Delays and Partial Agreements in Multi-Issue Bargaining*

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Abstract

We model a situation in which two players bargain over two pies, one of which can only be consumed starting at a future date. Suppose the players value the pies asymmetrically: one player values the existing pie more than the future one, while the other player has the opposite valuation. We show that players may consume only a fraction of the existing pie in the first period, and then consume the remainder of it, along with the second pie, at the date at which the second pie becomes available. Thus, our model features a special form of bargaining delay, in which agreements take place in multiple stages. Such *partial agreements* arise when players are patient enough, when they expect the second pie to become available soon, and when the asymmetry in their valuations is large enough.

JEL Classification Codes: C73, C78

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1 Introduction

In many real life situations, parties negotiate over multiple issues. While bargaining over an issue today, the parties involved may know that in the future they will come back to the negotiating table and bargain over a new issue altogether. In this paper we construct a bilateral bargaining model that captures such a situation. The model features two pies, X and Y. Pie X is already on the table, while pie Y can only be consumed starting at a random future date. Players cannot commit to a division of the second pie until it is available. The assumption that the second pie is available only in the future may be interpreted as a physical constraint on the environment, or may capture the idea that some issues are not yet ripe for discourse. We assume that players have opposite valuations for the two pies: player 1 values pie X more than pie Y, while player 2 values pie Y more than pie X.

A key feature that distinguishes our model from standard bargaining games *a la* Rubinstein (1982) is that we allow players to make *partial agreements*: they may divide and consume only a fraction of each available pie, leaving the remainder for future consumption. The possibility of such partial agreements arises naturally in many settings. For instance, legislators in Congress may pass a law that leaves some aspects of the issue unresolved, with the idea of dealing with the unresolved aspects at some future date. A firm negotiating a contract with a supplier may also choose to leave parts of their agreement unspecified at the start of their business relationship, with the intention of specifying them in the future.

We show that our game has a unique subgame perfect equilibrium (SPE) outcome. In the SPE outcome, players may reach a partial agreement on the first pie at the beginning of the game, consuming the remainder of it (along with the second pie) at the date at which the second pie becomes available. Thus, our model features a special form of bargaining delay, in which agreements take place in multiple stages. These partial agreements arise when players are patient enough, when they expect the second pie to be available soon, and when the asymmetry in their valuations of the pies is large enough. Otherwise, the players consume the entirety of the first pie in the first period, and the entirety of the second pie when it becomes available.

The intuition behind our partial agreement result is as follows. If players come to a complete agreement on pie X (i.e., the first pie), then they will split the second pie Y

evenly as soon as it is available. However, players can reach a more efficient agreement by saving a fraction of pie X until pie Y is available: by doing this, player 1 will get what remains of this pie, and in return he will consume less of pie Y. If the fraction that remains of pie X is small, player 1 will never accept an offer that gives him a zero share of pie Y. Hence, player 1 will consume a share of the pie he values less, regardless of who is proposer. In this case, saving marginally more of pie X until pie Y is available always leads to a more efficient allocation. On the other hand, if the fraction that remains of pie X is above a threshold $\overline{\lambda} \in (0,1)$, player 1 will accept offers that give him most of what remains of this pie, and none of the other one, when he is responder. The marginal benefit of saving more of pie X is therefore lower when the fraction of it that remains is larger than $\overline{\lambda}$: in this case, any additional savings of pie X will only lead to a more efficient allocation when player 1 is proposer, but not when he is responder.

In equilibrium, players trade off the benefit from saving a fraction of the first pie against the delay cost of waiting until the second pie becomes available. When the benefit is larger than the cost, players reach a partial agreement on the first pie, consuming $1-\overline{\lambda}$ of it, and they complete this agreement when the second pie becomes available. Otherwise, if the cost of delay is larger than the benefit, players reach a complete agreement on the first pie immediately, and then consume the second pie as soon as it is available. Finally, note that the benefit of saving a fraction of the first pie is large when the asymmetry in the players' valuations is large, while the cost of delay is small when players are more patient and the second pie is expected to be available soon.

Our paper relates to the literature on delay and inefficiencies in bargaining.[1] In particular, it relates to Compte and Jehiel (2004), who construct a bargaining model with history-dependent outside options and show that in this setting players will make gradual concessions until reaching an agreement. However, in their model agreement (i.e. consumption) takes place at a single stage, while in our model agreement takes place in multiple stages. Because of the presence of the second issue, our paper also relates to the literature on repeated bargaining (e.g., Muthoo, 1995) and to the literature on multi-issue bargaining.[2] Finally, our proof of existence and uniqueness of SPE adapts the arguments of Shaked and Sutton (1984) to our setting with partial agreements.

[1]For bargaining models featuring delay see, for instance, Kennan and Wilson (1993), Merlo and Wilson (1995, 1998), Abreu and Gul (2000) and Yildiz (2004).

[2]See, for instance, Fershtman (1990), Hortsmann (1997), Inderst (1998), Lang and Rosenthal (2001) and In and Serrano (2003, 2004).

