

# Online Appendix for “Explaining Preferences from Behavior: A Cognitive Dissonance Approach”

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## Abstract

This online appendix presents a version of the the partisanship application mentioned in footnote 4 of the paper titled “Explaining Preferences from Behavior: A Cognitive Dissonance Approach” in which the parties strategically choose to run on their ideal policy positions.

Two parties labeled  $L$  and  $R$  have ideal policies  $(0, 0)$  and  $(1, 1)$  respectively; in particular, if  $(x^w, y^w)$  denotes the policy chosen by the winning party that comes to power, then the payoff to each party  $L$  and  $R$  is given by

$$\begin{aligned}v^L(x^w, y^w) &= -\mathbf{1}_{\{x^w \neq 0\}} - \lambda^L \mathbf{1}_{\{y^w \neq 0\}} \\v^R(x^w, y^w) &= -\mathbf{1}_{\{x^w \neq 1\}} - \lambda^R \mathbf{1}_{\{y^w \neq 1\}}\end{aligned}$$

where  $\lambda^L > 0$  and  $\lambda^R > 0$  are parameters that weight the second issue relative to the first, and  $\mathbf{1}_{\{\cdot\}}$  is the indicator function, which takes value 1 if the event described in brackets is satisfied and 0 otherwise. Each party runs on one of the four possible policies. Denote by  $(x^j, y^j)$  the policy that party  $j = L, R$  runs on. Parties are committed to the policies they run on and choose their platforms strategically, so they need not run on their ideal points. (However, we will show that in equilibrium they do.)

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Although voters are initially evenly distributed across the four possible ideal points, each voter may change his or her ideal point in the course of deciding which party to support. A voter with initial ideal point  $(x^o, y^o)$  may change her ideal point to a new one  $(x^n, y^n)$ . If  $(x^n, y^n) = (x^o, y^o)$ , then the voter has chosen not to change her initial ideal point. The voter's payoff from supporting party  $j = L, R$  and adopting the ideal point  $(x^n, y^n)$  is

$$u((x^n, y^n), (x^j, y^j) \mid (x^o, y^o)) = -(\mathbf{1}_{\{x^n \neq x^j\}} + \kappa \mathbf{1}_{\{x^n \neq x^o\}}) - \gamma(\mathbf{1}_{\{y^n \neq y^j\}} + \kappa \mathbf{1}_{\{y^n \neq y^o\}})$$

where  $\gamma > 0$  and  $\kappa > 0$  are parameters. If a voter supports a party  $j$  whose position differs from hers on the first issue  $x$ , then she experiences a cognitive dissonance cost of 1. If her position differs on the second issue then she experiences a cognitive dissonance cost of  $\gamma$ .  $\kappa$  is the psychological cost of changing her ideal point on the first issue and  $\gamma\kappa$  is the analogous cost of changing her ideal point on the second issue.

Note that the two issues are separable and  $\gamma$  represents the salience of the second issue vis-à-vis the first. While we assume (for parsimony) that the parameter  $\kappa$  is constant across the electorate, voters may differ in the parameter  $\gamma$ , which varies on an interval  $[\underline{\gamma}, \bar{\gamma}]$  with  $0 < \underline{\gamma} < 1 < \bar{\gamma}$ . Thus, some voters may consider the first issue more important than the second, while others consider the second issue to be more important than the first. We also assume that  $\kappa < 1$ . This assumption enables us to focus on the interesting case where individuals change their preferences in response to the positions taken by the two parties.<sup>1</sup>

The joint distribution of  $\gamma$  and voter ideal points depends on a state variable, which we denote  $\theta \in \{\theta_\ell, \theta_r\}$ . For each  $\theta$  the marginal distribution of  $\gamma$  is continuous on  $[\underline{\gamma}, \bar{\gamma}]$ . Given the state  $\theta$ , let  $\phi_{(x,y)}^-[\theta]$  denote the fraction of voters with ideal point  $(x, y)$  for whom  $\gamma < 1$ , and  $\phi_{(x,y)}^+[\theta]$  the fraction with ideal point  $(x, y)$  for whom  $\gamma > 1$ . By the assumption that the marginal distribution of  $\gamma$  is continuous in both states, the fraction of voters for whom  $\gamma = 1$  is always zero; therefore,  $\phi_{(x,y)}^-[\theta] + \phi_{(x,y)}^+[\theta] = 1$  for all  $(x, y)$  and both realizations of  $\theta$ . We assume that the two states  $\theta_\ell$  and  $\theta_r$  are equally likely, and that:

