Information Aggregation Failure in a Model of Social Mobility*

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Abstract

I study a model in which high and low income voters must decide between continuing with a status quo policy or switching to a different policy, which is more redistributive. Under the status quo policy, low income voters may get an opportunity to become high income earners. Such opportunities are expected to arise infrequently, so in expectation these voters prefer the more redistributive policy. Nevertheless, there is an equilibrium in which the vast majority of them cast their ballots in favor of re-electing the less redistributive status quo. Although these voters are fully strategic and correctly assess their chances for upward mobility, they vote exactly *as if* they are naive voters who over-estimate their chances of becoming rich. As the size of the electorate grows, the status quo policy can be re-elected with certainty, and the election fails to fully aggregate information.

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1 Introduction

This paper demonstrates the failure of elections to aggregate private information in an environment where strategic voters have redistributive preferences and get opportunities for social mobility. This result stands in contrast to existing results by Feddersen and Pesendorfer (1997) and much of the subsequent literature in which elections were shown to fully aggregate private information.

The model in which I derive this result has two groups of voters—high and low income—who must collectively decide whether to re-elect a status quo policy or switch to an alternative policy, which is more redistributive. Under the status quo policy, low income voters may get an opportunity to become high income earners, but will succeed in climbing the economic ladder only if they are talented. Low income voters do not know whether or not they are talented unless they receive the opportunity, in which case they learn about their talent from either their success or failure. Low income voters are uncertain about the probability with which they can expect to receive the opportunity for social mobility under the status quo, but they expect it to be low. Consequently, these voters prefer the more redistributive policy to the status quo. The status quo policy is in effect for one period, after which an election is held to decide whether to continue with it in the next period or to switch to the more redistributive policy.

I characterize the set of equilibria of the model in the limit as the number of voters grows to infinity. If the probability that a voter is talented is high, then in this limit there are generically three equilibria. One of these is a pure strategy equilibrium in which several low income voters, who are fully strategic and correctly assess their chances of upward mobility, cast their ballots exactly as if they were naive (sincere) voters who hold mistakenly optimistic beliefs about their mobility prospects. The voters who behave this way are precisely the vast majority of low income voters who did not receive the economic opportunity in the first period. All of these voters vote to re-elect the status quo policy despite knowing, at the time they vote, that the more redistributive policy gives them a strictly higher expected payoff.

The logic behind this result is as follows. In any equilibrium of the model, high income voters (including those low income voters who climbed the economic ladder in the first period) vote to re-elect the status quo policy, while low income voters who got the opportunity and learned that they are untalented vote for the more redistributive policy. If all low income voters who did not receive the economic opportunity in the first period vote *en masse* to re-elect the status quo, then the election should not be close: the status quo should win almost surely when the size of the electorate is large. But if the election is close, and a low income voter is pivotal, it can only be because nearly half of the electorate consists of low income voters who received the economic opportunity, learned that they were untalented, and voted for the more redistributive policy. For so many low income voters to learn that they are untalented, it must be that economic opportunities are plentiful under the status quo. When this is the case, and low income voters believe that they are likely enough to be talented, it is optimal for them to vote for the status quo. The status quo is re-elected almost surely.

Under such behavior, the election fails to fully aggregate information. This means that there is a positive *ex ante* probability that in a complete information version of the same voting game—i.e., one in which all voters knew the likelihood of getting the economic opportunity—the status quo policy would *not* be re-elected in limit as the number of voters gets large. This happens precisely when the likelihood of getting the economic opportunity is low, in which case the low income voters who did not get the opportunity in the first period would want to elect the more redistributive policy. With complete information, the law of large numbers implies that status quo policy wins almost surely as the size of the electorate gets large. Instead, in the model with incomplete information, there is an equilibrium in which the status quo policy wins with strictly positive probability in the large electorate limit, implying that the election fails to fully aggregate information.[1]

The observation that strategic voters may not vote sincerely goes back to the seminal papers of Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). This paper uses their approach to address the classical political economy question of whether it could be rational for the poor to vote against redistribution, thereby creating a link between the classical political economy literature on redistributive politics and the strategic voting literature that studies models in which voters are privately informed. I answer the classical political economy question in the affirmative, and show that the seemingly irrational behavior of low income voters can in fact be rationalized without adding issue dimensions, such as social issues, or assuming that voters have non-material (ideological) interests. Voters in my model are fully rational and strategic, but their behavior is observationally indistinguishable from that of naive voters who are overly optimistic about their social mobility prospects.

[1]The described result holds in an equilibrium that exists when the probability that any given voter is talented is high, but in this case there also exists an equilibrium that fully aggregates information. On the other hand, if the probability that a voter is talented is low, then there is a unique equilibrium that fully aggregates information, while if this probability is in an intermediate range then there is a unique equilibrium that fails to fully aggregate information.

2 The Political Environment

There are two periods and two policies. In the first period, a status quo policy is in effect. Then, an election is held to decide whether to continue with it in the next period, or to switch to a different policy, which is more redistributive. The policy that gets a majority of votes is implemented in the second period.

There are $2n + 1$ voters. At the start of each period, each voter's income is drawn independently. With probability $\lambda \in (0, \frac{1}{2})$ a voter has a high income, and with remaining probability $1 - \lambda$ he has a low income.[2] Under the more redistributive policy, each high income voter receives a payoff ψh, where $\psi < 1$ and $h > 0$, while each low income voter receives a payoff of 0. Under the status quo policy, each high income voter has payoff h, while each low income voter has probability δ of receiving an opportunity to become a high income voter. If a low income voter receives the opportunity to climb the economic ladder and he is *talented*, then he becomes a high income voter and receives the payoff h. If he is not talented or if he does not receive the opportunity, then his payoff is $-l < 0$. Each low income voter has prior probability $p \in (0, 1)$ of being talented. If a voter becomes a high income earner in the first period, he remains a high income earner in the second period. Voters who receive an opportunity to become high income earners in the first period and are unsuccessful learn that they are untalented, and would be unsuccessful also in the following period. Voters who do not receive the opportunity in the first period do not learn whether or not they are talented.

All voters know the success rate p, but they are uncertain about δ. This uncertainty can be taken to reflect uncertainty about whether the status quo policy "works well:" if δ is high, then opportunities for social mobility are plentiful and the status quo policy works well, whereas if δ is low then opportunities are scarce and the policy does not work well. In particular, if the status quo policy is re-elected, then the value of δ in the second period is the same as in the first. Each voter experiences the consequences of the first period policy only for himself; thus, each voter is uninformed about who (and how many others) received the opportunity to climb the economic ladder, and whether or not they succeeded.

[2] This assumption implies that in a large electorate (i.e., as $n \to \infty$) high income voters are a fraction λ of the population and are outnumbered by low income voters—a common assumption. In a finite electorate, on the other hand, the fraction of high income voters is random, and with positive probability the rich will outnumber the poor. Since all of the main results are about large electorates, I maintain the assumption that each voter's income-type is an independent random draw, which simplifies the analysis. A pervious working paper version of this paper (Acharya 2013) analyzed the case in which the share of each income-type is fixed even in finite electorates.

Let F denote the distribution of δ. I assume that F has a continuous density f, and $f(\delta) \geq \nu$ for some $\nu > 0$ and all $\delta \in [0,1]$. This assumption serves the same role in my analysis as Feddersen and Pesendorfer's (1997) Assumption 2 does in theirs.[3] It ensures that as the electorate grows, pivotal beliefs about the state get concentrated on the value that brings the probability of drawing a vote for either alternative closest to $\frac{1}{2}$. Let $\mathbb{E}[\delta]$ denote the expected value of δ according to the prior F.

Given δ, the expected payoff from the status quo policy for a low income voter is

$$y(\delta) = -(1-\delta p)l + \delta p h = -l + \delta p(l+h) \tag{1}$$

which is linear and increasing in δ. Therefore, the unconditional expected payoff to re-electing the status quo policy for a low income voter who did not receive the economic opportunity is simply $y(\mathbb{E}[\delta])$. I assume that

$$y(\mathbb{E}[\delta]) < 0 < y(1) \tag{2}$$

which says that low income voters do not expect the status quo policy to work well, and therefore they prefer the more redistributive policy; however, if they thought that the status quo policy did work well, then they would prefer that policy to the more redistributive one.

Since $y(\delta)$ is strictly increasing in δ, there is a cutoff

$$\bar{\delta} = \frac{1}{p}\left(\frac{l}{l+h}\right) \tag{3}$$

such that a low income voter who does not yet know whether or not he is talented would prefer the status quo policy if the expected value of δ were above $\bar{\delta}$, and would prefer the more redistributive policy if it were below $\bar{\delta}$. Note that the assumption $0 < y(1)$ made in (2) implies $p > l/(l+h)$, which means that $\bar{\delta} < 1$.

3 Voting Behavior and Information Aggregation

A type-contingent strategy for a voter maps his type to the probability with which he votes to re-elect the status quo policy. To analyze the model, I study type-symmetric Bayes Nash equilibrium in weakly undominated strategies, which I refer to simply as

[3] Feddersen and Pesendorfer (1997) also assume that the density of the state is bounded above.

"equilibrium" (Feddersen and Pesendorfer 1997). A type-symmetric equilibrium is one in which all voters of the same type use the same type-contingent strategy. There are four types of voters in the model: (i) high income earners, H, (ii) low income earners that became high income earners in the first period, L^+, (iii) low income earners that got the opportunity in the first period but did not succeed, L^-, and (iv) low income earners that did not get the opportunity in the first period, L^0. Note that H and L^+ voters have a weakly dominant strategy to vote to re-elect the status quo, while L^- voters have a weakly dominant strategy to vote for the more redistributive policy.[4]

In equilibrium, the L^0 voters must vote as if they are pivotal; that is, they condition on the hypothetical event that apart from their own vote, the election is tied (see, e.g., Austen-Smith and Banks 1996). Since our equilibrium is type-symmetric all of these L^0 voters vote for the status quo with the same probability. Call that probability x. The probability of drawing a vote for the status quo policy from a random voter, conditional on δ, is then

$$\pi(\delta, x) = \lambda + (1-\lambda)[\delta p + (1-\delta)x] \tag{4}$$

The probability of a voter being pivotal is

$$\beta(\delta|x, n) = \binom{2n}{n}(\pi(\delta, x))^n(1-\pi(\delta, x))^n. \tag{5}$$

For an L^0 voter, the distribution of δ conditional on his type and the pivotal event is

$$f^{piv}(\delta|x, n, L^0) = \frac{\beta(\delta|x, n)(1-\delta)f(\delta)}{\int_0^1 \beta(\tilde{\delta}|x, n)(1-\tilde{\delta})f(\tilde{\delta})d\tilde{\delta}}, \tag{6}$$

which is well-defined because $0 < \pi(\delta, x) < 1$ for all (δ, x). Therefore, conditional on being pivotal and being an L^0 type, the expected value of δ is

$$\mathbb{E}^{piv}[\delta|x, n, L^0] = \int_0^1 \delta f^{piv}(\delta|x, n, L^0)d\delta. \tag{7}$$

In equilibrium, L^0 voters vote to re-elect the status quo policy if $\mathbb{E}^{piv}[\delta|x, n, L^0] > \bar{\delta}$ and vote for the more redistributive alternative if $\mathbb{E}^{piv}[\delta|x, n, L^0] < \bar{\delta}$. They mix between the two only if $\mathbb{E}^{piv}[\delta|x, n, L^0] = \bar{\delta}$. I use these facts to establish the existence of an

[4] In a Poisson game formulation *a la* Myerson (1998) these types of voters would have *strictly* dominant strategies. I do not adopt the Poisson game formulation so as to make my results more directly comparable to those of Feddersen and Pesendorfer (1997).

equilibrium for all values of n. The following proposition records this result, along with the equilibrium behavior of voters of type H, L^+ and L^-.

