Sincere Voting in Large Elections*

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Abstract

Austen-Smith and Banks (1996) showed that sincere voting is not typically an equilibrium of the Condorcet voting model when the electorate is large. Here, we reverse their finding by adding to the Condorcet model a third type of voter—one that receives no information in favor of either of the alternatives—as well as global uncertainty about the probability that each voter is such a "no evidence type." The expected number of no evidence type voters can be arbitrary small; nevertheless, if the electorate is large enough, then each of the two standard Condorcet types votes sincerely in every nondegenerate type-symmetric equilibrium.

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1 Introduction

In the seminal paper on voting games with private information, Austen-Smith and Banks (1996, hereafter A-SB) revisited the setting first studied by the Marquis of Condorcet (1785) in which a bench of jurors, each with private information about the guilt or innocence of a particular suspect, must vote either to convict or acquit. A-SB laid out the provocative finding that with a large electorate, there is almost always a behavioral discrepancy between "sincere," or "informative," voting and "strategic" (i.e., "equilibrium") voting. In other words, A-SB showed that in the Condorcet environment, rational voters must sometimes vote against their private information.

A-SB's finding launched a research agenda aimed at studying different *institutions*, or voting rules, to determine when information is aggregated by equilibrium behavior despite the fact that not everyone is voting informatively/sincerely (see, e.g., Feddersen and Pesendorfer 1997). Very little attention has been paid, however, to the robustness and descriptive validity of the original *behavioral* finding of the Condorcet model, namely the discrepancy between sincere and strategic voting. This is surprising given that few empirical scholars of elections are willing to embrace the idea that real-life voters would vote against their private information in large elections.[1] While many of these scholars find it plausible that strategic calculations can lead voters to vote against their private information in small electorates like juries, committees, and clubs, the discrepancy between sincere/informative voting and equilibrium voting has been harder to digest for large electorates, where intuition suggests that real-life voters who show up at the ballots vote with their evidence, not against it. In contrast to this intuition, voting theory has predicted the exact opposite: that equilibrium cannot support sincere voting with large electorates, but may support it in small electorates.

In this paper we offer a method of reconciling the discrepancy between sincere and strategic voting in the Condorcet model by introducing a third type of voter to the model: one who receives no evidence that the suspect is either innocent or guilty. We call this type the "no evidence type." The standard Condorcet model, whose equilibrium properties were studied by A-SB, has only two types of voters: those who received

[1] In fact, given the perceived implausibility of insincere voting in large real-life elections, many scholars have proposed alternative theories of voting, such as expressive voting (Tullock, 1971, Brennan and Hamlin, 1988, Hamlin and Jennings, 2011, Kamenica and Egan Brad, 2012), or they conjecture that voters have non-instrumental motivations in deciding who to vote for, and whether to vote at all (Green and Shapiro, 1996; Feddersen and Sandroni, 2013).

evidence in favor of guilt, and those who received evidence in favor of innocence. Accordingly, we call these two types of voters "Condorcet types." We assume that all voters are uncertain as to what fraction of voters in the population are not Condorcet types. These beliefs are subject to very mild constraints, and the motivating case of very little uncertainty is covered. If the Condorcet types vote sincerely while the no-evidence types vote close to random, then being pivotal in a large election would imply that the fraction of no-evidence type voters is large. This conclusion is drawn based on the pivotal event and is independent of how unlikely it is that the fraction of no-evidence types is large *ex ante*, so long as a large fraction is possible. At this point, however, the pivotal event provides very little information about the suspect's innocence or guilt. The Condorcet types are then willing to go with their private evidence. Establishing that this is an equilibrium description therefore only requires showing that when the Condorcet types vote sincerely, the no-evidence types are willing to mix close enough to random.

After establishing the existence of a sincere voting equilibrium, the last step of our argument shows that our model rationalizes sincere voting in a strong sense: We show that with a large enough electorate the two Condorcet types vote sincerely in *every* equilibrium in which voting is non-degenerate, i.e. when it is not the case that all types of voters vote for the same alternative.[2]

Although our perturbation is special, the steps we take to reconcile the discrepancy between sincere and equilibrium voting yield a more general insight. To reconcile this discrepancy by perturbing only the *information structure* of the Condorcet model—and not the preferences of voters—it must be that a voter learns significantly less from the pivotal event than from her private signal. In particular, voters must infer, conditional on being pivotal, that the votes of others are basically random.[3] Our perturbation to the information structure involves exactly this form of conditional beliefs across *all* nondegenerate equilibria. We comment further on this observation in Section 4.1. In the next three sections, we lay out the model, main result and its proof.

[2]This result stands in contrast to the analysis in Mandler (2012) where aggregate uncertainty about signal qualities does not restore sincere voting. The key difference is that in Mandler's model every voter is a Condorcet type (i.e., there is no no-evidence type) and the pivotal event leads voters to draw inferences only about the signal qualities.

[3]They do not actually have to believe that the behavior of others is mostly random. They have to believe that, conditional on being pivotal, the behavior of others is mostly random. This point—that what matters is the connection between others' votes and the state, conditional on being pivotal—distinguishes the equilibria here from constructions in the standard Condorcet model in which a nontrivial fraction of voters randomize in such a way as to give others the incentive to vote sincerely.

2 Model

We consider a majoritarian election in which $2n+1$ voters must each vote for one of two alternatives, $a \in \{0,1\}$. There is a state of the world, denoted $s = (\omega, \alpha)$, that is drawn randomly from a distribution φ over state space $S = \{0,1\} \times [0,1]$. The state determines the distribution of voter types; in particular, conditional on the state s, each voter's type is drawn independently from the set $T = \{\emptyset, 0, 1\}$ and the probability of type $t \in T$ is given by

$$\Pr\big(t \mid s = (\omega, \alpha)\big) = \begin{cases} \alpha & \text{if } t = \emptyset \\ (1-\alpha)q_\omega & \text{if } t = \omega \\ (1-\alpha)(1-q_\omega) & \text{if } t = -\omega \end{cases} \tag{1}$$

where q_0 and q_1 are parameters and, as usual, $-\omega$ indicates 1 if $\omega = 0$ and 0 if $\omega = 1$. Moreover, only the first component of the state ω is payoff relevant, and determines which alternative is superior for all voters. The payoff $u(t, a, s)$ for a voter of type $t \in T$ from electing $a \in \{0,1\}$ when the state is $s \in S$ is

$$u\big(t, a, s = (\omega, \alpha)\big) = \begin{cases} 1 & \text{if } a = \omega \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Thus, all voters strictly prefer to elect alternative a when the state is $\omega = a$. This completes the description of the basic structure of our model.

With a few assumptions it is natural to interpret our model as an extension of the Condorcet model studied by A-SB in which we have included a third type of voter, type $t = \emptyset$. In the remainder of this section, we introduce these assumptions as well as some definitions that will be useful in the analysis.

2.1 Assumptions

We introduce four assumptions on the information structure of our model. The first of these states that the distribution of type $t = \emptyset$ is independent of the payoff relevant state. The second implies that types $t \in \{0,1\}$ can be interpreted as types that have evidence that the payoff relevant state is $\omega = t$.

Assumption 1 (independence) *φ distributes ω and α independently.*

Assumption 2 (evidence) $q_0, q_1 \in \left(\frac{1}{2}, 1\right)$.