2 Model

There are two players $i = 1, 2$ and two perfectly divisible pies X and Y. Time is discrete and indexed by $t = 0, 1, 2, ...$ If (x_{1t}, y_{1t}) and (x_{2t}, y_{2t}) are the shares of pies X and Y that players 1 and 2 consume in period t respectively, then the players' payoffs in period t are $u_1(x_{1t}, y_{1t}) = x_{1t} + ry_{1t}$ for player 1 and $u_2(x_{2t}, y_{2t}) = rx_{2t} + y_{2t}$ for player 2, where $0 < r < 1$ is player 1's marginal rate of substitution between pies and $\infty > 1/r > 1$ is the corresponding quantity for player 2. Therefore, player 1 values pie X more than pie Y, while player 2 values pie Y more than pie X. An *outcome* is denoted $\{((x_{1t}, x_{2t}), (y_{1t}, y_{2t}))\}_{t=0}^{\infty}$. Player i's preferences over (possibly random) outcomes are represented by $E\left[\sum_{t\geq 0} \delta^t u_i(x_{it}, y_{it})\right]$, where $\delta \in (0, 1)$ is a common discount factor and the expectation is taken over the realizations of the random outcome.

In every period t, each player is recognized with probability 1/2 to be *proposer*. The proposer offers nonnegative consumption shares $((x_{1t}, x_{2t}), (y_{1t}, y_{2t}))$. Importantly, we allow these consumption shares to be *partial offers*; that is, the proposer may offer to consume a fraction of the available pies, leaving the remainder for future consumption. The other player, the *responder*, must then either accept or reject the offer. If the offer is accepted, then the proposed shares are consumed and the period ends; if the offer is rejected, then the players consume 0 of each pie and the period ends.

The only restriction on offers is that they be feasible, and feasibility depends on the state. In each period the state is given by (j, s) where $j = 1, 2$ is the identity of the proposer and s determines the fractions of pies X and Y that can be consumed in that period.[3] We assume that pie X exists beginning in period 0, and thus part or all of it may be consumed starting in the first period. On the other hand, pie Y arrives stochastically: if it has not arrived by period t, then it arrives at the beginning of period $t+1$ with probability $p \in (0, 1)$. Players cannot consume any fraction of pie Y before it arrives. Moreover, they cannot commit to a division of this pie before it arrives.

Let λ_t be the fraction of pie X not yet consumed by t. Thus $\lambda_0 = 1$ and $\lambda_t = 1 - \sum_{\tau<t}(x_{1\tau} + x_{2\tau})$ for all $t > 0$. We will later show that in the period in which pie Y arrives, the players will come to an agreement over all of pie Y and all of the fraction of pie X that has not yet been consumed. Thus, at this point the game will effectively end. In other words, if pie Y arrives in period t and λ_t is the fraction of pie X not yet

[3] We sometimes abuse terminology and refer only to component s as the state.

consumed by that period, then in equilibrium the players will consume the total shares $x_{1t} + x_{2t} = \lambda_t$ and $y_{1t} + y_{2t} = 1$ in period t. This implies that the only relevant periods are those up to (and including) the period in which pie Y arrives. For such periods t, the state s_t can be written either as $\lambda_t X$ if pie Y has not yet arrived, or $\lambda_t XY$ if pie Y arrived in period t. Feasibility of offers at time t with state $s_t = \lambda_t X, \lambda_t XY$ requires

$$x_{1t} + x_{2t} \leq \lambda_t \text{ and } y_{1t} + y_{2t} \leq \psi_t = \begin{cases} 0 & \text{if } s_t = \lambda_t X \\ 1 & \text{if } s_t = \lambda_t XY. \end{cases} \tag{1}$$

Consider states $s_t = \lambda_t X, \lambda_t XY$. If $s_t = 0X$, then we say that the players are *waiting* for pie Y to arrive (having already consumed all of X). We say that players reach a *complete agreement* at state $s_t, s_t \neq 0X$, if they consume $x_{1t} + x_{2t} = \lambda_t$ and $y_{1t} + y_{2t} = \psi_t$. We say that the players *delay* at state $s_t \neq 0X$, if they consume $x_{1t} + x_{2t} = 0$ and $y_{1t} + y_{2t} = 0$. We say that the players reach a *partial agreement* at state s_t in all other cases, i.e. if they do not delay, are not waiting, or do not reach a complete agreement.

3 Equilibrium

This section characterizes the subgame perfect equilibrium (SPE) of the model. Our first result shows that states λXY are terminal: in any SPE, players consume all of pie Y and all λ of pie X in these states. Before stating the lemma, define $\overline{\lambda}(\delta, r) \equiv \frac{r\delta}{2-\delta} \in (0, r)$.

Lemma 1. *In every SPE, the players reach a complete agreement when the state is λXY. Moreover, the SPE payoffs in these states are unique. For $j = 1, 2$, let $((x_1^j, x_2^j), (y_1^j, y_2^j))$ be the offer that player j makes at state λXY. These quantities are as follows:*

1. *If $\lambda \geq \overline{\lambda}(\delta, r)$, then*

$$(x_1^1, y_1^1) = \left(\lambda, \tfrac{2(1-\delta)(2-\delta(1+r\lambda))}{4(1-\delta)+\delta^2(1-r^2)}\right) \qquad (x_1^2, y_1^2) = \left(\lambda - \tfrac{2(1-\delta)(\lambda(2-\delta)-r\delta)}{4(1-\delta)+\delta^2(1-r^2)}, 0\right).$$

2. *If, on the other hand, $\lambda \leq \overline{\lambda}(\delta, r)$, then*

$$(x_1^1, y_1^1) = \left(\lambda, 1 - \tfrac{\delta(r+\lambda)}{2r}\right) \qquad (x_1^2, y_1^2) = \left(\lambda, \tfrac{r\delta-\lambda(2-\delta)}{2r}\right).$$

And, in both cases, $x_2^j = \lambda - x_1^j$ and $y_2^j = 1 - y_2^j$ for $j = 1, 2$.

