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An optimization-based decision support framework for coupled pre- and post-earthquake infrastructure risk management

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A R T I C L E I N F O

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A B S T R A C T

In the context of infrastructure resilience and risk management, key decisions occur in the aftermath of adverse events (e.g., immediate response and later repair of damage), but preemptive decisions must be made under uncertainty about the specific disaster realization to face in the future (e.g., strengthening components, or allocating resources for post-event activities). This paper proposes an optimization framework to address problems in which preemptive decisions are coupled with those eventually required to respond to an uncertain adverse event. Specifically, strategic decisions are pursued regarding whether to proactively retrofit or reactively repair bridges in a transportation network under seismic hazards, with the objective of minimizing the cost of maintaining a target network performance metric throughout a set of possible adverse scenarios. A two-stage stochastic programming approach is presented, which relates pre- and post-event decisions, accounting for the uncertainty throughout scenarios. The proposed approach implies a decomposition of the optimization problem that enables the analysis of large sets of scenarios, which is advantageous when dealing with complex networks as the ones addressed in infrastructure engineering practice. The methodological framework is presented along with an analysis of the San Francisco Bay Area transportation network, as an instance of a realistic, complex infrastructure network. Results evidence the potential of the approach to provide risk-informed decision support, while being able to deal with large sets of components and scenarios under an exact optimization approach, and solving problems with large number of variables and constraints.

1. Introduction

This paper proposes a decision support framework that integrates probabilistic risk assessment of complex infrastructure networks and stochastic programming to determine cost-optimal actions for pre- and post-disaster stages, while guaranteeing acceptable performance throughout a set of scenarios describing hazards. Instances of risk management problems that motivate a framework for coupled decisions in two stages include: first, how to allocate relief resources before a disastrous event in order to maximize the efficiency for their distribution when a disaster occurs, thus, contributing to prompt recovery [1]; and second, how to determine which network components require preemptive investments (e.g., enhancing arcs' fragility, or flow capacity), and which ones may be intervened only when disastrous events occur, without compromising pre-specified performance targets.

The proposed framework provides computational support for these problems when dealing with large infrastructure networks, for which weighing costs of individual investments versus acceptable system performance throughout many scenarios becomes highly expensive. To illustrate the proposed framework, this paper focuses on whether to preemptively retrofit bridges or repair damage *a posteriori* (which implies actions of higher cost than retrofitting, but which occur with lower probability). An optimization problem is formulated with the objective of minimizing the cost of retrofit actions, and the expected cost of repair actions (and other expected consequences), while constraining a set of performance metrics within a pre-specified acceptable range.

Three complexity drivers make the proposed problem challenging: first, the size of the network and the intricacy of its connections, which makes performance assessments computationally expensive; second, the many possible combinations of decision options that are being evaluated, since their effect on performance must be assessed; and third, the number of scenarios describing the hazard(s) of interest, all of which require performance evaluations involving the previous two items as well. Since our focus is on integrating optimization and comprehensive risk assessment, it will be critical to address the complexity related to the number of scenarios. A stochastic programming

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framework is proposed in order to deal with the relationship of actions in the present and the consequences observed in possible future scenarios. An exact optimization approach is adopted because, unlike heuristics alone, it provides a guarantee of optimality (or information about optimality gaps).

The key contribution of this research is in providing decision support strategies for risk management of complex, real-world networks, which are integrated with sophisticated probabilistic descriptions of potential disaster scenarios derived from seismic hazards. The core of the proposed approach is in the development of stochastic programming procedures [2–4] in which exact optimization can be performed throughout a set of potential disaster scenarios, described by magnitudes and rates of occurrence identified through seismic risk assessment procedures [5]. These capabilities are tested on complex, realistic networks, for which seismic risk assessments are available.

The remainder of this paper is organized as follows: Section 2 discusses literature on optimization techniques and other decision support tools for infrastructure resilience and risk management. Section 3 presents the decision problem for risk management of transportation networks and discusses its integration with the risk assessment methodology in [5], with a focus on decisions about retrofit and repair actions. Section 4 introduces key concepts of stochastic programming and proposes a formulation to decompose an optimization problem into separate yet coupled Mixed Integer Programs (MIP), thus, enabling the analysis and optimization of larger networks and sets of scenarios. Section 5 provides an example regarding the San Francisco Bay Area transportation network to illustrate the proposed methodology. Finally, Section 6 provides conclusions for the paper and ideas for future work.

2. Related work

Resilience has become a widespread topic in the literature on infrastructure operation [6–9] and in practice [10,11] under the challenges imposed by increasing urban concentration, sustainable development, lifeline interdependencies, and multiple evolving hazards. Although definitions vary throughout disciplines, Bruneau et al. [12] consider the following components in the measurement of resilience in technical, organizational, social, and economic dimensions: (i) *reduced failure probabilities*, (ii) *reduced consequences*, and (iii) *reduced time to recovery*. The proposed framework contributes to resilience by enabling optimal retrofit policies throughout a hazard consistent set of scenarios, aiming to guarantee acceptable performance levels for infrastructure systems in the aftermath of disasters.

Related efforts in addressing decision problems within risk assessment and management of complex infrastructure networks include research by: Ouyang et al. [7] regarding a framework to analyze infrastructure resilience, which includes comprehensive literature revision; Lim et al. [13] regarding efficient reliability assessment for complex infrastructure networks; Xu et al. [6] regarding the scheduling of response actions for power infrastructure under seismic hazards by means of genetic algorithms; Baroud et al. [9] regarding stochastic measures of network resilience; Hu et al. [14] regarding optimal management of large-scale transportation networks; and Nogal et al. [15] regarding the study of transportation network resilience under extreme events.