Proposition 1. *For every finite n, an equilibrium to the voting game exists. In every equilibrium voters of type H and L^+ vote for the status quo policy, while voters of type L^- vote for the more redistributive policy.*

Proof. See Appendix A. □

Given that the behavior of types H, L^+ and L^- is fixed for all n, I will henceforth abuse terminology and identify an equilibrium with the probability x with which each L^0 type voter votes to re-elect the status quo policy.

3.1 Analysis of Large Electorates

To understand the behavior of the L^0 types of voters, it helps to understand the properties of $\mathbb{E}^{piv}[\delta|x, n, L^0]$—in particular, when is this conditional expectation of δ below the threshold $\bar{\delta}$ defined in (3) and when is it above? Proposition 2 below shows that even though $\mathbb{E}^{piv}[\delta|x, n, L^0]$ is a complicated object for finite n, it is straightforward to characterize its limit in n for every value of x. This makes it possible to characterize the behavior of the L^0 voters in large electorates.

Fix any $x \neq p$, and note that the probability $\pi(\delta, x)$ that a randomly drawn voter votes for the status quo is strictly monotonic in δ, so there is a unique value of δ that minimizes $|\pi(\delta, x) - \frac{1}{2}|$. Denote this value by $\delta^*(x)$. This is the value of δ that brings the probability of drawing a vote for either alternative closest to $\frac{1}{2}$, i.e., the value that is most likely to deliver a tied election. The proposition below then states three facts: (i) $\mathbb{E}^{piv}[\delta|p, n, L^0] = \mathbb{E}[\delta]$, (ii) $\mathbb{E}^{piv}[\delta|x, n, L^0]$ converges to $\delta^*(x)$ for all $x \neq p$, and (iii) if $x \neq p$, then $\mathbb{E}^{piv}[\delta|\cdot, n, L^0]$ can be made arbitrarily close to $\delta^*(x)$ in a neighborhood of x by choosing n large enough and the neighborhood small enough.

Proposition 2. *If $x = p$ then $\mathbb{E}^{piv}[\delta|x, n, L^0] = \mathbb{E}[\delta]$ for all n. If $x \neq p$, then for all $\epsilon > 0$, there exists $\rho > 0$ and a number N such that $n \geq N$ implies*

$$\left|\mathbb{E}^{piv}[\delta|\tilde{x}, n, L^0] - \delta^*(x)\right| \leq \epsilon \qquad \forall \tilde{x} \in \mathcal{B}_\rho(x) := \{\tilde{x} \in [0,1] : |x - \tilde{x}| \leq \rho\}.$$

Proof. See Appendices B and C. □

The proof of the proposition in Appendix C relies on a general result presented in Appendix B. I show that if $x \neq p$, then the conditional distribution $f^{piv}(\cdot|x, n, L^0)$ converges to a Dirac mass at $\delta^*(x)$ as n gets large. In this sense, Proposition 1 belongs to a class of results that trace their origins to an argument in statistics by Bayes (1763) himself, and to a result in the voting literature by Good and Mayer (1975) and Chamberlain and Rothschild (1981).[5] Various extensions and applications of these results appear in several more recent papers, including Feddersen and Pesendorfer (1997), Mandler (2012) and Krishna and Morgan (2012). There are many ways to prove Proposition 2. I adopt the approach of Feddersen and Pesendorfer (1997).

3.2 Graphical Analysis

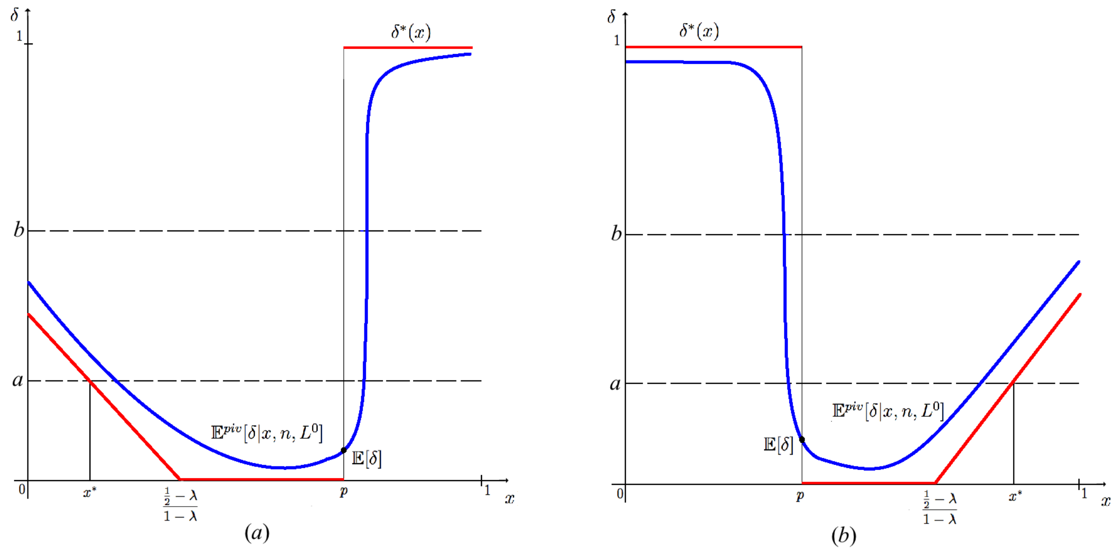
Although $\mathbb{E}^{piv}[\delta|\cdot, n, L^0]$ is a complicated object, Proposition 2 suggests that when n is large, we can study the properties of $\mathbb{E}^{piv}[\delta|\cdot, n, L^0]$ by studying the properties of $\delta^*(\cdot)$, which is a much less complicated object. I now solve explicitly for $\delta^*(\cdot)$ and use it to provide some graphical intuition for the formal equilibrium analysis that follows.

Since $\pi(\cdot, x)$ in (4) is a linear function of δ with slope $(1-\lambda)(p-x)$, it is strictly increasing in δ for $x < p$, strictly decreasing in δ for $x > p$, and constant in δ for $x = p$. Also, note that $\pi(1, x) = \lambda + (1-\lambda)p$, which is independent of x, and consider first the case where $\pi(1, x) = \lambda + (1-\lambda)p > \frac{1}{2}$. If $x > \frac{\frac{1}{2}-\lambda}{1-\lambda}$ then $\pi(\delta, x)$ remains above $\frac{1}{2}$ for all δ. In particular, if $\frac{\frac{1}{2}-\lambda}{1-\lambda} < x < p$ then $\pi(\delta, x)$ is upward sloping so $\pi(\delta, x)$ gets closest to $\frac{1}{2}$ at $\delta = 0$; thus $\delta^*(x) = 0$ for $\frac{\frac{1}{2}-\lambda}{1-\lambda} < x < p$. Similarly, if $p < x \leq 1$ then $\pi(\delta, x)$ is downward sloping so $\pi(\delta, x)$ gets closest to $\frac{1}{2}$ at $\delta = 1$; thus $\delta^*(1) = 1$ for $p < x \leq 1$. On the other hand, if $0 \leq x \leq \frac{\frac{1}{2}-\lambda}{1-\lambda}$, then $\pi(\delta, x)$ does intersect $\frac{1}{2}$ meaning that $|\pi(\delta, x) - \frac{1}{2}|$ takes a minimum value of 0. Thus, solving $\pi(\delta, x) = \frac{1}{2}$ for δ gives us $\delta^*(x)$ in the case where $0 \leq x \leq \frac{\frac{1}{2}-\lambda}{1-\lambda}$. The solution is

$$\delta^*(x) = \frac{1}{p-x}\left(\frac{\frac{1}{2}-\lambda}{1-\lambda} - x\right) \tag{8}$$

Therefore, as Figure 1(a) depicts, $\delta^*(x)$ is strictly and continuously decreasing from $\delta^*(0) = \frac{1}{p}\frac{\frac{1}{2}-\lambda}{1-\lambda}$ to 0 on the interval $[0, \frac{\frac{1}{2}-\lambda}{1-\lambda}]$, constantly equal to 0 on the interval $(\frac{\frac{1}{2}-\lambda}{1-\lambda}, p)$, and constantly equal to 1 on the interval $(p, 1]$. Analogously, if we consider the case where $\pi(1, x) = \lambda + (1-\lambda)p < \frac{1}{2}$, then we find that $\delta^*(x)$ is constantly equal to 1 on the

[5] Good and Mayer (1975) and Chamberlain and Rothschild (1981) used Bayes's original argument to show that if F is a distribution on $[0,1]$ with continuous density f, then $\lim_{N\to\infty} N\int_0^1 \binom{2N}{N}\alpha^N(1-\alpha)^N dF(\alpha) = \frac{1}{2}f(\frac{1}{2})$. I thank David Myatt and Vijay Krishna both for telling me about this.



Figures 1(a) and (b): Both depict the function $\delta^*(\cdot)$ in red and the function $\mathbb{E}^{piv}[\delta|\cdot, n, L^0]$ in blue for large n. (a) depicts the case where the probability that a random voter votes for the status quo when $\delta = 1$ is $\pi(1, x) = \lambda + (1-\lambda)p > \frac{1}{2}$. (b) depicts the case where $\pi(1, x) = \lambda + (1-\lambda)p < \frac{1}{2}$.

interval $[0, p)$, constantly equal to 0 on the interval $(p, \frac{\frac{1}{2}-\lambda}{1-\lambda})$, and strictly and continuously increasing from 0 to $\delta^*(1) = \frac{1}{1-p}\frac{1/2}{1-\lambda}$ on the interval $[\frac{\frac{1}{2}-\lambda}{1-\lambda}, 1]$. Figure 1(b) depicts this case.