Proof. See Appendix A. □

Lemma 1 shows that players reach a complete agreement at states λXY. At such states, the proposer j makes an offer $((x_1^j, x_2^j), (y_1^j, y_2^j))$ that maximizes his payoff, subject to the constraint that the offer is acceptable to i. Thus, the proposer's offer solves:

$$\max_{x_1^j, x_2^j, y_1^j, y_2^j \geq 0} u_j\left(x_j^j, y_j^j\right) \text{ subject to}$$
$$u_i\left(x_i^j, y_i^j\right) \geq \delta\left(\frac{1}{2}u_i\left(x_i^j, y_i^j\right) + \frac{1}{2}u_i\left(x_i^i, y_i^i\right)\right)$$
$$x_1^j + x_2^j \leq \lambda, \text{ and } y_1^j + y_2^j \leq 1 \tag{2}$$

Lemma 1 states the offers that solve (2) for $j = 1, 2$. These offers depend on the fraction λ of pie X that is available. When $\lambda = 0$ each player obtains approximately 1/2 of pie Y (if δ is large). As λ increases, the players are able to reach a more efficient allocation: player 1 obtains what remains of pie X, and in return he gets less of pie Y. For small values of λ, player 1 never accepts an offer that gives him nothing of pie Y when he is responder. Thus, in this case he always consumes some of pie Y, regardless of who makes offers. On the other hand, for large values of λ player 1 will accept offers that give him most of what remains of pie X, and none of pie Y, when he is responder. The threshold $\overline{\lambda}(\delta, r)$ defines the point below which player 1 consumes pie Y regardless of the identity of the proposer. Note that the marginal benefit of saving more of pie X is discontinuous at $\overline{\lambda}$. For $\lambda < \overline{\lambda}$, saving more of pie X implies that player 1 will consume less of pie Y regardless of the identity of the proposer. For $\lambda > \overline{\lambda}$, saving more of pie X implies that player 1 will consume less of pie Y only when he is proposer.

The threshold $\overline{\lambda}(\delta, r)$ is increasing in r and δ. Intuitively, $\overline{\lambda}(\delta, r)$ depends on the magnitude of the proposer advantage: if player 2's proposer advantage is large, his offer at states λXY will be such that he will consume all of pie Y even when the fraction λ that remains of pie X is small. As usual, the proposer advantage is large when players are impatient (i.e., δ small). Moreover, in this setting the proposer advantage is also large when the asymmetry in valuations is large (i.e., r small). The reason for this is that the responder's incentives to delay are weaker when r is small: the responder's benefit of delaying is that this may enable him to obtain a portion of the pie he values less if he is proposer next period, while the cost is that he has to delay consumption of the pie he values more. Clearly, the benefit is decreasing in r, while the cost is independent of r.

Define the function $\phi(p, \delta, r) = p - \frac{1-\delta}{\delta}\frac{2r^2}{1-r^2}$. This function takes larger values when players are patient (δ large), when pie Y is expected to arrive soon (p large), and when the asymmetry in the players' valuations is large (r small). Note that r is a measure of the efficiency gains that arise from saving pie X: for any fraction λ of pie X that remains, the efficiency gains from saving this fraction until pie Y arrives are large when r is small. On the other hand, δ and p measure the delay cost of waiting until pie Y arrives. Thus, the function $\phi(p, \delta, r)$ quantifies the net benefit of delay.

The function $\phi(\cdot)$ partitions the parameters space into two sets: the set of parameters $(p, \delta, r) \in (0, 1)^3$ for which $\phi(p, \delta, r) \geq 0$, and the set of parameters $(p, \delta, r) \in (0, 1)^3$ for which $\phi(p, \delta, r) < 0$. Note that these two sets of parameters have nonempty interiors. The next result characterizes the equilibrium outcomes on the two regions separated by the function $\phi(\cdot)$.

Theorem 1. *There are unique SPE payoffs. Moreover,*

1. *If players are impatient enough, if the second pie is expected to arrive late, and if the asymmetry in valuations is small enough (i.e., if $\phi(p, \delta, r) < 0$), then in every SPE the players reach a complete agreement in the first period.*

2. *If players are patient enough, if the second pie is expected to arrive soon, and if the asymmetry in valuations is large enough (i.e., if $\phi(p, \delta, r) > 0$), then in every SPE the players reach a partial agreement in the first period, $t = 0$, consuming $x_1 + x_2 = 1 - \overline{\lambda}(\delta, r)$. Players then complete the agreement in the period in which pie Y arrives.*[4]

Proof. See Appendix B. □

The results on Theorem 1 can be best understood in terms of the relation between the cost and benefit of saving pie X. To see this, consider first the case with $\phi(p, \delta, r) > 0$ and suppose that the state is λX with $\lambda \leq \overline{\lambda}(\delta, r)$. Lemma 2 in the Appendix shows that players will always delay at these states. The reason for this is that when $\phi(p, \delta, r) > 0$ the delay cost of waiting until pie Y arrives is small. Moreover, by our discussion above, the marginal benefit from saving pie X is large when $\lambda \leq \overline{\lambda}(\delta, r)$, since in this case player 1 will always consume a portion of pie Y when it arrives. Combining these two effects, at these states the marginal cost of saving pie X is lower than the marginal benefit, so players delay making an agreement on what remains of pie X until pie Y arrives.

[4] The knife-edge case $\phi(p, \delta, r) = 0$ supports equilibria with both partial and complete agreements.

