Equilibrium False Consciousness[*]

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Abstract

I present a model in which high and low income voters must decide between continuing with a status quo policy or switching to a different policy, which is more redistributive. Under the status quo policy, low income voters may get an opportunity for upward social mobility. However, such opportunities are expected to arise infrequently, so these voters actually prefer the more redistributive policy to the status quo. Nevertheless, there is an equilibrium in which the vast majority of them cast their ballots in favor of re-electing the status quo. Although these voters are fully strategic and correctly assess their chances for upward mobility, they cast their ballots as if they are naive voters who over-estimate their chances of becoming rich. In this sense, these voters appear to have a *false consciousness* about their mobility prospects. Moreover, as a result of their votes, the status quo policy is re-elected even when the vast majority of voters, including them, would have preferred the more redistributive policy. In other words, the election fails to aggregate information.

JEL Classification Codes: D72, P16

Key words: redistributive politics, social mobility, strategic voting, information aggregation

1 Introduction

This paper develops a simple model of redistributive politics in which high and low income voters must decide whether to continue with the status quo policy or switch to an alternative policy, which is more redistributive. Under the status quo policy, low income voters may receive an opportunity to become high income earners, but will succeed in climbing the economic ladder only if they are talented. Low income voters do not know whether they are talented unless they receive the opportunity, in which case they learn about their talent from either their success or failure. Low income voters are uncertain about the probability with which they can expect to receive the opportunity for social mobility under the status quo policy, but they expect the probability to be low. Consequently, these low income voters prefer the more redistributive policy to the status quo. The status quo policy is in effect for one period, after which an election is held to decide whether to continue with it in the next period or to switch to the more redistributive policy.

The model has two results. The first and main result of the model is the following behavioral equivalence result: there exists an equilibrium in which several low income voters, who are fully strategic and correctly assess their chances of upward mobility, cast their ballots exactly as if they were naive (sincere) voters who hold mistakenly optimistic beliefs about their mobility prospects. The voters who behave this way are precisely the vast majority of low income voters who did not receive the economic opportunity in the first period. All of these voters vote to re-elect the status quo policy despite knowing that the more redistributive policy gives them a strictly better payoff. This behavior remains part of an equilibrium of the game as the size of the electorate goes to infinity, which leads to the second result of the paper: the failure of this equilibrium to fully aggregate information.

The behavioral equivalence result is noteworthy considering the wide skepticism that much of the empirically observed behavior of the American electorate could be consistent with voter rationality.[1] The result implies that if one had to draw inferences only from data on low income individuals who overestimate their chance of climbing the economic ladder and vote naively, one could not rule out the possibility that these individuals are actually strategic voters who correctly assess their chances

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[1]See, for example, Bartels (2008a), Achen and Bartels (2004), Healy, Malhotra and Mo (2010) and Wolfers (2002), among others.

for upward mobility. I refer to the behavioral equivalence result as "equilibrium *false consciousness*," borrowing a term used by Bénabou and Ok (2001) to describe the overly optimistic beliefs that some voters hold about their mobility prospects.[2] The naive appearance of false consciousness among the voters of my model is, however, an equilibrium phenomenon that arises from the strategic considerations of rational agents who actually hold correct beliefs.

The logic behind the behavioral equivalence result is as follows. In any equilibrium of the model, high income voters (including those low income voters who climbed the economic ladder in the first period) vote to re-elect the status quo policy, while low income voters who got the opportunity and learned that they are untalented vote for the more redistributive policy. If all low income voters who did not receive the economic opportunity in the first period vote *en masse* to re-elect the status quo, then the election should not be close: the status quo should win almost surely when the size of the electorate is large. But if the election is close, and a low income voter is pivotal, it can only be because half of the electorate consists of low income voters who received the economic opportunity, learned that they were untalented, and voted for the more redistributive policy. For so many low income voters to learn that they are untalented, it must be that economic opportunities are plentiful under the status quo. When these opportunities are plentiful, and low income voters believe that they are likely to be talented, it is optimal for them to vote for the status quo. The status quo is re-elected almost surely, and the election fails to aggregate information.

The failure of information aggregation is also noteworthy given the many positive results on information aggregation beginning with the seminal work of Feddersen and Pesendorfer (1997). Nevertheless, the findings of a few other recent papers also suggest that aggregation failure may be a more common phenomenon than was initially suspected. Bhattacharya (2013), in particular, provides weak conditions for aggregation failure in a jury-type setting with two states of the world. He shows that if voter preferences fail to satisfy a condition he calls "strong preference monotonicity," then there exists an equilibrium that fails to aggregate information.[3] Similarly, Mandler

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[2] The term *false consciousness*, which goes back to Friedrich Engels in his 1893 "Letter to Franz Mehring," is used by Marxist scholars to describe how capitalist processes mislead the proletariate, more generally.

[3] Strong preference monotonicity is satisfied if the likelihood that a voter switches to favoring the alternative that has more support in a particular state under full information is higher than the probability he switches to the other alternative, following a small increase in his beliefs.

(2012) studies a jury model with uncertainty about uncertainty, and demonstrates the failure of information aggregation for his model. In contrast to these papers, my model produces aggregation failure with a continuum of states and only first order uncertainty; so, in this sense, it is closer to the original work of Feddersen and Pesendorfer (1997) and the few other models that exhibit aggregation failure.[4] I demonstrate that the logic behind Bhattacharya's (2013) strong preference monotonicity condition applies also to the case of a continuum of states. However, despite this theoretical contribution, the central contribution of my model is the behavioral equivalence result, which shows that the seemingly irrational behavior of low income voters can in fact be rational under assumptions that are both parsimonious and natural. Indeed, this is the first paper to my knowledge that utilizes strategic voting in a genuine political economy model of redistribution and social mobility.[5]

The paper is organized as follows. Section 2 presents the political environment. Section 3 then develops the main analysis, leading to the behavioral equivalence result. It also characterizes the set of equilibria of the model as the size of the electorate grows to infinity. Section 4 studies the aggregation properties of these equilibria. Section 5 offers some remarks about the model's key assumptions. Section 6 provides a discussion of how the paper relates to the literatures on social mobility, redistribution and voter rationality. Section 7 concludes.

2   The Political Environment

There are two periods and two policies. In the first period, a status quo policy is in effect. Then, an election is held to decide whether to continue with it in the next period, or to switch to a different policy, which is more redistributive. The policy that wins the election is implemented in the second period. There are $n + 1$ low income voters and $\lambda(n + 1)$ high income voters, and all voters must vote for one of the two policies. Assumption 1 below states that low income voters form the majority. Assumption 2 states that λ and n are chosen so that total population is odd.

ASSUMPTION 1: $0 < \lambda < 1$

ASSUMPTION 2: $n \in \mathcal{N} \equiv \{\tilde{n} \in \mathbb{N} : (1 + \lambda)(\tilde{n} + 1) \text{ is odd}\}$

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[4]See, for example, Gul and Pesendorfer (2009) and the setup described in Section 6 of Feddersen and Pesendorfer (1997).

[5]Other political economy models of redistribution and social mobility, such as Bénabou and Ok (2001) and Piketty (1995), are discussed in Section 6.

Assumption 2 implies that the election cannot end in an exact tie. Also, note that the assumption does not restrict us to the case of small electorates since $\mathcal{N}$ is an unbounded set whenever it is nonempty.[6]

Under the more redistributive policy, each high income voter receives a payoff y_R^h while each low income voter receives a payoff y_R^l. Under the status quo policy, each high income voter has payoff y_Q^h, while each low income voter has probability δ of receiving an opportunity to become a high income voter. If a low income voter receives the opportunity to climb the economic ladder, and he is *talented*, then he becomes a high income voter and receives a payoff y_Q^h. If he is not talented, or if he does not receive the opportunity, then his payoff is y_Q^l. Each low income voter has prior probability p of being talented. If a voter becomes a high income earner in the first period, he remains a high income earner in the second period. Voters who receive an opportunity to become high income earners in the first period and are unsuccessful learn that they are untalented, and would also be unsuccessful in the following period. Voters who do not receive the opportunity in the first period do not learn whether or not they are talented.

While all voters know the success rate p, they are uncertain about δ. This uncertainty can be taken to reflect uncertainty about whether the status quo policy works well. If δ is high, then the status quo policy works well, whereas if δ is low then it does not. In particular, if the status quo policy is re-elected, then the value of δ in the second period is the same as in the first. I assume that each voter experiences the consequences of the first period policy only for himself; thus, each voter is uninformed about who (and how many others) received the opportunity to climb the economic ladder, and whether or not they succeeded. Let F denote the distribution of δ. The following assumption states that this prior is "non-doctrinaire."

ASSUMPTION 3: F has continuous density f, and $f(\delta) > 0$ for all $\delta \in (0, 1)$.

Assumption 3 serves the same role in my analysis as Feddersen and Pesendorfer's (1997) Assumption 2 does in theirs, though my assumption is considerably weaker.[7] In particular, this assumption ensures that conditional on a vote being pivotal, beliefs converge to the state that is most likely to deliver a tie in the limit as the size of the electorate goes to infinity.