Both Figures 1(a) and (b) also depict hypothetical curves for $\mathbb{E}^{piv}[\delta|x, n, L^0]$ drawn close to $\delta^*(x)$. Since Proposition 2 implies that $\mathbb{E}^{piv}[\delta|x, n, L^0]$ converges to $\delta^*(x)$ for all $x \neq p$, the figures are intended to represent $\mathbb{E}^{piv}[\delta|x, n, L^0]$ for large n. Note that $\mathbb{E}^{piv}[\delta|x, n, L^0]$ is continuous and $\mathbb{E}^{piv}[\delta|p, n, L^0] = \mathbb{E}[\delta]$.

In light of Proposition 3, the figures are suggestive of which values of x are limit equilibria. For example, in a large electorate, voting for the status quo policy is a best response if $\delta^*(x) \geq \bar{\delta}$; thus $x = 1$ is a limit equilibrium when $\delta^*(1) > \bar{\delta}$. Similarly, voting for the more redistributive policy is a best response if $\delta^*(x) < \bar{\delta}$; thus $x = 0$ is a limit equilibrium when $\delta^*(0) < \bar{\delta}$. Any $x \in (0, 1)$ is a limit equilibrium only if $\delta^*(\cdot)$ cuts $\bar{\delta}$ at x.

With this in mind, consider the case of Figure 1(a) where $\pi(1, x) = \lambda + (1-\lambda)p > \frac{1}{2}$, and suppose that $\bar{\delta} < \delta^*(0)$. This is equivalent to $\frac{l}{l+h} < \frac{\frac{1}{2}-\lambda}{1-\lambda}$, which follows because $\bar{\delta} = \frac{1}{p}\frac{l}{l+h}$ from (3), and $\delta^*(0) = \frac{1}{p}\frac{\frac{1}{2}-\lambda}{1-\lambda}$. The figure depicts this as the case where $\bar{\delta}$ equals a number a that is smaller than $\delta^*(0)$. The figure suggests that in this case $x = 1$ is an equilibrium since $\delta^*(1) > \bar{\delta}$. It also suggests that there are equilibria close to $x = p$ and to $x = x^*$ where

$$x^* := \left[\frac{\frac{1}{2}-\lambda}{1-\lambda} - \frac{l}{l+h}\right] \Big/ \left[1 - \frac{1}{p}\frac{l}{l+h}\right] \tag{9}$$

which is calculated by solving $\delta^*(x) = \bar{\delta}$ for x, knowing that $\bar{\delta}$ is given by (3) and $\delta^*(x)$ is given by (8) when $x \in [0, \frac{\frac{1}{2}-\lambda}{1-\lambda}]$. The reason that the figure suggests that there are equilibria close to these points is because these points are close to the intersection points of $\mathbb{E}^{piv}[\delta|x, n, L^0]$ with $\bar{\delta}$ when $\bar{\delta} = a$. Similarly, if $\bar{\delta}$ equals a number b that is higher than $\delta^*(0) = \frac{1}{p}\frac{\frac{1}{2}-\lambda}{1-\lambda}$, then Figure 1(a) suggests that $x = 0$ and $x = 1$ are equilibria, and that there is also an equilibrium close to $x = p$.

Now examine Figure 1(b). Suppose that $\bar{\delta}$ equals a number a that is smaller than $\delta^*(1) = \frac{1}{1-p}\frac{1/2}{1-\lambda}$. That is, suppose that

$$p > \frac{\left[\frac{l}{l+h}\right]\left[\frac{1-\lambda}{1/2}\right]}{1 + \left[\frac{l}{l+h}\right]\left[\frac{1-\lambda}{1/2}\right]} \tag{10}$$

Then again there appear to be three equilibria, one at $x = 1$, one close to $x = p$, and another close to $x = x^*$, where x^* is again given by (9). Note that these are the same values of x as in the case of Figure 1(a) above when $\bar{\delta} < \delta^*(0)$. If $\bar{\delta}$ equals a number b larger than $\delta^*(1)$ so that (10) holds with reverse inequality, then there appears to be only one equilibrium close to $x = p$.

3.3 Limit Equilibria

I now formalize the graphical analysis of the previous section, and provide a full characterization of the equilibrium set for large values of n. First, I review two familiar concepts. I say that x is a *large electorate equilibrium* if there exists N such that $n \geq N$ implies that x is an equilibrium of the voting game when the population parameter is n. I say that x_∞ is a *limit equilibrium* if it is an accumulation point of a sequence of equilibria corresponding to a sequence of games indexed by n.[6] Note that when the size of the electorate is large, every equilibrium to the game has to be close to a limit equilibrium.[7]

[6] More formally, denote the game by G_n. For every sequence of voting games $\{G_n\}$ there exists a sequence of equilibria $\{x_n\}$ where each x_n denotes the probability with which the L^0 voters vote for the the status quo policy in game G_n. x_∞ is an accumulation point of such a sequence $\{x_n\}$ if the sequence has a subsequence that converges to x_∞. Since $\{x_n\} \subset [0, 1]$, the existence of limit equilibria follows from the existence of equilibria for games G_n, established in Proposition 1.

[7] Suppose, for the sake of contradiction, that a sequence of equilibria $\{x_n\}$, each corresponding to the game with population parameter n, has a subsequence that remains bounded away from all of the limit equilibria. That subsequence would in turn contain a sub-subsequence that converges to a limit that is

Also, note that every large electorate equilibrium is a limit equilibrium, but the converse is not true.

Proposition 3. *Suppose that $\lambda + (1-\lambda)p > \frac{1}{2}$ holds, or $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) both hold. Then, there are generically three limit equilibria: (i) $x_\infty = \max\{0, x^*\}$, where x^* is defined in (9), (ii) $x_\infty = p$, and (iii) $x_\infty = 1$. When $x_\infty \in \{0, 1\}$ is a limit equilibrium, it is also a large electorate equilibrium. If $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) holds with reverse inequality, then $x_\infty = p$ is the unique limit equilibrium.*

Proof. See Appendix D. $\square$

The key result in Proposition 3 is that $x_\infty = 1$ is a limit equilibrium (and also a large electorate equilibrium) when $\lambda + (1-\lambda)p > \frac{1}{2}$, or when $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) both hold, which means that the probability of being talented, p, cannot be too small. If $p > \frac{1}{2}$, meaning that individuals are more likely to be talented than untalented, then $\lambda + (1-\lambda)p > \frac{1}{2}$ and we are in a case where $x_\infty = 1$ is a limit equilibrium. In this case, if the electorate is large, then a large fraction of low-income voters are poor not because they are intrinsically untalented, but because opportunities are scarce.

The right hand side of inequality (10) has the following comparative statics. It is increasing in l and decreasing in h and λ. In particular, as l increases, all else the same, then the redistributive policy becomes more attractive, and the inequality is harder to satisfy since the right hand side of the inequality increases. As h increases, the redistributive policy becomes less attractive, and the inequality becomes becomes easier to satisfy since the right hand side decreases. As λ decreases, the right hand side of the inequality increases, making it harder to satisfy.

I now provide some additional intuition for the pure strategy large electorate equilibria mentioned in Proposition 3. If $x_\infty = 0$, then none of the L^0 voters cast their ballots for the status quo policy. Therefore, the election is not likely to be close unless many low income voters received the economic opportunity, became rich, and voted for the status quo. This means that an L^0 type's expectation of δ, conditional on being pivotal, cannot be small. When $\bar{\delta}$ is larger than this conditional expectation of δ, the L^0 voters are happy to vote for the more redistributive policy. But when it is smaller, they would actually like to switch their votes to the status quo policy.

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bounded away from each limit equilibrium. But since this sub-subsequence is also a subsequence of $\{x_n\}$, its limit would have to be a limit equilibrium. Contradiction. Thus, when n is large, every equilibrium of the game has to be close to one of the limit equilibria.

Now consider the case of $x_\infty = 1$. If p is large and all voters in L^0 vote to re-elect the status quo policy, the election is unlikely to be close: the status quo policy should win by a large margin. But if the election does turn out to be close, then a large fraction of the electorate must have voted for the redistributive policy. Since only the untalented voters in L^- vote for this policy, it must be that a large number of voters discovered that they are untalented. They could only discover this by receiving the economic opportunity. In other words, for so many voters to discover that they are untalented, it must be that δ is large. Therefore, in this case, it makes sense—from the perspective of the L^0 voters—to vote for the status quo policy. On the other hand, if p is small, then the election is close when not as many low income voters get the opportunity. In this case, the L^0 voters may want to switch their votes from the status quo policy to the more redistributive one.

3.4 Information Aggregation

I now study the information aggregation properties of the limit equilibria characterized in the previous section. I say that a limit equilibrium x_∞ *aggregates information* if for almost every possible realization of δ, every subsequence of equilibria that converges to x has a limiting distribution over electoral outcomes that equals a limiting distribution over equilibrium outcomes of an otherwise identical subsequence of games, but in which δ is common knowledge in all of the games in this sequence. A more formal definition is given in Appendix E. The following proposition identifies which limit equilibria aggregate information and which do not.

Proposition 4. *If $\lambda + (1-\lambda)p > \frac{1}{2}$, then of the three possible limit equilibria mentioned in Proposition 3, $x_\infty = \max\{0, x^*\}$ aggregates information, while $x_\infty = p$ and $x_\infty = 1$ fail to aggregate information. If $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) holds, then none of the three limit equilibria mentioned in Proposition 3 aggregates information. If $\lambda + (1-\lambda)p < \frac{1}{2}$, and (10) holds with reverse inequality, then the unique limit equilibrium, $x_\infty = p$, aggregates information.*

Proof. See Appendix E. □

Proposition 4 covers three cases. In the first case, where p is large, there is a limit equilibrium, namely $x_\infty = \max\{0, x^*\}$, that aggregates information. But the two other limit equilibria, namely $x_\infty = p$ and $x_\infty = 1$, fail to aggregate information. Alternatively, there is a case where *all* limit equilibria fail to aggregate information. This is the case where $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) both hold. Finally, when p is very small so that

$\lambda + (1-\lambda)p < \frac{1}{2}$ and the reverse of (10) holds, there is a third case in which the unique limit equilibrium aggregates information.

The existence of equilibria in which information fails to aggregate is noteworthy given the many positive results on information aggregation beginning with the seminal work of Feddersen and Pesendorfer (1997). Nevertheless, the findings of a few other recent papers also suggest that aggregation failure may be a more common phenomenon than was initially suspected. Bhattacharya (2013), for example, shows that having voters with diverse preferences, as in this model, can lead to the existence of equilibria that fail to aggregate information, but typically there will exist at least one equilibrium that aggregates information.[8] Bhattacharya (2013) studies a jury-type setting with two states of the world, and shows that if voter preferences fail to satisfy a condition he calls "strong preference monotonicity," then there exists an equilibrium that fails to aggregate information. Although Bhattacharya (2013) studied a model with binary states, and the current paper has a continuum of states, an analogue to his strong preference monotonicity condition fails in my model when $\lambda + (1-\lambda)p > \frac{1}{2}$.[9] In contrast to these results that show the existence of *some* equilibria that fail to aggregate information, I show that in the case where $\lambda + (1-\lambda)p < \frac{1}{2}$ and (10) holds, *all* equilibria fail to aggregate information.[10]

Models in which information aggregation failure takes place with a continuum of signals include Mandler (2012), Gul and Pesendorfer (2009) and Section 6 of Feddersen and Pesendorfer (1997). However, the source of information aggregation failure in these models is not preference diversity. Instead, these models share the common feature that there is second order uncertainty, i.e. uncertainty about the the distribution of voter preferences. Therefore, the source of aggregation failure in these models is qualitatively different from the one here.