From an optimization perspective, Frangopol and Bocchini [17] propose the use of resilience as an optimization criterion for bridge rehabilitation, considering the maximization of the transportation network resilience as well as the minimization of the total rehabilitation cost, relying on bi-objective genetic algorithms for the construction of an efficient frontier (i.e., a set of solutions not outperformed by others). Pereira et al. [18] use multi-objective evolutionary algorithms for resilient routing configurations that are robust to changes in the traffic demands and able to maintain performance in the presence of arc failure events. While heuristic approaches are compelling because of their speed, it is important to push the boundaries of exact methods, which can be enhanced by means of decomposition techniques. Furthermore, their guarantee of optimality is valuable for risk management problems that already deal with high uncertainty in the assessment process. In addition to responding to the needs in risk management for complex infrastructure networks, the proposed stochastic programming framework is in line with existing challenges for operational research in disaster management, as highlighted in [16].

The work in [19] illustrates a novel mathematical optimization approach for the case of intentional attacks; in contrast, the methodology in this paper addresses risk related to natural hazards for large scale networks under extensive scenarios. The work in [20] studies resilience for the San Francisco Bay Area transportation network based on a sequential game strategy from the perspective of users; while such approach is complementary to the one provided in this paper, our focus is on integrated seismic risk assessment and performance optimization. The work in [21] proposes a stochastic programming approach for pre-disaster investments to improve performance in a transportation network based on fixed failure probabilities. Fan et al. [22] propose a stochastic programming model to prioritize retrofit actions on highway network bridges; their approach relies on a flow-based mathematical formulation, and deals with instances with limited network size and scenarios. A distinctive feature of this paper is how the optimization relies on extensive correlated ground motions and fragility curves to assess risk in a region of interest, along with traffic simulation through extensive damage scenarios, to inform retrofit strategies.

Regarding the use of decomposition techniques for exact optimization methods, Starita and Scaparra [23] minimize the impact of worst case disruptions, using a bi-level mixed-integer model along with a Benders decomposition approach. Viswanath and Peeta [24] minimize total travel time over selected routes for earthquake response while maximizing the total population covered, using a geographical simplification and a branch-and-cut strategy for an integer programming model. While these decomposition approaches are similar to the one in this paper, they rely on worst case scenario analysis and geographical simplification approaches, which are limited compared to the full probabilistic risk assessment considered here.

Miller-Hooks et al. [8] deal with resilience in freight transportation networks, accounting for the impact of pre-disaster decisions on recovery related decisions with the objective of maximizing the flow throughout the network; their approach relies on stochastic programming but is not integrated with probabilistic risk assessment for realistic complex networks. Mete and Zabinsky [25] also use a two-stage stochastic program for a problem of warehouse location and transportation of aid materials, in which the random variable is the demand, rather than the condition of bridges as is the case of this paper. Furthermore, approaches to infrastructure management often seek to determine which network components should be activated to achieve a certain performance goal, whereas the proposed approach seeks to invest in modifying components' attributes (e.g., the fragility of a highway bridge) to improve performance, which is only possible due to the integration with sophisticated risk assessment methodologies.

In this paper, the decision support strategy is based on exact optimization, and stochastic behavior is accounted for through probabilistic seismic risk assessment, rather than median or worst case scenario analysis. It is worth noting that the use of stochastic programming to address preemptive and post-disaster decisions is complementary to existing approaches such as Bayesian pre-posterior analysis [26,27], or Markov Decision Processes [28], which in turn may rely on dynamic programming [29], or stochastic dynamic programming [30].

This paper builds upon previous research by Miller and Baker [31,5] regarding probabilistic risk assessment for complex infrastructure networks, including statistical models for seismic activity which allows the simulation of correlated ground motions in a region of interest [32], the structural response of infrastructure network components via fragility curves, a model for traffic simulation that captures users' demand and preferences, and the use of network analysis tools to integrate the previous phases into a system level socio-economic metric [33]. The major novelty in this paper is in aggregating all these analysis layers within an exact optimization framework for large scale networks.

3. Decision making framework for disaster preparedness and response

Improving infrastructure resilience implies making decisions at several stages (i.e., disaster preparedness, early response, restoration, long-term recovery), in which actions at a certain stage often depend on preceding actions, which may limit or potentiate future decisions. In infrastructure engineering, a typical decision is whether to invest in strengthening system components in order to avoid negative consequences in case of disaster (e.g., repair costs, socio-economic impact of system unavailability). We consider a decision problem that seeks to minimize the investments on preparedness actions along with the expected future consequences associated to such investment choices:

$$\min(\mathit{investments} + E[\mathit{consequence}]) \tag{1}$$

In this paper, investments and consequences are related to retrofit and repair actions, respectively, aimed at maintaining network performance, although they may also include business interruption, or human and economic loss, depending on specific problem requirements. The problem formulation incorporates constraints in which performance metrics of interest are computed as a function of the investments to be made, and are then compared to pre-specified target values to be satisfied.

$$\mathit{metric}_{(p,t)}(\mathit{investments}) \geqslant \mathit{target}_{(p,t)} \tag{2}$$

where p is an index for different performance metrics of interest, and t is an index for different stages in time. The performance index may denote metrics that are different in nature, and/or a single metric evaluated for different sub-systems of interest. The time index is valuable for resilience oriented decision support, for instance: in an immediate post-earthquake stage (e.g., within one day), a high-performance target could be imposed on system components granting access to hospitals, whereas for less critical components, targets may be lower for a few more stages. The latter constitutes a powerful decision support feature, since investments can be determined aiming to pursue a certain recovery curve. The inequality in Eq. (2) stands for generic performance metrics for which higher values are preferable; in this paper, we will consider travel time between origins and destinations as a performance metric, thus, using the opposite sense for the inequality (since smaller values are preferable).

The proposed problem structure is general and suits a variety of applications. Specifically, we will consider the decision whether to retrofit, repair, or do nothing for a specific component in a transportation network, depending on how performance metrics (travel time) respond to the applied sequence of actions for that and other components for a set of scenarios describing the hazard(s) of interest.