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[6]The set $\mathcal{N}$ is always nonempty and unbounded when λ is the ratio of even an odd integers.

[7]Feddersen and Pesendorfer (1997) assume that f satisfies the following property: there exists $\alpha > 0$ such that $1/\alpha > f(\delta) > \alpha$ for all $\delta \in [0, 1]$.

I now make some assumptions regarding voter preferences. Let $\bar{\delta}$ denote the expected value of δ according to the prior F. Conditional on δ, the expected payoff from the status quo policy for a low income voter is given by

$$y(\delta) = (1 - \delta p)y_Q^l + \delta p y_Q^h \tag{1}$$

Therefore, the unconditional expected payoff to re-electing the status quo policy for a low income voter who did not receive the economic opportunity is $y(\bar{\delta})$. His expected payoff to electing the more redistributive policy is simply y_R^l. The following assumption states, mainly, that low income voters do not expect the status quo policy to work well, and therefore they prefer the more redistributive policy; however, if they thought that the status quo policy did work well, then they would prefer that policy to the more redistributive one.

ASSUMPTION 4: (i) $y_Q^l < y_R^l \leq y_R^h < y_Q^h$ and (ii) $y(\bar{\delta}) < y_R^l < y(1)$

Assumption $4(i)$ states that the more redistributive policy is in fact more redistributive than the status quo policy. In particular, voters who remain low income earners in the first period receive a higher payoff under the more redistributive policy than under the status quo policy. On the other hand, high income voters receive a lower payoff under the redistributive policy, but their payoff is always at least as large as the payoff of low income voters. Thus, low income voters who learn that they are not talented prefer the more redistributive policy while high income voters prefer the less redistributive status quo. Assumption $4(ii)$ states that a low income voter who did not receive the economic opportunity in the first period prefers greater redistribution because the status quo policy does not provide the economic opportunity with high enough probability. However, if he expected to receive the opportunity with high enough probability, then he would actually prefer the status quo. Since $y(\delta)$ is strictly increasing in δ, there is a cutoff

$$\delta^* = \left(\frac{y_R^l - y_Q^l}{y_Q^h - y_Q^l}\right)\frac{1}{p} \tag{2}$$

such that a low income voter who does not yet know whether or not he is talented would prefer the status quo policy if the expected value of δ were above δ^*, and would prefer the more redistributive policy if it were below δ^*. Note that δ^* is increasing in y_R^l; thus, as the redistributive policy becomes more redistributive, such low income

voters prefer it to the status quo for a larger range of δ. Also note that the assumption $y_R^l < y(1)$, which guarantees that $\delta^* < 1$, implies $p > \frac{y_R^l - y_Q^l}{y_Q^h - y_Q^l}$ so the probability that a voter is talented is not small. In fact, the next assumption states that it is larger than $1/2$, but not equal to 1.

ASSUMPTION 5: $1/2 < p < 1$

The assumption that $p \neq 1$, so that some voters are untalented, will be crucial for generating both the behavioral equivalence and aggregation failure results that are at the core of this paper. In addition, the assumption that $p > 1/2$ (along with the previous assumptions) implies that in large electorates the majority of low income voters are poor not because they are intrinsically untalented, but because opportunities are scarce.

3   Voting Behavior

Let $\theta = (y_Q^l, y_Q^h, y_R^l, y_R^h, \lambda, F, p)$ denote the tuple of all parameters of the model besides n, and denote the voting game by $G(n, \theta)$. A strategy for a voter is the probability with which he votes to re-elect the status quo policy. The relevant equilibrium concept is symmetric equilibrium in weakly undominated strategies, which I refer to simply as equilibrium. In what follows, I first establish the existence of equilibrium for the game $G(n, \theta)$ when (n, θ) satisfies Assumptions 1–5. Then I present an analytical result that is useful in characterizing the set of all equilibria. Finally, I conclude the section by using this result to characterize the set of equilibria in large electorates.

3.1   Existence of Equilibrium

Call the set of voters that started off as high income earners H. These voters have a weakly dominant strategy to vote for the status quo policy. This implies that the status quo policy is re-elected if and only if more than

$$n \times q_n \equiv \frac{(1 - \lambda)(n + 1) - 1}{2} \tag{3}$$

low income voters vote for it. Equation (3) implicitly defines the quantity q_n. Observe that $\{q_n\}_{\mathcal{N}}$ is a strictly increasing sequence that converges to

$$q_\infty \equiv \frac{1 - \lambda}{2}. \tag{4}$$

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Next, partition the set of low income voters into three categories, L^+, L^- and L^0. Voters in category L^+ are those who became high income earners in the first period. These voters remain high income earners in the second period, so like voters in H, they too have a weakly dominant strategy to vote for the status quo policy. On the other hand, voters in L^- are low income earners who learned that they are not talented. These voters have a weakly dominant strategy to vote for the more redistributive policy. Finally, voters in L^0 are those who did not get the opportunity to climb the economic ladder in the first period. These voters would be interested in knowing what the value of δ is, but they are unable to observe it. I use x^i to denote the probability with which voter $i \in L^0$ votes to continue with the status quo policy. Since equilibrium is symmetric, this means that $x^i = x$ for all $i \in L^0$. The following theorem establishes the existence of equilibrium to the game $G(n, \theta)$, and it records the equilibrium behavior of voters in H, L^+ and L^- described above.

Proposition 1. *For every parameter profile (n, θ) satisfying Assumptions $1-5$, an equilibrium to the game $G(n, \theta)$ exists.*

In every equilibrium, voters in H and L^+ vote for the status quo policy, while voters in L^- vote for the more redistributive policy.

Proof. The previous paragraph argued that voters in H and L^+ find it weakly dominant to vote for the status quo policy while voters in L^- find it weakly dominant to vote for the more redistributive alternative. Therefore, given a symmetric strategy x for voters in L^0, a random low income voter casts his ballot for the status quo policy with probability

$$\pi(\delta, x) = \delta p + (1 - \delta)x. \tag{5}$$

Voters in L^0 are fully rational and vote strategically, so they vote as if they are pivotal. Given x and n, the probability that a low income voter is pivotal is

$$\varphi(\delta | x, n) = \binom{n}{nq_n} (\pi(\delta, x))^{nq_n} (1 - \pi(\delta, x))^{n(1-q_n)}. \tag{6}$$

Conditional on being pivotal, and being an L^0 voter, the distribution of δ is given by

$$f^{piv}(\delta | x, n) = \frac{\varphi(\delta | x, n)(1 - \delta)f(\delta)}{\int_0^1 \varphi(\omega | x, n)(1 - \omega)f(\omega)d\omega}, \tag{7}$$

which is well-defined because $0 < \pi(\delta, x) < 1$ for all (δ, x). Therefore, conditional on being pivotal, the expected value of δ is

$$\bar{\delta}^{piv}(x, n) = \int_0^1 \delta f^{piv}(\delta | x, n) d\delta \tag{8}$$

Note that $\bar{\delta}^{piv}(x, n)$ is continuous in x. Recall that (2) defines the threshold δ^* that determines whether an L^0 voter prefers the status quo policy or the more redistributive policy. In particular, if $\bar{\delta}^{piv}(0, n) \leq \delta^*$ then $x = 0$ is an equilibrium; if $\bar{\delta}^{piv}(1, n) \geq \delta^*$ then $x = 1$ is an equilibrium; and if $\bar{\delta}^{piv}(0, n) > \delta^* > \bar{\delta}^{piv}(1, n)$ then the intermediate value theorem implies that there is a number $\tilde{x} \in (0, 1)$ such that $\bar{\delta}^{piv}(\tilde{x}, n) = \delta^*$. Clearly $\tilde{x}$ is an equilibrium. □

Since Proposition 1 establishes the behavior of voters in H, L^+ and L^- for all equilibria, an equilibrium can be identified with the behavior of voters in L^0, i.e. its value for x. So, throughout the remainder of the paper, I will abuse terminology and refer to an equilibrium by its value for x.

3.2   Analytical Result for Large Electorates

In this section, I present an analytical result that will enable me to fully characterize equilibria (and their properties) in large electorates. Let x be a symmetric strategy. If $x \neq p$ then $\pi(\delta, x)$ is strictly monotonic in δ, so there is a unique value of δ that minimizes $|\pi(\delta, x) - q_\infty|$. Denote this value by $\delta^\dagger(x)$. The following proposition is the key mathematical result that I use to study voting behavior in large electorates.