[8] Kim and Fey (2007) also study a model with preference diversity and show that there are equilibria that fail to aggregate information. However, they also allow voters to abstain as in Feddersen and Pesendorfer (1996).

[9] Strong preference monotonicity fails here because $\pi(\delta, 0) = \lambda + (1-\lambda)\delta p$ is increasing in δ while $\pi(\delta, 1) = \lambda + (1-\lambda)(\delta p + 1 - \delta)$ is decreasing in δ, and $x = 0$ is the optimal value of x in the case where the state is known to be any $\delta < \bar{\delta}$ while $x = 1$ is the optimal value of x in the case were the state is known to be any $\delta > \bar{\delta}$.

[10] In this case, McLennan's (1998) sufficient condition for information aggregation fails: there is no feasible symmetric strategy profile that aggregates information.

4 Discussion

4.1 A "Behavioral Equivalence" Interpretation

In the $x_\infty = 1$ limit equilibrium, the expected value of δ conditional on being pivotal is nearly 1. In other words, conditional on being pivotal, low income voters who did not receive the opportunity to climb the economic ladder in the first period believe that they are nearly certain to receive the opportunity in the next period, so they vote to re-elect the status quo policy. A naive voter who does not behave strategically, but has overly optimistic beliefs about his chances for upward mobility (e.g. one who incorrectly estimates the unconditional expectation of δ to be nearly equal to 1) would vote exactly in the same way as the strategic L^0 voter who conditions his vote on the pivotal event. In fact, the observable behavior of these two kinds of voters is identical, since conditional on being pivotal, the strategic L^0 voter also believes that δ is nearly 1. This "behavioral equivalence" is noteworthy considering the wide skepticism that much of the empirically observed behavior of the American electorate could be consistent with voter rationality.[11] It implies that if one had to draw inferences only from data on the voting patterns of low income individuals who appear to overestimate their chance of climbing the economic ladder and vote naively, one could not rule out the possibility that these are individuals who actually correctly assess their chances for upward mobility and vote strategically.

It is also worth noting that in the $x_\infty = 1$ equilibrium, conditional on being pivotal an L^0 voter believes not only that δ is large in expectation, but also that half of the electorate—specifically, those who support the redistributive policy—are untalented. The fact that this voter votes as if he simultaneously believes that he lives in a land of plentiful opportunities, and in which there are several untalented other voters who voted for the left wing redistributive policy, is an ironic but perhaps realistic feature of the model.

[11] See, e.g., Bartels (2008*a*), Achen and Bartels (2004), Healy, Malhotra and Mo (2010) and Wolfers (2002), among others. For example, Bartels (2008*a*) writes: "At the individual level ... psychological pressures produce unrealistic optimism about one's own prospects, and an illusion of control over uncontrollable events." And in response to Shenkman's (2008) accusation that "The consensus in the political science profession is that voters are rational," Bartels (2008*b*) writes, "Well, no. A half-century of scholarship provides plenty of grounds for pessimism about voters' rationality." In response to this skepticism, the behavioral equivalence result of this paper shows that seemingly unsophisticated behavior may in fact be observationally identical to equilibrium behavior. Moreover, when $\bar{\delta} < \delta^*(0)$, it is the case that in all limit equilibria the low income voters who did not receive the economic opportunity in the first period vote for the status quo policy with positive probability. So, despite the existence of multiple equilibria, there is a strong sense in which the model rationalizes the behavior of agents who know that the more redistributive policy is better for them, but nevertheless end up voting for the less redistributive policy.

4.2 Equilibrium in Pure Strategies

A feature of the model that sets it apart from other voting models with private information is the fact that when p is large enough, there always exists a large electorate equilibrium in pure strategies: $x = 1$ is an equilibrium of the voting game when the electorate is large. In fact, in the case where $\lambda + (1-\lambda)p > \frac{1}{2}$ and $\bar{\delta} < \delta^*(0)$ (so that the redistributive policy is still better for low income voters than the status quo, but not that much better) or when $\lambda + (1-\lambda)p < \frac{1}{2}$ and $\bar{\delta} < \delta^*(1)$ (which is equivalent to (10)), this is the only strict equilibrium. The other limit equilibria are in totally mixed strategies and are not strict.

4.3 Probability of Being Talented

When (10) holds, the assumption that $p \neq 1$ is crucial for generating the result that $x = 1$ is an equilibrium of the model for large electorates, as well as for generating the result that there is an equilibrium that fails to aggregate information. In particular, under the assumption that $p = 1$, all low income voters are certain to be talented, and the model has a unique limit equilibrium, $x_\infty = x^*$, which aggregates information. Therefore, the model requires uncertainty both about the probability of getting the economic opportunity and whether or not a given voter is talented.

However, it is also interesting to investigate the case where the parameter p is also unknown to the voters, in addition to the parameter δ being unknown. That is, voters are not only uncertain of how well the status quo policy works, they are also uncertain of how likely any given one of them is to be talented. Assume that p and δ are independently distributed, and suppose that the distribution of p has a continuous density g on the support $[\underline{p}, 1]$. Further, suppose that there is a number $\mu > 0$ such that $g(p) \geq \mu$ for all $p \in [\underline{p}, 1]$. Then we have the following result.

Proposition 5. *If*

$$\underline{p} \times \min\left\{1, \frac{1}{1-\underline{p}}\left(\frac{1/2}{1-\lambda}\right)\right\} > \frac{l}{l+h}$$

then $x_\infty = 1$ is a limit equilibrium (and a large electorate equilibrium) that fails to aggregate information. In the limit as $\lambda \to \frac{1}{2}$, the unique limit equilibrium (and large electorate equilibrium) is $x_\infty = 0$, which aggregates information.

Proof. See Appendix F. □

4.4 Redistributive Policy First

The results of the model also hinge on the assumption that the status quo policy is less redistributive than the alternative. If the more redistributive policy is implemented first, or if there is only one period and voters must choose between the two policies without first experiencing either policy, then the more redistributive policy always wins the election. In light of this, one may think that the model is one of "status quo bias" rather than "right wing bias."

Meltzer and Richard (1978) have suggested that it may be difficult to separate status quo bias from right wing bias. They argue that in the past century, the United States progressively adopted left wing policies to replace status quo policies that were defended by right wing politicians. Meltzer and Richard point to Roosevelt's New Deal and Johnson's Great Society—both examples of policies that saw intense opposition when they were first introduced, but whose programs, such as Social Security and Medicare, enjoy popular support today. In light of these facts, the assumption that the right wing policy comes first is consistent with the political competition over policies in recent American history. Consequently, it may be quite challenging to distinguish status quo bias from right wing bias not just in this model, but also in reality.[12]

4.5 Mobility Under the More Redistributive Policy

It is worth noting that I have not assumed that the more redistributive policy does not give low income voters a chance for upward social mobility. In particular, one can take the payoff to low income voters from the redistributive policy (here, normalized to 0) to be an *expected* payoff, arising from an underlying assumption about the extent to which the redistributive policy produces opportunities for upward mobility. It is sufficient for the conclusions of the model that the probability of receiving opportunities for upward mobility under the status quo policy is independent of the probability of receiving opportunities for upward mobility under the more redistributive policy. In this case, one can treat the payoff to low income voters under the redistributive policy as a constant

[12] Another model of status quo bias is that of Fernandez and Rodrik (1991), who argue that if a policy reform creates winners and losers and it is not possible to ex ante identify who the winners and losers will be, then even if it is ex post in the interest of a majority, it may be opposed by a majority. Both this paper and theirs builds upon the intuition that since voters do not know how a policy affects them individually, they may support or oppose it even when doing the opposite would be better for them. However, in Fernandez and Rodrik, the wedge is between ex ante and ex post welfare, while in this paper voters may strategically vote against a policy that they *know* is worse for them in expectation.

across all possible realizations of δ in the first period, since the voter cannot obtain any information about whether or not the redistributive policy works well from experiencing only the status quo policy.[13]

4.6 Other Works on Social Mobility and Redistribution

Where does this paper sit in the literature on social mobility and redistribution? Two prominent contributions are those of Piketty (1995) and Benabou and Ok (2001).

Piketty (1995) develops a model in which individuals use their and their ancestors' social mobility experiences to make inferences about the relative importance of luck and effort in determining economic success. He shows that some dynasties converge to the right wing belief that luck is relatively unimportant in determining success, and come to oppose redistribution, while other dynasties converge to the left wing belief that luck is important, and come to support redistribution. Piketty's rational agent is not purely individualistic; instead, his preferences also contain an ideological component. On the other hand, all agents in my model are purely individualistic, caring only about their material payoff.

Bénabou and Ok (2001) address the question of whether purely self-interested low income voters with rational expectations about their mobility prospects could vote against redistribution. They show that if the income transition function between periods is strictly concave, then such behavior is possible. However, Bénabou and Ok's story is considerably different from the one in this paper. While in my model, there is a tension between a voter's true (naive) preferences and his equilibrium behavior, no such tension exists in the Bénabou-Ok model. Relatedly, the Bénabou-Ok model does not exhibit information aggregation failure, which is one of the important features of my model.[14]

[13] Actually, the results of the model only rely on the existence of a threshold $\bar{\delta} < 1$ such that L^0 voters prefer the status quo policy when they expect δ to be greater than $\bar{\delta}$. In particular, this condition can be satisfied even when the probability for receiving an opportunity for upward mobility under the redistributive policy is perfectly correlated with the same probability under the status quo policy. For example, if the probability of receiving the opportunity under the more redistributive policy is $\kappa\delta$, then the critical threshold $\bar{\delta} < 1$ exists when $\kappa < \frac{l+h}{\psi h} - \frac{1}{p}\frac{l}{\psi h}$.

[14] In addition, Bénabou and Ok also allow for downward social mobility. They take their model to data, and argue that their findings indicate that the "[mobility] effect is probably dominated by the demand for social insurance." So, they conclude by issuing some skepticism that opposition to redistributive policies can be explained by beliefs concerning social mobility.

5 Conclusion

Many low income voters oppose increased redistribution even when they stand to gain from it. A longstanding question in political economy asks whether such opposition could ever be rational.