3.1. Risk assessment methodology

The proposed framework adopts work by Miller and Baker [31,5] for probabilistic risk assessment of complex infrastructure networks exposed to seismic hazards. The work in [5] includes the simulation of correlated ground motion intensities for the San Francisco Bay Area, and the creation of hazard-consistent scenarios, capturing the magnitudes and occurrence rates that can be expected for the network of interest. Let r_ξ represent the occurrence rate of scenario ξ, which constitutes a primary input for the decision support model. Each scenario from [5] is comprised of an occurrence rate (in terms of annual exceedance), associated to a specific earthquake magnitude, and a set of comprehensive ground motions (expressed in terms of spectral acceleration) throughout the geographical area of interest. The size of the set can be adjusted using the methodology in [34] (refined in [31]), in which a large, risk-consistent set of scenarios (calibrated to historic data) can be reduced to a smaller set through a supplemental optimization problem that re-calculates the occurrence rates for a subset of the scenarios in order to represent risk accurately with significantly less scenarios.

The subsequent step in [5] is to compute the damage state of network bridges as a function of their fragility and the ground motion intensities observed for each scenario ξ. Fragility curves are evaluated to determine whether each bridge is functional under each scenario ξ given the ground motion intensity observed at its specific location. As a result, one version of the network is obtained for each scenario ξ, in which certain network arcs connecting nodes are affected (i.e., unavailable) due to the damage on bridges. Let graph G_ξ represent the network under seismic scenario ξ, occurring with probability r_ξ, and with travel time $t_{a,\xi}$ for each arc a in the network. The performance of the network is then assessed by running a traffic simulation for each network realization, in which trips are assigned using an iterative process following the User Equilibrium method [35], providing arc travel times for all earthquake scenarios. We seek to find the minimum cost retrofit strategy that would preserve travel times between selected origin-destination (OD) pairs within pre-specified acceptable ranges for every damage realization.

3.2. Optimization problem

Consider the following sets: $\mathscr{N}$ of network nodes, $\mathscr{A}$ of network arcs, Ξ of damage scenarios, $\mathscr{B}$ of bridges, and $\mathscr{P}$ of paths connecting origin-destination pairs of interest. Let $\mathscr{B}_\mathbf{p}$ be a subset of arcs that belong to path $\mathbf{p} \in \mathscr{P}$ and are directly related to network bridge(s), while $\mathscr{A}_\mathbf{p}$ is the subset of remaining arcs in path $\mathbf{p} \in \mathscr{P}$ (i.e., those not directly related to bridges). While $\mathscr{N}$, $\mathscr{A}$, $\mathscr{B}$ and Ξ are given by the problem, $\mathscr{P}$ may be chosen in different ways, producing different $\mathscr{B}_\mathbf{p}$ and $\mathscr{A}_\mathbf{p}$; we discuss possibilities and implications of such choices later in the paper.

Let decision variable $x_a \in \{0, 1\}$ denote whether a retrofit action is applied on bridge $a \in \mathscr{B}$, while variable $y_{a,\xi} \in \{0, 1\}$ denotes whether bridge $a \in \mathscr{B}$ must be repaired in scenario $\xi \in \Xi$ in order to comply with target performance metrics for the observed damage realization. Finally, auxiliary variable $\tau_{\mathbf{p},\xi} \geqslant 0$ captures the travel time through bridges on path $\mathbf{p} \in \mathscr{P}$ of the network under scenario $\xi \in \Xi$ (only for aesthetic purposes in the manuscript; not an actual decision).

The parameters in the model are described below:

- $I_{b,a}$: whether bridge b is related to arc $a \in \mathscr{A}$
- $t_{a,0}$: travel time for arc $a \in \mathscr{A}$ under no network damage
- $t_{a,\xi}$: travel time for arc $a \in \mathscr{A}$ under damage scenario $\xi \in \Xi$
- $t_\mathbf{p}$: target travel time for path $\mathbf{p} \in \mathscr{P}$ (under no network damage)
- $\epsilon_{\mathbf{p},\xi}$: allowed travel time increase on path $\mathbf{p} \in \mathscr{P}$ under scenario $\xi \in \Xi$
- c_a: cost of a retrofit action on arc/bridge $a \in \mathscr{B}$
- $q_{a,\xi}$: cost of a repair action on arc/bridge $a \in \mathscr{B}$ under scenario $\xi \in \Xi$
- ω_ξ: indirect consequences under damage scenario $\xi \in \Xi$
- m: available resources
- γ_a: resources necessary to retrofit arc/bridge $a \in \mathscr{B}$

Parameters $t_{a,0}$, $t_{a,\xi}$, and $t_\mathbf{p}$ are computed directly from running the traffic simulation without any retrofit/repair actions applied, while $I_{b,a}$ is derived from the graph construction process (see Section 5). Parameters $\epsilon_{\mathbf{p},\xi}$, c_a, $q_{a,\xi}$, ω_ξ, m, and γ_a are conceived as user inputs; in Section 5 we provide the values used in our analysis. The mathematical formulation of the proposed optimization model is presented below:

$$\min \sum_{a\in\mathscr{B}} c_a x_a + \mathrm{E}_\xi\left[\sum_{a\in\mathscr{B}} q_{a,\xi} y_{a,\xi} + \omega_\xi\right] \tag{3}$$

s.t.:

$$\tau_{\mathbf{p},\xi} = \sum_{a \in \mathscr{B}_\mathbf{p}} [t_{a,0}(x_a + y_{a,\xi}) + t_{a,\xi}(1 - x_a - y_{a,\xi})],$$
$$\forall\ \mathbf{p} \in \mathscr{P}, \xi \in \Xi \quad (4)$$

$$\tau_{\mathbf{p},\xi} + \sum_{a \in \mathscr{A}_\mathbf{p}} t_{a,0} \leqslant (1 + \epsilon_{\mathbf{p},\xi})t_\mathbf{p}, \forall\ \mathbf{p} \in \mathscr{P}, \xi \in \Xi \quad (5)$$