Next, consider states λX with $\lambda > \overline{\lambda}(\delta, r)$. Lemma 3 in the Appendix shows that if $\phi(p, \delta, r) > 0$, then the proposer will make an offer to consume a total $\lambda - \overline{\lambda}(\delta, r)$ of pie X. The reason for this is that the marginal benefit of saving more of pie X is lower at states λX with $\lambda > \overline{\lambda}(\delta, r)$. Therefore, even if $\phi(p, \delta, r) > 0$, at these states the marginal benefit of saving pie X is lower than the marginal cost, so players are better off consuming a positive fraction of pie X than delaying an agreement. The fraction $\overline{\lambda}(\delta, r)$ that they leave unconsumed is exactly the point at which the marginal benefit from saving more of pie X becomes larger.

The fraction $\lambda - \overline{\lambda}(\delta, r)$ of pie X that players consume at states λX with $\lambda > \overline{\lambda}(\delta, r)$ does not depend on the identity of the proposer. Intuitively, the fraction of pie X that players consume depends on the cost and benefit of saving more of this pie for future consumption; and these costs and benefits are independent on who is making offers today. Note that the fraction $\lambda - \overline{\lambda}(\delta, r)$ of pie X that players consume is smaller when δ is large and when r is large.

In Appendix C we show how the players divide the fraction $\lambda - \overline{\lambda}(\delta, r)$ of pie X that they consume at states λX with $\lambda > \overline{\lambda}(\delta, r)$, depending on the identity of the proposer. Let (x_1^i, x_2^i) be the shares that player i proposes at such states and let $x_j = \frac{1}{2}(x_j^1 + x_j^2)$ be the expected share that player j consumes. In Appendix C we show that $x_1 > x_2$, so player 1 consumes a larger fraction than player 2. Moreover, the fraction of $\lambda - \overline{\lambda}(\delta, r)$ that player 1 consumes is increasing in δ and p. Intuitively, if player 1 chooses to delay an agreement at states λX, pie Y may arrive next period. If this occurs, player 1 will get most of what remains of pie X. Therefore, player 1 has a greater incentive to delay when δ and p are large, so in this case he bargains from a stronger position.

Consider next the case where $\phi(p, \delta, r) < 0$, so that delaying the consumption of pie X until pie Y arrives is costly. In this case, the proposer will always find it optimal to make an offer such that players consume all of the remainder of pie X at states λX, regardless of whether $\lambda > \overline{\lambda}(\delta, r)$ or $\lambda \leq \overline{\lambda}(\delta, r)$. Intuitively, when $\phi(p, \delta, r) < 0$ the marginal benefit of saving more of pie X is smaller than the cost of waiting until pie Y arrives, regardless of the fraction of pie X that remains. Hence, players always reach a complete agreement on pie X at the beginning of the game, and then consume all of pie Y as soon as it arrives.

4 Final Remarks

We end the paper with three comments about our model. First, throughout the paper we assume that players cannot commit to a division of pie Y until it arrives. This assumption is crucial for the results in Theorem 1. If players could commit to a future division of pie Y, then at $t = 0$ they would effectively bargain over the two issues together. Then, by arguments similar to those in Lemma 1, players would reach an agreement over the division of the two pies at the beginning of the game.

Another assumption we made throughout the paper is that pies X and Y have the same size. The results of Theorem 1 generalize to settings in which the two pies have different sizes. Let pie X be of size 1 and pie Y be of size $\rho > 0$. In this case, the agreement that players reach at states λXY will again depend on the fraction λ of pie X that remains. In particular, in this setting there exists a threshold $\tilde{\lambda}(\delta, r, \rho)$ such that player 1 consumes a positive share of pie Y regardless of the identity of the proposer at states λXY with $\lambda < \tilde{\lambda}(\delta, r, \rho)$. Moreover, $\tilde{\lambda}(\delta, r, \rho) = \rho\overline{\lambda}(\delta, r)$, where $\overline{\lambda}(\delta, r)$ is the threshold in Lemma 1. If waiting until pie Y arrives is costly, players will reach a partial agreement on pie X at the start of the game, consuming a fraction $1 - \rho\overline{\lambda}(\delta, r)$, and they will then complete this agreement when pie Y arrives. That is, in this setting players want to consume pie X until the proportion of what remains of pie X to all of pie Y equals $\overline{\lambda}(\delta, r)$. Note that the fraction of pie X that players consume at the start of the game is decreasing in ρ. Moreover, if ρ is large enough (i.e., if $1 \leq \rho\overline{\lambda}(\delta, r)$), then players will completely delay the consumption of pie X until pie Y arrives.

Finally, the model in this paper is one in which players can make *partial* offers. A previous version of this paper also analyses the case where players are constrained to make complete offers; i.e., all offers must satisfy the constraints in (1) with equality. There, we show that there exists a function $\hat{\phi}(p, \delta, r)$, which is increasing in δ and p and decreasing in r, such that: (i) if $\hat{\phi}(p, \delta, r) < 0$ players reach a complete agreement on pie X at $t = 0$, and then reach a complete agreement on pie Y as soon as it arrives; and (ii) if $\hat{\phi}(p, \delta, r) > 0$, players delay an agreement on pie X until pie Y arrives. Moreover, we show that $\hat{\phi}(p, \delta, r) < \phi(p, \delta, r)$ for all p, δ, r, and that there is an open and nonempty set of parameters such that $\hat{\phi}(p, \delta, r) < 0 < \phi(p, \delta, r)$. Therefore, there are settings in which players would reach a complete agreement on pie X at $t = 0$ if they were constrained to make complete offers, but would reach a partial agreement if they could.

