MRI Signal Calculations (hopefully) Made Easy

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Purpose

- Overview of Bloch/Matrix simulations
  - Exact simulation of many cases
  - Numerical simulations often reasonable
- In-depth explanation of EPG method
  - Extended phase graph
  - Simulates "dephased states"
  - Common in many MRI sequences

Matrix Simulations

- Single Spin
- RF rotation
- Gradient- or off-resonance-induced rotation
- Relaxation

Jaynes 1955 "The Matrix Treatment of Nuclear Induction"

Rotations

RF (x) Rotation:

$$\mathbf{M}' = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{M}$$

- RF can rotate about any transverse axis
- Rotations due to precession are just about z

Any rotation is just a matrix multiplication

Magnetization Propagation

Relaxation: *(Jaynes – 1955)*

$$\mathbf{M}' = \begin{bmatrix} e^{-\tau/T_2} & 0 & 0 \\ 0 & e^{-\tau/T_2} & 0 \\ 0 & 0 & e^{-\tau/T_1} \end{bmatrix} \mathbf{M} + m_0 \begin{bmatrix} 0 \\ 0 \\ 1 - e^{-\tau/T_1} \end{bmatrix}$$

Can represent any propagation in the form $\boxed{\mathbf{M}' = \mathbf{A}\mathbf{M} + \mathbf{B}}$

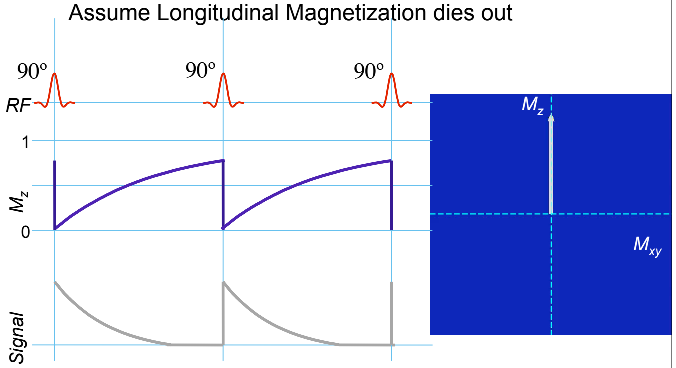
Magnetization Expressions

- Matrix expression for magnetization propagation: $\mathbf{M}_{k+1} = \mathbf{A}\mathbf{M}_k + \mathbf{B}$
- Steady state solution: $\mathbf{M}_{SS} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$

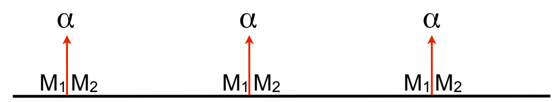
If we let $\Delta\mathbf{M}_k = \mathbf{M}_k - \mathbf{M}_{SS}$

then $\boxed{\Delta\mathbf{M}_{k+1} = \mathbf{A} \times \Delta\mathbf{M}_k}$

Example: T_1 Weighting



Example: T1-Weighted Imaging



$$M_2 = \begin{bmatrix} cos\alpha & 0 & sin\alpha 0 \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix} M_1$$

$$M_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_1 \end{bmatrix} M_2 + \begin{bmatrix} 0 \\ 0 \\ m_0(1-E_1) \end{bmatrix}$$

$$M_1 = E_1 cos\alpha\, M_1 + m_0(1-E_1)$$

$$M_1 = \frac{m_0(1-E_1)}{1-E_1 cos\alpha}$$

Comments

- 3x3 Matrix reduced to 1D
- Other examples reduce to 2D
- Sometimes analytic solution is reasonable

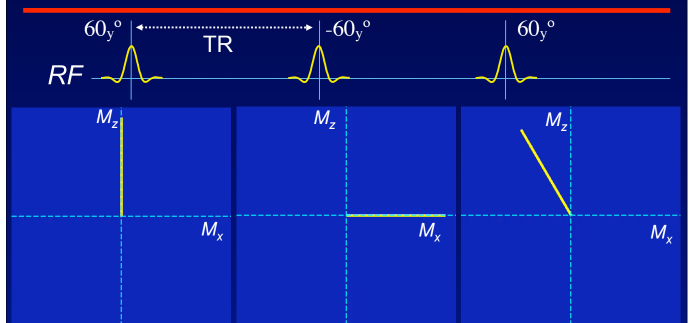
Zur Y, Stokar S, Bendel P. An analysis of fast imaging sequences with steady-state transverse magnetization refocusing. Magn Reson Med 1988; 6:175–193.