$$\sum_{a \in \mathscr{B}} \gamma_a x_a \leqslant m \quad (6)$$

$$x_a \in \{0, 1\}\ \forall\ a \in \mathscr{B} \quad (7)$$

$$y_{a,\xi} \in \{0, 1\}\ \forall\ a \in \mathscr{B}, \xi \in \Xi \quad (8)$$

3.2.1. Objective function

The objective of the problem (Eq. 3) is to minimize a function of the costs and consequences derived from the strategy to be implemented: the first term accounts for the aggregate cost of implemented retrofit actions at a cost c_a when binary decision variable $x_a = 1$ indicates a retrofit action on arc a; the second term accounts for the expected value of the cost of implemented repair actions $y_{\xi,a} = 1$ throughout all scenarios ξ at a cost $q_{a,\xi}$; the third term refers to a consequence ω_ξ that captures any saving or expense that occurs at a scenario ξ as a result of having implemented actions x_a. In this paper, the consequence will be associated to repair expenses only (i.e., no ω_ξ). Ongoing research explores other consequences to capture in ω_ξ, such as economic loss due to system unavailability, the impact of retrofit actions on post-disaster logistics, or imposing a penalty if the number of repair actions exceeds available personnel or resources, which could be achieved via additional constraints that account for such quantities.

3.2.2. Performance enforcement constraint

The performance constraint is at the core of the proposed approach since it captures the impact of decisions. In contrast to [5], in which the aggregate travel time is used as a scalar metric, we use the travel time between selected origin–destination (OD) pairs as a performance metric. This provides flexibility in the sense that the analysis can focus on a single OD pair, a critical subset of them, or the complete set (with corresponding computational costs). Furthermore, different performance targets (travel time to be met) can be imposed for different OD pairs according to their priority, or other decision-maker criteria.

Eq. (5) implements the performance constraint for each selected OD pair for every damage scenario, guaranteeing that the aggregate travel time across a path connecting an OD pair does not exceed its target value by more than a tunable percentage ϵ. Auxiliary variable $\tau_{\mathbf{p},\xi}$ (Eq. 4) captures the sum of travel times through arcs that belong to path $\mathbf{p}$ *and* are associated to bridges (note that $\tau_{\mathbf{p},\xi}$ values are dependent on the damage scenario). The term that is added to $\tau_{\mathbf{p},\xi}$, in Eq. (5), aggregates travel times through arcs in path $\mathbf{p}$ that are *not* related to bridges (and do not depend on the damage scenario). The sum of arc travel times between an OD pair is forced to be within $\epsilon_{\mathbf{p},\xi}$% of the target value (travel time under no damage). If no retrofit or repair actions are taken on a bridge, the post-disaster travel time on its related arcs will be that of the corresponding damage scenario ($t_{a,\xi}$), whereas a pre- or post-disaster action sets those arcs to a no damage travel time (denoted by a dummy scenario with index 0 in $t_{a,0}$). Note that the proposed model addresses the general case in which arcs are not necessarily paired one-to-one with bridges; the case in which bridges are paired with arcs can be achieved by making $\mathscr{B} = \mathscr{A}$, and hence $A_P = \varnothing$.

The performance constraint operates under the assumption that retrofit/repair actions will only affect the travel times on arcs directly associated to bridges (i.e., it does not account for cross-effects of bridge condition on travel times of arcs beyond their immediate neighborhood). While this is often not the case in practice, accounting for the effect of any bridge on any arc is prohibitive for mathematical modeling since it responds to a traffic simulation with no closed-form expression for each of the many crossed effects (likely producing a non-convex space). This assumption, however, only holds during the optimization phase of the methodology: once retrofit decisions are obtained, they are simulated in the traffic model by replacing the fragility curves of retrofitted bridges (by stronger ones) and re-computing travel times with full cross-effects for all damage scenarios under the obtained retrofit policy. In this sense, the traffic dynamics is simplified into a set of realizations of the graph, which allow to evaluate decision alternatives within a mathematical optimization framework. While numerical results can be subject to error due to current assumptions, the methodology opens research possibilities based on mathematical optimization (e.g., iterative simulation-optimization approaches that incorporate relevant crossed-effects without their full enumeration).

The set of performance constraints for each scenario is comprised of as many equations as OD pairs are being studied. Alternatively, the analyst may use a global performance metric (e.g., aggregate travel time, median travel time, maximum travel time), in which case there would be less constraints but the calculation of the metric itself might be as demanding. Furthermore, the eventual inclusion of time indices for parameter $\epsilon_{\mathbf{p},\xi}$ (along with performance constraints replicated over time) would allow to impose gradually tighter performance targets for OD pairs through different stages of the recovery process, mimicking a resilience-oriented framework. However, such feature implies a considerable increase in the model complexity.

As described throughout this section, the contribution of this framework relative to previous work lies on being able to exploit the several layers of information and processing from solid probabilistic risk assessment methodologies [31,5], and integrate them into an exact optimization strategy that enables decision support for risk management of complex infrastructure networks. In this sense, contrasting the proposed framework with often faster heuristic approaches on the sole speed criterion is not the main purpose of this contribution.

3.2.3. Side constraints

Eq. (6) limits the number of possible retrofit actions as a function of available resources (e.g., budget). Additionally, an intuitive condition is to enforce that x_a and $y_{a,\xi}$ must not take value at the same time. This was not explicitly stated in the implementation, and is achieved via the objective function (i.e., avoiding to invest twice in a component to achieve the same effect). However, we have considered including such constraints as they may provide a tighter formulation that improves the convergence of the integer problem.

4. Stochastic optimization strategies

In this section, relevant aspects of stochastic programming are introduced, followed by a decomposition of the optimization problem from Section 3, which allows to deal with the large number of scenarios that typically arise in practical risk management problems.