Appendix

A. Proof of Lemma 1

Suppose $s = \lambda XY$. Let $\underline{v}_i(j, \lambda XY)$ and $\overline{v}_i(j, \lambda XY)$ be the infimum and supremum SPE payoffs to player $i = 1, 2$ when the state is $(j, \lambda XY)$, $j = 1, 2$, and let $\underline{w}_i(\lambda XY) = \frac{1}{2}\underline{v}_i(1, \lambda XY) + \frac{1}{2}\underline{v}_i(2, \lambda XY)$ and $\overline{w}_i(\lambda XY) = \frac{1}{2}\overline{v}_i(1, \lambda XY) + \frac{1}{2}\overline{v}_i(2, \lambda XY)$. To prove Lemma 1, let $i \neq j$ and consider player j's problem of choosing an offer that maximizes his payoff, subject to the constraint that player i's payoff from accepting this offer equals $\delta\overline{w}_i(\lambda XY)$. Since $\delta < 1$, at the solution to this problem player j will make an offer such that he and player i consume all λ of pie X and all of pie Y. The payoff that player j receives when this offer is accepted is a lower bound on his SPE payoff, since player i always accepts such an offer. Similarly, consider player j's problem of choosing an offer that maximizes his payoff, subject to the constraint that player i's payoff from accepting this offer equals $\delta\underline{w}_i(\lambda XY)$. Again, at the solution to this problem, player j makes an offer such that he and player i consume all λ of pie X and all of pie Y. Moreover, the payoff that j gets if this offer is accepted is an upper bound to his SPE payoff, since this is the worst offer that player i would accept. Using these bounds on payoffs, one can apply arguments similar to those in Shaked and Sutton (1984) to show that SPE payoffs at states $s = \lambda XY$ are unique, and that these payoffs are attained by a strategy profile in which the proposer makes an offer to consume all λ of pie X and all of pie Y. Finally, the offers that players make at states λXY are the solution to problem (2) for $j = 1, 2$. □

B. Proof of Theorem 1

Outline. The proof is organized as follows. After establishing some preliminaries, we begin by considering the case where $\phi(p, \delta, r) > 0$. Lemma 2 shows that, in this case, players delay in states λX with $\lambda \leq \overline{\lambda}(\delta, r)$. Lemma 3 shows that at states λX with $\lambda > \overline{\lambda}(\delta, r)$, the proposer finds it optimal to make an offer to consume a total $\lambda - \overline{\lambda}(\delta, r)$ of pie X. Lemma 4 uses these results to derive upper and lower bounds on the players' SPE payoffs at states λX with $\lambda > \overline{\lambda}(\delta, r)$. Using these bounds, Lemma 5 adapts the arguments in Shaked and Sutton (1984) to show that SPE payoffs are unique. Finally, Lemma 6 provides a sketch of the argument for the case where $\phi(p, \delta, r) < 0$. In what follows, we suppress notation by writing ϕ and $\overline{\lambda}$ in place of $\phi(p, \delta, r)$ and $\overline{\lambda}(\delta, r)$.

Preliminaries. We first introduce some definitions. Note that player i's payoff at state $(j, \lambda XY)$ is $v_i(j, \lambda XY) = x_i^j + r y_i^j$, where $\left(\left(x_1^j, x_2^j\right), \left(y_1^j, y_2^j\right)\right)$ is the offer that player j makes. Player i's expected payoff is $w_i(\lambda XY) = \frac{1}{2} v_i(1, \lambda XY) + \frac{1}{2} v_i(2, \lambda XY)$. Using the expressions for $\left(\left(x_1^j, x_2^j\right), \left(y_1^j, y_2^j\right)\right)$ in Lemma 1, we get

$$w_1(\lambda XY) = \begin{cases} \frac{2r(1-\delta)+\lambda(2-\delta-r^2\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda \geq \overline{\lambda}(\delta, r), \\ \frac{r+\lambda}{2} & \text{if } \lambda \leq \overline{\lambda}(\delta, r). \end{cases}$$

$$w_2(\lambda XY) = \begin{cases} \frac{(2-\delta-r^2\delta)+2\lambda r(1-\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda \geq \overline{\lambda}(\delta, r), \\ \frac{r+\lambda}{2r} & \text{if } \lambda \leq \overline{\lambda}(\delta, r). \end{cases} \tag{B.1}$$

Let $W(\lambda XY) = r w_1(\lambda XY) + w_2(\lambda XY)$ be the *total (normalized) payoff*. Using (B.1),

$$W(\lambda XY) = \begin{cases} \frac{2r(1-\delta)(r+\lambda)+(1+r\lambda)(2-\delta-r^2\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda > \overline{\lambda}(\delta, r), \\ \frac{(1+r^2)(r+\lambda)}{2r} & \text{if } \lambda \leq \overline{\lambda}(\delta, r). \end{cases} \tag{B.2}$$