- (i)  $\phi_{(0,1)}^-[\theta_\ell] + \phi_{(1,0)}^+[\theta_\ell] > 1$  and  $\phi_{(0,1)}^-[\theta_r] + \phi_{(1,0)}^+[\theta_r] < 1$
- (ii)  $\phi_{(0,0)}^-[\theta_\ell] + \phi_{(1,1)}^+[\theta_\ell] > 1$  and  $\phi_{(0,0)}^-[\theta_r] + \phi_{(1,1)}^+[\theta_r] < 1$

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<sup>1</sup>We could solve the model without this assumption but there would be several more cases to handle, some of which duplicate our main result. The others are substantively uninteresting.

These assumptions imply that state  $\theta_\ell$  unambiguously favors party  $L$  and state  $\theta_r$  favors party  $R$ . Since we assumed that the two states are equally likely, neither party has an advantage *ex ante*.

The timing of events is as follows:

1. Parties announce their positions  $(x^L, y^L)$  and  $(x^R, y^R)$ .
2. The state  $\theta \in \{\theta_\ell, \theta_r\}$  is drawn, which determines the initial distribution of voter ideal points.
3. Each voter takes as given his or her initial ideal point  $(x^o, y^o)$  and decides what position to take on each of the two issues  $(x^n, y^n)$  and which party to support.
4. Voters vote expressively for the party that they support. That is, a voter with initial ideal point  $(x^o, y^o)$  votes for party  $L$  if

$$\max_{(x^n, y^n)} u((x^n, y^n), (x^L, y^L) \mid (x^o, y^o)) > \max_{(x^n, y^n)} u((x^n, y^n), (x^R, y^R) \mid (x^o, y^o))$$

and votes for party  $R$  if the reverse inequality holds. Voters for whom the two sides are equal may vote for either of the two parties.

Since we do not take the voters to be strategic players (i.e., voting is expressive as in the Hotelling-Downs model) the game is effectively a simultaneous move game for the parties. Therefore, an equilibrium of the game is one in which each voter's choice of the party she supports and the new ideal position that she adopts is sequentially optimal given the platforms of the two parties, and given this behavior, the pair of platforms  $((x^L, y^L), (x^R, y^R))$  adopted by the parties is such that for each  $j = L, R$ ,  $(x^j, y^j)$  maximizes

$$\text{Prob}[(x^w, y^w) = (x^j, y^j)]v^j(x^j, y^j) + \text{Prob}[(x^w, y^w) = (x^{-j}, y^{-j})]v^j(x^{-j}, y^{-j})$$

where  $(x^{-j}, y^{-j})$  is set to equal  $(x^R, y^R)$  if  $j = L$ , and  $(x^L, y^L)$  if  $j = R$ .

**Proposition OA1.** *There is an essentially unique equilibrium in which parties  $L$  and  $R$  run on their ideal policies  $(x^L, y^L) = (0, 0)$  and  $(x^R, y^R) = (1, 1)$ . Voters with initial ideal points  $(0, 0)$  and  $(1, 1)$  do not change their ideal points. Voters with initial ideal point  $(0, 1)$  and preference parameter  $\gamma < 1$  change their ideal points to  $(0, 0)$  while those with parameter  $\gamma > 1$  change their ideal points to  $(1, 1)$ . Voters with initial ideal point  $(1, 0)$  and preference parameter  $\gamma < 1$  change their ideal point to  $(1, 1)$  while those with parameter  $\gamma > 1$  change their ideal point to  $(0, 0)$ .*

**Proof:** Suppose  $(x^L, y^L) = (0, 0)$  and  $(x^R, y^R) = (1, 1)$  so that both parties announce their ideal policies as their positions. To show that this is an equilibrium, we first prove the claim that voters do as the proposition describes. Clearly, voters with initial ideal point  $(x^o, y^o) = (0, 0)$  will support party  $L$  and not change their ideal point. Voters with initial ideal point  $(x^o, y^o) = (1, 1)$  will support party  $R$  and not change their ideal point.