Assumption 1 says that we introduce type $t = \emptyset$ to the Condorcet model in a way that does not interfere (i.e., is not correlated) with the payoff relevant state ω, while Assumption 2 is the standard evidence assumption of the Condorcet framework. Conveniently, Assumption 1 enables us define the distributions of α and ω separately: we write F for the CDF of α and η for the prior probability that $\omega = 1$. The posterior probability that $\omega = 1$ for a voter of type $t \in T$ is then

$$\eta^t = \begin{cases} \eta & \text{if } t = \emptyset \\ \frac{\eta(1-q_1)}{(1-\eta)q_0+\eta(1-q_1)} & \text{if } t = 0 \\ \frac{\eta q_1}{(1-\eta)(1-q_0)+\eta q_1} & \text{if } t = 1 \end{cases} \tag{3}$$

and it is readily verified that Assumption 2 implies

$$\eta^0 < \eta < \eta^1. \tag{4}$$

Thus, voters of type $t = 0$ observe signals that are stronger evidence of the state being $\omega = 0$ while voters of type $t = 1$ observe signals that are stronger evidence of the state being $\omega = 1$. For voters of type $t = \emptyset$, their posterior belief about the payoff relevant state ω is equal to the prior, so we refer to type $t = \emptyset$ as the *no-evidence type.* By viewing the two values of the payoff-relevant state $\omega = 0, 1$ as corresponding to the guilty and innocent states of the Condorcet model, we can interpret types $t \neq \emptyset$ as *Condorcet types.* Our third assumption, stated below, guarantees that there is no discrepancy between sincere and informative voting for the two Condorcet types.

Assumption 3 (sufficiently agnostic prior) $\frac{1-q_0}{(1-q_0)+q_1} < \eta < \frac{q_0}{(1-q_1)+q_0}$.

Because Assumption 2 implies $\frac{1-q_0}{(1-q_0)+q_1} < \frac{1}{2} < \frac{q_0}{(1-q_1)+q_0}$, Assumption 3 simply states that η is not too far from $\frac{1}{2}$. The main implication of this assumption is that $\eta^0 < \frac{1}{2} < \eta^1$, which rules out the case where the prior η is so heavily biased in favor of one alternative that there is a discrepancy even between sincere and informative voting for the Condorcet types.[4] Because of Assumption 3, sincere voting requires each type $t \in \{0, 1\}$ to vote for alternative $a = t$ with probability 1.

Our last assumption says that voters can rule out the possibility that everyone is certain to be a no-evidence type. It says that the distribution of the probability of

[4] In the literature, informative voting refers to the case where all voters naively vote for the alternative for which they have received evidence, while sincere voting refers to the case where all voters naively vote for the alternative that, if elected, would give them the highest payoff.

being a no-evidence type has support $[0, \alpha^*]$ where α is a large enough number less than 1. Thus, it rules out the possibility that nearly all voters are no-evidence types, but allows the possibility that a large enough fraction of them have no evidence.[5] Note, however, that the assumption places no other restrictions on the distribution F besides this restriction on the support; in particular, the expected value of α according to F can be arbitrarily close to 0.

Assumption 4 (support restriction) *F has support $[0, \alpha^*]$ where*

$$\frac{\max\left\{(q_0 - \frac{1}{2}), (q_1 - \frac{1}{2})\right\}}{(q_0 - \frac{1}{2}) + (q_1 - \frac{1}{2})} < \alpha^* < 1$$

2.2 Formal Definitions

To summarize the formalism, the primitives of the model are $\mathcal{M} \equiv \big(S, T, \{0, 1\}, u(\cdot)\big)$, which consists of the sets of states S, types T, and alternatives $\{0, 1\}$, as well as the payoff function $u(\cdot)$. Its informational environment is a tuple $\xi \equiv (\eta, F, q_0, q_1)$. Note that all four of the assumptions above are assumptions on the informational environment ξ. A special informational environment that satisfies these four assumptions is the *Condorcet environment*, which is one in which F places unit mass on $\alpha = 0$.[6] These parameters determine the distribution of types in the population, according to (1). Together with the parameter n, which determines the number of voters $2n + 1$, the primitives and the informational environment define a voting *game* $(\mathcal{M}, \xi, n)$.

Given a game $(\mathcal{M}, \xi, n)$, a type symmetric strategy is a function $\sigma : T \to [0, 1]$, where $\sigma(t)$ the probability that a voter votes for alternative 1 when her type is t. Throughout the paper, we identify symmetric strategy profiles with the symmetric strategy σ that is used by all voters. In light of Assumption 3, we say that the Condorcet types $t = 0, 1$ vote *sincerely* in a symmetric strategy σ if $\sigma(0) = 0$ and $\sigma(1) = 1$. A symmetric strategy σ is an *equilibrium* of the voting game if the strategy profile in which all voters use strategy σ is a Bayes Nash equilibrium in weakly undominated strategies. We say

[5] In our analysis, a Condorcet type voter will believe that, conditional on being pivotal, the fraction of no evidence types approaches α^* as $n \to \infty$. Thus, it will be necessary for sufficiently many other voters to have no evidence (i.e., for α^* to be large enough) for any given Condorcet type voter to want to vote sincerely. This is why we require a lower bound on α^*. We rule out the case that $\alpha^* = 1$ to avoid a particular degeneracy. We discuss the role of Assumption 4 in more detail in the text after equation (15).

[6] The key feature of the Condorcet environment is that there are no no-evidence types; the only types that occur with positive probability are the two Condorcet types.

that an equilibrium σ is *degenerate* if either $\sigma(t) = 1$ for all t or $\sigma(t) = 0$ for all t. In such equilibria, no voter is ever pivotal. Following the literature, we will be interested in *nondegenerate* equilibria. As usual, we say that a particular property is satisfied if *n is large enough* (or *for all n large enough*) if there exists a finite N such that the property is satisfied for all $n \geq N$.

3 Main Result

The benchmark for the main result of this paper is the observation, stemming from the A-SB analysis, that that there is no equilibrium of the Condorcet game with large n in which both of the Condorcet types vote sincerely. Although this is not the key result that A-SB emphasize in their paper, this observation has driven much of the literature on equilibrium voting theory. We record the precise statement as follows.

Proposition 0 (Austen-Smith and Banks 1996) *Let ξ be any Condorcet environment. For all n large enough the game $(\mathcal{M}, \xi, n)$ has an equilibrium in which both Condorcet types vote sincerely, if and only if $q_0 = q_1$.*

Many scholars interpret this observation as implying that the failure of sincere or informative voting in large elections is a robust phenomenon, because they implicitly treat the relevant parameter space as $\mathcal{Q} := \left\{(q_0, q_1) : (q_0, q_1) \in \left(\frac{1}{2}, 1\right)^2\right\}$. In the space $\mathcal{Q}$, the necessary and sufficient condition $q_0 = q_1$ that appears in Proposition 0 is knife-edge. Moreover, for any pair of parameters, $q_0 \neq q_1$, it is the case that for all parameters $(\tilde{q}_0, \tilde{q}_1) \in \mathcal{Q}$ that are in a neighborhood of (q_0, q_1) the condition fails. Under this perspective, the failure of sincere voting in large elections is robust since it holds for *every* perturbation of (q_0, q_1) in $\mathcal{Q}$. Our main result below suggests that the opposite is true if one allows for the possibility of even a small amount of uncertainty about the presence of the no-evidence type.