Proposition 2. *Let $\{(n, \theta)\}_{\mathcal{N}}$ be any sequence of parameter profiles each satisfying Assumptions 1–5. If $x = p$, then $\bar{\delta}^{piv}(x, n) = \bar{\delta}$ for all n. If $x \neq p$, then for all $\epsilon > 0$, there exists $\rho > 0$ and a number N such that $n \geq N$ implies*

$$|\bar{\delta}^{piv}(\tilde{x}, n) - \delta^\dagger(x)| \leq \epsilon \qquad \forall \tilde{x} \in B_\rho(x) \equiv \{\tilde{x} \in [0, 1] : |x - \tilde{x}| \leq \rho\}.$$

Proof. See the Appendix. □

The proposition implies that $\bar{\delta}^{piv}(x, n)$ converges "quasi-uniformly" to $\delta^\dagger(x)$, except at the point $x = p$. Quasi-uniform convergence is a property that is weaker than locally uniform convergence: If the number ρ were independent of ϵ and N, then
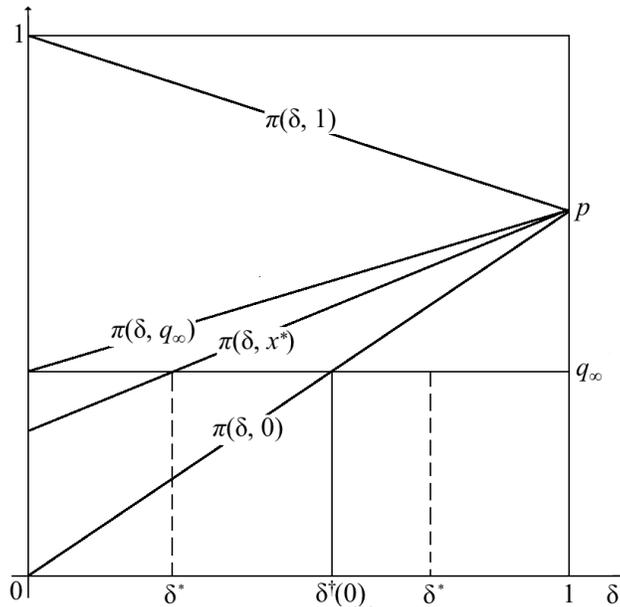
Figure 1: $\pi(\cdot, x)$ for various values of x

the convergence would be locally uniform. But the statement of the proposition allows ρ to depend on these quantities, so the convergence may not be locally uniform. Nevertheless, quasi-uniform convergence will suffice for the subsequent analysis.

The proof of the proposition in the Appendix actually establishes the stronger result that if $x \neq p$, then the conditional distribution $f^{piv}(\cdot|x, n)$ converges to a Dirac mass at $\delta^\dagger(x)$ as n gets large. In this sense, Proposition 1 belongs to a class of results that trace their origins to an argument in statistics by Bayes (1763) himself, and to a result in the voting literature by Good and Mayer (1975) and Chamberlain and Rothschild (1981).[8] Various extensions and applications of these results appear in several more recent papers, including Feddersen and Pesendorfer (1997), Mandler (2012) and Krishna and Morgan (2012). Though there are many ways to prove Proposition 2, I adopt the approach of Feddersen and Pesendorfer (1997). However, before applying their techniques, I first establish a uniform convergence result that is crucial for proving the quasi-uniform convergence result of the proposition. The need for this extra step arises because the thresholds of vote fractions, q_n, needed to re-elect

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[8]Good and Mayer (1975) and Chamberlain and Rothschild (1981) used Bayes's original argument to show that if F is a distribution on $[0, 1]$ with continuous density f, then $\lim_{N \to \infty} N \int_0^1 \binom{2N}{N} \alpha^N (1 - \alpha)^N dF(\alpha) = \frac{1}{2} f(\frac{1}{2})$. I thank David Myatt and Vijay Krishna for telling me about this.

the status quo policy are themselves varying in n—a consequence of assuming that the population share of high income earners $\lambda/(1+\lambda)$ is deterministically constant.[9]

3.3 Pure Strategy Equilibrium

I now use Proposition 2 to show that in large electorates there is always a pure strategy equilibrium to the game $G(n,\theta)$. Remarkably, $x = 1$ is always an equilibrium when the electorate is large. In other words, it is an equilibrium for all low income voters who did not receive the economic opportunity in the first period to vote *en masse* for the status quo. On the other hand, $x = 0$ is not an equilibrium in large electorates if δ^* is low enough (specifically, when $\delta^* < \delta^\dagger(0) = q_\infty/p = (1-\lambda)/2p$) but it is an equilibrium when δ^* is high enough. This means that $x = 0$ is not an equilibrium if the unconditional expected payoff to the status quo policy is not much lower than the payoff to the more redistributive policy. These results are proven next.

Proposition 3. *Let $\{(n,\theta)\}_\mathcal{N}$ be any sequence of parameter profiles each satisfying Assumptions 1–5. Then there is a number N such that $n \geq N$ implies*

(i) $x = 1$ is an equilibrium to the game $G(n,\theta)$

(ii) $x = 0$ is an equilibrium to the game $G(n,\theta)$ if $\delta^* > \delta^\dagger(0)$, and is not an equilibrium to the game $G(n,\theta)$ if $\delta^* < \delta^\dagger(0)$.

Proof. Note that $\pi(\delta,1)$ is strictly decreasing in δ and $\pi(1,1) = p > 1/2 > q_\infty$. Therefore, $\delta^\dagger(1) = 1$. By Proposition 2 and Assumption 4, it follows that if n is large, then $x = 1$ is an equilibrium to the game $G(n,\theta)$. Next, note that $\pi(\delta,0)$ is strictly increasing in δ over the entire interval $[0,1]$, and ranges from 0 to p. (See, e.g., Figure 1.) Therefore, if $\delta^\dagger(0) < \delta^*$, Proposition 2 implies that $x = 0$ is an equilibrium to the game $G(n,\theta)$ for large n. On the other hand, if $\delta^\dagger(0) > \delta^*$, then Proposition 2 implies that $x = 0$ cannot be an equilibrium to the game $G(n,\theta)$ when n is large. $\square$

Proposition 3 is a trivial consequence of the analytical result established in Proposition 2. In explaining the logic behind this result, it is instructive to begin with the

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[9]This step can be avoided if one is willing to assume instead that there is an ex ante stage in which voters are identical, and then each voter becomes a high income earner with probability $\lambda/(1+\lambda)$ and a low income earner with complementary probability. In this case, the law of large numbers implies that low income voters would form a majority with probability 1 as n approaches infinity, but for a finite electorate, there is no guarantee that the rich are a minority.

case of $x = 0$. Observe that in light of Proposition 2, the condition $\delta^* < \delta^\dagger(0)$ states that if the electorate is large and the election is close, then a voter's conditional expectation of δ is higher than δ^*. This inequality is likely to be satisfied when the alternative policy is only slightly more redistributive than the status quo, hence when δ^* is low. If $x = 0$, then none of the L^0 voters cast their ballots for the status quo policy. Therefore, the election is not likely to be close unless many low income voters received the economic opportunity, became rich, and voted for the status quo. But this means that the conditional expectation of δ cannot be too small. When δ^* is larger than this conditional expectation, L^0 voters are happy to vote for the more redistributive policy. But when it is smaller, they would actually like to switch their votes to the status quo policy.

The main result of this paper is that $x = 1$ is always an equilibrium when the electorate is large. When all voters in L^0 vote to re-elect the status quo policy, the election is unlikely to be close: the status quo policy should win by a large margin. But if the election does turn out to be close, then a large fraction of the electorate must have voted for the redistributive policy. Since only the untalented voters in L^- vote for this policy, it must be that a large number of voters discovered that they are untalented. But they could only discover this by receiving the economic opportunity. In other words, for so many voters to discover that they are untalented, it must be that δ is large. Therefore, in this case, it makes sense—from the perspective of the L^0 voters—to vote for the status quo policy. The conclusion is that $x = 1$ is always an equilibrium if the size of the electorate is large enough.

In the $x = 1$ equilibrium, the expected value of δ conditional on being pivotal is nearly 1. In other words, conditional on being pivotal, low income voters who did not receive the opportunity to climb the economic ladder in the first period believe that they are nearly certain to receive the opportunity in the next period; so they vote to re-elect the status quo policy. A naive voter who does not behave strategically, but has overly optimistic beliefs about his chances of upward mobility (e.g. one who believes that δ is nearly 1) would vote exactly in the same way as the strategic L^0 voter who conditions his vote on being pivotal: these two kinds of voters are behaviorally identical, since conditional on being pivotal, the strategic L^0 voter also believes that δ is nearly 1. Therefore, as Marxist scholars refer to the naive optimistic voter as having a "false consciousness" about his mobility prospects, one might say that the
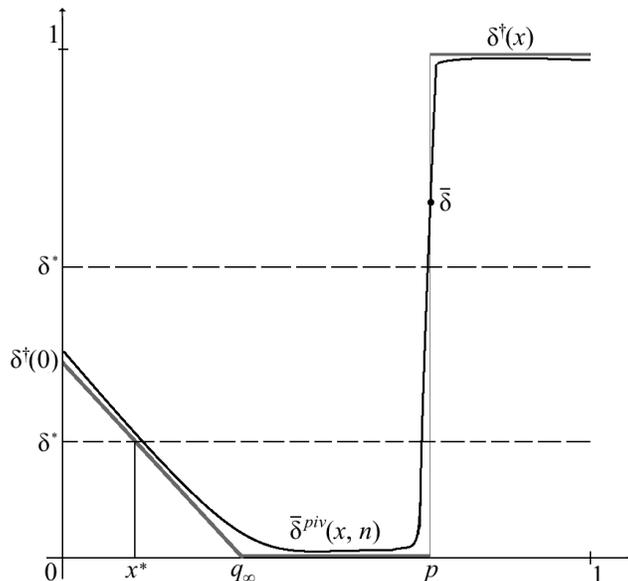
Figure 2: $\delta^\dagger(x)$ and $\bar\delta^{piv}(x,n)$ for some large n.

strategic L^0 voters of this model have an "equilibrium false consciousness" about their mobility prospects. In the $x = 1$ equilibrium, these voters have correct beliefs, but they behave as if they are nearly certain to receive the opportunity for upward mobility in the next period if the status quo policy is kept in place.