Now let the state be λX and normalize $t = 0$ to be the period in which this state is reached. A *consumption path* is a sequence $\{(x_{1t}, x_{2t})\}_{t=0}^{\infty}$ with the interpretation that x_{it} is the share of pie X that player i consumes in period t conditional on the event that pie Y has not arrived. Define the *consumption sequence* $\{\mu_t\}_{t=0}^{\infty}$ associated with the consumption path $\{(x_{1t}, x_{2t})\}_{t=0}^{\infty}$ by $\mu_t = x_{1t} + x_{2t}$ for all t. The *total (normalized) payoff* from consumption path $\{(x_{1t}, x_{2t})\}$ is

$$U\left(\{(x_{1t}, x_{2t})\}\right) \equiv E\left[r \sum_{t=0}^{\infty} \delta^t u_1\left(x_{1t}, y_{1t}\right) + \sum_{t=0}^{\infty} \delta^t u_2\left(x_{2t}, y_{2t}\right)\right], \tag{B.3}$$

where the shares x_{it} are given by the consumption path $\{(x_{1t}, x_{2t})\}$ until pie Y arrives, and are determined in equilibrium along with the shares y_{it} when pie Y arrives. If pie Y has not arrived by period $t-1$, it arrives in period t with probability p and players come to an agreement over all of what is left of pie X and all of pie Y (by Lemma 1); with probability $1-p$ pie Y does not arrive in period t, so players consume x_{1t} and x_{2t} as determined by the consumption path. For any period t prior to the arrival of pie Y, we have $r u_1(x_{1t}, y_{1t}) + u_2(x_{2t}, y_{2t}) = r\mu_t$. On the other hand, if pie Y arrives in period $t > 0$ then $r u_1(x_{1t}, y_{1t}) + u_2(x_{2t}, y_{2t}) = W(\lambda_t XY)$, where $\lambda_t = \lambda - \sum_{\tau=0}^{t-1} \mu_\tau$ is the fraction of pie X left in period t and $W(\lambda XY)$ is given by (B.2) (recall that the state at $t = 0$ is λX). Therefore, (B.3) becomes

$$U\left(\{(x_{1t}, x_{2t})\}\right) = r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t W\left(\left(\lambda - \sum_{\tau=0}^{t} \mu_\tau\right) XY\right)\right] \tag{B.4}$$

Define $\xi \equiv \frac{r(2-\delta-r^2\delta)+2r(1-\delta)}{4(1-\delta)+\delta^2(1-r^2)}$, $\zeta \equiv \frac{r}{2}+\frac{1}{2r}$, and $\alpha \equiv \delta p \sum_{t=0}^{\infty} \delta^t (1-p)^t = \frac{\delta p}{1-\delta(1-p)}$. The number α is the "effective discount rate" of waiting until pie Y arrives. The following facts will be repeatedly used in the proof.

Fact 1: $W((\lambda-\kappa)XY) = \begin{cases} W(\lambda XY) - \xi\kappa & \text{if } \overline{\lambda} \leq \lambda-\kappa \leq \lambda \leq 1, \\ W(\lambda XY) - \zeta\kappa & \text{if } 0 \leq \lambda-\kappa \leq \lambda \leq \overline{\lambda}. \end{cases}$

Fact 2: $r - \alpha\xi > 0$ for all δ, p, r. Moreover, $\phi > 0$ iff $r - \alpha\zeta < 0$.

The quantities in Fact 2 measure the net benefit of consuming marginally more of pie X at state λX before the arrival of pie Y. To see this, note that when $\lambda > \overline{\lambda}$ consuming marginally more of pie X increases the players' normalized total surplus by r today, but it decreases it by ξ in the period in which pie Y arrives. Since α is the effective discount rate of waiting until pie Y arrives, the quantity $r-\alpha\xi$ measures the net benefit of consuming marginally more of pie X at states λX with $\lambda > \overline{\lambda}$. Similarly, when $\lambda \leq \overline{\lambda}$ consuming more of pie X also increases the players' total surplus by r today, but it decreases it by ζ in the period in which pie Y arrives. Thus, $r - \alpha\zeta$ measures the net benefit of consuming marginally more of pie X at states λX with $\lambda \leq \overline{\lambda}$. The first part of Fact 2 says that the net benefit of consuming marginally more of pie X is positive at states λX with $\lambda > \overline{\lambda}$ for all parameter values. On the other hand, the second part of Fact 2 says that, at states λX with $\lambda \leq \overline{\lambda}$, this net benefit is positive if and only if $\phi < 0$.

Using Fact 1 in (B.4) and rearranging the terms in the sum yields

$$U(\{(x_{1t}, x_{2t})\}) = \begin{cases} (r-\alpha\zeta)\sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) & \text{if } \lambda_t \leq \overline{\lambda} \forall t, \\ (r-\alpha\xi)\sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) & \text{if } \lambda_t \geq \overline{\lambda} \forall t. \end{cases} \tag{B.5}$$

Lemma 2. *Let $\phi > 0$. Then in every SPE the players delay in states λX with $\lambda \leq \overline{\lambda}$.*

Proof. Fix an equilibrium strategy profile. Note that every strategy profile generates a probability distribution over consumption paths. Let $\{(x_{1t}, x_{2t})\}$ be a consumption path in the support of the distribution generated by this strategy profile, and let $\{\mu_t\}$ be the associated consumption sequence. Assume for the sake of contradiction that players don't delay, so that $\lambda \geq \mu_0 > 0$. By equation (B.5), the total normalized payoff from the consumption path $\{(x_{1t}, x_{2t})\}$ is

$$U(\{(x_{1t}, x_{2t})\}) = (r-\alpha\zeta)\sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) < \alpha W(\lambda XY),$$

where the inequality follows from Fact 2 and $\mu_0 > 0$. Therefore, the total payoff from this consumption path is strictly smaller than $\alpha W(\lambda XY)$, which is the total payoff of delaying all consumption until pie Y arrives. This means that at least one player receives a payoff strictly lower than the payoff he would get if there were delay. Since that player can always delay unilaterally (either by rejecting an offer or by making an unacceptable proposal), it cannot be that players consume $\mu_0 > 0$ in states λX with $\lambda \leq \overline{\lambda}$. □