For voters with initial ideal points  $(x^o, y^o) = (0, 1)$  the payoff from not changing their preference and supporting party  $L$  is  $-\gamma$ . The payoff from not changing their preference and supporting party  $R$  is  $-1$ . If these voters are to support party  $R$  and change their ideal point, then the most profitable ideal point to choose is  $(x^n, y^n) = (1, 1)$  in which case their payoff is  $-\kappa$ . If they are to support party  $L$  and change their ideal point, then the most profitable ideal point to choose is  $(x^n, y^n) = (0, 0)$ . The payoff from this is  $-\kappa\gamma$ . Now, because of our assumption that  $\kappa < 1$  we know that  $-\kappa\gamma > -\gamma > -1$  and  $-\kappa\gamma > -\kappa$  whenever  $\gamma < 1$ . This means that voters with initial ideal point  $(x^o, y^o) = (0, 1)$  and parameter  $\gamma < 1$  support party  $L$  and change their ideal point to  $(x^n, y^n) = (0, 0)$ . On the other hand, if  $\gamma > 1$  then  $-\kappa > -1 > -\gamma$  and  $-\kappa > -\kappa\gamma$ . This means that voters with initial ideal point  $(x^o, y^o) = (0, 1)$  change their ideal points to  $(x^n, y^n) = (1, 1)$  and support party  $R$ .

Therefore, to summarize, among voters with initial ideal point  $(x^o, y^o) = (0, 1)$  those with  $\gamma < 1$  change their ideal point to  $(x^n, y^n) = (0, 0)$  and support party  $L$  while those with  $\gamma > 1$  change their ideal point to  $(x^n, y^n) = (1, 1)$  and support party  $R$ . Those with  $\gamma = 1$  are a measure zero set so to compute the parties' vote shares from this group of voters we do not need to specify what they do.

Analogously we can show that among voters with initial ideal point  $(x^o, y^o) = (1, 0)$  those with  $\gamma < 1$  change their ideal point to  $(x^n, y^n) = (1, 1)$  and support party  $R$  while those with  $\gamma > 1$  change their ideal point to  $(x^n, y^n) = (0, 0)$  and support party  $L$ . Again, voters with  $\gamma = 1$  are a measure zero set so we ignore them.

Party  $L$ 's vote share given state  $\theta$  is then  $\frac{1}{4} \left( 1 + \phi_{(0,1)}^-[\theta] + \phi_{(1,0)}^+[\theta] \right)$  while party  $R$ 's vote share is one minus this quantity. By assumptions (i) and (ii) party  $L$ 's vote share is larger than  $1/2$  at state  $\theta = \theta_\ell$  and smaller than  $1/2$  at state  $\theta = \theta_r$ . Because the two states are equally likely, this means that parties  $L$  and  $R$  both have equal probability of winning the election.

We now use these facts to show that it is an equilibrium for the two parties to run on their ideal policies. If party  $R$  announces  $(x^R, y^R) = (1, 1)$  and party  $L$  stays at  $(x^L, y^L) = (0, 0)$  then  $L$ 's expected payoff is  $-\frac{1}{2}(0) - \frac{1}{2}(1 + \lambda^L)$ . If party  $L$  deviates to  $(1, 1)$  then its expected payoff is  $-(1 + \lambda^L)$  so the deviation would not be profitable. If  $L$  deviates