4.1. Stochastic programming overview

A common practice in optimization under uncertainty is to solve deterministically for a specific circumstance; for instance, the expected value, the worst case, or a tailored set of scenarios. Alternatively, deterministic optimization (either exact or heuristic methods) can be integrated into a simulation loop (e.g., via Monte Carlo based approaches). The term *stochastic programming* refers to the inclusion of uncertainty into the parameters that define an optimization problem. Often, the optimization problem is solved by evaluating variables throughout discrete potential future scenarios. The main benefit of such an exact approach, as opposed to usually faster heuristic alternatives, is the capability to prove optimality (or quantify optimality gaps), while the major drawback is the limited size of the instances it can manage (due to either large networks or extensive scenarios).

Since the size of the problem may grow quickly with the number of scenarios, decomposition approaches are used to separate the original problem into a master problem and a set of independent problems per scenario which depend only on scenario-specific variables. Specifically, the Benders and L-shaped decompositions [3,4,2,36,37] exploit a special problem structure that allows to treat independent scenarios as separate (yet coupled) optimization problems. This family of techniques remains to be used and refined in current day for stochastic programming, allowing to solve problems with uncertainty described by numerous scenarios.

4.2. Decomposition strategy for the proposed problem

A decomposition approach is implemented for the proposed decision support framework in order to take scenarios into account within the exact optimization approach. The problem from Section 3 is then divided into one master problem and a set of sub-problems, one per scenario.

4.2.1. Sub-problems per scenario

Each term of the expected value in the objective function (Eq. 3) becomes the single objective function for the sub-problem associated to each scenario ξ:

$$\phi_\xi = \min \sum_{a \in \mathscr{A}} q_a y_{\xi,a} + \omega_\xi \tag{9}$$

where ϕ_ξ captures the value of the objective function observed for scenario ξ after solving for a fixed value (x_a^*) of retrofit variables given by the current (temporary) solution of the master problem. The objective function minimizes the cost of necessary repair actions by solving the sub-problem with x_a^* as a constant input in the performance constraint (Eqs. 4 and 5), specifically for scenario $\xi \in \Xi$. Each sub-problem is fully described by Eqs. (4), (5), (8) and (9) for the corresponding scenario.

4.2.2. Master problem

The objective function for the master problem (which considers preemptive actions) is:

$$\min\left(\sum_{a \in \mathscr{A}} c_a x_a + \sum_{\xi \in \Xi} r_\xi \theta_\xi\right) \tag{10}$$

where each θ_ξ is an artificial variable introduced in the master problem in order to iteratively estimate the value of objective function of the sub-problem associated to scenario ξ by means of constraints referred to as *optimality cuts* (Eq. 11), which relate the values of master problem variables with the outcome obtained at secondary problems (occurring separately). These cuts have the following form:

$$\theta_\xi \geqslant \phi_\xi - (\phi_\xi - L)\left[\sum_{a \notin \mathscr{O}} (1 - x_a) + \sum_{a \in \mathscr{O}} x_a\right], \forall\, \xi \in \Xi \tag{11}$$

where L represents a lower bound for the objective function; and $\mathscr{O}$ is the set of first-stage decision variables that were not active in the iteration (i.e., those $x_a^* = 0$). These are known as combinatorial Benders cuts [38], and are often read as: the sub-problem solution will be ϕ_ξ unless some x_a are modified. For instance, a master solution with no retrofit actions is likely to produce large consequences (i.e., many repair actions). Then, a constraint will be added to motivate some master problem variables to be turned on (i.e., apply retrofit actions) in the next iteration (if preferable to accepting ϕ_ξ). Note that, for each iteration, up to $|\Xi|$ cuts (constraints) are added to the master problem (one per scenario). An alternative approach is to average sub-problem solutions into a single value, Φ, and aggregate their value into a single cut, using only one auxiliary variable θ (instead of one per scenario), as explored in [8], where an L-Shaped approach is followed using the following cut (instead of that in Eq. (11)):

$$\theta - (\Phi - L)\left[\sum_{a \in \mathscr{O}} x_a + \sum_{a \notin \mathscr{O}} x_a\right] \geqslant (\Phi - L)(|\mathscr{O}| - 1) + L \tag{12}$$

The lower bound L is computed based on the first stage objective (deterministic costs plus current value of variables θ_ξ estimating second stage problems). An upper bound (necessary to monitor convergence) is computed based on deterministic costs ($\sum_{a \in \mathscr{B}} c_a x_a^*$), plus actual weighted objectives of all sub-problems. It is important to note that cuts are only added when estimator variables θ_ξ (or θ for the L-Shaped case) under-estimate the objective functions for their corresponding sub-problems. In the Benders case, second stage objectives are compared to their corresponding estimator variables prior to introducing cuts. In the L-Shaped method, the unique estimator variable for second stage objectives is compared to the aggregated second stage objective.

As observed, the L-Shaped approach often results in fewer cuts (at most one per iteration) and, thus, slower growth of the master problem. However, it does not always dominate Benders cuts, which tend to show narrower gaps between lower and upper bounds through iterations, thus, providing better confidence if accepting solutions prior to optimality. In addition to optimality cuts, so-called *feasibility cuts* are often used in stochastic programming to correct infeasibility in second stage problems; these are not considered in this paper since the proposed sub-problems are never infeasible. The master problem is fully characterized by Eqs. (6), (8) and (10), plus the iteratively added optimality cuts (either Eq. (11) or (12)). These ideas can be extended to other pre- and post-disaster decision problems, such as adding redundancy, updating flow capacity, or pre-positioning resources for disaster response.