This paper developed a model of social mobility and redistribution in which numerous low income voters who care only about their economic payoff may, in a particular equilibrium, vote with certainty for a right wing status quo policy that they believe is worse for them than a competing left wing alternative. This occurs because voters are strategic, so they condition their vote on being pivotal: In the event that every vote for the wining policy matters, these voters believe that the status quo policy gives them a greater chance of upward mobility than it actually does. An important consequence of their behavior is the failure of the equilibrium to aggregate information.

These results are particularly relevant to democratic theorists who are concerned that a lack of voter sophistication causes democracy to fail on many dimensions, including preference– and information– aggregation. The aggregation failure result of this paper shows that even when voters behave rationally, they may collectively fail to elect the policy that is in the interest of the majority. In particular, these results imply that for democracy to succeed, we might have to place some peculiar demands on voters: that they not be overly optimistic and naive, but that they also not be fully rational.

One important issue that the paper does not address is what would happen if strategic candidates could strategically choose the left and right wing policies to win the support of voters. To my knowledge, there are few papers that study models of strategic voting with strategic candidates. One paper by Gul and Pesendorfer (2009), for example, analyzes such a model but they have only one strategic candidate choosing policy in an economic environment that is sparser than the one here. The study of strategic voting with two strategic candidates remains an open and fruitful area for future research.

Another question that the paper leaves unanswered is whether the equilibrium behavior of the voters in this model can be sustained over time. In particular, it would be interesting to study an infinite horizon extension of the model and ask whether there is an equilibrium path in which the behavioral equivalence between rational strategic voting and optimistic naive voting holds through the long run. I conjecture that it may not, because the optimistic voter has the potential to eventually learn that his beliefs were too optimistic. On the other hand, the rational voter has less to learn: his expectations were correct to begin with. Thus, aggregation failure may continue to hold in the long run if voters are fully rational, but not if they are optimistic and naive.

Appendix

A. Proof of Proposition 1

Note that $\mathbb{E}^{piv}[\delta|x, n, L^0]$ is continuous in x, and recall that (3) defines the threshold $\bar{\delta}$ that determines whether an L^0 voter prefers the status quo policy or the more redistributive policy. In particular, if $\mathbb{E}^{piv}[\delta|0, n, L^0] \leq \bar{\delta}$ then $x = 0$ is an equilibrium; if $\mathbb{E}^{piv}[\delta|1, n, L^0] \geq \bar{\delta}$ then $x = 1$ is an equilibrium; and if $\mathbb{E}^{piv}[\delta|0, n, L^0] > \bar{\delta} > \mathbb{E}^{piv}[\delta|1, n, L^0]$ then the intermediate value theorem implies that there is a number $\tilde{x} \in (0, 1)$ such that $\mathbb{E}^{piv}[\delta|\tilde{x}, n, L^0] = \bar{\delta}$, making $\tilde{x}$ an equilibrium. □

B. Feddersen-Pesendorfer Approach in Multiple Dimensions

Consider a majoritarian election where voters have private information about an uncertain state variable. Feddersen and Pesendorfer (1997) proved that as the size of the electorate grows to infinity, pivotal beliefs about the state must get concentrated around certain "critical values." These are values of the state that bring the probability of a vote for either alternative closest to $\frac{1}{2}$ (see their Lemma 1). Their result was for the case where the state is one-dimensional. However, in Proposition 5 of the main text of this paper, we allow for uncertainty about both δ and p, which makes the state (δ, p) of this extension to our model two-dimensional. Therefore, to analyze the model and this extension, I generalize the Feddersen-Pesendorfer result to multiple dimensions.

The notation of this section of the Appendix is independent of the rest of the paper. There are $2n + 1$ voters who must each vote in a majoritarian election for one of two alternatives, 0 or 1. Each voter has a privately known type drawn from a common type space $\Theta = \{\theta_1, ..., \theta_T\}$. There is an underlying state variable s that determines the distribution over types

$$q(s) = (q_1(s), ..., q_T(s)) \tag{11}$$

Here, $q_t(s)$ represents the probability of a voter being type θ_t when the state is s. (So $\sum_{t=1}^{T} q_t(s) = 1$ for all s.) I assume that s is distributed by the probability measure φ over a set $S \subset \mathbb{R}^Z$ that is the product of K intervals and $Z - K$ finite sets

$$S = [a_1, b_1] \times ... \times [a_K, b_K] \times \{d^1_{K+1}, d^2_{K+1}, ...\} \times ... \times \{d^1_Z, d^2_Z, ...\} \tag{12}$$

where $K \in \{1, .., Z\}$. (By allowing $K = Z$, I allow S to be the product of only intervals.) A generic state is denoted $s = (s_1, ..., s_Z)$. Let $||\cdot||$ denote the box-distance, i.e. $||s - s'|| < \epsilon$ if and only if $|s_z - s'_z| < \epsilon$ for all $z = 1, ..., Z$. Then, for $\epsilon > 0$ and $s \in S$, define

$$E_\epsilon(s) = \{s' \in S : ||s - s'|| < \epsilon\} \tag{13}$$

to be an ϵ-neighborhood of s. I say that a subset $S' \subseteq S$ is *admissible* if the first L component sets are nonempty intervals. I then make the following assumptions.

Assumption A.1: $q(s)$ is continuously differentiable in $(s_1, ..., s_K)$.

Assumption A.2: For all $t = 1, ..., T$, and every admissible $S' \subseteq S$ there is an admissible subset $S'' \subseteq S'$ such that $\inf_{s \in S''} q_t(s) > 0$.

Assumption A.3: All components $s_1, ..., s_Z$ of the state $s \in S$ are independent.

Assumption A.4: The marginal distribution of φ on the k^{th} interval $[a_k, b_k]$ has a continuous density f_k, $k = 1, ..., K$. Moreover, there exists a number $\nu > 0$ such that

$$f_k(s_k) > \nu \qquad \forall s_k \in [a_k, b_k] \qquad \forall k = 1, ..., K$$

Assumption A.5: Let $\gamma_z(s_z)$ denote the marginal probability of a point $s_z \in \{d^1_z, d^2_z, ...\}$. Then $\gamma_z(s_z) > 0$ for all $s_z \in \{d^1_z, d^2_z, ...\}$, $z = K + 1, ..., Z$.

Consider a type-symmetric strategy profile, and let x_t denote the probability that voter type θ_t votes for alternative 0. Let $x = (x_1, ..., x_T)$ denote the profile of voting probabilities. The probability of casting a vote for alternative 0 when the state is s is

$$\pi(s, x) = x \cdot q(s) \tag{14}$$

The probability that a voter is pivotal, given n and the profile of voting probabilities x can be viewed as a function of s. That probability is

$$\beta(s|x, n) = \binom{2n}{n} (\pi(s, x))^n (1 - \pi(s, x))^n \tag{15}$$

For each $t = 1, ..., T$, define

$$X_t = \left\{ x \in [0,1]^T \;\middle|\; \int_S \beta(s|x,n) q_t(s) d\varphi(s) > 0 \;\forall n \right\} \tag{16}$$

If $x \in X_t$, then the distribution of the state $s = (s_1, ..., s_Z)$, given that a voter is pivotal and is of type θ_t, is well-defined. That distribution is given by

$$\begin{aligned} f^{piv}(s|x,n,\theta_t) &= \frac{\beta(s|x,n)q_t(s)d\varphi(s)}{\int_S \beta(\tilde{s}|x,n)q_t(\tilde{s})d\varphi(\tilde{s})} \\ &= \frac{\beta(s|x,n)q_t(s)f_1(s_1)\cdots f_K(s_K)\gamma_{K+1}(s_{K+1})\cdots\gamma_Z(s_Z)}{\int_S \beta(\tilde{s}|x,n)q_t(\tilde{s})d\varphi(\tilde{s})} \end{aligned} \tag{17}$$

which follows from Assumption A.3. Because Assumption A.1 implies that $q(s)$ is continuous in s, $\pi(s,x)$ is continuous in its arguments. Therefore, for all x, the following problem has a solution:

$$\min_{s \in S} \left|\pi(s,x) - \frac{1}{2}\right| \tag{18}$$

Let $S^*(x)$ denote the set of solutions to this problem. For all $\epsilon > 0$, define

$$S^*_\epsilon(x) = \bigcup_{s \in S^*(x)} E_\epsilon(s) \tag{19}$$

Let $\mathcal{B}_\rho(x) := \{\tilde{x} \in X : ||x - \tilde{x}|| < \rho\}$ be the ρ-neighborhood of x. We then have:

Theorem A.1: *Assume A.1-A.5. For every type θ_t, $t = 1, ..., T$, every $x \in X_t$, and every $\epsilon > 0$ there is number $\rho > 0$ for which*

$$\lim_{n \to \infty} \int_{S^*_\epsilon(x)} f^{piv}(s|\tilde{x},n,\theta_t) ds = 1 \qquad \forall \tilde{x} \in \mathcal{B}_\rho(x) \tag{20}$$

Proof. Define the function $h : [0,1] \to \mathbb{R}$ by

$$h(\tilde{\pi}) = (\tilde{\pi})^{\frac{1}{2}}(1-\tilde{\pi})^{\frac{1}{2}} \tag{21}$$

and note that h is a continuous, strictly concave function that is maximized on any compact set $Y \subseteq [0,1]$ by the value of $\tilde{\pi}$ that minimizes $|\tilde{\pi} - \frac{1}{2}|$ on Y. Since h and π are continuous, $h(\pi(\cdot,x))$ is continuous on the compact set S, so it has a nonempty set of maximizers $S^*(x) \subseteq S$. Let $h^* = h(\pi(s,x))$ for any $s \in S^*(x)$ so that $h^* = h(\pi(s,x))$ for all $s \in S^*(x)$.