to  $(0, 1)$  then its vote share would be  $\frac{1}{4} \left( 1 + \phi_{(0,0)}^-[\theta] + \phi_{(1,1)}^+[\theta] \right)$ , which follows from an argument analogous to the one above in which we derived the (purported) equilibrium vote share. This vote share is larger than  $1/2$  when  $\theta = \theta_r$  and smaller than  $1/2$  when  $\theta = \theta_\ell$ , by assumption (ii). Therefore, party  $L$ 's probability of winning is  $1/2$ . This means that  $L$ 's expected payoff from the deviation is  $-\frac{1}{2}(\lambda^L) - \frac{1}{2}(1 + \lambda^L)$ , so again the deviation would not be profitable. Finally, suppose that party  $L$  were to deviate to  $(1, 0)$ . In this case, its vote share would be  $\frac{1}{4} \left( 1 + \phi_{(0,0)}^+[\theta] + \phi_{(1,1)}^-[\theta] \right)$ . This quantity is larger than  $1/2$  when  $\theta = \theta_\ell$  and smaller than  $1/2$  when  $\theta = \theta_r$ , by assumption (i). Therefore, the probability that  $L$  wins is again  $1/2$ . Consequently, the expected payoff from the deviation is  $-\frac{1}{2}(1) - \frac{1}{2}(1 + \lambda^L)$ . Therefore this deviation is also not profitable.

*Uniqueness claim:* We now establish the uniqueness claim of the proposition by ruling out all other possible equilibria, one by one.

If party  $R$  announces  $(x^R, y^R) = (1, 1)$  and party  $L$  announces  $(x^L, y^L) = (0, 1)$ , then  $L$ 's probability of winning is  $\frac{1}{2}$ , which means that  $L$ 's expected payoff is  $-\frac{1}{2} - \lambda^L$ . If  $L$  deviates to  $(0, 0)$ , its expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (1, 1)$  and party  $L$  announces  $(x^L, y^L) = (1, 0)$ , then  $L$ 's probability of winning is  $\frac{1}{2}$ , which means that  $L$ 's expected payoff is  $-1 - \frac{\lambda^L}{2}$ . If  $L$  deviates to  $(0, 0)$ , its expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $L$  announces  $(x^L, y^L) = (0, 0)$  and party  $R$  announces at  $(x^R, y^R) = (1, 0)$ , then voters with initial ideal point  $(0, 0)$  vote for  $L$  and those with initial ideal point  $(1, 0)$  vote for  $R$ . For voters with initial ideal point  $(0, 1)$  the payoff from not changing their preference and supporting party  $L$  is  $-\gamma$ , the payoff from not changing their preference and supporting party  $R$  is  $-1 - \gamma$ . If they are to support party  $L$  and change their ideal point, then the most profitable ideal point is  $(x^n, y^n) = (0, 0)$ , giving them a payoff of  $-\gamma\kappa$ . If they are to support party  $R$  and change their ideal point, then the most profitable ideal point would be  $(x^n, y^n) = (1, 0)$ , giving them a payoff of  $-\kappa - \gamma\kappa$ . Since  $-1 - \gamma < -\gamma < -\gamma\kappa$  and  $-\kappa - \gamma\kappa < -\gamma\kappa$ , voters with initial ideal point  $(0, 1)$  will change to  $(0, 0)$  and support party  $L$ . Following an analogous argument we can show that voters with initial ideal point  $(1, 1)$  will change to  $(1, 0)$  and support party  $R$ . Thus party  $R$ 's probability of winning is  $\frac{1}{2}$ , which means that  $R$ 's expected payoff is  $-\frac{1}{2} - \lambda^R$ . If  $R$  deviates to  $(1, 1)$ , its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{1}{2} - \frac{\lambda^R}{2}$ . Therefore, the deviation is profitable.