Proposition 1 (sufficient conditions for sincere voting) *Let ξ be an informational environment that satisfies Assumptions 1-4. Then, for all n large enough (i) game $(\mathcal{M}, \xi, n)$ has an equilibrium in which both Condorcet types vote sincerely, and (ii) in every nondegenerate equilibrium of the game $(\mathcal{M}, \xi, n)$ both Condorcet types vote sincerely.*

4 Proof of Main Result

Here we prove Proposition 1. Before starting the proof, we first introduce some convenient notation. Fix the primitives $\mathcal{M}$ and an environment ξ satisfying the conditions of the proposition. For a symmetric strategy profile σ, the *ex ante* probability of drawing a vote for alternative $a = 1$ at state (ω, α) is

$$\pi(\sigma, \omega, \alpha) = \begin{cases} \alpha\sigma(\emptyset) + (1-\alpha)\left[(1-q_0)\sigma(1) + q_0\sigma(0)\right] & \text{in states } (0,\alpha) \\ \alpha\sigma(\emptyset) + (1-\alpha)\left[q_1\sigma(1) + (1-q_1)\sigma(0)\right] & \text{in states } (1,\alpha). \end{cases} \tag{5}$$

In a nondegenerate equilibrium to a game $(\mathcal{M}, \xi, n)$, voters vote as if they are pivotal. The ex ante probability that a voter is pivotal is a function of the state (ω, α), the strategy profile σ, and the parameter n. To write this probability, first let

$$\beta(\sigma, \omega, \alpha) = (\pi(\sigma, \omega, \alpha))(1 - \pi(\sigma, \omega, \alpha)). \tag{6}$$

Then, the probability that a voter is pivotal, ignoring her own type, is $\binom{2n}{n}[\beta(\sigma, \omega, \alpha)]^n$. This implies that voting for alternative $a = 1$ is a best response for a voter of type $t \in T$ to the strategy profile σ if and only if

$$\lambda(\sigma, t, n) := \left(\frac{\eta}{1-\eta}\right)\left[\frac{\int_0^1 \binom{2n}{n}[\beta(\sigma, 1, \alpha)]^n \Pr[t|(1,\alpha)]dF(\alpha)}{\int_0^1 \binom{2n}{n}[\beta(\sigma, 0, \alpha)]^n \Pr[t|(0,\alpha)]dF(\alpha)}\right] \geq 1 \tag{7}$$

where $\Pr[t|(\omega, \alpha)]$ is given by (1). Voting for alternative $a = 0$ is a best response if the reverse inequality holds. Equation (7) says that, conditional on the voter's private information and the event that she is pivotal, the payoff-relevant state variable is more likely to be $\omega = 1$ than $\omega = 0$. We refer to $\lambda(\sigma, t, n)$ as the *odds ratio* for type t. Also, let $\tau(\alpha) \in \{\alpha, 1-\alpha, 1\}$, and define the function

$$\mu(\sigma, \tau(\alpha), n) = \left(\frac{\eta}{1-\eta}\right)\frac{\int_0^1 \binom{2n}{n}[\beta(\sigma, 1, \alpha)]^n \tau(\alpha)dF(\alpha)}{\int_0^1 \binom{2n}{n}[\beta(\sigma, 0, \alpha)]^n \tau(\alpha)dF(\alpha)}. \tag{8}$$

Note that $\mu(\sigma, \tau(\alpha), n)$ is not a function of α since equation (8) involves integrating out α. The second argument is meant to capture explicitly the dependence on the function $\tau(\cdot)$ of α that is placed in the integrands of both the numerator and denominator. Next, given that $\tau(\alpha) \in \{\alpha, 1-\alpha, 1\}$, note the relationship

$$\lambda(\sigma, t, n) = \begin{cases} \mu(\sigma, \alpha, n) & \text{if } t = \emptyset \\ \frac{1-q_1}{q_0}\mu(\sigma, 1-\alpha, n) & \text{if } t = 0 \\ \frac{q_1}{1-q_0}\mu(\sigma, 1-\alpha, n) & \text{if } t = 1. \end{cases} \tag{9}$$

With this relationship in mind, we will work with the function $\mu(\sigma, \tau(\alpha), n)$ evaluated at $\tau(\alpha) \in \{\alpha, 1-\alpha, 1\}$ rather than with the odds ratios $\lambda(\sigma, t, n)$.

For any pair (σ, ω) define

$$\Delta(\sigma, \omega) := \min_{\alpha \in [0,\alpha^*]} \left|\pi(\sigma, \omega, \alpha) - \tfrac{1}{2}\right| \tag{10}$$

which is the distance to $\frac{1}{2}$ from the closest point on the function $\pi(\sigma, \omega, \cdot)$ defined over the support of F. Let $\alpha^\dagger(\sigma, \omega)$ denote a solution to the minimization problem in (10), which may or may not be unique. We will repeatedly use the following lemma, which follows from a well-known result by Good and Mayer (1975).

Lemma 1. *Suppose that the support of F is $[0, \alpha^*]$ for some $\alpha^* < 1$, and that $\tau(\alpha) \in \{\alpha, 1-\alpha, 1\}$. For each n, there exist numbers $\alpha_n(0), \alpha_n(1) \in [0, \alpha^*]$ such that*

$$\mu(\sigma, \tau(\alpha), n) = \left[\frac{\tau(\alpha_n(1))}{\tau(\alpha_n(0))}\right] \mu(\sigma, 1, n). \tag{11}$$

For each $\omega = 0, 1$, if $\alpha^\dagger(\sigma, \omega)$ is unique then $\alpha_n(\omega) \to \alpha^\dagger(\sigma, \omega)$ as $n \to \infty$. If $\Delta(\sigma, 0) < \Delta(\sigma, 1)$ then $\lim_{n\to\infty} \mu(\sigma, \tau(\alpha), n) = 0$. If $\Delta(\sigma, 0) > \Delta(\sigma, 1)$ then $\lim_{n\to\infty} \mu(\sigma, \tau(\alpha), n) = +\infty$.

Proof: The existence of the numbers $\alpha_n(0), \alpha_n(1) \in [0, \alpha^*]$ follows from the mean value theorem of integration. The property that $\alpha_n(\omega)$ converges to $\alpha^\dagger(\sigma, \omega)$ when $\alpha^\dagger(\sigma, \omega)$ is unique follows directly from Good and Mayer (1975). By definition of $\mu(\sigma, \tau(\alpha), n)$,

$$\frac{1-\eta}{\eta} \mu(\sigma, \tau(\alpha), n) = \frac{\int_0^1 \binom{2n}{n} [\beta(\sigma, 1, \alpha)]^n \tau(\alpha) dF(\alpha)}{\int_0^1 \binom{2n}{n} [\beta(\sigma, 0, \alpha)]^n \tau(\alpha) dF(\alpha)}. \tag{12}$$

Again, by Good and Mayer (1975) the right hand side of (12) converges to 0 as $n \to \infty$ when $\Delta(\sigma, 0) < \Delta(\sigma, 1)$ and diverges to $+\infty$ as $n \to \infty$ when $\Delta(\sigma, 0) > \Delta(\sigma, 1)$. □

We now prove the existence and uniqueness parts of the proposition separately. The existence claim is part (i) of the proposition's statement. To prove this, we must show that for all n large enough $(\mathcal{M}, \xi, n)$ has an equilibrium σ^s in which $\sigma^s(0) = 0$ and $\sigma^s(1) = 1$; that is, both Condorcet types vote sincerely.

The uniqueness claim is part (ii). To prove this, we must show that if n is large enough, then in every non-degenerate equilibrium σ of the game $(\mathcal{M}, \xi, n)$ both Condorcet types vote sincerely. This means that if there are multiple nondegenerate equilibria they differ only in the behavior of the no-evidence type. The joint statement of the propositino then follows by taking n sufficiently large.