Sekihara K. Steady-state magnetizations in rapid NMR imaging using small flip angles and short repetition intervals. IEEE Trans Med Imaging 1987; 6:157–164.

van der Meulen P, Groen JP, Tinus AMC, Bruntink G. Fast field echo imaging: An overview and contrast calculations. Magn Reson Imaging 1988; 6:355–368.

Buxton RB, Fisel CR, Chien D, Brady TJ. Signal intensity in fast NMR imaging with short repetition times. J Magn Reson 1989; 83:576–585.

Example: Balanced SSFP (on-resonance)

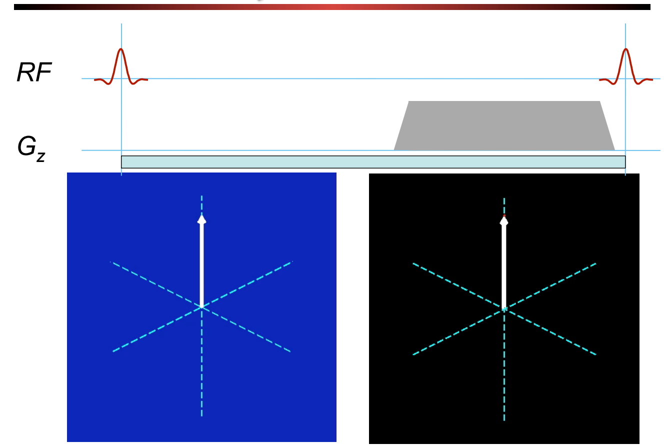


- Same periodic steady state
- Transient paths differ based on initial state

Matrix Calcs: Other Sequences

- Inversion Recovery
- Driven Equilibrium
- Fat-Saturation
- Transient magnetization preparation
- Spin-echo trains

Gradient Spoilers or Crushers

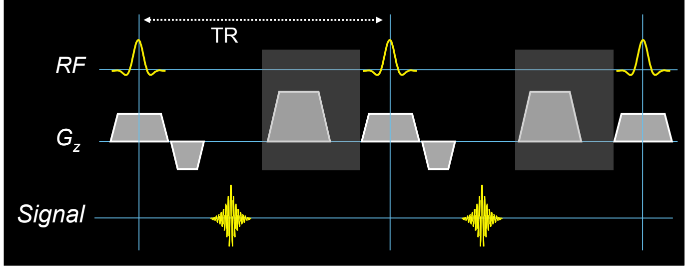


Gradient-Spoiling?

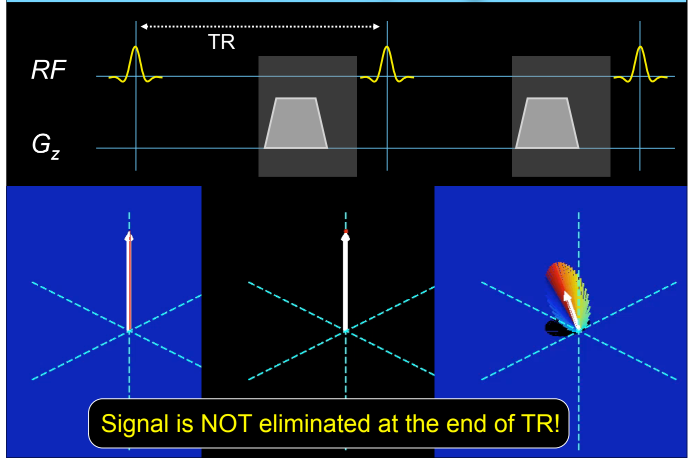
Does a spoiler gradient completely eliminate transverse signal at the end of each TR?