Lemma 3. *Let $\phi > 0$ and $s = \lambda X$ with $\lambda > \overline{\lambda}$. Consider proposer j's problem of choosing an offer that maximizes his expected discounted payoff subject to the constraint that the responder's expected discounted payoff be equal to $w_i \leq u_i(\lambda - \overline{\lambda}, 0) + \alpha w_i(\overline{\lambda} XY)$, where $w_i(\overline{\lambda} XY)$ is given by (B.1). At the solution to this problem, the proposer makes an offer such that he and the responder consume a total fraction $\lambda - \overline{\lambda}$ of pie X.*

Proof. We prove Lemma 3 for $j = 1$ and $i = 2$. The proof for $j = 2$ and $i = 1$ is symmetric and omitted. Suppose player 1 makes an offer (x_1, x_2) with $x_1 + x_2 = \lambda - \overline{\lambda}$. Player 2's discounted payoff from accepting this offer is $rx_2 + \alpha w_2(\overline{\lambda} XY)$, since Lemma 2 implies that after such an offer is accepted the players will delay until pie Y arrives. Let x_2 be the consumption share that gives player 2 a value of w_2, i.e. $rx_2 + \alpha w_2(\overline{\lambda} XY) = w_2$. Player 1's payoff from this offer is $\lambda - x_2 + \alpha w_1(\overline{\lambda} XY) = (\lambda - \overline{\lambda} - \frac{1}{r} w_2) + \alpha(\frac{1}{r} w_2(\overline{\lambda} XY) + w_1(\overline{\lambda} XY))$. Multiplying this quantity by r, player 1's *normalized* payoff from player 2 accepting this offer is

$$r(\lambda - \overline{\lambda}) - w_2 + \alpha(w_2(\overline{\lambda} XY) + rw_1(\overline{\lambda} XY)) = r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - w_2 \equiv U^* - w_2$$

which implicitly defines the quantity U^*. On the other hand, suppose player 1 makes any other offer that gives player 2 an expected discounted payoff of w_2, and which leads to a (possibly random) consumption path $\{(x_{1t}, x_{2t})\}$ with associated consumption sequence $\{\mu_t\} = \{x_{1t} + x_{2t}\}$. Player 1's expected *normalized* payoff from this offer is $E[U(\{(x_{1t}, x_{2t})\})] - w_2$, where $U(\{(x_{1t}, x_{2t})\})$ is given by (B.4) and the expectation is over consumption paths. This follows since player 2's expected discounted payoff must be w_2. We now show that $U^* > U(\{(x_{1t}, x_{2t})\})$ for all other possible consumption paths $\{(x_{1t}, x_{2t})\}$. This will prove the lemma, since it implies that player 1 is better off making an offer (x_1, x_2) with $x_1 + x_2 = \lambda - \overline{\lambda}$.

There are two cases to consider: (1) $\sum_{t=0}^{\infty} \mu_t \leq \lambda - \overline{\lambda}$, and (2) there exists $t' \geq 0$ such that $\sum_{t=0}^{t'} \mu_t > \lambda - \overline{\lambda}$ (and $\sum_{t=0}^{\tau} \mu_t \leq \lambda - \overline{\lambda}$ for all $\tau < t'$). Consider case (1). Note that in this case, $\lambda_t = \lambda - \sum_{\tau=0}^{t-1} \mu_\tau \geq \overline{\lambda}$ for all t. Equation (B.5) then implies that

$$
\begin{aligned}
U(\{(x_{1t}, x_{2t})\}) - U^* &= (r - \alpha\xi) \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) - r(\lambda - \overline{\lambda}) - \alpha W(\overline{\lambda} XY) \\
&< -\alpha\xi(\lambda - \overline{\lambda}) + \alpha W(\lambda XY) - \alpha W(\overline{\lambda} XY) = 0
\end{aligned}
$$

where the inequality follows by Fact 2 and since $\sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t < \sum_{t=0}^{\infty} \mu_t \leq \lambda - \overline{\lambda}$, and the last equality is a consequence of Fact 1. Thus $U^* > U(\{(x_{1t}, x_{2t})\})$.

Consider case (2). By Lemma 2, we have $\mu_t = 0$ for all $t > t'$, since for any such t the state would be $\lambda_t X$ with $\lambda_t < \overline{\lambda}$ (and hence players will delay). Let $\{\tilde{\mu}_t\}$ be such that $\tilde{\mu}_t = \mu_t$ for all $t \neq t'$, $\tilde{\mu}_{t'} = \lambda - \overline{\lambda} - \sum_{t=0}^{t'-1} \mu_t$. Note that $\sum_{t=0}^{t'} \tilde{\mu}_t = \sum_{t=0}^{\infty} \tilde{\mu}_t = \lambda - \overline{\lambda}$. Let $\{(\tilde{x}_{1t}, \tilde{x}_{2t})\}$ satisfy $\tilde{x}_{1t} + \tilde{x}_{2t} = \tilde{\mu}_t$ for all t. Since $\sum_{t=0}^{\infty} \tilde{\mu}_{t'} = \lambda - \overline{\lambda}$, the arguments above imply that $U^* > U(\{(\tilde{x}_{1t}, \tilde{x}_{2t})\})$. Note that $\lambda - \sum_{t=0}^{t'} \mu_t = \overline{\lambda} - (\mu_{t'} - \tilde{\mu}_{t'})$. Then,