Now fix a type θ_t, $x \in X_t$ and $\epsilon > 0$. If $S^*_\epsilon(x) = S$ then the result is trivially true, since $\int_S f^{piv}(s|x,n,\theta_t) = 1$. So assume $S^*_\epsilon(x) \subsetneq S$. Since $h \circ \pi$ is continuous, there exists $\eta \in (0,1)$ such that

$$h^* - 2\eta > \sup_{s \notin S^*_\epsilon(x)} h(\pi(s,x)) \tag{22}$$

Since Assumption A.1 says that $q(\cdot)$ is continuously differentiable in $(s_1, ..., s_L)$, $h \circ \pi$ is uniformly continuous, which implies that there exists $\alpha > 0$ such that

$$|h(\pi(s,\tilde{x})) - h(\pi(s,x))| < \eta \qquad \forall (s,\tilde{x}) \in S \times \mathcal{B}_\alpha(x) \tag{23}$$

This implies that

$$\begin{aligned} \eta &\geq \sup_{s \notin S^*_\epsilon(x)} |h(\pi(s,\tilde{x})) - h(\pi(s,x))| \\ &\geq \sup_{s \notin S^*_\epsilon(x)} h(\pi(s,\tilde{x})) - \sup_{s \notin S^*_\epsilon(x)} h(\pi(s,x)) \qquad \forall \tilde{x} \in \mathcal{B}_\alpha(x) \end{aligned} \tag{24}$$

Combining (22) with (24), we have

$$\begin{aligned} h^* - \eta &= (h^* - 2\eta) + \eta \\ &> \sup_{s \notin S^*_\epsilon(x)} h(\pi(s,x)) + \eta \geq \sup_{s \notin S^*_\epsilon(x)} h(\pi(s,\tilde{x})) \qquad \forall \tilde{x} \in \mathcal{B}_\alpha(x) \end{aligned} \tag{25}$$

Now, for the η fixed above, define

$$\begin{aligned} A^*_\epsilon(\tilde{x}) &:= \{s \in S^*_\epsilon(x) : |h^* - h(\pi(s,\tilde{x}))| \leq \eta/2\} \\ A^{**}_\epsilon(x) &:= \{s \in S^*_\epsilon(x) : |h^* - h(\pi(s,x))| \leq \eta/4\} \end{aligned} \tag{26}$$

Since $h \circ \pi$ is uniformly continuous, there exists $\alpha' > 0$ such that

$$|h(\pi(s,x)) - h(\pi(s,\tilde{x}))| < \eta/4 \qquad \forall (s,\tilde{x}) \in S \times \mathcal{B}_{\alpha'}(x) \tag{27}$$

So for all $s \in A^{**}_\epsilon(x)$,

$$\begin{aligned} |h^* - h(\pi(s,\tilde{x}))| &\leq |h^* - h(\pi(s,x))| + |h(\pi(s,x)) - h(\pi(s,\tilde{x}))| \\ &\leq \eta/4 + \eta/4 = \eta/2 \qquad \forall \tilde{x} \in \mathcal{B}_{\alpha'}(x) \end{aligned} \tag{28}$$

This proves that $A_\epsilon^{**}(x) \subseteq A_\epsilon^*(\tilde{x})$ for all $\tilde{x} \in \mathcal{B}_{\alpha'}(x)$. Since every maximizer of $h(\pi(\cdot, s))$ is in $A_\epsilon^{**}(x)$, this implies that it is also in $A_\epsilon^*(\tilde{x})$ for every $\tilde{x} \in \mathcal{B}_{\alpha'}(x)$. Fix any such maximizer $s^* = (s_1^*, ..., s_Z^*)$ and let

$$B_\epsilon^* = [s_1^* - \epsilon, s_1^* + \epsilon] \times ... \times [s_K^* - \epsilon, s_K^* + \epsilon] \times \{s_{K+1}^*\} \times ... \times \{s_Z^*\} \tag{29}$$

Collecting these facts, we have

$$\emptyset \neq A_\epsilon^*(\tilde{x}) \cap B_\epsilon^* \subseteq S_\epsilon^*(x) \subsetneq S \qquad \forall \tilde{x} \in \mathcal{B}_{\alpha'}(x) \tag{30}$$

By the uniform continuity of $h \circ \pi$, there are intervals $I_1, ..., I_K$ that contain $s_1^*, ..., s_K^*$ respectively such that

$$R(\tilde{x}) := I_1 \times ... \times I_K \times \{s_{K+1}^*\} \times ... \times \{s_Z^*\} \subseteq A_\epsilon^*(\tilde{x}) \cap B_\epsilon, \ \forall \tilde{x} \in \mathcal{B}_{\alpha'}(x) \tag{31}$$

Since $R(\tilde{x})$ is an admissible subset of S for all $\tilde{x} \in \mathcal{B}_{\alpha'}(x)$, Assumption A.2 implies that there is an admissible set

$$R^*(\tilde{x}) := I_1^* \times ... \times I_K^* \times \{s_{K+1}^*\} \times ... \times \{s_Z^*\} \subseteq R(\tilde{x}) \subseteq A_\epsilon^*(\tilde{x}) \cap B_\epsilon^* \tag{32}$$

such that $\underline{q}_t(\tilde{x}) := \inf_{s \in R^*(\tilde{x})} q_t(s) > 0$. Let $\mu(\tilde{x})$ denote the length of the shortest interval $I_1^*, ..., I_K^*$ that are the first K components of $R^*(\tilde{x})$.

Now, for all $\tilde{x} \in \mathcal{B}_{\min\{\alpha, \alpha'\}}(x)$, $s \in S \backslash S_\epsilon^*(x)$ and $s' \in A_\epsilon^*(\tilde{x}) \cap B_\epsilon^*$, we have

$$\frac{\beta(s|\tilde{x}, n)}{\beta(s'|\tilde{x}, n)} = \left(\frac{h(\pi(s, \tilde{x}))}{h(\pi(s', \tilde{x}))}\right)^{2n} \leq \left(\frac{h^* - \eta}{h^* - \eta/2}\right)^{2n} \leq \left(\frac{1 - \eta}{1 - \eta/2}\right)^{2n} \tag{33}$$

The first inequality follows from (25) and the definition of $A_\epsilon^*(\tilde{x})$ in (26). The second follows from the fact that the left side is increasing in h^*.

We also have

$$\begin{aligned} \int_{A_\epsilon^*(\tilde{x}) \cap B_\epsilon^*} q_t(s) d\varphi(s) &\geq \int_{R^*(\tilde{x})} q_t(s) d\varphi(s) \\ &\geq \underline{q}_t(\tilde{x}) \left(\gamma_{K+1}(s_{K+1}^*) \cdots \gamma_Z(s_Z^*) \cdot \int_{I_1^*} f_1(s_1) ds_1 \cdots \int_{I_K^*} f_K(s_K) ds_K\right) \\ &\geq \underline{q}_t(\tilde{x}) \cdot \gamma_{K+1}(s_{K+1}^*) \cdots \gamma_Z(s_Z^*) \cdot \big(\mu(\tilde{x}) \cdot \nu\big)^K > 0 \end{aligned} \tag{34}$$

The first inequality follows from (32). The second follows from the definition of $\underline{q}_t(\tilde{x})$ above, the fact that $s_1, ..., s_Z$ are independent by Assumption A.3, and the definition of $R^*(\tilde{x})$ in (32). The third inequality follows from Assumption A.4 and the definition of $\mu(\tilde{x})$ as being the length of the shortest interval $I_1^*, ..., I_K^*$. The last inequality follows from Assumption A.5 and the fact that $\underline{q}_t(\tilde{x})$, $\mu(\tilde{x})$ and ν are all positive.

To prove the theorem, we have for all $\tilde{x} \in \mathcal{B}_{\min\{\alpha,\alpha'\}}(x)$

$$\begin{aligned}
\int_{S\setminus S_\epsilon^*(x)} f^{piv}(s|\tilde{x}, n, \theta_t)ds &= \frac{\int_{S\setminus S_\epsilon^*(x)} \beta(s|\tilde{x}, n) q_t(s) d\varphi(s)}{\int_S \beta(s|\tilde{x}, n) q_t(s) d\varphi(s)} \\
&\leq \frac{\int_{S\setminus S_\epsilon^*(x)} \beta(s|\tilde{x}, n) q_t(s) d\varphi(s)}{\int_{A_\epsilon^*(\tilde{x})\cap B_\epsilon^*} \beta(s|\tilde{x}, n) q_t(s) d\varphi(s)} \\
&\leq \frac{\sup_{s\in S\setminus S_\epsilon^*(x)} \beta(s|\tilde{x}, n)}{\inf_{s\in A_\epsilon^*(\tilde{x})\cap B_\epsilon^*} \beta(s|\tilde{x}, n)} \frac{\int_{S\setminus S_\epsilon^*(x)} q_t(s) d\varphi(s)}{\int_{A_\epsilon^*(\tilde{x})\cap B_\epsilon^*} q_t(s) d\varphi(s)} \\
&\leq \left(\frac{1-\eta}{1-\eta/2}\right)^{2n} \frac{1}{\underline{q}_t(\tilde{x}) \cdot \gamma_{K+1}(s_{K+1}^*) \cdots \gamma_Z(s_Z^*) \cdot \big(\mu(\tilde{x}) \cdot \nu\big)^K}
\end{aligned} \tag{35}$$

The first inequality follows from $A_\epsilon^*(\tilde{x}) \cap B_\epsilon^* \subsetneq S$ in (30). The second follows from applying the appropriate bounds to the numerator and denominator on the left side. The third follows from (33) and (34), and the fact that $\int_{S\setminus S_\epsilon^*(x)} q_t(s) d\varphi(s) \leq 1$. The result then follows from choosing $\rho = \min\{\alpha, \alpha'\}$ and because (35) implies that for all $\tilde{x} \in \mathcal{B}_\rho(x)$,

$$\lim_{n\to\infty} \int_{S\setminus S_\epsilon^*(x)} f^{piv}(s|\tilde{x}, n, \theta_t)ds = 0$$

$\square$

C. Proof of Proposition 2

The proof of Proposition 2 follows from Theorem A.1, presented Appendix B, for the case where $Z = K = 1$, $s_1 = \delta$ and $[a_1, b_1] = [0, 1]$. The assumptions are satisfied because $0 < \pi(\delta, x) < 1$ for all (δ, x). For $x \neq p$ and all $\epsilon > 0$, the set $S_\epsilon^*(x)$ is a neighborhood of $\delta^*(x)$. Therefore, for all $\tilde{x}$ in some neighborhood $\mathcal{B}_\rho(x)$ of x, write

$$\mathbb{E}^{piv}[\delta|\tilde{x}, n, L^0] = \int_{\delta\notin S_\epsilon(x)} \delta f^{piv}(\delta|\tilde{x}, n, L^0)d\delta + \int_{\delta\in S_\epsilon(x)} \delta f^{piv}(\delta|\tilde{x}, n, L^0)d\delta \tag{36}$$

and observe that for n large enough, this quantity is bounded above by $\epsilon + (\delta^*(x) + \epsilon) = \delta^*(x) + 2\epsilon$ and bounded below by $(\delta^*(x) - \epsilon)(1 - \epsilon) > \delta^*(x) - 2\epsilon$, both of which follow from Theorem A.1. Thus for all $\tilde{x}$ in some neighborhood $\mathcal{B}_\rho(x)$ of x, the conditional expectation $\mathbb{E}^{piv}[\delta|\tilde{x}, n, L^0]$ is within 2ϵ of $\delta^*(x)$. □

D. Proof of Proposition 3

Consider the case where $\lambda + (1 - \lambda)p > \frac{1}{2}$. Since $\pi(\delta, 1)$ is strictly decreasing in δ and $\pi(1, 1) = \lambda + (1 - \lambda)p > \frac{1}{2}$, we have $\delta^*(1) = 1$. By the assumption in (2) that $0 < y(1)$ we know that $\bar{\delta} < 1$. Thus if n is large enough then Proposition 2 implies that $x = 1$ is an equilibrium, making it a large-electorate equilibrium. Similarly, $\pi(\delta, 0)$ is strictly increasing in δ and ranges from λ to $\lambda + (1 - \lambda)p$ on the interval $[0, 1]$. Therefore, if $\delta^*(0) < \bar{\delta}$ then for n large enough, Proposition 2 implies that $x = 0$ is a large electorate equilibrium. Similarly, if $\lambda + (1 - \lambda)p < \frac{1}{2}$, then because $\pi(\delta, 1)$ is strictly decreasing in δ, Proposition 2 also implies that for n large enough, $x = 1$ is an equilibrium if and only if $\delta^*(1) > \bar{\delta}$, i.e. if and only if $\frac{1}{1-p}\frac{1/2}{1-\lambda} > \frac{1}{p}\frac{l}{l+h}$, which rearranges to make (10).