Finally, note that in the $x = 1$ equilibrium, conditional on being pivotal, an L^0 voter believes not only that δ is nearly equal to 1, but also that half of the electorate (specifically, those who support the redistributive policy) are untalented. The fact that this voter votes as if he simultaneously believes that he lives in a land of plentiful opportunities, and in which there are several untalented other voters who squandered their opportunities and voted for the left wing redistributive policy, is an ironic, but perhaps realistic, feature of the model.

3.4 Limit Equilibrium

I now characterize the set of equilibria of the game $G(n,\theta)$ when n is large. To do this, I first define a "limit equilibrium."

Proposition 1 implies that under Assumptions 1–5, the game $G(n,\theta)$ has an equilibrium. Let $\{(n,\theta)\}_{\mathcal{N}}$ denote a sequence of parameter profiles each satisfying these assumptions. Say that a number $z \in [0,1]$ is a *limit equilibrium* of the sequence of

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games $\{G(n,\theta)\}_\mathcal{N}$ if there exists a sequence $\{x_n\}_\mathcal{N}$ of corresponding equilibria (meaning that x_n is a voting equilibrium to the game $G(n,\theta)$ for all $n \in \mathcal{N}$) that has a subsequence that converges to z. Immediately, we know that limit equilibria exist since every sequence of equilibria $\{x_n\}_\mathcal{N}$ is contained in the unit interval. (Also, Proposition 3 implies that $x = 1$ is a limit equilibrium.) The main result of this section characterizes the entire set of limit equilibria.

Before stating this result, I provide some graphical intuition. Examine Figure 2, which depicts $\delta^\dagger(x)$ as a function of x and is derived from Figure 1. Here, I have defined $\delta^\dagger(p) = \bar{\delta}$ and included the point $(p, \bar{\delta})$. Also, since $\bar{\delta}^{piv}(x,n)$ is continuous in x for all n, the figure also depicts the curve $\bar{\delta}^{piv}(x,n)$ as a function of x for a large fixed value of n. Since n is large, this curve is drawn close to $\delta^\dagger(x)$. (Recall that Proposition 2 states that $\bar{\delta}^{piv}(x,n)$ converges quasi-uniformly to $\delta^\dagger(x)$.) Furthermore, one possible value for δ^* is depicted by the upper dotted line. In this case, $\delta^* > \delta^\dagger(0)$ and both $x = 0$ and $x = 1$ are equilibria. Notice, however, that there is another equilibrium at the point where $\bar{\delta}^{piv}(x,n)$ curve line crosses the dotted line. In this equilibrium, the value of x is close to p. Next, consider the situation where δ^* is given by the lower dotted line, so that $\delta^* < \delta^\dagger(0)$. In this case, $x = 1$ is the only pure strategy equilibrium. But, there are also two other mixed strategy equilibria, which correspond to the two points where the dotted line crosses the function $\bar{\delta}^{piv}(x,n)$. Consequently, Figure 2 suggests that with large electorates, there are generically three equilibria of the kind described above. I will show that these equilibria are (very close to) the limit equilibria of the game.

Before stating the proposition, define

$$x^* = \begin{cases} \frac{q_\infty - \delta^* p}{1 - \delta^*} & \text{if } \delta^* < \delta^\dagger(0) = q_\infty/p \\ 0 & \text{if } \delta^* > \delta^\dagger(0) = q_\infty/p \end{cases} \tag{9}$$

The proposition shows that except for knife-edge cases, the set of limit equilibria of the model is $\{x^*, p, 1\}$.

Proposition 4. *Let $\{(n,\theta)\}_\mathcal{N}$ be any generic sequence of parameter profiles each satisfying Assumptions 1–5. Then the set of limit equilibria of the sequence of games $\{G(n,\theta)\}_\mathcal{N}$ is $\{x^*, p, 1\}$.*

Proof. The claims that 1 is always a limit equilibrium, and $x^* = 0$ is a limit equilibrium when $\delta^* > \delta^\dagger(0)$, both follow from Proposition 3. Therefore, we must

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prove the following three claims: (i) p is always a limit equilibrium, (ii) $x^* = \frac{q_\infty - \delta^* p}{1 - \delta^*}$ is a limit equilibrium when $\delta^* < \delta^\dagger(0)$, and (iii) there are no other limit equilibria.

(i) Define $\delta^\dagger(p) = \bar{\delta}$ as in Figure 2. Then fix a small number $\epsilon > 0$. Proposition 2 implies that $\{\delta^{piv}(x,n)\}_\mathcal{N}$ converges pointwise to $\delta^\dagger(x)$. That means $\bar{\delta}^{piv}(p - \epsilon, n)$ converges to 0 while $\bar{\delta}^{piv}(p + \epsilon, n)$ converges to 1. Therefore, since $\delta^* \in (0,1)$, there is a number n large enough such that $\bar{\delta}^{piv}(p - \epsilon, n) < \delta^*/2$ and $\bar{\delta}^{piv}(p + \epsilon, n) > (1 + \delta^*)/2$. But since $\bar{\delta}^{piv}(x,n)$ is continuous in x for all n, the intermediate value theorem implies that for n large enough, there exists a number $x \in [p - \epsilon, p + \epsilon]$ such that $\bar{\delta}^{piv}(x,n) = \delta^*$. Thus x is an equilibrium to the game $G(n, \theta)$.

Now start with $\epsilon > 0$ very small, and consider the sequence $\{\epsilon/k\}_{k=1}^\infty$. By the procedure above, we can associate with each ϵ/k, a number N_k such that for all $n \geq N_k$, there is an equilibrium x_n to the game $G(n, \theta)$ that is within ϵ/k of p. Moreover, we can use the procedure to construct a sequence of equilibria $\{x_n\}_\mathcal{N}$ where the kth element of this sequence is associated with ϵ/k. Then, by construction, this sequence of equilibria converges to p since the sequence $\{\epsilon/k\}$ converges to 0. Therefore, p must be a limit equilibrium.

(ii) The proof that $x^* = \frac{q_\infty - \delta^* p}{1 - \delta^*}$ is also a limit equilibrium when $\delta^* < \delta^\dagger(0)$ is exactly analogous to the argument above, and omitted.

(iii) Finally, there are no other limit equilibria besides the ones reported in the proposition. Indeed, suppose that there were another limit equilibrium, and call it z. Suppose $\delta^* > \delta^\dagger(0)$, so that $z \in (0,1)$, $z \neq p$. Since $\delta^* \in (\delta^\dagger(0), 1)$, for $\epsilon > 0$ small enough we know that $\delta^\dagger(z)$ is bounded away from δ^*; in particular

$$|\delta^\dagger(z) - \delta^*| > 2\epsilon. \tag{10}$$

(See the graph of $\delta^\dagger(x)$ in Figure 2.) Since z is a limit equilibrium, there is a sequence of equilibria $\{x_n\}_\mathcal{N}$ that has a subsequence that converges to z. Denote that subsequence by $\{x_k\}_{\mathcal{K} \subseteq \mathcal{N}}$, where $\mathcal{K}$ is an infinite set. Then, by Proposition 2, there exists $\rho > 0$ and N such that $n \geq N$ implies

$$|\bar{\delta}^{piv}(x,n) - \delta^\dagger(z)| \leq \epsilon \qquad \forall x \in B_\rho(z). \tag{11}$$

The inequalities (10) and (11) imply that for all $n \geq N$

$$|\delta^* - \bar{\delta}^{piv}(x,n)| > \epsilon \qquad \forall x \in B_\rho(z). \tag{12}$$

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This in turn implies that there is an index $k \in \mathcal{K}$, $k \geq N$, such that $x_k \in (0, 1)$ and $|\delta^* - \bar{\delta}^{piv}(x_k, k)| > \epsilon$. But then x_k cannot be an equilibrium to the game $G(k, \theta)$, establishing the contradiction.