1) Yes

2) No



Gradient Spoiling



Matrix Simulation: Gradient Spoiling

- Signal is net-sum of different dephased spins
- Similar to crushers in spin echo
- Diffusion is an extension, but more spins!

- Quickly these problems become intractable

Extended Phase Graphs

- Treat a group of spins under constant gradients
- Decompose spin system into "dephased states"
  - Transverse states F_k and F_{-k}
  - Longitudinal states Z_k (k>=0)

Hennig J. Multiecho imaging sequences with low refocusing flip angles. J Magn Reson 1988; 78:397–407.

Weigel M, Schwenk S, Kiselev V, Scheffler K, Hennig J. Extended phase graphs with anisotropic diffusion. J Magn Reson 2010; 205:276–285.

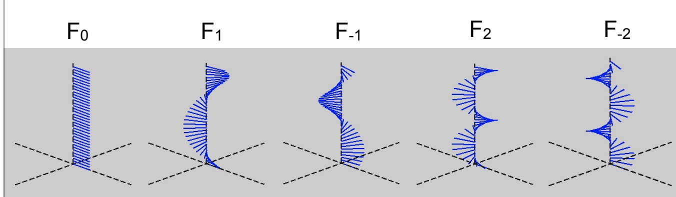
Weigel M - ISMRM Educational Sessions 2010 and 2011

Miller KL - ISMRM Educational Sessions 2010 and 2011

Fourier Interpretation

- $F^+ = DFT\{ M_x + i*M_y \}$
- $F^- = DFT\{ M_x - i*M_y \}$
- $Z = DFT\{ M_z \}$

There are some minor subtleties to be exact

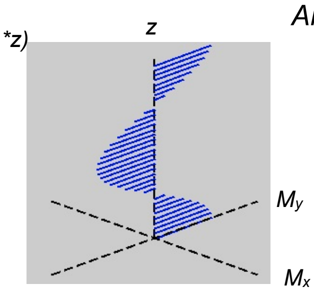


Z states are just sinusoids of M_z magnetization

Review Question

- What are the F states this magnetization represents?

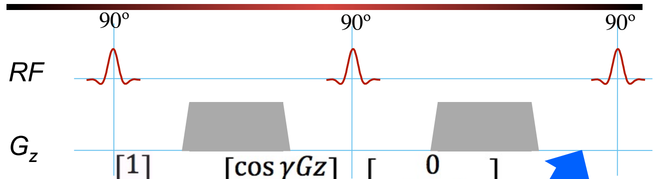
$M_x(z) = 0$
$M_y(z) = \cos(2pi *z)$



Answers:

A) $F_1 + F_{-1}$
B) $F_1 + iF_{-1}$
C) $i(F_1 + F_{-1})$

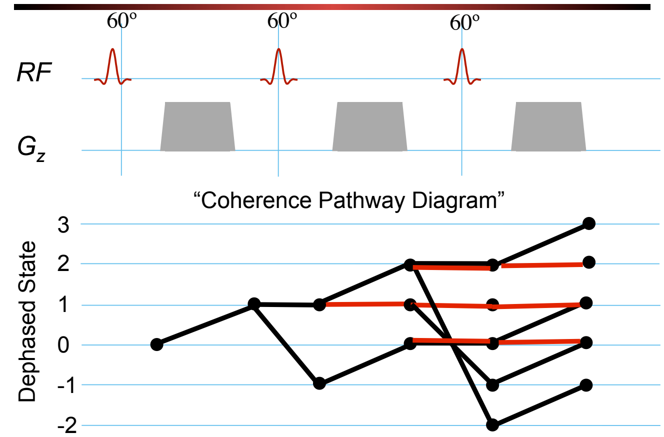
Phase Graph Example



$$\begin{bmatrix}1\\0\\0\end{bmatrix} \quad \begin{bmatrix}\cos\gamma Gz\\ \sin\gamma Gz\\ 0\end{bmatrix} \quad \begin{bmatrix}0\\ \sin\gamma Gz\\ -\cos\gamma Gz\end{bmatrix}$$

$$\begin{bmatrix}\sin\gamma Gz\ \sin\gamma Gz\\ \sin\gamma Gz\cos\gamma Gz\\ -\cos\gamma Gz\end{bmatrix} = \begin{bmatrix}0.5(1-\cos 2\gamma Gz)\\ 0.5(\sin\gamma Gz)\\ -\cos\gamma Gz\end{bmatrix} \quad S = \begin{bmatrix}0.5\\0\\-\end{bmatrix}$$