We have completed the cases where $x_\infty = 0, 1$ are large electorate equilibria. Therefore, we must prove the following three claims: (i) p is always a limit equilibrium, (ii) x^* is a limit equilibrium if $\lambda + (1 - \lambda)p > \frac{1}{2}$ and $\bar{\delta} < \delta^*(0)$ hold, or if $\lambda + (1 - \lambda)p < \frac{1}{2}$ and $\bar{\delta} < \delta^*(1)$ hold, and (iii) there are no other limit equilibria.

(i) Define $\delta^*(p) = \mathbb{E}[\delta]$ and fix $\epsilon > 0$. Proposition 2 implies that $\{\mathbb{E}^{piv}[\delta|\cdot, n, L^0]\}_n$ converges pointwise to $\delta^*(\cdot)$. Consider first the case where $\lambda + (1 - \lambda)p > \frac{1}{2}$. Then $\mathbb{E}^{piv}[\delta|p - \epsilon, n, L^0]$ converges to 0 while $\mathbb{E}^{piv}[\delta|p + \epsilon, n, L^0]$ converges to 1. Since $\mathbb{E}^{piv}[\delta|x, n, L^0]$ is continuous in x for all n and since $\bar{\delta} \in (0, 1)$, there is a number n large enough such that $\mathbb{E}^{piv}[\delta|p - \epsilon, n, L^0] < \bar{\delta}/2$ and $\mathbb{E}^{piv}[\delta|p + \epsilon, n, L^0] > (1 + \bar{\delta})/2$. But since $\mathbb{E}^{piv}[\delta|x, n, L^0]$ is continuous in x for all n, the intermediate value theorem implies that for n large enough, there exists a number $x \in [p - \epsilon, p + \epsilon]$ such that $\mathbb{E}^{piv}[\delta|x, n, L^0] = \bar{\delta}$. This number x is an equilibrium if n is large enough.

Now start with $\epsilon > 0$ very small, and consider the sequence $\{\epsilon/k\}_{k=1}^\infty$. By the procedure above, we can associate with each ϵ/k, a number N_k such that for all $n \geq N_k$, there is an equilibrium x_n to the game with population parameter n that is within ϵ/k of p. Moreover, we can use the procedure to construct a sequence of equilibria $\{x_n\}_\mathcal{N}$ for an increasing index set $\mathcal{N} \subseteq \mathbb{N}$, where the kth element of this sequence is an equilibrium x_n of a game with population parameter $n \geq N_k$ and is within ϵ/k of p. Then, by construction, this is a subsequence of equilibria of a sequence of games, and this subsequence converges to p since the sequence $\{\epsilon/k\}$ converges to 0. Therefore, p must be a limit equilibrium.

The case where $\lambda+(1-\lambda)p<\frac{1}{2}$ is exactly analogous, except that in this case $\mathbb{E}^{piv}[\delta|p-\epsilon,n,L^0]$ converges to $\delta^*(p-\epsilon)=1$ and $\mathbb{E}^{piv}[\delta|p+\epsilon,n,L^0]$ converges to $\delta^*(p+\epsilon)=0$ for small $\epsilon>0$. Nevertheless, since $\bar{\delta}\in(0,1)$ the intermediate value theorem applies to establish the existence of a number $x\in[p-\epsilon,p+\epsilon]$ such that $\mathbb{E}^{piv}[\delta|x,n,L^0]=\bar{\delta}$. Then, the above procedure can be applied again to show that p must be a limit equilibrium.

(ii) The proof that x^* is also a limit equilibrium if $\lambda+(1-\lambda)p>\frac{1}{2}$ and $\bar{\delta}<\delta^*(0)$, or if $\lambda+(1-\lambda)p<\frac{1}{2}$ and $\bar{\delta}<\delta^*(1)$, is exactly analogous to the argument above, and omitted.

(iii) Finally, there are no other limit equilibria besides the ones reported in the proposition. Indeed, suppose that there were another limit equilibrium, and call it $x_\infty=z$. Consider the case where $\lambda+(1-\lambda)p>\frac{1}{2}$ and $\bar{\delta}>\delta^*(0)$, so that $z\in(0,1)$, $z\neq p$. Since $\bar{\delta}\in(\delta^*(0),1)$, we know that $\delta^*(z)$ is bounded away from $\bar{\delta}$; in particular, for some $\epsilon>0$ small enough

$$|\delta^*(z)-\bar{\delta}|>2\epsilon. \tag{37}$$

Since z is a limit equilibrium, there is a sequence of equilibria $\{x_n\}$ that has a subsequence that converges to z. Denote that subsequence by $\{x_k\}_{\mathcal{K}}$, where $\mathcal{K}\subseteq\mathbb{N}$ is an infinite set. Then, by Proposition 2, there exists $\rho>0$ and N such that $n\geq N$ implies

$$\left|\mathbb{E}^{piv}[\delta|x,n,L^0]-\delta^*(z)\right|\leq\epsilon \qquad \forall x\in\mathcal{B}_\rho(z). \tag{38}$$

The inequalities (37) and (38) imply that for all $n\geq N$

$$\left|\bar{\delta}-\mathbb{E}^{piv}[\delta|x,n,L^0]\right|>\epsilon \qquad \forall x\in\mathcal{B}_\rho(z). \tag{39}$$

This in turn implies that there is an index $k\in\mathcal{K}$, $k\geq N$, such that $x_k\in(0,1)$ and $\left|\bar{\delta}-\mathbb{E}^{piv}[\delta|x_k,k,L^0]\right|>\epsilon$. But then x_k cannot be an equilibrium to the game with population parameter $n=k$, establishing the contradiction. We can use an analogous argument in the other cases as well. □

E. Information Aggregation and Proof of Proposition 4

Denote the game described in the main text by G_n, where n is the population parameter. Consider all of the assumptions of this game, except that now suppose that the realization of δ is publicly revealed to the voters. Call this modified game G_n°. Then, in any equilibrium to the game G_n°, all voters not in L^0 vote exactly as before. In particular, members of H and L^+ vote to keep the status quo while members of L^- vote for the more redistributive policy. Members of L^0, however, vote for the more redistributive policy if $\delta < \bar{\delta}$ and for the status quo policy if $\delta > \bar{\delta}$. As with the equilibria of the game G_n, we can identify an equilibrium of the game G_n° with the probability $x^\circ(\delta)$ that a voter in L^0 votes for the status quo policy, written as a function of δ. Then, for all sequences of games $\{G_n^\circ\}$, and all realizations $\delta \neq \bar{\delta}$, there exists a unique sequence of equilibria $\{x_n^\circ(\delta)\}$ corresponding to the sequence of games $\{G_n^\circ\}$. Denote by $\{E_n^\circ(\delta)\}$ the corresponding sequence of equilibrium distributions over the two possible electoral outcomes.

Now, suppose that x_∞ is a limit equilibrium of a sequence of games $\{G_n\}$ in which δ is not revealed to the voters. This means that corresponding to this sequence of games, there is a sequence of equilibria $\{x_n\}$ that has a subsequence $\{x_k\}_{\mathcal{K}\subseteq\mathbb{N}}$ that converges to x_∞. Associated with every such subsequence is a sequence of equilibrium distribution functions $\{E_k(\delta)\}_{\mathcal{K}}$ over the two electoral outcomes, again viewed as functions over possible realizations of δ. If, for every such subsequence of every sequence of voting equilibria, and almost every realization of δ, we have

$$\lim_{k\to\infty} E_k^\circ(\delta) = \lim_{k\to\infty} E_k(\delta)$$

then the limit equilibrium x_∞ is said to *aggregate information.* (Here, I am taking both limits along the index set $\mathcal{K}$.) In words, x_∞ aggregates information if in large electorates where voters do not know δ, and play equilibria close to x_∞, the distribution over electoral outcomes is almost surely the same as it would have been had δ been publicly revealed.

Proof of Proposition 4. The proof relies on an application of the law of large numbers. Suppose $\{x_k\}_{\mathcal{K}\subseteq\mathbb{N}}$ is a subsequence of equilibria that converges to limit equilibrium x_∞.

Consider first the case where $\lambda + (1-\lambda)p > \frac{1}{2}$ so that $\max\{0, x^*\}$, p and 1 are limit equilibria. Suppose that $x_\infty \in \{p, 1\}$. Then, we have $\lim \pi(\delta, x_\infty) > \frac{1}{2}$ for all δ. Therefore, for all δ, the status quo policy wins the election almost surely in the game G_k as $k \to \infty$. But for all $\delta < \min\{\delta^*(0), \bar{\delta}\}$, the redistributive policy wins almost surely in the game G_k° as $k \to \infty$. (When $\delta < \min\{\delta^*(0), \bar{\delta}\}$ and $k \to \infty$, all L^0 voters vote for the redistributive policy in the game G_k° and the votes of high income voters in L^+ and H are almost surely insufficient to re-elect the status quo.) Therefore, as $k \to \infty$, the equilibrium distribution over electoral outcomes in the games G_k and G_k° are different when $\delta < \min\{\delta^*(0), \bar{\delta}\}$. So the limit equilibrium $x_\infty \in \{p, 1\}$ fails to aggregate information.

Next, let $x_\infty = \max\{0, x^*\}$. Begin with the case where $\pi(\delta, \max\{0, x^*\}) < \frac{1}{2}$. In this case, $\delta < \bar{\delta}$. So, in both games G_k and G_k°, the redistributive policy wins almost surely as $k \to \infty$. Now consider the case $\pi(\delta, \max\{0, x^*\}) > \frac{1}{2}$, where $\delta > \bar{\delta}$. If $\bar{\delta} < \delta^*(0)$ then in both games G_k and G_k°, the status quo policy wins almost surely as $k \to \infty$. If, on the other hand, $\bar{\delta} > \delta^*(0)$ then $\max\{0, x^*\} = 0$, so $\delta > \bar{\delta} > \delta^*(0)$ implies $\pi(\delta, 0) = \lambda + (1-\lambda)\delta p > \frac{1}{2}$. In this case, for both games G_k and G_k° just the L^+ and H votes are sufficient to guarantee that the status quo policy wins almost surely as $k \to \infty$.