We can use an analogous argument in the case of $\delta^* < \delta^\dagger(0)$ to show that there can be no other limit equilibria besides x^*, p and 1. $\qquad \square$

An important implication of Proposition 4 is that when the size of the electorate is large, every equilibrium to the game $G(n, \theta)$ has to be close to a limit equilibrium. To understand why, ask the following question: Can a sequence of equilibria $\{x_n\}_{\mathcal{N}}$ have a subsequence that remains bounded away from all of the limit equilibria? The answer is no, because that subsequence would in turn contain a sub-subsequence that converges to a limit that is bounded away from each limit equilibrium. But since this sub-subsequence is also a subsequence of $\{x_n\}_{\mathcal{N}}$, its limit would have to be a limit equilibrium. Contradiction. This implies that when n is large, every equilibrium of the game $G(n, \theta)$ has to be close to one of the limit equilibria.

To summarize the result of this section, p and 1 are always limit equilibria. When the alternative policy is relatively more redistributive than the status quo, so that δ^* is high, then $x^* = 0$ is also limit equilibrium, meaning that the voters who did not receive the economic opportunity in the first period all vote against the status quo. On the other hand, if the alternative policy is only slightly more redistributive than the status quo, then δ^* is low, and $x^* = \frac{q_\infty - \delta^* p}{1 - \delta^*}$ is a limit equilibrium. In this case, when the electorate is large, some voters who did not receive the economic opportunity in the first period will end up voting to re-elect the status quo policy.

4 Information Aggregation

Consider all of the assumptions as before, except now suppose that the realization of δ is publicly revealed to the voters. Call this modified game $G^\circ(n, \theta)$. Then, in any equilibrium to the game $G^\circ(n, \theta)$, all voters not in L^0 vote exactly as before. In particular, members of H and L^+ vote to keep the status quo while members of L^- vote for the more redistributive policy. Members of L^0, however, vote for the more redistributive policy if $\delta < \delta^*$ and for the status quo policy if $\delta > \delta^*$. As with equilibria of the game $G(n, \theta)$, we can identify an equilibrium of the game $G^\circ(n, \theta)$ with the probability x° that a voter in L^0 votes for the status quo policy.

Then, for all sequences of parameter profiles $\{(n, \theta)\}_{\mathcal{N}}$ satisfying Assumptions 1–5, and all realizations $\delta \neq \delta^*$, there exists a unique sequence of equilibria $\{x_n^\circ(\delta)\}_{\mathcal{N}}$, written as functions of δ, corresponding to the sequence of games $\{G^\circ(n, \theta)\}_{\mathcal{N}}$. Denote by $\{E_n^\circ(\delta)\}_{\mathcal{N}}$ the corresponding sequence of equilibrium distributions over the two possible electoral outcomes.

Now, suppose that z is a limit equilibrium of the sequence of games $\{G(n, \theta)\}_{\mathcal{N}}$ in which δ is not revealed to the voters. This means that corresponding to this sequence of games, there is a sequence of equilibria $\{x_n\}_{\mathcal{N}}$ that has a subsequence $\{x_k\}_{\mathcal{K} \subseteq \mathcal{N}}$ that converges to z. Associated with every such subsequence is a sequence of equilibrium distribution functions $\{E_k(\delta)\}_{\mathcal{K}}$ over the two electoral outcomes, again viewed as functions over possible realizations of δ. If, for every such subsequence of every sequence of voting equilibria, and almost every realization of δ, we have

$$\lim_{k \to \infty} E_k^\circ(\delta) = \lim_{k \to \infty} E_k(\delta)$$

then the limit equilibrium z is said to *aggregate information*. (Here, I am taking both limits along the index set $\mathcal{K}$.) Put in words, z aggregates information if in large electorates where voters do not know δ, and play equilibria close to z, the distribution over electoral outcomes is almost surely the same as it would have been had δ been publicly revealed. We are interested in knowing which values of z aggregate information, and which do not. This question is answered by the following proposition.

Proposition 5. *Let $\{(n, \theta)\}_{\mathcal{N}}$ be any generic sequence of parameter profiles each satisfying Assumptions 1–5. Of the three limit equilibria $\{x^*, p, 1\}$ of the sequence of games $\{G(n, \theta)\}_{\mathcal{N}}$, only the limit equilibrium x^* aggregates information.*

Proof. I sketch the proof, which relies on a simple application of the law of large numbers. Suppose $\{x_k\}_{\mathcal{K} \subseteq \mathcal{N}}$ is a subsequence of equilibria that converges to z.

First, let $z = p, 1$. Then, we have $\lim \pi(\delta, x_k) > q_\infty$ for all δ. Therefore, for all δ, the status quo policy wins the election almost surely in the game $G(k, \theta)$ as $k \to \infty$. But for all $\delta < \min\{\delta^\dagger(0), \delta^*\}$, the redistributive policy wins almost surely in the game $G^\circ(k, \theta)$ as $k \to \infty$. (When $\delta < \min\{\delta^\dagger(0), \delta^*\}$ and $k \to \infty$, all L^0 voters vote for the redistributive policy in the game $G^\circ(k, \theta)$ and the votes of high income voters in L^+ and H are almost surely insufficient to re-elect the status quo.) Therefore, as $k \to \infty$, the equilibrium distribution over electoral outcomes in the games $G(k, \theta)$

and $G^\circ(k, \theta)$ are different when $\delta < \min\{\delta^\dagger(0), \delta^*\}$. So the limit equilibrium $z = p, 1$ fails to aggregate information.

Next, let $z = x^*$. Begin with the case where $\pi(\delta, x^*) < q_\infty$. In this case, $\delta < \delta^*$. So, in both games $G(k, \theta)$ and $G^\circ(k, \theta)$, the redistributive policy wins almost surely as $k \to \infty$. Now consider the case $\pi(\delta, x^*) > q_\infty$. If $\delta^* < \delta^\dagger(0)$ then $\delta > \delta^*$ so in both games $G(k, \theta)$ and $G^\circ(k, \theta)$, the status quo policy wins almost surely as $k \to \infty$. If, on the other hand, $\delta^* > \delta^\dagger(0)$ then $x^* = 0$, so the assumption implies that $\delta p > q_\infty$. In this case, for both games $G(k, \theta)$ and $G^\circ(k, \theta)$ just the L^+ and H votes are sufficient to guarantee that the status quo policy wins almost surely as $k \to \infty$. $\qquad \square$

To summarize Proposition 5, there are generically three limit equilibria, two of which (p and 1) do not aggregate information. Importantly, the pure strategy equilibrium $x = 1$ in large electorates fails to aggregate information.

Finally, it is worth mentioning that an attractive and novel feature of the model that sets it apart from other voting models with private information is the fact that there always exists an equilibrium in pure strategies, i.e. the $x = 1$ equilibrium, when the electorate is large. In fact, in the case where $\delta^* < \delta^\dagger(0)$ (so that the redistributive policy is still better for low income voters than the status quo, but not that much better), this is the only strict equilibrium. The other limit equilibria, $x = x^*, p$, are in totally mixed strategies and are not strict. Hence, only the $x = 1$ equilibrium survives the usual perturbation-based refinements that rule out all but strict equilibria (e.g. the practice of perturbing each agent's conjecture about how others will play to determine whether it is still optimal for him to play his exact equilibrium strategy). Moreover, in the case where $\delta^* < \delta^\dagger(0)$, it is a fact that in all limit equilibria the low income voters who did not receive the economic opportunity in the first period vote for the status quo policy with positive probability. So, despite the existence of multiple equilibria, there is a strong sense in which the model rationalizes the behavior of agents who know that the more redistributive policy is better for them, but nevertheless end up voting for the less redistributive policy.

5 Remarks about the Assumptions

5.1 Mobility Under the More Redistributive Policy

One might (rightly) object that it is unreasonable to assume that the redistributive policy does not give low income voters an opportunity for upward social mobility.