Phase Graph Example



Phase Graph Basic Definitions

- Different "dephasing" states
- Each has transverse and longitudinal M
- RF pulse (generally) for state k:
  - Produces signal in longitudinal state k and transverse states k and -k.

    (180 pulses may be an exception)
- Gradient dephaser for state k:
  - Moves transverse magnetization to k+1
  - Does not affect longitudinal magnetization

Phase Graph "States"

- F+ and F- states for transverse magnetization:
  - Examples:
    - F_2 state is 2nd order, positive
    - F_{-1} state is 1st order, negative
    - $F_0^- = \mathrm{conj}(F_0^+)$
- Z states for longitudinal magnetization:
    - Z_3 is 3rd order state
- Can represent as a matrix:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & \\ F_0^* & F_{-1} & F_{-2} & \dots \\ Z_0 & Z_1 & Z_2 & \end{bmatrix}$$

Phase Transitions: RF

- RF Pulses can invert state (e.g. F_3 to F_{-3}) or can transfer between F and Z states.
- Simple pre-multiplication P' = RP
- Example 90_y pulse:

$$R = \begin{bmatrix} 0.5 & -0.5 & 1 \\ -0.5 & 0.5 & 1 \\ -0.5 & -0.5 & 0 \end{bmatrix}$$

Phase Transitions: Gradient

- Increase number of states by 1
- Replace all F_k states with F_{k-1}
- (Replace F_0 in first row using F_0^* from second)
- Do not change Z states

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & & 0 \\ F_0^* & F_{-1} & F_{-2} & \dots & 0 \\ Z_0 & Z_1 & Z_2 & & 0 \end{bmatrix}$$

Phase Transitions: Relaxation

- Transverse:
  - All states attenuated by E2 = exp(-T/T2)
- Longitudinal:
  - All states attenuated by E1 = exp(-T/T1)
  - Z_0 state ***only*** has recovery of m_0(1-E1)

Phase Transitions: Diffusion

- F and Z states "increase" diffusion attenuation exponentially with state number and time
- F state transition is adjusted to model diffusion during a gradient

- Basic story: You can model diffusion!
- Full story:

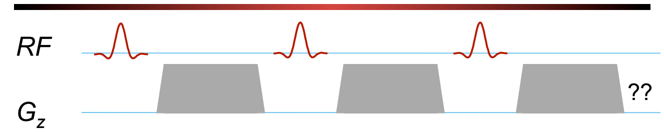
Weigel M, Schwenk S, Kiselev V, Scheffler K, Hennig J. Extended phase graphs with anisotropic diffusion. J Magn Reson 2010; 205:276–285.

Matlab Examples

- Transition functions:
  - epg_RF.m Applies RF to P matrix
  - epg_grad.m Applies gradient to P matrix
  - epg_grelax.m Gradient, relaxation and diffusion
- Helper functions:
  - epg_trim.m Reduce states w/ threshold
  - epg_plot.m Plot states

  - epg_spins2FZ Convert M vectors to F,Z states
  - epg_FZ2spins Convert F,Z states to M vectors