4.1 Proof of the Existence Claim

Let $\underline{x} := \frac{1}{\alpha^*}\left[\frac{1}{2} - (1-\alpha^*)q_1\right] \in \left(0, \frac{1}{2}\right)$ and let $\underline{\sigma}^s$ be the strategy profile in which the two Condorcet types vote sincerely while $\underline{\sigma}^s(\emptyset) = \underline{x}$. In this case, $\pi(\underline{\sigma}^s, \alpha, 0) < \frac{1}{2}$ for all α, whereas $\pi(\underline{\sigma}^s, \alpha^*, 1) = \frac{1}{2}$. Consequently, $\Delta(\underline{\sigma}^s, \alpha, 0) > 0$ while $\Delta(\underline{\sigma}^s, \alpha, 1) = 0$, so by Lemma 1 we have

$$\lim_{n\to\infty} \mu(\underline{\sigma}^s, \alpha, n) = \infty. \tag{13}$$

By the relationship in (9), this means that $\lim_{n\to\infty} \lambda(\underline{\sigma}^s, \emptyset, n) = \infty$, so voters of type $t = \emptyset$ have a profitable deviation to $\underline{\sigma}^s(\emptyset) = 1$ for all values of n large enough. Analogously, we can show that for strategy profile $\overline{\sigma}^s$ in which the Condorcet types vote sincerely while $\overline{\sigma}^s(\emptyset) = \overline{x}$ where $\overline{x} := \frac{1}{\alpha^*}\left[\frac{1}{2} - (1-\alpha^*)(1-q_0)\right] \in \left(\frac{1}{2}, 1\right)$, then $\lim_{n\to\infty} \lambda(\overline{\sigma}^s, \emptyset, n) = 0$ and thus type $t = \emptyset$ has a profitable deviation to $\overline{\sigma}^s(\emptyset) = 0$ for all n large enough.

Given a symmetric strategy profile σ^s in which the Condorcet types vote sincerely and $\sigma^s(\emptyset) = x$, it is clear that $\lambda(\sigma^s, \emptyset, n)$ is continuous in x for all n. Therefore, the results above for $\underline{\sigma}^s$ and $\overline{\sigma}^s$, along with the intermediate value theorem, imply that for large enough values of n, there exists a number $x_n \in [\underline{x}, \overline{x}]$ such that $\lambda(\sigma^s, \emptyset, n) = 1$. This makes $\sigma^s(\emptyset) = x_n$ a best response to σ^s. Thus, denote by σ_n^s the symmetric strategy in which the Condorcet types vote sincerely and type $\emptyset$ votes for alternative 1 with probability x_n. It remains to show that for n large enough, sincere voting by each Condorcet type is a best response to σ_n^s as well.

Because Assumption 2 implies $\frac{q_1}{1-q_0} > 1 > \frac{1-q_1}{q_0}$, it follows that for all n large enough, sincere voting by each Condorcet type is a best response to σ_n^s if

$$\lim_{n\to\infty} \mu(\sigma_n^s, 1-\alpha, n) = 1 \tag{14}$$

which follows from the relationship in (9). To see why this limit holds, note that Lemma 1 implies that there exist four numbers $\hat{\alpha}_n(0)$, $\hat{\alpha}_n(1)$, $\check{\alpha}_n(0)$, $\check{\alpha}_n(1) \in [0, \alpha^*]$, all of which converge to $\alpha^* = \alpha^\dagger(\sigma_n^s, 0) = \alpha^\dagger(\sigma_n^s, 1)$ as $n \to \infty$ (by Lemma 1), such that

$$\begin{aligned}
\mu(\sigma_n^s, 1-\alpha, n) &= \left(\frac{\eta}{1-\eta}\right)\left[\frac{1-\hat{\alpha}_n(1)}{1-\hat{\alpha}_n(0)}\right] \frac{\int_0^1 \binom{2n}{n}[\beta(x_n,1,\alpha)]^n dF(\alpha)}{\int_0^1 \binom{2n}{n}[\beta(x_n,0,\alpha)]^n dF(\alpha)} \\
&= \left(\frac{\eta}{1-\eta}\right)\left[\frac{1-\hat{\alpha}_n(1)}{1-\hat{\alpha}_n(0)}\right]\left[\frac{1/\check{\alpha}_n(1)}{1/\check{\alpha}_n(0)}\right] \frac{\int_0^1 \binom{2n}{n}[\beta(x_n,1,\alpha)]^n \alpha dF(\alpha)}{\int_0^1 \binom{2n}{n}[\beta(x_n,0,\alpha)]^n \alpha dF(\alpha)} \\
&= \left[\frac{1-\hat{\alpha}_n(1)}{1-\hat{\alpha}_n(0)}\right]\left[\frac{1/\check{\alpha}_n(1)}{1/\check{\alpha}_n(0)}\right] \mu(\sigma_n^s, \alpha, n).
\end{aligned} \tag{15}$$

We can apply Lemma 1 here because Assumption 4 implies the conditions of the lemma that were necessary to invoke the result of Good and Mayer (1975). Note that $\alpha^\dagger(\sigma_n^s, 0) = \alpha^\dagger(\sigma_n^s, 1) = \alpha^*$ is implied by $\alpha^\dagger(\underline{\sigma}^s, 0) = \alpha^\dagger(\overline{\sigma}^s, 1) = \alpha^*$, for which we need $1 - q_0 < \underline{x}$ and $q_1 > \overline{x}$. Substituting $\underline{x}$ and $\overline{x}$ into these two inequalities and rearranging gives us $\alpha^* > \frac{q_1 - \frac{1}{2}}{q_1 - (1-q_0)}$ and $\alpha^* > \frac{\frac{1}{2} - (1-q_0)}{q_1-(1-q_0)}$. These inequalities are implied by the lower bound on α^* in Assumption 4, stated as the requirement that α^* exceed the maximum of the two right hand sides of these inequalities. The condition $\alpha^* < 1$ also stated in Assumption 4 enables us to avoid having to characterize the limit of (15) as $n \to \infty$ when the ratio of limits is 0/0.[7] Since $\mu(\sigma_n^s, \alpha, n) = \lambda(\sigma_n^s, \emptyset, n) = 1$ for all n large enough (by construction and the relationship in (9), and because all of the four numbers $\hat{\alpha}_n(0), \hat{\alpha}_n(1), \check{\alpha}_n(0), \check{\alpha}_n(1)$ converge to α^*) the limit in (14) holds. □

To explain the logic of the above argument, let us interpret condition (14) as saying that the odds ratio for a "fictional type" converges to 1. This fictional type is a type that receives an informative signal (thus she is not a no-evidence type) but then forgets the content of the signal, remembering only that she is one of the two Condorcet types. If, conditional on the pivotal event, this fictional type is indifferent between the two alternatives, then the best response for each Condorcet type is to vote sincerely. The proof above shows that the limiting indifference for the fictional type emerges due to the limiting indifference for the no-evidence type. If the electorate is large and type $t = \emptyset$ mixes in the right way, then conditional on any type of voter being pivotal, type $t = \emptyset$ appears very likely in the population. In fact, the conditional distribution of α puts almost the entire probability mass on a neighborhood of α^* as n goes to ∞. When α^* is close to 1, this implies that, conditional on being pivotal, voters believe that nearly every other voter is a no-evidence type. Because the likelihood of the no-evidence type is the same in both states, the pivotal event does not provide enough information about the payoff relevant state to trump the private information of the Condorcet types. Consequently, conditional on being pivotal, the Condorcet types believe that a large fraction of the population has no evidence, and are willing to vote their signals.