Now consider the case where $\lambda + (1-\lambda)p < \frac{1}{2}$. Suppose that $\bar{\delta} > \delta^*(1)$. Note that if $\delta < \bar{\delta}$ then only the H and L^+ types vote for the status quo policy in games G_k°, so the limiting share of votes for the status quo is almost surely $\lambda + (1-\lambda)\delta p < \frac{1}{2}$. On the other hand, if $\delta > \bar{\delta}$ then the H, L^+ and L^0 type voters vote for the status quo policy in these games, so the limiting share of votes for the status quo is almost surely $\lambda + (1-\lambda)(\delta p + 1 - \delta)$. This share of votes is larger than $\frac{1}{2}$ if and only if $\delta < \frac{1}{1-p}\frac{1/2}{1-\lambda} = \delta^*(1)$, which is not possible under the hypothesis that $\delta > \bar{\delta} > \delta^*(1)$. Therefore, the redistributive policy wins almost surely in the game G_k° as $k \to \infty$ for all values of δ. When $\bar{\delta} > \delta^*(1)$, the unique limit equilibrium of the game G_k is $x_\infty = p$. Then as $k \to \infty$ the share of votes for the status quo policy in this game is almost surely $\lambda + (1-\lambda)\delta p < \frac{1}{2}$. Therefore, the redistributive policy wins almost surely in the game G_k as $k \to \infty$. This implies that the limit equilibrium $x_\infty = p$ aggregates information.

Now suppose that $\lambda + (1-\lambda)p < \frac{1}{2}$ and $\bar{\delta} < \delta^*(1)$. In this case, if $\delta > \bar{\delta}$ then only the H, L^+ and L^0 type voters vote for the status quo policy in the games G_k°, so the share of votes for the status quo policy as $k \to \infty$ is almost surely $\lambda + (1-\lambda)(\delta p + 1 - \delta)$. The status quo wins almost surely in this limit if this share of votes is larger than $\frac{1}{2}$, or in other words if $\delta < \frac{1}{1-p}\frac{1/2}{1-\lambda} = \delta^*(1)$. If, on the other hand, $\delta > \delta^*(1)$, then the redistributive policy wins almost surely in the same limit. If $\delta < \bar{\delta}$ then only the H and L^+ type voters vote for the status quo policy in the game G_k° so the share of votes for the status quo policy in this game is almost surely $\lambda + (1-\lambda)\delta p < \frac{1}{2}$ as $k \to \infty$. This means that the redistributive policy wins almost surely in the limit. Thus, to summarize, for realizations of δ smaller than $\bar{\delta}$ or larger than $\delta^*(1)$, the redistributive policy wins almost surely in the game G_k° as $k \to \infty$; otherwise, for realizations of $\delta \in (\bar{\delta}, \delta^*(1))$ the status quo policy wins almost surely in the limit.

If $x_\infty = x^*$, then the status quo policy wins almost surely in the game G_k as $k \to \infty$ if $\pi(\delta, x^*) > \frac{1}{2}$, which holds when $\delta < \bar{\delta}$; and the redistributive policy wins almost surely in the limit if $\delta > \bar{\delta}$. Thus, this limit equilibrium fails to aggregate information. If $x_\infty = p$ then $\pi(\delta, p) = \lambda + (1-\lambda)p < \frac{1}{2}$ so the redistributive policy wins almost surely in the game G_k as $k \to \infty$. So again this limit equilibrium fails to aggregate information. Finally, if $x_\infty = 1$ then the status quo policy wins almost surely in the game G_k as $k \to \infty$ when $\pi(\delta, 1) > \frac{1}{2}$, i.e. when $\delta < \delta^*(1)$, and loses almost surely in this limit when $\delta > \delta^*(1)$. Therefore, this limit equilibrium also fails to aggregate information. □

F. Proof of Proposition 5

Again in this case, the H and L^+ voters have a weakly dominant strategy to vote for the status quo policy while the L^- voters have a weakly dominant strategy to vote for the redistributive policy. Let $\mathbb{E}^{piv}[\delta p | x, n, L^0]$ denote the expectation of δp conditional on being an L^0 voter in an electorate of size $2n+1$ when each L^0 voters vote for the status quo policy with probability $x \in [0,1]$ and the other types of voters play their equilibrium strategies. Note that Theorem A.1 in Appendix B applies with $Z = K = 2$, $(s_1, s_2) = (\delta, p)$, $[a_1, b_1] = [0, 1]$ and $[a_2, b_2] = [\underline{p}, 1]$. A straightforward implication of the theorem is that

$$\forall \epsilon > 0 \; \exists N \text{ s.t. } n \geq N \Rightarrow \min_{(\delta,p) \in S^*(x)} \delta p - \varepsilon < \mathbb{E}^{piv}[\delta p | x, n, L^0] < \max_{(\delta,p) \in S^*(x)} \delta p + \varepsilon \qquad (40)$$

where $S^*(x)$, defined in Appendix B, is the set of states $s = (\delta, p)$ that minimize the difference $|\pi(x, s) - \frac{1}{2}|$.

Consider the case of $x = 1$ and first suppose that $\underline{p} < \frac{\frac{1}{2}-\lambda}{1-\lambda}$ so that $\frac{1}{1-\underline{p}} \frac{1/2}{1-\lambda} < 1$. Then $S^*(1) = \{(\delta, p) : \underline{p} \leq p \leq \frac{\frac{1}{2}-\lambda}{1-\lambda}, \, \delta = \frac{1}{1-p} \frac{1/2}{1-\lambda}\}$ so $\min_{(\delta,p) \in S^*(1)} \delta p = \frac{\underline{p}}{1-\underline{p}} \frac{1/2}{1-\lambda}$. Therefore, by (40) and the assumption made in the statement of the proposition, for all n large enough we have $\mathbb{E}^{piv}[\delta p | 1, n, L^0] > \frac{l}{l+h}$, which implies that $x = 1$ is a large electorate equilibrium. Now, on the other hand, if $\underline{p} > \frac{\frac{1}{2}-\lambda}{1-\lambda}$ then $\frac{1}{1-\underline{p}} \frac{1/2}{1-\lambda} > 1$ so $\min_{(\delta,p) \in S^*(x)} \delta p = \underline{p}$. Therefore, again (40) and the assumption of the proposition imply that $\mathbb{E}^{piv}[\delta p | 1, n, L^0] > \frac{l}{l+h}$ for all n large enough. This again establishes that $x = 1$ is a large electorate equilibrium.

To see why this limit equilibrium does not aggregate information, note that in the limit of a sequence of games indexed by n the status quo policy wins almost surely if $\lambda + (1-\lambda)(\delta p + 1 - \delta) > \frac{1}{2}$, i.e. if $\delta(1-p) < \frac{1/2}{1-\lambda}$, and loses almost surely if $\delta(1-p) > \frac{1/2}{1-\lambda}$. In the limit of a corresponding sequence of games where δ is common knowledge, the status quo wins almost surely if $\delta p > \frac{l}{l+h}$ and $\delta(1-p) < \frac{1/2}{1-\lambda}$, or if $\delta p < \frac{l}{l+h}$ and $\delta p > \frac{\frac{1}{2}-\lambda}{1-\lambda}$. It loses almost surely in the limit if $\delta p > \frac{l}{l+h}$ and $\delta(1-p) > \frac{1/2}{1-\lambda}$, or if $\delta p < \frac{l}{l+h}$ and

$\delta p < \frac{\frac{1}{2}-\lambda}{1-\lambda}$. Therefore, if the realization of (δ, p) is such that $p \geq \underline{p}$ and $\delta p < \min\left\{\frac{l}{l+h}, \frac{\frac{1}{2}-\lambda}{1-\lambda}\right\}$ then the status quo wins almost surely in the limit of the sequence of games with private information but loses almost surely in the limit of the sequence of corresponding games with complete information. Note that these inequalities define a set of positive measure. Thus, the limit equilibrium $x_\infty = 1$ fails to aggregate information.

Now consider the case of $x = 0$. If $\underline{p} < \frac{\frac{1}{2}-\lambda}{1-\lambda}$ then $S^*(0) = \left\{(\delta, p) : \frac{\frac{1}{2}-\lambda}{1-\lambda} \leq p \leq 1, \delta = \frac{1}{p}\frac{\frac{1}{2}-\lambda}{1-\lambda}\right\}$. On the other hand, if $\underline{p} > \frac{\frac{1}{2}-\lambda}{1-\lambda}$ then $S^*(0) = \left\{(\delta, p) : \underline{p} \leq p \leq 1, \delta = \frac{1}{p}\frac{\frac{1}{2}-\lambda}{1-\lambda}\right\}$. In either case, the set $\left\{\delta : (\delta, p) \in \lim_{\lambda \to \frac{1}{2}} S^*(0)\right\}$, which is the set of values of δ such that (δ, p) belongs to the set $S^*(0)$ in the limit as λ approaches its upper bound, is a singleton containing $\delta = 0$. This implies that $\lim_{\lambda \to \frac{1}{2}} \max_{(\delta,p) \in S^*(0)}(\delta p) = 0$. This together with (40) implies that for all n large enough $\mathbb{E}^{piv}[\delta p | 0, n, L^0] < \frac{l}{l+h}$, making $x = 0$ a large electorate equilibrium.

To see why this equilibrium aggregates information, note that in a sequence of games indexed by n, the status quo wins almost surely in the limit if $\lambda + (1-\lambda)(\delta p) > \frac{1}{2}$, i.e. if $\delta p > \frac{\frac{1}{2}-\lambda}{1-\lambda}$, and loses almost surely if $\delta p < \frac{\frac{1}{2}-\lambda}{1-\lambda}$. As above, in the limit of a corresponding sequence of games where δ is common knowledge, the status quo wins almost surely if $\delta p > \frac{l}{l+h}$ and $\delta(1-p) < \frac{1/2}{1-\lambda}$, or if $\delta p < \frac{l}{l+h}$ and $\delta p > \frac{\frac{1}{2}-\lambda}{1-\lambda}$. It loses almost surely in the limit if $\delta p > \frac{l}{l+h}$ and $\delta(1-p) > \frac{\frac{1}{2}}{1-\lambda}$, or if $\delta p < \frac{l}{l+h}$ and $\delta p < \frac{\frac{1}{2}-\lambda}{1-\lambda}$. But as $\lambda \to \frac{1}{2}$, we know that $\frac{\frac{1}{2}-\lambda}{1-\lambda} < \frac{l}{l+h}$. Therefore, the limiting electoral outcomes of the two sequences of games coincide for almost all values of (δ, p), which means that the limit equilibrium $x_\infty = 0$ aggregates information. $\square$

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