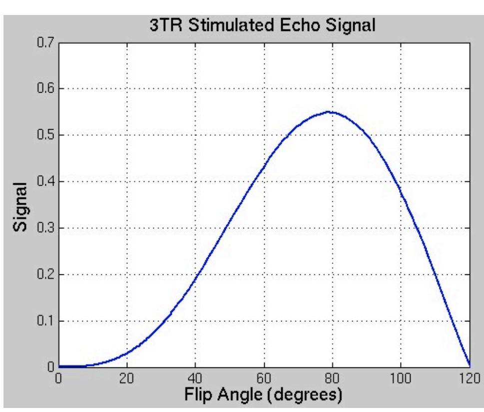
Stimulated Echoes: Code



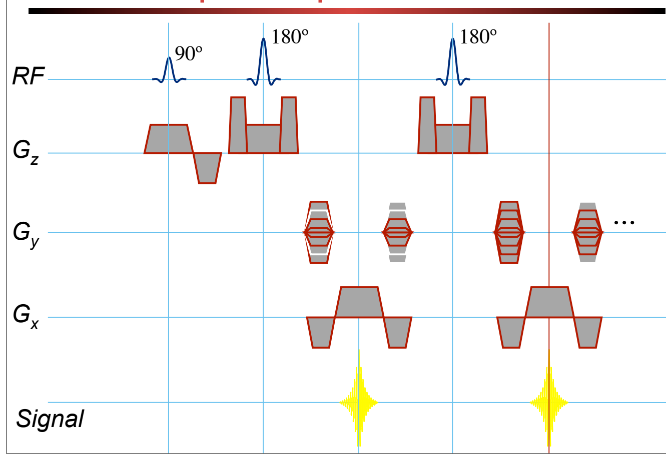

```
function [S,P] = epg_stim(flips)

P = [0 0 1]';                                      % Z0=1 (Equilibrium)
for k=1:length(flips)
  P = epg_rf(P,flips(k)*pi/180,pi/2);             % RF pulse
  P = epg_grelax(P,1,.2,0,1,0,1);                 % Gradient
end;
S = P(1,1);                                        % Signal from F0
```

fplot(epg_stim([x x x],[0,120])



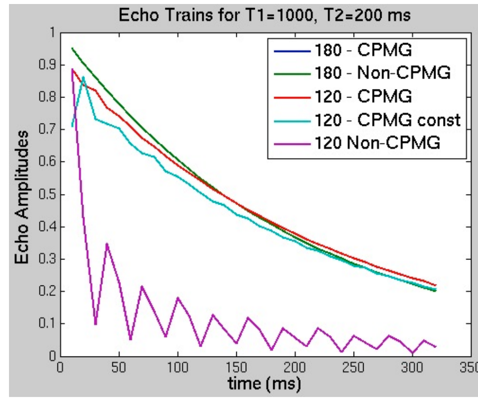
Example: Spin Echo Trains



Spin Echo Trains

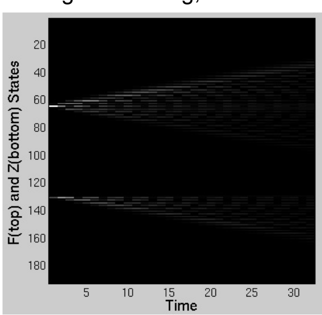
- 90_x 180_y 180_y 180_y 180_y .... (CPMG)
- 90_x 180_x 180_x 180_x 180_x .... (Non CPMG)
- 90_x 150_y 120_y 120_y 120_y .... (Prep CPMG)
- 90_x 120_y 120_y 120_y 120_y .... (Const CPMG)
- 90_x 120_x 120_x 120_x 120_x .... (Non CPMG)