Remark 1. Readers may be interested in a characterization of the equilibrium values of the no-evidence type's mixing probability x_n that appears in the proof of existence

[7] It is not clear how to characterize this limit when $\alpha^* = 1$. Thus, we imposed Assumption 4(i) to avoid getting the degeneracy of 0/0 when taking the ratio of limits. In light of this, it is possible that Assumption 4(i) is not necessary for our results, though it makes the analysis significantly tractable.

in Section 4.1. It follows from this proof that if α^* is close to 1, n is large, and the two Condorcet types vote sincerely, then the symmetric best response for the no-evidence type is to mix close to 50-50. More precisely, for all n large enough, we have

$$\lim_{\alpha^* \to 1} x_n = \frac{1}{2}.$$

This means that in the limit of full support for F, the no-evidence type behaves as if very naive: from the perspective of this type, both payoff relevant states appear equally likely, and the type mixes evenly between the two alternatives.

4.2 Proof of the Uniqueness Claim

We now prove the uniqueness claim of the proposition. The proof of this part proceeds in three steps.

Step 1. We first show that if $(\mathcal{M}, \xi, n)$ has another non-degenerate equilibrium σ in which at least one of the Condorcet types does not vote sincerely then either $\sigma(0) = 0$ and $0 < \sigma(1) < 1$, or $0 < \sigma(0) < 1$ and $\sigma(1) = 1$.

To see why, note that incentives are strictly ordered for the Condorcet types, i.e. $\lambda(\sigma, 0, n) < \lambda(\sigma, 1, n)$ for all n. This implies that $\sigma(0) \leq \sigma(1)$. Moreover, this ordering also implies that we cannot have indifference by both Condorcet types, so we cannot have $0 < \sigma(0) < 1$ and $0 < \sigma(1) < 1$. In addition, we cannot have $\sigma(0) = \sigma(1) = 0$ and $\sigma(0) = \sigma(1) = 1$ in a non-degenerate equilibrium because in these cases $\mu(\sigma, 1-\alpha, n) = 1$, and then from the relationship in (9) we see that given this profile the strict best response for each Condorcet type is to vote sincerely. This contradicts the conjecture that $\sigma(0) = 1$ or $\sigma(1) = 0$. Therefore, we must have one of the two cases stated above. (The other possible case where $\sigma(0) = 0$ and $\sigma(1) = 1$ is the case of sincere voting.)

Step 2. We now argue that if n is large enough, $(\mathcal{M}, \xi, n)$ does not have an equilibrium in which $\sigma(0) = 0$ and $0 < \sigma(1) < 1$. To see why, note that in a symmetric strategy profile of this form, we have

$$\pi(\sigma, \omega, \alpha) = \begin{cases} \alpha\sigma(\emptyset) + (1-\alpha)(1-q_0)\sigma(1) & \text{in states } (0, \alpha) \\ \alpha\sigma(\emptyset) + (1-\alpha)q_1\sigma(1) & \text{in states } (1, \alpha). \end{cases} \tag{16}$$

If $\Delta(\sigma, \omega) > \Delta(\sigma, \tilde{\omega})$, $\omega \neq \tilde{\omega}$, then by Lemma 1 (with the substitution of $\tau(\alpha) = 1 - \alpha$) and the relationship in (9), type $t = 1$ has a profitable deviation to a pure strategy. Specifically $\sigma(1) = 1$ is a profitable deviation if $\omega = 1$ and $\tilde{\omega} = 0$ and $\sigma(1) = 0$ is a profitable deviation if $\omega = 0$ and $\tilde{\omega} = 1$.

If $\Delta(\sigma, 0) = \Delta(\sigma, 1)$ then there are three cases given that $(1-q_0)\sigma(1) < q_1\sigma(1)$. (This inequality holds because $\sigma(1) > 0$ by hypothesis, and $1 - q_0 < \frac{1}{2} < q_1$ by assumption.) The three exhaustive cases are:

(i) $\sigma(\emptyset) \leq (1 - q_0)\sigma(1)$,

(ii) $q_1\sigma(1) < \sigma(\emptyset)$ and

(iii) $(1 - q_0)\sigma(1) < \sigma(\emptyset) \leq q_1\sigma(1)$.

Consider case (i). In this case, we have $\Delta(\sigma, 0) = \frac{1}{2} - (1 - q_0)\sigma(1)$ with $\alpha^\dagger(\sigma, 0) = 0$. Also, because $\sigma(\emptyset) < q_1\sigma(1)$ we have $\Delta(\sigma, 1) = \frac{1}{2} - q_1\sigma(1)$. Because $\Delta(\sigma, 0) = \Delta(\sigma, 1)$ by hypothesis, this implies that $\frac{1}{2} - (1 - q_0)\sigma(1) = \frac{1}{2} - q_1\sigma(1)$, which is impossible.

Now consider case (ii). Because type $t = 1$ is voting for alternative 0 with positive probability, we need $\lambda(\sigma, 1, n) \leq 1$. So for all $\varepsilon_1 > 0$ if n is large enough then

$$\left[\frac{q_1}{1 - q_0}\right]\left[\frac{1 - \alpha^\dagger(\sigma, 1)}{1 - \alpha^\dagger(\sigma, 0)}\right]\mu(\sigma, 1, n) \leq 1 + \varepsilon_1. \tag{17}$$

This follows because the expression on the left is the limiting value of $\lambda(\sigma, 1, n)$ as $n \to \infty$, due to Lemma 1 and the relationship in (9).

Then, because $\sigma(\emptyset) > 0$ by hypothesis, $\lambda(\sigma, \emptyset, n) \geq 1$. There are now two subcases:

First, suppose $\sigma(\emptyset) < \frac{1}{2}$. Then $\alpha^\dagger(\sigma, 0) = \alpha^\dagger(\sigma, 1) = \alpha^*$, so by Lemma 1 and the relationship in (9) we know that for all $\varepsilon_\emptyset > 0$, if n is large enough then $\mu(\sigma, \alpha, n) \leq \mu(\sigma, 1, n) + \varepsilon_\emptyset$. Since type $t = \emptyset$ is voting for alternative 1 with positive probability (by hypothesis of this case) we have $\lambda(\sigma, \emptyset, n) = \mu(\sigma, \alpha, n) \geq 1$. Therefore, we must have $\mu(\sigma, 1, n) \geq 1 - \varepsilon_\emptyset$. Combining this with (17), we have $\frac{q_1}{1-q_0}(1 - \varepsilon_\emptyset) \leq 1 + \varepsilon_1$, which does not hold when $\varepsilon_\emptyset$ and ε_1 are small enough.