However, I have actually not made this assumption. In particular, one can take y_R^l to be an *expected* payoff, arising from an underlying assumption about the extent to which the redistributive policy produces opportunities for upward mobility. It is sufficient for the conclusions of the model that the probability of receiving opportunities for upward mobility under the status quo policy is independent of the probability of receiving opportunities for upward mobility under the more redistributive policy. In this case, one can treat y_R^l as a constant across all possible realizations of δ in the first period, since the voter cannot obtain any information about whether or not the redistributive policy works well from experiencing only the status quo policy.[10]

5.2 Probability of Being Talented

The assumption that $p \neq 1$ is crucial for generating the result that $x = 1$ is an equilibrium of the model for large electorates. In particular, under the assumption that $p = 1$, all low income voters are certain to be talented and the model has a unique limit equilibrium for generic parameter values; this is precisely the limit equilibrium $x = x^*$, which aggregates information. However, the assumption that $p = 1$ is not compelling, since in the real world there is uncertainty both about the chances of receiving economic opportunities under specific policies, and about whether individual voters are able to convert those opportunities into success. On the other hand, the assumption that $p > 1/2$ is slightly stronger than necessary: all of the results would go through under the weaker assumption that $p > q_\infty = (1 - \lambda)/2$. To see this in a simple and transparent way, note that all I need for the results is that the point p in Figure 1 be above the q_∞ line so that $\delta^\dagger(x) = 1$ for all $x > p$, as depicted in Figure 2. Now, the assumption that all voters have equal probability p of being talented is, of course, less compelling, but it serves to keep the model parsimonious and is unlikely to be driving the main results. Finally, the assumption that there is uncertainty about how well the status quo policy really works (i.e. about δ) but not about a voter's probability of being talented (p) is important for the results. But

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[10]Actually, the results of the model only rely on the existence of a threshold $\delta^* < 1$ such that L^0 voters prefer the status quo policy when they expect δ to be greater than δ^*. In particular, this condition can be satisfied even when the probability for receiving an opportunity for upward mobility under the redistributive policy is perfectly correlated with the same probability under the status quo policy. For example, it is easy to verify that if the probability of receiving the opportunity under the more redistributive policy is $\kappa\delta$, then the critical threshold $\delta^* < 1$ exists when $\kappa < \frac{y_Q^h - y_Q^l}{y_R^h - y_R^l} - \frac{y_R^l - y_Q^l}{p(y_R^h - y_R^l)}$.

note that under this assumption there is only first order uncertainty both about an individual voter's talent, and about whether the status quo policy works well.

5.3 Redistributive Policy First

The results of the model also hinge on the assumption that the status quo policy is less redistributive than the alternative. It is easy to show that if the more redistributive policy is implemented first, or if there is only one period and voters must choose between the two policies without first experiencing either policy, then the more redistributive policy always wins the election. In light of this, one could criticize the model for being a model of "status quo bias" rather than a model of "right wing bias." Certainly, it may be the case that disapproval of left wing policies by American voters is a consequence of status quo bias. Meltzer and Richard (1978) argue, for example, that in the past century, the United States progressively adopted left wing policies to replace status quo policies that were defended by conservatives. Meltzer and Richard point to Roosevelt's New Deal and Johnson's Great Society—both examples of policy agendas that saw intense opposition when they were first introduced, but whose programs, such as Social Security and Medicare, enjoy popular support today. In light of these facts, the assumption that the right wing policy comes first is consistent with history. Moreover, the close association between conservatism and right wing ideology in the United States provides some additional justification for the premise that throughout history, voters are continuously choosing between the right wing status quo and left wing alternatives.[11] Consequently, it may be quite challenging to distinguish status quo bias from right wing bias not just in this model, but also in reality. The fact that this challenge arises both in the model and in reality should be viewed as a strength rather than limitation of the model.

6 Related Literature

Almost four decades ago, Albert Hirschman (1973) drew the connection between beliefs about social mobility and tolerance for income inequality by explaining what he called the *tunnel effect*. Hirschman described how a low income individual could make inferences about his own mobility prospects by observing the experiences of his

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[11]Conservatism, whose Latin root *conservare* means "to retain," is often interpreted as a preservationist ideology seeking specifically to retain the status quo. See, e.g., Jost et.al. (2003).

neighbors, relatives and friends. If this individual sees people around him climbing the economic ladder, he forms optimistic beliefs about his own chances, which in turn leads him to tolerate current inequality. By extending Hirschman's theory to its logical end, one can argue that such inferences may cause low income voters to oppose redistribution if such policies have some temporal persistence (in the sense that once instituted, they are difficult to reverse). Indeed, these voters may prefer the policy that in reality is more likely to make them worse off.

My model is obviously related to Hirschman's ideas, and other papers relating social mobility to redistribution. For example, Piketty (1995) provides a model in which individuals use their and their ancestors' social mobility experiences to make inferences about the relative importance of luck and effort in determining economic success. Piketty shows that some dynasties converge to the right wing belief that luck is relatively unimportant in determining success, and come to oppose redistribution, while other dynasties converge to the left wing belief that luck is important, and come to support redistribution.[12]

Piketty's (1995) rational agent is not purely individualistic; instead, his preferences also contain an ideological component. Perhaps more in line with the agenda of this paper is a paper by Bénabou and Ok (2001), which addresses the question of whether purely self-interested low income voters with rational expectations about their mobility prospects could vote against redistribution. These authors show that if the income transition function between periods is strictly concave, then such behavior is possible. However, Bénabou and Ok's story is considerably different from the one in this paper. While in the current paper, there is a tension between a voter's true (naive) preferences and his equilibrium considerations, no such tension exists in the Bénabou-Ok model. Relatedly, the Bénabou-Ok model does not exhibit information aggregation failure, which is one of the important features of my model.[13]

My paper also speaks to a number of informal arguments in the literature on social mobility and redistribution. Many years ago Jennifer Hochschild (1981) conducted in-depth interviews with regular Americans and found that those who expressed a

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[12]Alesina and La Ferrara (2005) provide some evidence in support of Piketty's conclusion that experiences of inter-generational upward mobility reduce support for redistribution.

[13]In addition, Bénabou and Ok (2001) also allow for downward social mobility. They take their model to data, and argue that their findings indicate that the "[mobility] effect is probably dominated by the demand for social insurance." So, they conclude by issuing some skepticism that opposition to redistributive policies can be explained by beliefs concerning social mobility.

strong belief in the "American dream" were also likely to oppose redistribution. Recently, the importance of such beliefs about social mobility for explaining attitudes towards redistribution has recaptured the interest of scholars and commentators alike, especially after the sociologist Thomas DiPrete (2007) produced some evidence suggesting that young Americans overestimate their chances of becoming rich (and the degree of overestimation is considerably large whether one uses subjective or objective definitions of the word "rich"). Not surprisingly, however, these findings have also generated a great deal of skepticism concerning the rationality of American voters. For example, Bartels (2008a) writes the following: "At the individual level ... psychological pressures produce unrealistic optimism about one's own prospects, and an illusion of control over uncontrollable events." And in response to Shenkman's (2008) accusation that "The consensus in the political science profession is that voters are rational," Bartels (2008b) writes, "Well, no. A half-century of scholarship provides plenty of grounds for pessimism about voters' rationality." In response to this skepticism, the behavioral equivalence result of this paper shows that seemingly unsophisticated behavior may in fact be observationally identical to equilibrium behavior, though the aggregation failure result implies that this is not a good thing. The results suggest that it may be democratic institutions (e.g. voting rules), rather than voter "irrationality," that stands in the way of information aggregation.

7   Conclusion

Many low income voters oppose increased redistribution even when they stand to gain from it. A longstanding question in political economy asks whether such opposition could ever be rational. Though some scholars have guessed that voters' beliefs about social mobility might be important in answering this question, many have been quick to dismiss the possibility.

In this paper, I offered a model of social mobility and redistribution in which numerous low income voters who care only about their economic payoff may, in a particular equilibrium, vote with certainty for a right wing status quo policy that they believe is worse for them than a competing left wing alternative. This occurs because voters are strategic, so they condition their vote on being pivotal: In the event that every vote for the wining policy matters, these voters believe that the status quo policy gives them greater chance of upward mobility than it actually does. Thus,

even fully rational and strategic voters behave exactly as if they were naive voters who have a "false consciousness" regarding their prospect for upward mobility. An important consequence of their behavior is the failure of the equilibrium to aggregate information.

These results are particularly relevant to democratic theorists who are concerned that a lack of voter sophistication causes democracy to fail on many dimensions, including information aggregation. Along with the other papers cited in the introduction, the behavioral equivalence and aggregation failure results of this paper show that even when voters behave rationally, they may collectively fail to elect the policy that is in the interest of the majority. In particular, these results imply that for democracy to succeed, we might have to place some peculiar demands on voters: that they not be overly optimistic and naive, but that they also not be fully rational.

One important question that the paper leaves unanswered is whether the equilibrium behavior of the voters in this model can be sustained over time. In particular, it would be interesting to study an infinite horizon extension of the model and ask whether there is an equilibrium path in which the behavioral equivalence between rational strategic voting and optimistic naive voting continues to hold in the long run. I conjecture that it may not, because the optimistic voter has the potential to eventually learn that his beliefs were too optimistic. On the other hand, the rational voter has less to learn: his expectations were correct to begin with. Thus, it may be the case that aggregation failure will continue to hold in the long run if voters are fully rational, but will not not hold if they are optimistic and naive. Nevertheless, analyzing strategic voting in dynamic elections has been challenging and this extension is clearly not straightforward.

Appendix: Proof of Proposition 2

If $x = p$, then $f^{piv}(\cdot|x, n)$ is equal to the unconditional distribution f for all $n \in \mathcal{N}$, since $\varphi(\delta|p, n)$ becomes constant in δ. So $\bar{\delta}^{piv}(p, n) = \bar{\delta}$ for all n.

To prove the remainder, define the functions $h^n : [0, 1] \to \mathbb{R}$, $n \in \mathcal{N} \cup \{\infty\}$, by

$$h^n(\tilde{\pi}) = \tilde{\pi}^{q_n}(1 - \tilde{\pi})^{1-q_n}.$$

I will make use of the following easy-to-verify facts.