Spin Echo Train Results 1

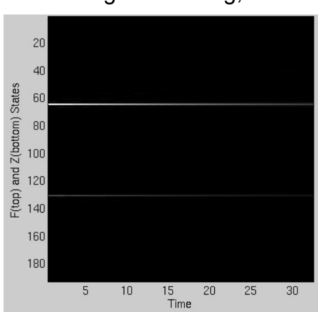


Spin Echo Train: F,Z States

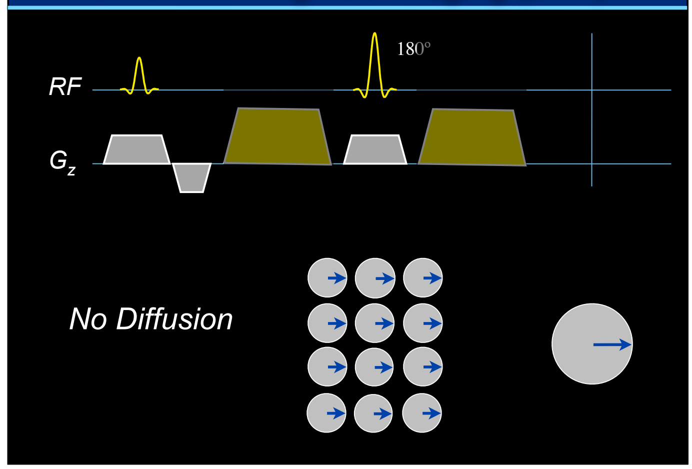
120-deg Refocusing, Non CPMG



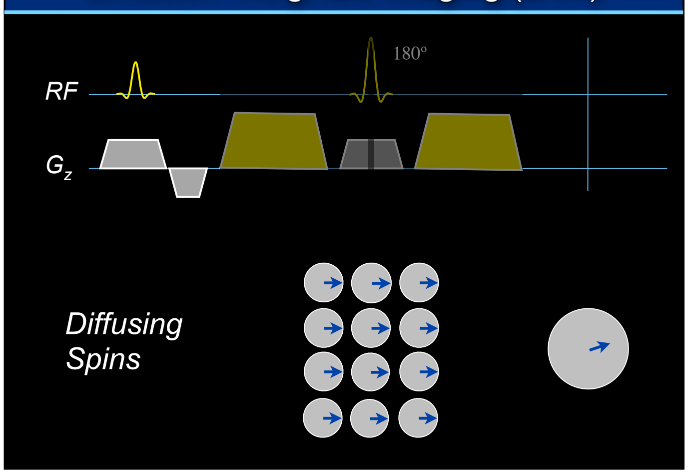
120-deg Refocusing, CPMG



Diffusion-Weighted Imaging (DWI)

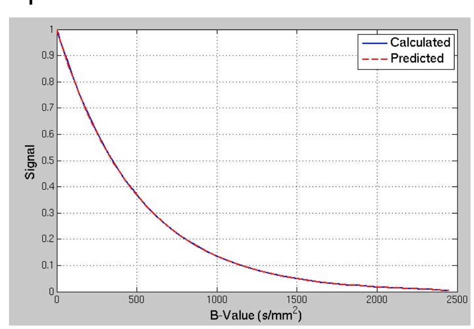


Diffusion-Weighted Imaging (DWI)



Diffusion-Weighted Imaging

- Simple Spin Echo Sequence
- Compare with theoretical diffusion attenuation

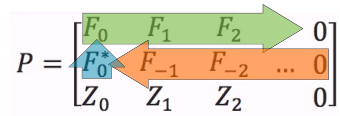


Steady-State EPG -- I

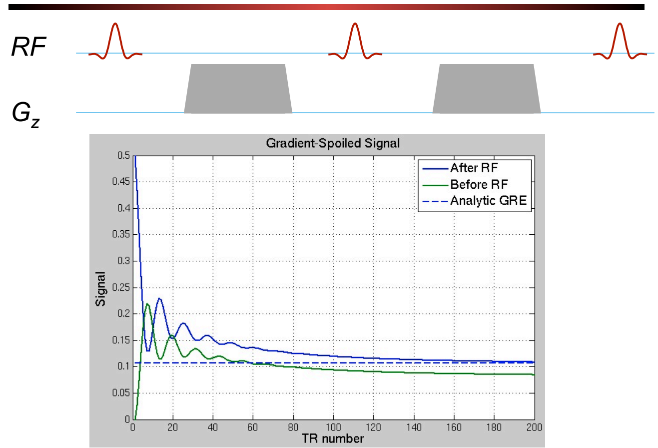
- Can calculate state propagation repeatedly
- Difficult, often unnecessary to do analytically
- Fast when number of states can be limited