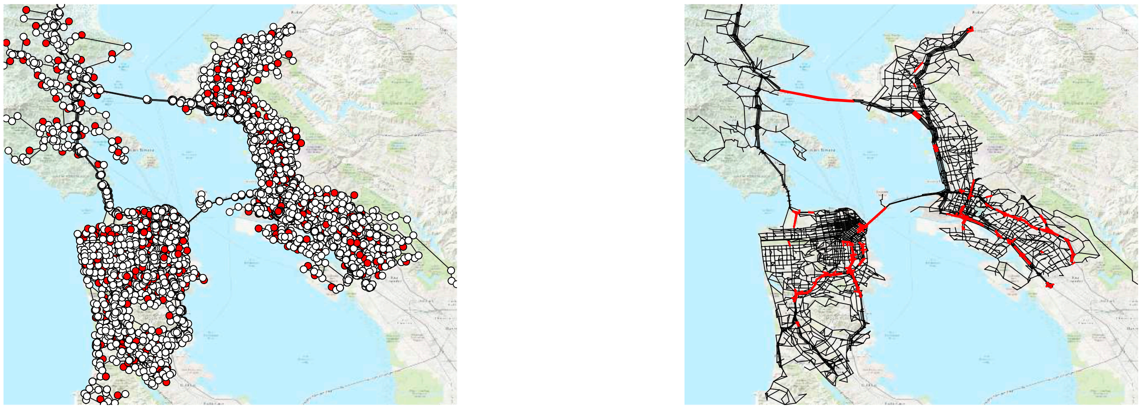
4.3. Considerations for implementation

The size of the optimization problem might lead to high computational burdens for the case of realistic infrastructure networks. In this sense, several strategies are incorporated in the implementation of the optimization model in order to overcome computational challenges. The first step in handling complexity is the decomposition of the proposed stochastic programming approach, which allows to solve sub-problems independently, thus, enabling a parallel treatment of disaster scenarios. In this sense, the remaining source of complexity is the number of variables and constraints in the master problem and each of the sub-problems, i.e., the number of components for which actions can be applied and the number of performance metrics that are being accounted for (in this case, the number of origin-destination pairs). Clustering techniques have been proposed [39–42] which address such problems of scale.

Further strategies to deal with slow convergence often rely on aiding the exact optimization solver by providing one or more initial feasible solutions, as well as a lower bound L for the problem optimum, usually obtained by means of heuristic algorithms. The most common approaches involve greedy algorithms, genetic and other evolutionary algorithms, as well as local search procedures (the reader is referred to [43,44] for exhaustive discussion on heuristic algorithms). It is worth highlighting that a strong lower bound L is critical for fast convergence of the stochastic programming strategy, since L influences optimality cuts (i.e., tight bounds make the most out of each iteration). Pre-processing computations towards this end are a wise effort, since weak bounds may even compromise the computational gains pursued through the decomposition approach.

5. Illustrative example

This section presents an example that considers several sub-networks of the San Francisco Bay Area transportation network in order to demonstrate the proposed methodology, test how it performs for problems of differing sizes, and identify possible limitations and



(a) (b)

Fig. 1. San Francisco/Oakland network: (a) nodes in origin-destination pairs (in red); and (b) bridges considered in the optimization problem (in red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

improvements. Section 5.1 presents an analysis of the San Francisco/Oakland portion of the network to demonstrate the impact of the retrofit/repair policies obtained after applying the proposed framework. Section 5.2 provides an analysis of computational complexity as a function of problem size (using sub-networks of different sizes).

5.1. Retrofit strategy for the San Francisco/Oakland network

The San Francisco/Oakland portion of the network is a key urban concentration, considering 65 bridges that are critical in connecting 5184 OD pairs in a network consisting of 3130 nodes and 5257 arcs. The set of OD pairs is chosen under two criteria: first, graph theoretical indices are used to obtain a sample of nodes that are highly central, as well as nodes that are highly peripheral, to have a broad representation of types of trips; and second, a sample of nodes in the vicinity of bridges in order to induce paths that incorporate bridges so that the effect of their failure is captured. The set of damage scenarios to analyze results from the optimization procedure described in sub-section 3.1, producing sets of 100, 150, and 250 scenarios out of an original set of nearly 2000; the set of 100 scenarios is used for the presented results. Fig. 1 provides a representation of the network including nodes associated to OD pairs, as well as bridges considered in the problem.

The optimization process is carried out as follows: the objective is to minimize the cost of retrofit actions and the expected cost of the repair actions throughout scenarios. From a decision making perspective, the challenge is in the valuation of investments to prevent damage that may not often occur. The proposed methodology provides a framework to support such decision processes. Fig. 2 shows the set of retrofitted bridges resulting from the optimization procedure.

The San Francisco network is urban and densely connected, hence, reasonably short detours are often available. However, significant travel time increases are observed in the traffic simulation when no retrofit actions are implemented, whereas notably smaller increases are observed for most arcs when using the proposed optimization strategy, except for arcs associated to the main bridges connecting separate parts of the network (which tend to have a bottleneck characteristic). It is worth noting that the target travel times pursued by the optimization problem trigger interventions in the network, but these are not always achieved when running the traffic simulation model for the retrofitted network, precisely because the optimization ignores cross-effects in the network, while the simulation does not (as discussed in Section 3).

For each scenario ξ (with occurrence rate r_ξ), the overall travel time can be computed as the sum of all arc travel times throughout all OD-pairs. By sorting the obtained per-scenario travel times, it is possible to

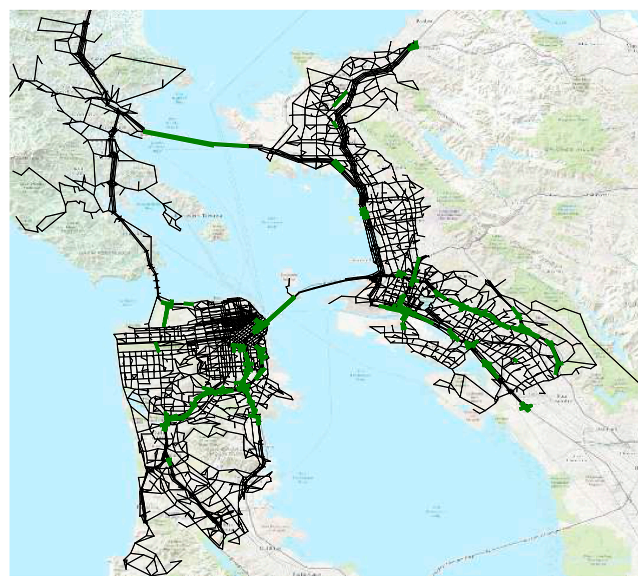


Fig. 2. Set of retrofitted bridges (in green) for the San Francisco/Oakland sub-network. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