If party  $L$  announces  $(x^L, y^L) = (0, 0)$  and party  $R$  announces  $(x^R, y^R) = (0, 1)$ , then voters with initial ideal point  $(0, 0)$  vote for  $L$  and voters with initial ideal point  $(0, 1)$  vote

for  $R$ . For voters with initial ideal point  $(1, 0)$ , the highest payoff comes from changing to  $(0, 0)$  and supporting  $L$ . For voters with initial ideal point  $(1, 1)$ , the highest payoff comes from changing to  $(0, 1)$  and supporting  $R$ . Thus,  $R$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-1 - \frac{\lambda^R}{2}$ . If  $R$  deviates to  $(1, 1)$ , its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{1}{2} - \frac{\lambda^R}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (0, 1)$  and party  $L$  announces  $(x^L, y^L) = (1, 0)$ , then voters with initial ideal point  $(1, 0)$  will support  $L$  and voters with initial ideal point  $(0, 1)$  will support  $R$ . For voters with initial ideal point  $(0, 0)$ , if they change the ideal point and support  $L$ , then the most profitable ideal point to choose is  $(1, 0)$ , which gives a payoff of  $-\kappa$ ; if they change the ideal point and support  $R$ , then the most profitable ideal point to choose is  $(0, 1)$ , which gives a payoff of  $-\gamma\kappa$ . When  $\gamma > 1$ , voters with initial ideal point  $(0, 0)$  change to  $(1, 0)$  and support  $L$ ; when  $\gamma < 1$ , they change to  $(0, 1)$  and support  $R$ . Using an analogous argument we can show that when  $\gamma > 1$ , voters with initial ideal point  $(1, 1)$  change to  $(0, 1)$  and support  $R$ ; when  $\gamma < 1$ , they change to  $(1, 0)$  and support  $L$ . Party  $L$ 's vote share given state  $\theta$  is then  $\frac{1}{4} \left( 1 + \phi_{(0,0)}^+[\theta] + \phi_{(1,1)}^-[\theta] \right)$ , which is bigger than  $\frac{1}{2}$  when  $\theta = \theta_r$  and smaller than  $\frac{1}{2}$  when  $\theta = \theta_\ell$  by assumption (ii). Therefore, party  $L$ 's probability of winning is  $\frac{1}{2}$ , which means that  $L$ 's expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . If  $L$  deviates to  $(0, 0)$ , then voters with initial ideal point  $(0, 0)$  support  $L$  and voters with initial ideal point  $(0, 1)$  support  $R$ . For voters with initial ideal point  $(1, 1)$ , the highest payoff comes from changing to  $(0, 1)$  and supporting  $R$ . For voters with initial ideal point  $(1, 0)$ , the highest payoff comes from changing to  $(0, 0)$  and support  $L$ . Thus  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (1, 0)$  and party  $L$  announces  $(x^L, y^L) = (0, 1)$ , then voters with initial ideal point  $(0, 1)$  will support  $L$  and voters with initial ideal point  $(1, 0)$  will support  $R$ . Voters with initial ideal point  $(0, 0)$  get a payoff of  $-\gamma$  by not changing their ideal point and supporting  $L$ , and  $-1$  by not changing their ideal point and supporting  $R$ . If they change the ideal point and support  $L$ , then the most profitable ideal point is  $(0, 1)$ , which gives a payoff of  $-\gamma\kappa$ ; if they change the ideal point and support  $R$ , then the most profitable ideal point is  $(1, 0)$ , which gives a payoff of  $-\kappa$ . When  $\gamma > 1$ ,  $-\gamma < -1 < -\kappa$  and  $-\gamma\kappa < -\kappa$ , which means voters with ideal point  $(0, 0)$  support  $R$  and when  $\gamma < 1$  they support  $L$ . Using an analogous argument we can show that when  $\gamma > 1$ , voters with ideal point  $(1, 1)$  support  $L$  and when  $\gamma < 1$  they support  $R$ . Party  $L$ 's vote share given state  $\theta$  is then  $\frac{1}{4} \left( 1 + \phi_{(0,0)}^-[\theta] + \phi_{(1,1)}^+[\theta] \right)$ , which is bigger than  $\frac{1}{2}$  when  $\theta = \theta_\ell$  and smaller than  $\frac{1}{2}$  when  $\theta = \theta_r$  by assumption (ii). Therefore, party  $L$ 's probability of

winning is  $\frac{1}{2}$ , which means that  $L$ 's expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . If  $L$  deviates to  $(0,0)$ , then its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{1}{2}$ . Therefore, the deviation is profitable.