Now suppose $\sigma(\emptyset) \geq \frac{1}{2}$. In this case, $\Delta(\sigma, 0) = 0$ so we must have $\Delta(\sigma, 1) = 0$. Moreover, $\alpha^\dagger(\sigma, 0) = \min\left\{\frac{1-2(1-q_0)\sigma(1)}{2[\sigma(\emptyset)-(1-q_0)\sigma(1)]}, \alpha^*\right\}$ and $\alpha^\dagger(\sigma, 1) = \min\left\{\frac{1-2q_1\sigma(1)}{2[\sigma(\emptyset)-q_1\sigma(1)]}, \alpha^*\right\}$. Note that $\alpha^\dagger(\sigma, 0) > 0$. Because in this subcase the no-evidence type votes for alternative with 1 with positive probability, we must again have $\lambda(\sigma, \emptyset, n) = \mu(\sigma, \alpha, n) \geq 1$. Lemma 1 implies that for all $\varepsilon_\emptyset > 0$, if n is large enough then

$$\left[\frac{\alpha^\dagger(\sigma, 1)}{\alpha^\dagger(\sigma, 0)}\right]\mu(\sigma, 1, n) \geq 1 - \varepsilon_\emptyset. \tag{18}$$

But note that because $\alpha^\dagger(\sigma, 1) \leq \alpha^\dagger(\sigma, 0)$ (which follows because $1 - q_0 < q_1$ and the hypothesis that $\sigma(\emptyset) \geq \frac{1}{2}$ in this subcase) (18) implies $\mu(\sigma, 1, n) \geq 1 - \varepsilon_\emptyset$. Also, the fact that $\alpha^\dagger(\sigma, 1) \leq \alpha^\dagger(\sigma, 0)$ means that (17) implies $\frac{q_1}{1-q_0}\mu(\sigma, 1, n) \leq 1 + \varepsilon_1$. Together, these imply $\frac{q_1}{1-q_0}(1-\varepsilon_\emptyset) \leq 1+\varepsilon_1$, which again does not hold when $\varepsilon_\emptyset$ and ε_1 are small enough (recall $q_1 > 1-q_0$).

Finally consider case (iii). There are again two sub-cases:

First let $\alpha^*\sigma(\emptyset) \geq \frac{1}{2}$. Then, we have $\Delta(\sigma,0) = \Delta(\sigma,1) = 0$, meaning that $q_1\sigma(1) \leq \frac{1}{2}$. Moreover since $\alpha^* < 1$, in this sub-case we need $\sigma(\emptyset) > \frac{1}{2}$. These two conclusions imply that $\sigma(\emptyset) > q_1\sigma(1)$, contradicting the hypothesis of case (iii) that $\sigma(\emptyset) \leq q_1\sigma(1)$.

Next, let $\alpha^*\sigma(\emptyset) < \frac{1}{2}$. In this case, we have $\alpha^\dagger(\sigma,0) = \alpha^*$. Therefore, because type $t = 1$ puts positive probability on voting for alternative 0, it must be that for all $\varepsilon_1 > 0$ if n is large enough then for some $\check{\alpha}_n(1) \in [0,\alpha^*]$ we have

$$\left[\frac{q_1}{1-q_0}\right]\left[\frac{1-\check{\alpha}_n(1)}{1-\alpha^*}\right]\mu(\sigma,1,n) \leq 1+\varepsilon_1. \tag{19}$$

Again, this follows from Lemma 1 and the relationship in (9). Similarly, because type $t = \emptyset$ puts positive probability on voting for alternative 1, it is also the case that for all $\varepsilon_\emptyset > 0$ if n is large enough then for some $\hat{\alpha}_n(1) \in [0,\alpha^*]$ we have

$$1-\varepsilon_\emptyset \leq \left[\frac{\hat{\alpha}_n(1)}{\alpha^*}\right]\mu(\sigma,1,n). \tag{20}$$

Combining the fact that $\hat{\alpha}_n(1) \leq \alpha^*$ for all n with (19) we have

$$\left[\frac{\hat{\alpha}_n(1)}{\alpha^*}\right]\mu(\sigma,1,n) \leq \left[\frac{1-q_0}{q_1}\right]\left[\frac{1-\alpha^*}{1-\check{\alpha}_n(1)}\right](1+\varepsilon_1). \tag{21}$$

Combining (21) with (20) we have $1-\varepsilon_\emptyset \leq \left[\frac{1-q_0}{q_1}\right]\left[\frac{1-\alpha^*}{1-\check{\alpha}_n(1)}\right](1+\varepsilon_1)$. Now, the right hand side of this inequality bounded above by the value it takes when $\check{\alpha}_n(1) = \alpha^*$ (since we know that $\check{\alpha}_n(1) \in [0,\alpha^*]$) in which case it equals $\frac{1-q_0}{q_1}(1+\varepsilon_1)$. Therefore, we need $\frac{1-\varepsilon_\emptyset}{1-\varepsilon_1} \leq \frac{1-q_0}{q_1}$. But this cannot be satisfied when $\varepsilon_\emptyset$ and ε_1 are small enough.

Step 3. The argument that rules out equilibria in which $0 < \sigma(0) < 1$ and $\sigma(1) = 1$ is analogous to Step 2, so we omit it. □

It is well known that when the number of voters is large, the Condorcet model studied by A-SB has an equilibrium in which one of the Condorcet types votes sincerely while the other mixes. But our uniqueness result above rules out this type of equilibrium.

To understand how, suppose that in the Condorcet game type 0 votes for alternative 1 with probability 0. That means that if type 1 votes for alternative 1 with probability $\sigma(1) \in (0,1)$, the ex ante probability of casting a vote for alternative 1 in state 0 is

$\pi(\sigma, 0, \alpha) = (1 - q_0)\sigma(1)$ and in state 1 is $\pi(\sigma, 0, \alpha) = q_1\sigma(1)$. Because $q_0 < \frac{1}{2}$, the expression $(1 - q_0)\sigma(1)$ can never equal $\frac{1}{2}$. For type 1 to be mixing in equilibrium for all n large enough, it has to be that $q_1\sigma(1)$ is equidistant from $\frac{1}{2}$ as $(1 - q_0)\sigma(1)$ is. This implies that the two expressions cannot both lie on the same side of $\frac{1}{2}$. This means that we need $\frac{1}{2} - (1 - q_0)\sigma(1) = q_1\sigma(1) - \frac{1}{2}$, or in other words $\sigma(1) = 1/(1 + q_1 - q_0)$. This is the large n limit of the equilibrium mixing probability of type 1 when $q_1 > q_0$.

Now consider a game that satisfies the conditions of Proposition 1. Suppose that type 0 and the no evidence type vote for alternative 0 and type 1 votes for alternative 1 with probability $\sigma(1) \in (0, 1)$. This means that the ex ante probability of casting a vote for alternative 1 in state 0 is $\pi(\sigma, 0, \alpha) = (1 - \alpha)(1 - q_0)\sigma(1)$ and in state 1 is $\pi(\sigma, 1, \alpha) = (1 - \alpha)q_1\sigma(1)$. The former can never equal $\frac{1}{2}$. For type 1 to be indifferent for all n large enough, the latter must never equal $\frac{1}{2}$ either. If $q_1\sigma(1) \geq \frac{1}{2}$ then there is a value of α that makes the latter expression equal $\frac{1}{2}$. Therefore, we need the two expressions to be on the same side of $\frac{1}{2}$, i.e. $\frac{1}{2} - (1 - \alpha)(1 - q_0)\sigma(1) = \frac{1}{2} - (1 - \alpha)q_1\sigma(1)$. This implies $1 - q_0 = q_1$, which is impossible.

Remark 2. To prove the existence and uniqueness claims of Proposition 1 we characterized the limit of a system of equalities and inequalities, such as (7), (13), (14), and (17) - (21). This enabled us to establish that the equilibrium incentive conditions are satisfied for large values of n away from the limit. The alternative approach of characterizing equilibrium for each finite n and then studying limiting properties can be quite tedious. Our approach avoids the tedium by relying on results like Lemma 1 that provide large n approximations of the relevant odds ratios.