determine the chance of exceeding specific travel times at the network level, given the risk consistent hazard assessment from [31]. Fig. 3 shows the annual exceedance rates for aggregate travel time in the network resulting from the described simulation procedure, showing that, as expected, very large increases in travel time (x-axis) are only observed for highly damaging events with low occurrence rates (y-axis). For reference, Fig. 3 shows the results obtained for the case in which all bridges are retrofitted (green), and the case with no retrofit actions (red). These results provide optimistic and pessimistic bounds, respectively, for what can be expected of risk management strategies.

Two policies based on the proposed optimization approach are shown in Fig. 3: one that limits the number of retrofit actions to 30% of the bridges in the network (blue); and a second that limits retrofit actions to 50% (purple). For comparison purposes, results are also shown for the average performance of greedy randomized retrofit strategies for the cases in which retrofits are limited to 30% of the bridges (cyan) and 50% of them (yellow). The optimization-based retrofit policies, in general, show better results than their heuristic counterparts. However, these trends seem not to hold for certain points related to large

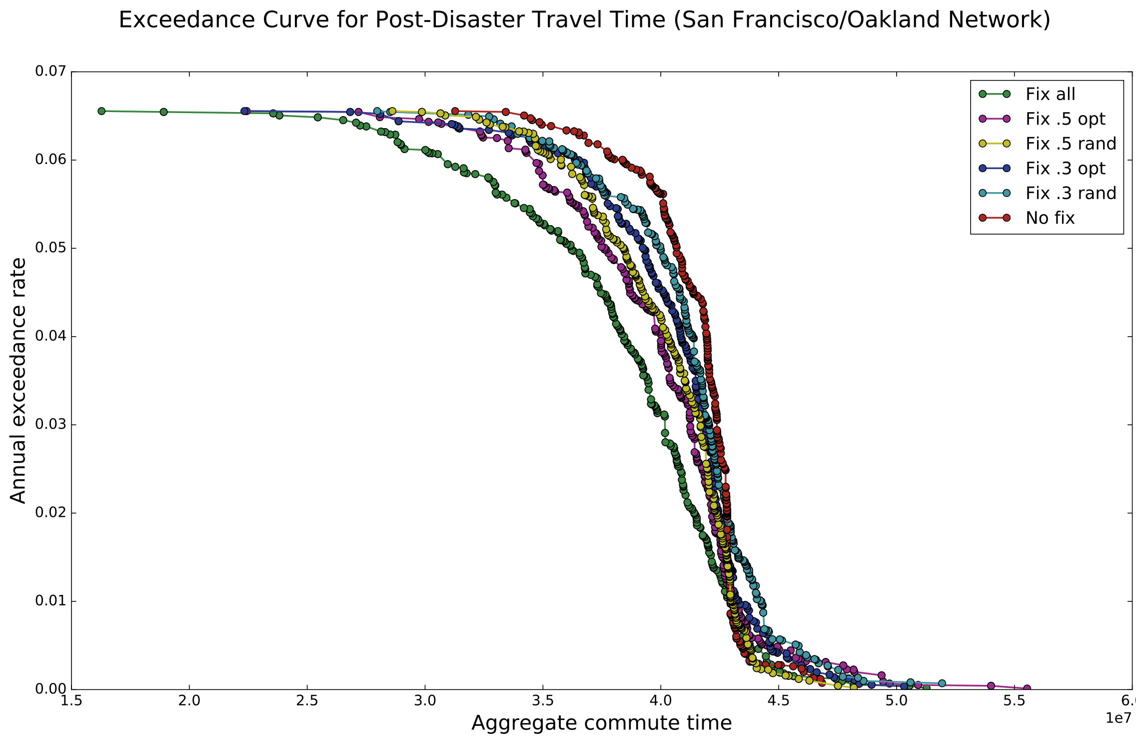


Fig. 3. Exceedance curves for aggregate users' travel time for the network: without intervention (red), with full intervention (green), with optimal intervention of 30% of bridges (blue), with optimal intervention of 50% of bridges (purple), with greedy randomized intervention of 30% of bridges (cyan), and with greedy randomized intervention of 50% of bridges (yellow). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

aggregate commute times (right-hand side of the figure), possibly due to the effect of extensive damage scenarios on trips assignment within the traffic simulation.

5.2. Complexity escalation with network size

In the San Francisco/Oakland network, retrofit and repair decisions can be made for 65 bridges, and performance constraints are enforced for a set of 5184 OD pairs and 100 damage scenarios. For the full optimization problem with no decomposition, this leads to 6565 variables (65 retrofit decisions and 6500 repair decisions) and 518400 constraints. The stochastic programming version, on the other hand, consists of one master problem and 100 auxiliary problems (one per scenario). The master problem includes 165 variables (65 related to retrofit actions and 100 additional variables that capture the objective function of the auxiliary problems). The number of variables for each sub-problem matches the number of bridges (i.e., 65) that could possibly be repaired under each scenario. The number of constraints for the master problem and all sub-problems is 5184, since the performance constraint must be met for all OD-pairs throughout scenarios.

The computational advantage results from solving 101 small problems (whose computation time can be added linearly) rather than solving a unique aggregated problem, for which the burden grows rapidly due to the combinatorial nature of the integer optimization problem. However, it is important to highlight that the 100 problems in the stochastic programming approach require a large number of iterations to include constraints (optimality cuts) that allow the optimizer to converge to a global optimum. In addition, each sub-problem implies a new optimization model, imposing an overhead cost for the decomposition approach (i.e., loading many models into memory, as well as running, updating, and coupling them).