Fact 1. The sequence $\{q_n\}$ is strictly increasing and converges to q_∞.

Fact 2. For all $n \in \mathcal{N} \cup \{\infty\}$, $n > 1$, the composite function $h^n(\pi(\cdot, \cdot)) : [0, 1]^2 \to \mathbb{R}$ is uniformly continuous.

Fact 3. For all $x \neq p$ and all $n \in \mathcal{N} \cup \{\infty\}$, the function $h^n(\pi(\cdot, x)) : [0, 1] \to \mathbb{R}$ is single-peaked and maximized by the value of $\delta \in [0, 1]$ that minimizes $|\pi(\delta, x) - q_n|$. Thus, $\delta^\dagger(x)$ maximizes $h^\infty(\pi(\delta, x))$.

Fact 4. $\varphi(\delta|x, n) = \binom{n}{nq_n} (h(\pi(\delta, x)))^n$.

Lemma 1. $\{h^n(\pi(\cdot, \cdot))\}_\mathcal{N}$ *converges uniformly to* $h^\infty(\pi(\cdot, \cdot))$.

Proof. This can be proven by piecewise applying *Dini's theorem* (see, e.g., Aliprantis and Border, 2006, Theorem 2.66). The need to apply the theorem piecewise arises because the pointwise convergence of $\{h^n(\pi(\cdot, \cdot))\}$ to $h^\infty(\pi(\cdot, \cdot))$ is only piecewise monotonic, as I will show.

First, I show that $\{h^n(\pi(\cdot, x))\}$ converges pointwise to $h^\infty(\pi(\cdot, x))$. Note that if $(\delta, x) = (0, 0)$ or $(0, 1)$, then $h^n(\pi(\delta, x)) = h^\infty(\pi(\delta, x))$ for all $n \in \mathcal{N}$. So let $(\delta, x) \neq (0, 0)$ or $(0, 1)$. Then, we have

$$|h^\infty(\pi(\delta, x)) - h^n(\pi(\delta, x))| = |(\pi(\delta, x))^{q_\infty}(1 - \pi(\delta, x))^{1-q_\infty} - (\pi(\delta, x))^{q_n}(1 - \pi(\delta, x))^{1-q_n}|$$

$$= (\pi(\delta, x))^{q_\infty}(1 - \pi(\delta, x))^{1-q_\infty} \left| 1 - \left(\frac{\pi(\delta, x)}{1 - \pi(\delta, x)} \right)^{q_n - q_\infty} \right|$$

But since $q_n \to q_\infty$ by Fact 1, we have

$$\lim \left(\frac{\pi(\delta, x)}{1 - \pi(\delta, x)} \right)^{q_n - q_\infty} = 1.$$

Therefore $\{h^n(\pi(\cdot, x))\}$ converges pointwise to $h^\infty(\pi(\cdot, x))$.

Next, I show that the convergence is (piecewise) monotonic. Define the sets

$$X_1 = \{(x, \delta) \in [0, 1]^2 : \pi(\delta, x) \leq 1/2\}$$
$$X_2 = \{(x, \delta) \in [0, 1]^2 : \pi(\delta, x) \geq 1/2\}$$

and the functions $h_1^n(\pi(\cdot,\cdot)) : X_1 \to \mathbb{R}$ and $h_2^n(\pi(\cdot,\cdot)) : X_2 \to \mathbb{R}$ such that $h_j^n(\pi(\delta,x)) \equiv h^n(\pi(\delta,x))$, $j = 1,2$, $n \in \mathcal{N} \cup \{\infty\}$. All of these functions are continuous, and I have shown above that $\{h_j^n(\pi(\cdot,x))\}$ converges pointwise to $h_j^\infty(\pi(\cdot,x))$, $j = 1,2$. Moreover, these sequences are monotonic:

$$\forall (x,\delta) \in X_1 \qquad h_1^n(\pi(\delta,x)) \geq h_1^{n'}(\pi(\delta,x)) \qquad n < n'$$
$$\forall (x,\delta) \in X_2 \qquad h_2^n(\pi(\delta,x)) \leq h_2^{n'}(\pi(\delta,x)) \qquad n < n'$$

The first line is a consequence of Fact 1 and $\pi(\delta,x) \leq 1/2$ for all $(x,\delta) \in X_1$, by definition of X_1. The second is a consequence of Fact 1 and $\pi(\delta,x) \geq 1/2$ for all $(x,\delta) \in X_2$. Therefore, Dini's theorem implies that $\{h_j^n(\pi(\cdot,\cdot))\}$ converges uniformly to $h_j^\infty(\pi(\cdot,\cdot))$, $j = 1,2$. But then $\{h^n(\pi(\cdot,\cdot))\}$ converges uniformly to $h^\infty(\pi(\cdot,\cdot))$. $\square$

Now recall that $B_\rho(x)$, defined in the statement of the proposition, refers to the subset of points in the unit interval that are within ρ of x. Also, define

$$\Delta_\epsilon(x) \equiv \{\delta \in [0,1] : |\delta - \delta^\dagger(x)| \leq \epsilon\}$$

to be an ϵ-neighborhood of $\delta^\dagger(x)$ intersecting with the unit interval.

Lemma 2. *Fix $x \neq p$ as in the hypothesis of the proposition. Fix $\epsilon > 0$ small enough so that $\Delta_\epsilon(x) \neq [0,1]$. Then there exists $\eta_\epsilon \in (0,1)$, $\mu > 0$ and a number M such that for all $n \geq M$*

$$h^\infty(\pi(\delta^\dagger(x),x)) - \eta_\epsilon > \sup_{\delta \notin \Delta_\epsilon(x)} h^n(\pi(\delta,\tilde{x})) \qquad \forall \tilde{x} \in B_\mu(x). \tag{13}$$

Furthermore, for all $n \in \mathcal{N} \cup \{\infty\}$, define the set

$$\Omega_\epsilon^n(\tilde{x}) = \{\omega \in \Delta_\epsilon(x) : |h^\infty(\pi(\delta^\dagger(x),x)) - h^n(\pi(\omega,\tilde{x}))| \leq \eta_\epsilon/2\}. \tag{14}$$

Then, there exist $\alpha > 0$, $\nu > 0$, and a number M' such that for all $n \geq M'$

$$\int_{\omega \in \Omega_\epsilon^n(\tilde{x})} (1 - \omega)f(\omega)d\omega \geq \alpha \qquad \forall \tilde{x} \in B_\nu(x) \tag{15}$$

Proof. When ϵ is small, the single-peakedness and continuity of $h^\infty(\pi(\cdot,x))$, together with Fact 3, implies that there is a number $\eta_\epsilon > 0$ such that

$$h^\infty(\pi(\delta^\dagger(x),x)) - 2\eta_\epsilon > \sup_{\delta \notin \Delta_\epsilon(x)} h^\infty(\pi(\delta,x)). \tag{16}$$

Since $1 \geq h^\infty(\pi(\delta,x)) \geq 0$ for all $(\delta,x) \in [0,1]^2$, we know that $\eta_\epsilon < 1$. Uniform convergence from Lemma 1 implies the existence of M such that for all $n \geq M$

$$|h^n(\pi(\delta,\tilde{x})) - h^\infty(\pi(\delta,\tilde{x}))| \leq \eta_\epsilon/2 \qquad \forall (\delta,\tilde{x}) \in [0,1]^2. \tag{17}$$

By the uniform continuity of $h^\infty(\pi(\cdot,\cdot))$ there exists $\mu > 0$ such that

$$|h^\infty(\pi(\delta,\tilde{x})) - h^\infty(\pi(\delta,x))| \leq \eta_\epsilon/2 \qquad \forall (\delta,\tilde{x}) \in [0,1] \times B_\mu(x). \tag{18}$$

Combining (17) and (18) using the triangle inequality implies that for $n \geq M$

$$|h^n(\pi(\delta,\tilde{x})) - h^\infty(\pi(\delta,x))| \leq \eta_\epsilon \qquad \forall (\delta,\tilde{x}) \in [0,1] \times B_\mu(x). \tag{19}$$

Thus, for all $n \geq M$

$$
\begin{aligned}
\eta_\epsilon &\geq \sup_{\delta \notin \Delta_\epsilon(x)} |h^n(\pi(\delta,\tilde{x})) - h^\infty(\pi(\delta,x))| \\
&\geq \sup_{\delta \notin \Delta_\epsilon(x)} h^n(\pi(\delta,\tilde{x})) - \sup_{\delta \notin \Delta_\epsilon(x)} h^\infty(\pi(\delta,x)) \qquad \forall \tilde{x} \in B_\mu(x)
\end{aligned} \tag{20}
$$

Combining (16) and (20) shows that for all $n \geq M$

$$
\begin{aligned}
h^\infty(\pi(\delta^\dagger(x),x)) - \eta_\epsilon &= \left(h^\infty(\pi(\delta^\dagger(x),x)) - 2\eta_\epsilon\right) + \eta_\epsilon \\
&> \sup_{\delta \notin \Delta_\epsilon(x)} h^\infty(\pi(\delta,x)) + \eta_\epsilon \geq \sup_{\delta \notin \Delta_\epsilon(x)} h^n(\pi(\delta,\tilde{x})) \qquad \forall \tilde{x} \in B_\mu(x)
\end{aligned}
$$

which establishes (13).

