = I^T G(w)^{-1} I \)
- average power: \( E P = E I^T G(w)^{-1} I = \text{Tr} G(w)^{-1} \Gamma \)

\[
\text{minimize} \quad \text{Tr} G(w)^{-1} \Gamma + \mu \sum_k l_k w_k \quad \text{(average power + \( \mu \cdot \text{area} \))}
\]

subject to \( w_k \geq 0 \)

- parameter \( \mu > 0 \) trades off average power, area
- nonlinear but \textbf{convex problem}, readily (globally) solved
- indirectly limits \( E j_k^2, V_j \)
Properties of solution

observation: many $w_k$’s are zero, i.e., many wires aren’t used
average power formulation can be used for P&G topology selection:

• start with lots of (potential) wires
• let average power formulation choose among them
• topology (given by nonzero $w_k$) independent of $\mu$

resulting current density and node voltages:

• RMS current density is equal in all (nonzero) segments
  in fact $\mu = \rho j_{\text{max}}^2$ yields $Ej_k^2 = j_{\text{max}}^2$ in all (nonzero) segments
• observation: $V_j$ are small
Example

- $10 \times 10$ grid, each node connected to neighbors (180 segments)
- 8 current sources, $I \in \mathbb{R}^8$ is random with three possible values
- 4 ground pins on the perimeter (at corner points)
design for constant currents (with same RMS values)

- a tree; each source connected to nearest ground pin
- RMS current density 1,
  area = 448,
  max. voltage = 7.7

design via average power formulation

- mesh, not a tree
- RMS current density 1,
  area = 347,
  max. voltage = 5.7
Barrier method

use Newton’s method to minimize

$$\text{Tr } G(w)^{-1} \Gamma + \mu l^T w - \beta^{(i)} \sum_k \log w_k$$

• barrier term $-\beta \sum_k \log w_k$ ensures $w_k > 0$
• solve for decreasing sequence of $\beta^{(i)}$
• can show $w^{(i)}$ is at most $n \beta^{(i)}$ suboptimal
• $O(n^3)$ cost per Newton step

works very well for $n < 1000$ or so; easy to add other convex constraints
Pruning

• often clear in few iterations which $w_k$ are converging to 0

• removing these $w_k$ early greatly speeds up convergence

• sizes 1000s of $w_k$s in minutes
Where $\Gamma$ comes from

- from simulation: $\Gamma = \frac{1}{T_{\text{sim}}} \int_0^{T_{\text{sim}}} I(t)I(t)^T \, dt$

- or, from block RMS currents and estimates of correlation:
  $$\Gamma_{ij} = \text{RMS}(I_i) \text{RMS}(I_j) \rho_{ij}$$

- can use eigenvalue decomposition to simplify $\Gamma$
  $$\Gamma = \sum_i \lambda_i q_i q_i^T, \quad \hat{\Gamma} = \sum_{i=1}^{r} \lambda_i q_i q_i^T$$

(reduced rank approximation speeds up avg. pwr. solution)
Conclusion

- P&G meshes outperform trees when current variation taken into account

- Average power formulation
  - yields tractable convex optimization problem
  - chooses topology
  - guarantees RMS current density limit
  - indirectly limits node voltages