5 Robustness

We now conclude with a discussion of the robustness of our results to (i) allowing for abstention, and (ii) replacing the no-evidence type with a "low-evidence type."

5.1 Abstention

Given the literature on strategic abstention started by Feddersen and Pesendorfer (1996), it is natural to ask whether our result is robust to allowing voters, in particular the no-evidence type, to abstain. Under the conditions of Proposition 1, if the game is augmented to allow abstention, and if n is large enough, then there is an equilibrium in which the Condorcet types vote sincerely and the no-evidence type votes with probability 1, randomizing between a vote for 0 and a vote for 1. To see this, consider a no-evidence type voter's decision under the conjecture that the Condorcet types are voting sincerely and all other no-evidence types are voting for alternative 1 with probability x_n defined in Section 4.1. We know from section 4.1 that conditional on being pivotal, the voter is indifferent between voting for the two alternatives. Moreover, the voter's expected payoff from abstaining is equal to the expected payoff from voting for either alternative as the lottery over choices that result from abstention is independent of the payoff-relevant state ω. The key difference with the Feddersen-Pesendorfer analysis lies in our assumption that the population is fixed and in the equilibrium conjecture that everyone else votes. Under these conditions, there is only one possible profile of votes in which a no-evidence type voter is pivotal, so the analysis of the model with abstention is identical to that of Section 4.1.

This conclusion represents a limit to the "swing voters curse" literature. Feddersen and Pesendorfer's (1996) Proposition 1—that indifference between the two alternatives implies that abstention is strictly best—does not to hold with uncertainty about α in our model. Assessing whether there are also equilibria in which the no-evidence type randomizes between abstention and voting for one candidate requires more work, however. Since our focus is on asymmetric signal qualities ($q_0 \neq q_1$) one would need to consider an extension to Feddersen and Pesendorfer with imperfect signals (see their footnote 15) and uncertainty about the fraction of the no evidence type. The extension is not straightforward, so we leave it for future work.

5.2 Low Evidence Type

As a robustness exercise, we may replace the "no-evidence type" with a "low-evidence type"—one that is slightly more likely in one of the payoff relevant states, say state $\omega = 1$. We call this a low evidence type because it possesses slight evidence in favor of one of the states, namely state 1. Suppose the distribution of this type, also called $\emptyset$, is given by

$$\Pr\big(t \mid s = (\omega, \alpha)\big) = \begin{cases} \alpha(1+\gamma) & \text{if } t = \emptyset, \omega = 1 \\ \alpha(1-\gamma) & \text{if } t = \emptyset, \omega = 0 \\ (1-\alpha)q_\omega & \text{if } t = \omega \\ (1-\alpha)(1-q_\omega) & \text{if } t = -\omega. \end{cases}$$

where γ is close to 0. This captures the idea that a signal of $\emptyset$ is slightly more likely if the state is $\omega = 1$. With this change, the key to replicating the equilibrium construction above is to show that it is still possible to make the low-evidence type indifferent between the two alternatives, conditional on being pivotal and the Condorcet types voting sincerely. The analogue to equation (5) is

$$\pi(\sigma, \omega, \alpha) = \begin{cases} \alpha(1-\gamma)\sigma(\emptyset) + (1-\alpha(1-\gamma))\left[(1-q_0)\sigma(1) + q_0\sigma(0)\right] & \text{in states } (0,\alpha) \\ \alpha(1+\gamma)\sigma(\emptyset) + (1-\alpha(1+\gamma))\left[q_1\sigma(1) + (1-q_1)\sigma(0)\right] & \text{in states } (1,\alpha). \end{cases}$$

$\Delta(\sigma, \omega)$ and $\alpha^{\dagger}(\sigma, \omega)$ are defined in the same way using this new $\pi(\sigma, \omega, \alpha)$ function. An analogue to Lemma 1 also exists. If γ is small enough and α^* large enough, it is possible to find new values of $\underline{x}$ and $\overline{x}$ to mimic the proof of existence in Section 4.1. These numbers now also depend on γ. We apply the intermediate value theorem to establish that for n large enough, the indifference condition $\lambda(\sigma^s, \emptyset, 1) = 1$ can be satisfied by a suitable choice of x_n. The rest of the argument follows. That is, the conclusion of Proposition 1 holds if α^* is large enough and γ small enough.

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Appendix

In this appendix we impose slightly more structure and define a topological space of environments. We show that the set of environments in which Proposition 1 obtains is open and dense in this space.

Let $Z_F \subset [0,1]$ denote the set of values of α at which F has a jump. If Z_F is empty or finite, then the set $X_F = [0,1]\backslash Z_F$ is a union of intervals. The assumption is that F has at most a finite number of jumps and the derivative of F, which we denote f, exists on X_F and is continuous at every point in this set.

Assumption 5 (well behaved distribution) *F has at most a finite number of jumps (i.e. Z_F is finite) and admits a continuous density f elsewhere (i.e. on X_F).*

In addition to studying particular environments, we also study properties of the set of *all* environments that satisfy Assumptions 1-5. This requires endowing the set of environments with an appropriate topology. Let $\Phi(S)$ denote the set of probability distributions over the set of states S, with generic element denoted φ. Thus, F and η denote the marginals of φ on the first and second components respectively. Let

$$\mathcal{E} = \{\xi \equiv (\varphi, q_0, q_1) \in \Phi(S) \times \mathbb{R}^2 : \xi \text{ satisfies Assumptions 1 - 5}\} \tag{22}$$

denote the set of informational environments of the model. In light of Assumption 1, we identify φ with its marginals (η, F), abusing notation and writing $\varphi \equiv (\eta, F)$ whenever Assumption 1 is satisfied.[8] We endow the first component space of $\mathcal{E}$ with a suitable metric $d_v(\cdot,\cdot)$ and the second and third component spaces with the standard Euclidean metric $d(\cdot,\cdot)$. We consider the standard box distance, which we denote $d_B((\cdot,\cdot,\cdot),(\cdot,\cdot,\cdot))$, on the product space $\mathcal{E}$.[9] We say that an environment $\tilde{\xi}$ is *ε-close* to another environment ξ if $d_B(\xi,\tilde{\xi}) < \varepsilon$. As usual, a *neighborhood* of an environment ξ is the set of environments $\mathcal{B}_\varepsilon(\xi) := \{\tilde{\xi} \in \mathcal{E} : d_B(\xi,\tilde{\xi}) < \varepsilon\}$ that are ε-close to it, for some value of $\varepsilon > 0$.

The choice of Euclidean distance for the second and third component spaces—the coordinates (q_0, q_1)—is natural, and without meaningful loss of generality. The choice of the metric d_v first the first component space is more consequential. Let us introduce a partial order $\succeq$ on two possible metrics d_v and $d_{\tilde{v}}$, and write that $d_{\tilde{v}} \preceq d_v$ if for all $\varepsilon > 0$, there exists $k > 0$ such that $d_v(\varphi,\tilde{\varphi}) \leq k\varepsilon$ implies $d_{\tilde{v}}(\varphi,\tilde{\varphi}) \leq \varepsilon$ for all $\varphi, \tilde{\varphi} \in \Phi(S)$. That is, convergence in the topology generated by d_v implies convergence in the topology generated by $d_{\tilde{v}}$. Note that $\succeq$ is reflexive. An important metric is the Lévy-Prokhorov metric, which metrizes the topology of weak convergence. Denote this metric by d_L.