Steady-State EPG -- II

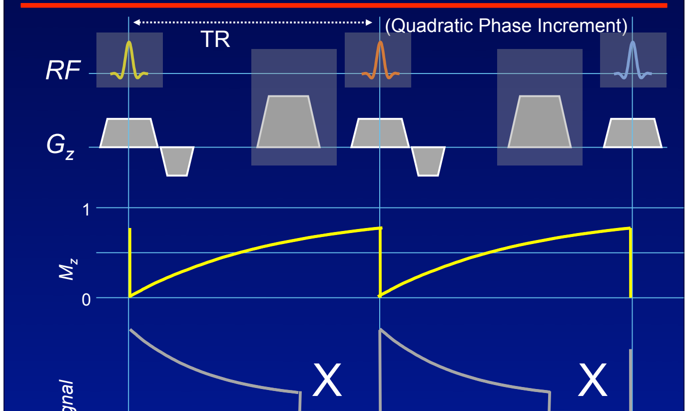
- Recall steady states: M' = AM + B
- Is there an EPG form?
  - Assuming a finite number (N) of states, yes!
  - Write each state as real + imaginary
  - Expand to vector of length 6N
  - RF rotations become block-diagonal matrices
  - Gradient transformation is mostly off-diagonal and diagonal 1's, except for F_0^* to F_0 state (conjugate)



Gradient-Spoiled EPG Simulation

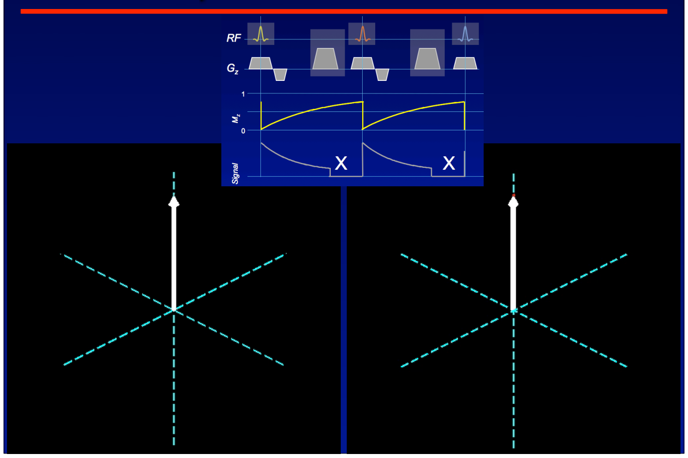


RF-Spoiled Gradient Echo

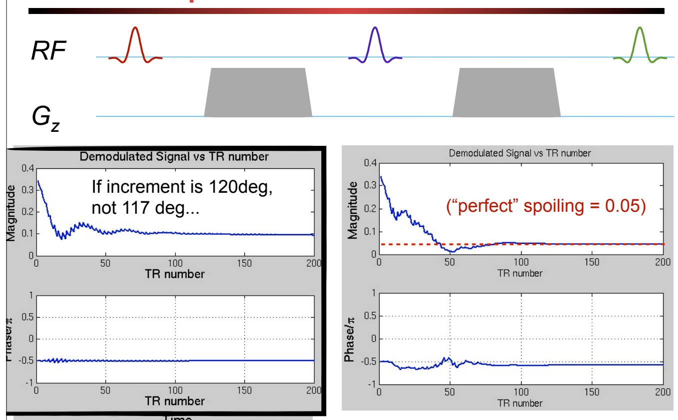


Eliminate transverse magnitization: T1 contrast

RF-Spoiled Gradient Echo



RF-Spoiled EPG Simulation



Other Comments

- Phase graphs work when gradient dephasing is in quantized units
- Phase graphs do not really describe bSSFP
- Nicely show how RF spoiling works

- Excellent utility in understanding spin echoes with reduced flip angles

Resources

- ISMRM Lectures (taped or live)
  - Karla Miller: Nice EPG description and how they are used for gradient echo sequences
  - Matthias Wiegel: EPG and diffusion

- Spin Simulations and EPG code:
  - bmr.stanford.edu (Software Link)