To prove the second statement, first define the following sets

$$\Omega_\epsilon^*(x) = \{\omega \in \Delta_\epsilon(x) : |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,x))| \leq \eta_\epsilon/8\}$$

$$\Omega_\epsilon^{**}(\tilde{x}) = \{\omega \in \Delta_\epsilon(x) : |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,\tilde{x}))| \leq \eta_\epsilon/4\}$$

Note how the constraints defining each of these sets differ. Since $h^\infty(\pi(\cdot,x))$ is continuous and single-peaked, and maximized by $\delta^\dagger(x)$, the set $\Omega_\epsilon^*(x)$ is a nonempty interval. Let $\gamma > 0$ denote its length.

Since $h^\infty(\pi(\cdot,\cdot))$ is uniformly continuous, there exists $\nu > 0$ such that

$$|h^\infty(\pi(\omega,x)) - h^\infty(\pi(\omega,\tilde{x}))| < \eta_\epsilon/8 \qquad \forall \tilde{x} \in B_\nu(x) \tag{21}$$

Combine this with the inequality that defines $\Omega_\epsilon^*(x)$ to get

$$
\begin{aligned}
\forall \omega \in \Omega_\epsilon^*(x) \qquad |h^\infty&(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,\tilde{x}))| \\
&\leq |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,x))| + |h^\infty(\pi(\omega,x)) - h^\infty(\pi(\omega,\tilde{x}))| \\
&\leq \eta_\epsilon/8 + \eta_\epsilon/8 = \eta_\epsilon/4 \qquad \forall \tilde{x} \in B_\nu(x).
\end{aligned}
$$

This proves that $\Omega_\epsilon^*(x) \subseteq \Omega_\epsilon^{**}(\tilde{x})$ for all $\tilde{x} \in B_\nu(x)$.

Since $\{h^n(\pi(\cdot,\cdot))\}$ converges uniformly to $h^\infty(\pi(\cdot,\cdot))$, there exists a number M' such that $n \geq M'$ implies

$$|h^\infty(\pi(\omega,\tilde{x})) - h^n(\pi(\omega,\tilde{x}))| \leq \eta_\epsilon/4 \qquad \forall (\omega,\tilde{x}) \in [0,1]^2.$$

Combining this with the inequality that defines $\Omega_\epsilon^{**}(\tilde{x})$ implies that for all $n \geq M'$

$$
\begin{aligned}
\forall \omega \in \Omega_\epsilon^{**}(\tilde{x}) \qquad & |h^\infty(\pi(\delta^\dagger(x), x)) - h^n(\pi(\omega, \tilde{x}))| \\
& \leq |h^\infty(\pi(\delta^\dagger(x), x)) - h^\infty(\pi(\omega, \tilde{x}))| + |h^\infty(\pi(\omega, \tilde{x})) - h^n(\pi(\omega, \tilde{x}))| \\
& \leq \eta_\epsilon/4 + \eta_\epsilon/4 = \eta_\epsilon/2 \qquad \forall \tilde{x} \in [0, 1].
\end{aligned}
$$

Thus we have proven that for all for all $n \geq M'$ and all $\tilde{x} \in B_\nu(x)$

$$
\Omega_\epsilon^*(x) \subseteq \Omega_\epsilon^{**}(\tilde{x}) \subseteq \Omega_\epsilon^n(\tilde{x}).
$$

Therefore, for all $n \geq M'$ and all $\tilde{x} \in B_\nu(x)$, the set $\Omega_\epsilon^n(\tilde{x})$ must contain an interval of length at least $\gamma > 0$, and in addition there exists a number α such that

$$
\inf_{\tilde{x} \in B_\nu(x)} \int_{\omega \in \Omega_\epsilon^n(\tilde{x})} (1 - \omega) f(\omega) d\omega \geq \inf_{\{[a,b] \subseteq [0,1]: b-a \geq \gamma\}} \int_a^b (1 - \omega) f(\omega) d\omega = \alpha > 0
$$

The first inequality holds because $\Omega_\epsilon^n(\tilde{x})$ contains an interval of length at least $\gamma > 0$ for all $\tilde{x} \in B_\nu(x)$, as we have shown. The second inequality, which states that α is strictly larger than 0, follows from the fact that $\gamma > 0$ and from Assumption 3, which states that the distribution f is non-doctrinaire.

Lemma 3. *Fix $x \neq p$ as in the hypothesis of the proposition. Fix $\epsilon > 0$. Then there exists $\rho > 0$, and N such that $n \geq N$ implies*

$$
\int_{\delta \in \Delta_\epsilon(x)} f^{piv}(\delta | \tilde{x}, n) d\delta > 1 - \epsilon \qquad \forall \tilde{x} \in B_\rho(x) \tag{22}
$$

Proof. If $\Delta_\epsilon(x) = [0, 1]$, then the result holds trivially since $\int_{\delta \in \Delta_\epsilon(x)} f^{piv}(\delta | \tilde{x}, n) d\delta = 1$ for all $\tilde{x}$. Therefore, suppose that $\Delta_\epsilon(x) \neq [0, 1]$. Then if μ, ν, M and M' are the numbers defined in Lemma 2, let $\rho = \min\{\mu, \nu\}$, so that for $n \geq \max\{M, M'\}$ and $\tilde{x} \in B_\rho(x)$, the following inequalities hold:

$$
\begin{aligned}
\int_{\delta \notin \Delta_\epsilon(x)} f^{piv}(\delta | \tilde{x}, n) d\delta &= \frac{\int_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta | \tilde{x}, n)(1 - \delta) f(\delta) d\delta}{\int_0^1 \varphi(\omega | \tilde{x}, n)(1 - \omega) f(\omega) d\omega} \\
&\leq \frac{\int_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta | \tilde{x}, n)(1 - \delta) f(\delta) d\delta}{\int_{\omega \in \Omega_\epsilon^n(\tilde{x})} \varphi(\omega | \tilde{x}, n)(1 - \omega) f(\omega) d\omega} \leq \frac{\sup_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta | \tilde{x}, n)}{\inf_{\omega \in \Omega^n(\tilde{x})} \varphi(\omega | \tilde{x}, n)} \frac{\int_{\delta \notin \Delta_\epsilon(x)} (1 - \delta) f(\delta) d\delta}{\int_{\omega \in \Omega^n(\tilde{x})} (1 - \omega) f(\omega) d\omega} \\
&\leq \frac{\sup_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta | \tilde{x}, n)}{\inf_{\omega \in \Omega^n(\tilde{x})} \varphi(\omega | \tilde{x}, n)} \frac{1}{\alpha} \leq \left(\frac{h^\infty(\pi(\delta^\dagger(x), x)) - \eta_\epsilon}{h^\infty(\pi(\delta^\dagger(x)), x) - \eta_\epsilon/2} \right)^n \frac{1}{\alpha} \leq \left(\frac{1 - \eta_\epsilon}{1 - \eta_\epsilon/2} \right)^n \frac{1}{\alpha}. \tag{23}
\end{aligned}
$$

The first inequality follows because $\Omega_\epsilon^n(\tilde{x}) \subseteq [0, 1]$ for all $\tilde{x} \in [0, 1]$. The second follows by bounding the integrals in the numerator and denominator. The third follows by

bounding the integrals in the numerator and denominator using the result of Lemma 2. The fourth follows by Fact 4 and Lemma 2. Finally, the fifth inequality follows by the fact that the left side is strictly increasing in h^∞. The result is proven by the fact that the term on the right hand side of the last inequality in (23) goes to 0 as n goes to infinity. $\qquad\square$

The proof of Proposition 2 now follows from Lemma 3. Write

$$\bar{\delta}^{piv}(\tilde{x}, n) = \int_{\delta \notin \Delta_\epsilon(x)} \delta f^{piv}(\delta|\tilde{x}, n)d\delta + \int_{\delta \in \Delta_\epsilon(x)} \delta f^{piv}(\delta|\tilde{x}, n)d\delta. \qquad (24)$$

and observe that for $n \geq N$ and $\tilde{x} \in B_\rho(x)$, this quantity is bounded above by $\epsilon + (\delta^\dagger(x) + \epsilon) = \delta^\dagger(x) + 2\epsilon$ and bounded below by $(\delta^\dagger(x) - \epsilon)(1 - \epsilon) > \delta^\dagger(x) - 2\epsilon$, both of which follow from Lemma 3. Thus for all $\tilde{x} \in B_\rho(x)$, the conditional expectation $\bar{\delta}^{piv}(\tilde{x}, n)$ is within 2ϵ of $\delta^\dagger(x)$. $\qquad\square$

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