Another metric that we consider is motivated by the restrictions that we have placed on $\varphi \equiv (\eta, F)$ in the assumptions above. Let $\Phi^*(S) \subset \Phi(S)$ be the subset of distributions that satisfy the five assumptions.[10] Recall that the conditional distribution F has at most a finite number of jumps, whose set is Z_F, and admits a continuous density f elsewhere on the set $X_F = [0,1] \backslash Z_F$. In light of this, define, for any distribution $\varphi \equiv (\eta, F)$ that is part of an environment $\xi = (\varphi, q_0, q_1) \in \mathcal{E}$, the function h_F by

$$h_F(\alpha) = \begin{cases} F(\alpha) - F(\alpha^-) & \text{if } \alpha \in Z_F \\ f(\alpha) & \text{if } \alpha \in X_F. \end{cases} \tag{23}$$

Therefore, $h_F(\alpha)$ is the size of the jump at α if α is a jump point of F, and is the value of the density of F at α if α is not a jump point. For all distributions F satisfying our assumptions, the function h_F is unique. We define the metric $d_M(\cdot,\cdot)$ on $\Phi^*(S)$ by

$$d_M\big(\varphi \equiv (\eta, F), \tilde{\varphi} \equiv (\tilde{\eta}, \tilde{F})\big) = \max\left\{|\eta - \tilde{\eta}|, \sup_{\alpha\in[0,1]} \left|h_F(\alpha) - h_{\tilde{F}}(\alpha)\right|\right\}. \tag{24}$$

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[8]This is also the motivation for abusing our notation for ξ, which in the main text we defined to be the tuple (η, F, q_0, q_1), but here it represents the tuple (φ, q_0, q_1).

[9]That is, $d_B((\varphi, q_0, q_1), (\tilde{\varphi}, \tilde{q}_0, \tilde{q}_1)) = \max\{d_v(\varphi,\tilde{\varphi}), d_B(q_0,\tilde{q}_0), d(q_1,\tilde{q}_1)\}$.

[10]More precisely, let $\Phi^*(S)$ be the set of distributions $\varphi \equiv (\eta, F)$ that satisfy Assumptions 1, 3, 4 and 5 for some values q_0 and q_1 that satisfy Assumption 2.

We now have the following result, which states that the set of games for for which sincere voting by the Condorcet types is the unique prediction, is dense in the space $\mathcal{E}$ endowed with the weak topology.

Proposition 2 (denseness of sincere voting) *Choose $d_v \preceq d_L$. For every $\varepsilon > 0$ and every environment $\xi \in \mathcal{E}$ (notably, including every Condorcet environment) there exists an environment $\xi^\varepsilon \in \mathcal{E}$ that is ε-close to ξ such that for all n large enough (i) game $(\mathcal{M}, \xi^\varepsilon, n)$ has an equilibrium in which both Condorcet types vote sincerely, and (ii) in every nondegenerate equilibrium of game $(\mathcal{M}, \xi^\varepsilon, n)$ both Condorcet types vote sincerely.*

Proof: Fix $\xi = (\eta, F, q_0, q_1) \in \mathcal{E}$. By Assumption 4, $F(\overline{\alpha}) = 1$ for some $\overline{\alpha} < 1$. Pick any such $\overline{\alpha}$. Pick any α^* that satisfies the condition in Proposition 1, and let F_{α^*} denote the uniform distribution on $[0, \alpha^*]$. Fix $\delta > 0$ and construct $\xi^\delta = (\varphi^\delta, q_0^\delta, q_1^\delta) \in \mathcal{E}$ by setting $q_0^\delta = q_0$, $q_1^\delta = q_1$, $\eta^\delta = \eta$ and $F^\delta(\alpha) = (1-\delta)F(\alpha) + \delta F_{\alpha^*}(\alpha)$. Note that, by construction, for all $\delta > 0$, the distribution F^δ has support $[0, \alpha^*]$ where α^* satisfies the condition in Proposition 1. Moreover, for all $\delta > 0$, the environment ξ^δ satisfies Assumptions 1 - 5 (because ξ satisfies these assumptions) so it belongs to $\mathcal{E}$. Therefore ξ^δ satisfies the conditions of Proposition 1 for all $\delta > 0$. By choosing δ small enough we can guarantee that ξ^δ is ε-close to ξ for any value of $\varepsilon > 0$ and any choice of metric $d_v \preceq d_L$. The result follows because ξ^ε satisfies the conditions in Proposition 1 for all $\delta > 0$. □

We now show that in topologies that are at least as fine as those generated by the metric d_M, the *failure* of sincere voting is knife-edge.

Proposition 3 (genericity of sincere voting) *Choose $d_v \preceq d_M$. For all $\varepsilon > 0$ and all environments $\xi \in \mathcal{E}$ (notably, including every Condorcet environment) there exists an environment $\xi^\varepsilon \in \mathcal{E}$ that is ε-close to ξ, and which has a neighborhood $\mathcal{B}_{\tilde{\varepsilon}}(\xi^\varepsilon)$ such that for every environment $\tilde{\xi} \in \mathcal{B}_{\tilde{\varepsilon}}(\xi^\varepsilon)$, if n is large enough then (i) game $(\mathcal{M}, \tilde{\xi}, n)$ has an equilibrium in which both Condorcet types vote sincerely and (ii) in every non-degenerate equilibrium of game $(\mathcal{M}, \tilde{\xi}, n)$ both Condorcet types vote sincerely.*

Proof: Fix $\xi = (\eta, F, q_0, q_1) \in \mathcal{E}$ and construct ξ^δ as in the proof of Proposition 2 above. Note that $d_M \preceq d_L$. Therefore, by choosing δ small enough, we can make ξ^δ ε-close to ξ for any choice of metric $d_v \preceq d_M$. Let $d_v = d_M$ and pick any such environment, calling it $\xi^\varepsilon = (\eta^\varepsilon, F^\varepsilon, q_0^\varepsilon, q_1^\varepsilon)$. We claim that for all environments $\tilde{\xi} = (\tilde{\eta}, \tilde{F}, \tilde{q}_0, \tilde{q}_1)$ in a neighborhood $\mathcal{B}_{\tilde{\varepsilon}}(\xi^{\varepsilon}) \subseteq \mathcal{E}$ the condition of Proposition 1 is satisfied when $\tilde{\varepsilon}$ is small enough. If $\tilde{\varepsilon} < \varepsilon/2$ then the density of $\tilde{F}$ is positive at exactly the points at which the density of F^{ε} is positive. Thus, if $\tilde{\varepsilon}$ is small enough, the support of $\tilde{F}$ coincides with the support of F^{ε}. Since F^{ε} satisfies the condition of Proposition 1 by construction (as in the proof of Proposition 2) so does $\tilde{F}$. The result then follows for all $d_v \preceq d_M$. □

The message of Proposition 3 is the opposite of the message of Proposition 0. The result suggests that whether a phenomenon like sincere voting by the Condorcet types is knife-edge, or whether the failure of sincere voting is knife-edge, depends very much on the space one considers. According to the proposition, the set of environments that do not have an equilibrium in which both Condorcet types vote sincerely in a large electorate is knife-edge in the topologies that we consider. The proposition implies that the set of environments that have equilibria in which the two Condorcet types vote sincerely has a subset that is both open and dense in $\mathcal{E}$. Thus, the result suggests that every environment in $\mathcal{E}$ can be "robustly" perturbed so that in every nondegenerate equilibrium of the perturbed environment both Condorcet types vote sincerely if the size of the electorate is large enough.