Eight problems of increasing sizes were built in order to test complexity escalation of the proposed framework. The increase in size was achieved by adding larger sets of bridges and OD pairs. The models were run on a personal computer with an *Intel Core-i5* CPU @ 2.5 GHz and 8 GB of RAM. The code was implemented in Python 2.7 and run using Gurobi as a solver for both master and sub-problems.

An incidental insight obtained from solving for several sizes of OD sets is that the optimization solution is not as good when considering small OD sets. This occurs because the retrofit policy focuses on few bridges, thus, when the simulation model is run, desired travel times become affected by poor performance in the rest of the network. The "post-simulation performance" of the optimization strategies improves notoriously when considering larger sets of OD pairs.

Fig. 4 shows the evolution of the computation time as a function of problem size (given by the product of variables and constraints). The first points for the stochastic programming curve (green line with squares) evidence a fixed cost associated to the decomposition procedures (i.e., the overhead cost of having many models), which does not make it compelling for small problems in which the full MIP can still do well. However, when the exponential growth of the MIP becomes notorious, the decomposition becomes more competitive and starts compensating for the overhead costs. The combinatorial versions of the Benders and L-Shaped decompositions are known to show slow convergence, as their cuts are considerably poor relative to the non-integer versions. Furthermore, it should be noted that Gurobi's state-of-the-art features induce a competitive advantage for the MIP with respect to our implementation of the decomposition approaches, namely: while the MIP uses Gurobi's solver (implemented in C as a final commercial product), the decomposition approaches use our academic implementations of Python callbacks, which most certainly slow down the overall process. Only for demonstrative purposes, this effect can be mitigated by disabling some of Gurobi's automatic features (e.g., heuristics) to have more comparable performance; evidently, this is not desirable in practice.

6. Conclusions

An optimization framework was introduced in the context of infrastructure risk management with the objective of addressing coupled

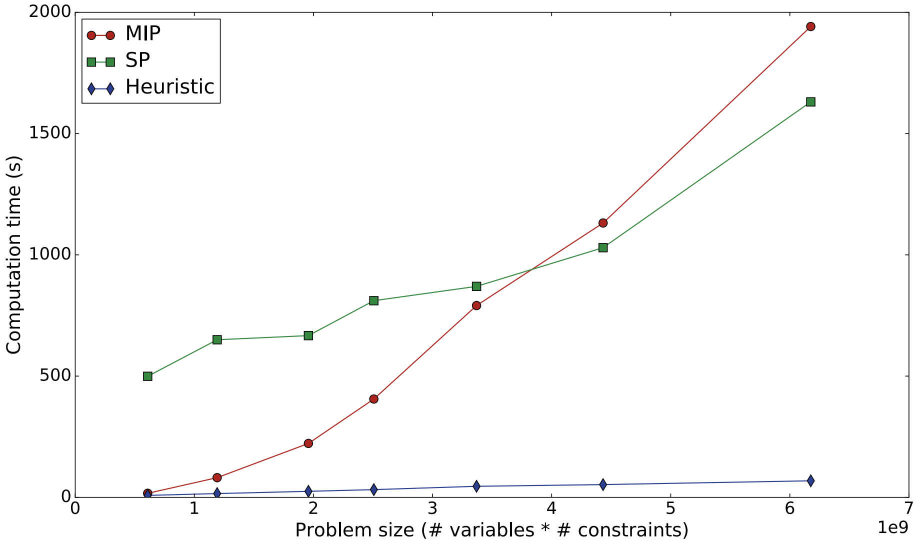


Fig. 4. Evolution of computational complexity with problem size for the Mixed Integer Program (MIP) with no decomposition (red); the Stochastic Programming (SP) with the proposed decomposition (green); and a greedy heuristic (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

pre- and post-disaster decisions in transportation networks. In particular, we propose a methodology that integrates sophisticated risk assessment methodologies and optimization techniques in order to inform preemptive risk management decisions by considering the potential consequences observed throughout extensive disaster scenarios.

The proposed methodology relies on ground motion simulation, structural evaluation of physical network components (i.e., bridges), and traffic simulation, to obtain a set of scenarios, each consisting of an occurrence rate and a realization of network travel times for a specific disaster. An optimization model is formulated, which minimizes the cost of retrofit actions on bridges and the expected cost of post-disaster consequences, subject to ensuring acceptable travel time increases in the network throughout disaster scenarios. Finally, Benders and L-Shaped decomposition techniques are applied to the optimization problem to address the complexity resulting from extensive scenario evaluation.

The main contribution of this paper lies in the integration of probabilistic seismic risk assessment, network performance evaluation, and advanced optimization methods, which provide a powerful decision support framework for risk assessment and management for critical infrastructure systems. The methodology is sufficiently general to fit applications involving different types of networks, hazards, or performance metrics, and can be extended to capture multiple stages. The developed framework can be used for risk-informed prioritization of investments, and can be run iteratively to test sensitivity of parameters, incorporate risk preferences, or evaluate scenarios not related to natural hazards (e.g., population growth).

In computational terms, although decomposition techniques offer a compelling possibility to incorporate extensive disaster scenarios, our current academic implementation (in Python) is not convincingly superior to the state-of-the-art features of commercial software. In addition to immediate improving steps such as migrating to a C implementation, or exploiting parallelization, ongoing research is devoted to non-integer versions of the problem and regularization techniques for the decompositions (as well as the use of stronger cuts). However, in spite of current limitations, the stochastic programming strategy allows to deal with a large number of scenarios within an exact optimization framework.

Future work is directed towards relaxing some of the assumptions in the optimization model. Currently, performance is monitored through pre-specified sets of origin-destination paths; this may be improved by automatic re-routing within the optimization model. Similarly, the crossed-effect of bridges on arc travel times is not accounted for; this can be improved by allowing bi-directional communication between the optimization and the traffic simulation.

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