If party  $L$  announces  $(x^L, y^L) = (1,0)$  and  $R$  stays  $(x^R, y^R) = (1,0)$ , then voters with any initial ideal point are indifferent between choosing either party. Thus each party's probability of winning is  $\frac{1}{2}$ , which means party  $L$ 's expected payoff is  $-\lambda^L$ . If  $L$  deviates to  $(0,0)$ , voters with initial ideal point  $(1,0)$  will support  $L$  and voters with initial ideal point  $(1,1)$  will support  $R$ . Voters with initial ideal point  $(0,0)$  get the highest payoff by changing to  $(1,0)$  and supporting  $L$ . Voters with initial ideal point  $(0,1)$  get the highest payoff by changing to  $(1,1)$  and supporting  $R$ . Thus party  $R$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{\lambda^R}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (0,1)$  and party  $L$  announces  $(x^L, y^L) = (0,1)$ , then voters with any initial ideal point are indifferent between choosing either party. Thus each party's probability of winning is  $\frac{1}{2}$ , which means party  $L$ 's expected payoff is  $-\lambda^L$ . If  $L$  deviates to  $(0,0)$ , then  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (1,1)$  and party  $L$  announces  $(x^L, y^L) = (1,1)$ , then voters with any initial ideal point are indifferent between choosing either party. Thus each party's probability of winning is  $\frac{1}{2}$ , which means party  $L$ 's expected payoff is  $-1 - \lambda^L$ . If  $L$  deviates to  $(0,1)$ , then its expected payoff is  $-\frac{1}{2} - \lambda^L$ . Therefore, the deviation is profitable.

If party  $L$  announces  $(x^L, y^L) = (0,0)$  and party  $R$  announces  $(x^R, y^R) = (0,0)$ , then voters with any initial ideal point are indifferent between choosing either party. Thus each party's probability of winning is  $\frac{1}{2}$ , which means party  $R$ 's expected payoff is  $-1 - \lambda^R$ . If  $R$  deviates to  $(0,1)$ , then voters with initial ideal point  $(0,1)$  will support  $R$  and voters with initial ideal point  $(0,0)$  will support  $L$ . Voters with initial ideal point  $(1,1)$  get the highest payoff by changing to  $(0,1)$  and supporting  $R$ . Voters with initial ideal point  $(1,0)$  get the highest payoff by changing to  $(0,0)$  and supporting  $L$ . Thus party  $R$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-1 - \frac{1}{2}\lambda^R$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (0,0)$  and party  $L$  announces  $(x^L, y^L) = (1,1)$ , then voters with initial ideal point  $(0,0)$  will support  $R$  and voters with initial ideal point  $(1,1)$  will support  $L$ . Voters with initial ideal point  $(1,0)$  get  $-\gamma$  by not changing their ideal point and supporting  $L$  and  $-1$  by not changing their ideal point and supporting  $R$ . If they change their ideal point and support  $L$ , the most profitable ideal point is

$(x^n, y^n) = (1, 1)$ , which gives them a payoff of  $-\gamma\kappa$ . If they change their ideal point and support  $R$ , the most profitable ideal point is  $(0, 0)$ , which gives them a payoff of  $-\kappa$ . When  $\gamma > 1$ ,  $-\gamma < -1 < -\kappa$  and  $-\gamma\kappa < -\kappa$ , so voters with initial ideal point  $(1, 0)$  will change their ideal point to  $(0, 0)$  and support  $R$ . When  $\gamma < 1$ ,  $-1 < -\gamma < -\gamma\kappa$  and  $-\kappa < -\gamma\kappa$ , they will change their ideal point to  $(1, 1)$  and support  $L$ . Using an analogous argument, we can show that when  $\gamma > 1$ , voters with ideal point  $(0, 1)$  will change their ideal point to  $(1, 1)$  and support  $L$ ; when  $\gamma < 1$ , they will change their ideal point to  $(0, 0)$  and support  $R$ . Thus party  $L$ 's vote share is  $\frac{1}{4} \left( 1 + \phi_{(0,1)}^+[\theta] + \phi_{(1,0)}^-[\theta] \right)$ , which is bigger than  $\frac{1}{2}$  when  $\theta = \theta_r$  and smaller than  $\frac{1}{2}$  when  $\theta = \theta_\ell$  by assumption (i). It means that party  $L$ 's expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . If  $L$  deviates to  $(0, 1)$ , voters with initial ideal point  $(0, 1)$  will support  $L$  and voters with initial ideal point  $(0, 0)$  will support  $R$ . Voters with initial ideal point  $(1, 1)$  get the highest payoff by changing to  $(0, 1)$  and supporting  $L$ ; voters with initial ideal point  $(1, 0)$  get the highest payoff by changing to  $(0, 0)$  and supporting  $R$ . Thus party  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (0, 1)$  and party  $L$  announces  $(x^L, y^L) = (1, 1)$ , then voters with initial ideal point  $(0, 1)$  will support  $R$  and voters with initial ideal point  $(1, 1)$  will support  $L$ . Voters with initial ideal point  $(1, 0)$  get the highest payoff by changing to  $(1, 1)$  and supporting  $L$ ; voters with initial ideal point  $(0, 0)$  get the highest payoff by changing to  $(0, 1)$  and supporting  $R$ . Thus party  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{1}{2} - \lambda^L$ . If  $L$  deviates to  $(0, 1)$ , its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\lambda^L$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (1, 0)$  and party  $L$  announces  $(x^L, y^L) = (1, 1)$ , then voters with initial ideal point  $(1, 0)$  will support  $R$  and voters with initial ideal point  $(1, 1)$  will support  $L$ . Voters with initial ideal point  $(0, 1)$  get the highest payoff by changing to  $(1, 1)$  and supporting  $L$ ; voters with initial ideal point  $(0, 0)$  get the highest payoff by changing to  $(1, 0)$  and supporting  $R$ . Thus party  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-1 - \frac{1}{2}\lambda^L$ . If  $L$  deviates to  $(0, 1)$ , its probability of winning is  $\frac{1}{2}$ , which means that  $L$ 's expected payoff is  $-\frac{1}{2} - \frac{\lambda^L}{2}$ . Therefore, the deviation is profitable.

If party  $L$  announces at  $(x^L, y^L) = (0, 1)$  and party  $R$  announces  $(x^R, y^R) = (0, 0)$ , then voters with initial ideal point  $(0, 0)$  will support  $R$  and voters with initial ideal point  $(0, 1)$  will support  $L$ . Voters with initial ideal point  $(1, 1)$  get the highest payoff by changing to  $(0, 1)$  and supporting  $L$ ; voters with initial ideal point  $(1, 0)$  get the highest payoff by changing to  $(0, 0)$  and supporting  $R$ . Thus party  $R$ 's probability of winning is  $\frac{1}{2}$ ,



which means its expected payoff is  $-1 - \frac{1}{2}\lambda^R$ . If  $R$  deviates to  $(0, 1)$ , then its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-1$ . Therefore, the deviation is profitable.

If party  $R$  announces  $(x^R, y^R) = (0, 0)$  and party  $L$  announces  $(x^L, y^L) = (1, 0)$ , then voters with initial ideal point  $(0, 0)$  will support  $R$  and voters with initial ideal point  $(1, 0)$  will support  $L$ . Voters with initial ideal point  $(0, 1)$  get the highest payoff by changing to  $(0, 0)$  and supporting  $R$ ; voters with initial ideal point  $(1, 1)$  get the highest payoff by changing to  $(1, 0)$  and supporting  $L$ . Thus party  $L$ 's probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $-\frac{1}{2}$ . If  $L$  deviates to  $(0, 0)$ , its probability of winning is  $\frac{1}{2}$ , which means its expected payoff is  $0$ . Therefore, the deviation is profitable.  $\square$