$$
\begin{aligned}
\tfrac{U(\{(\tilde{x}_{1t}, \tilde{x}_{2t})\}) - U(\{(x_{1t}, x_{2t})\})}{\delta^{t'}(1-p)^{t'}} &= r\left(\tilde{\mu}_{t'} - \mu_{t'}\right) + \alpha\left[W\left(\overline{\lambda} XY\right) - W\left(\left(\overline{\lambda} - (\mu_{t'} - \tilde{\mu}_{t'})\right) XY\right)\right] \\
&= (r - \alpha\zeta)(\tilde{\mu}_{t'} - \mu_{t'}) > 0
\end{aligned}
$$

where the first equality follows since $\tilde{\mu}_t = \mu_t$ for all $t < t'$ and $\tilde{\mu}_t = \mu_t = 0$ for all $t > t'$ and since $\lambda - \sum_{t=0}^{t'} \mu_t = \overline{\lambda} - (\mu_{t'} - \tilde{\mu}_{t'})$, the second equality follows from Fact 1, and the inequality follows from Fact 2 and $\tilde{\mu}_{t'} < \mu_{t'}$. It then follows that $U^* > U(\{x_{1t}, x_{2t}\})$. $\square$

Lemma 4. *Let $\underline{v}_i(j, \lambda X)$ and $\overline{v}_i(j, \lambda X)$ be the infimum and supremum SPE payoffs for player i when the state is $(j, \lambda X)$, and define $\overline{w}_i(\lambda X) = \frac{1}{2}\overline{v}_i(1, \lambda X) + \frac{1}{2}\overline{v}_i(2, \lambda X)$ and $\underline{w}_i(\lambda X) = \frac{1}{2}\underline{v}_i(1, \lambda X) + \frac{1}{2}\underline{v}_i(2, \lambda X)$. If $\lambda > \overline{\lambda}$ and $\phi > 0$, then we have*

1. $\underline{v}_i(j, \lambda X) \geq \delta(p w_i(\lambda XY) + (1-p)\underline{w}_i(\lambda X))$ *for* $i, j = 1, 2, j \neq i$

2. $\overline{v}_i(j, \lambda X) \leq \delta(p w_i(\lambda XY) + (1-p)\overline{w}_i(\lambda X))$ *for* $i, j = 1, 2, j \neq i$

3. $r\underline{v}_1(1, \lambda X) \geq r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - \delta(p w_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X))$

4. $r\overline{v}_1(1, \lambda X) \leq r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - \delta(p w_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$

5. $\underline{v}_2(2, \lambda X) \geq r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - r\delta(p w_1(\lambda XY) + (1-p)\overline{w}_1(\lambda X))$

6. $\overline{v}_2(2, \lambda X) \leq r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - r\delta(p w_1(\lambda XY) + (1-p)\underline{w}_1(\lambda X))$

where $w_i(\lambda XY)$, $i = 1, 2$, are the functions defined in (B.1).

Proof. Claims (1) and (2) are immediate, and the arguments for (5) and (6) are the same as the arguments for (3) and (4). We therefore only prove (3) and (4).

We start by proving (3). Suppose the state is λX with $\lambda > \overline{\lambda}$. Suppose player 1 offers $(\lambda - \overline{\lambda} - \overline{x}_2, \overline{x}_2)$ where $0 \leq \overline{x}_2 \leq \lambda - \overline{\lambda}$. (Note that this offer leaves fraction $\overline{\lambda}$ of pie X for future consumption.) By Lemma 2, if player 2 accepts this offer then players delay an agreement on the remainder of pie X until pie Y arrives. Therefore, the payoff to player 2 from accepting this offer is $r\overline{x}_2 + \alpha w_2(\overline{\lambda}XY)$. Note that if $r\overline{x}_2 + \alpha w_2(\overline{\lambda}XY) = \delta\left(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)\right)$, then player 2 will accept the offer $(\lambda - \overline{\lambda} - \overline{x}_2, \overline{x}_2)$. Importantly, one can show that this value of $\overline{x}_2$ that gives player 2 this payoff is smaller than $\lambda - \overline{\lambda}$, so that $(\lambda - \overline{\lambda} - \overline{x}_2, \overline{x}_2)$ is in fact a feasible offer.[5] The payoff player 1 gets if this offer is accepted is $\lambda - \overline{\lambda} - \overline{x}_2 + \alpha w_1(\overline{\lambda}XY)$. Using $r\overline{x}_2 + \alpha w_2(\overline{\lambda}XY) = \delta\left(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)\right)$, it follows that

$$r\underline{v}_1(1, \lambda X) \geq r(\lambda - \overline{\lambda}) - \delta(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)) + \alpha(rw_1(\overline{\lambda}XY) + w_2(\overline{\lambda}XY))$$

which establishes (3) since $rw_1(\overline{\lambda}XY) + w_2(\overline{\lambda}XY) = W(\overline{\lambda}XY)$.

Next, we prove (4). To show this, consider player 1's problem of making an offer that maximizes his discounted payoff subject to the constraint that player 2's payoff is equal to $w_2 = \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$. By definition, such an offer is the worst offer that player 2 would ever accept. Moreover, note that $w_2 \leq r\overline{x}_2 + \alpha w_2(\overline{\lambda}XY) \leq r(\lambda - \overline{\lambda}) + \alpha w_2(\overline{\lambda}XY)$, since $\underline{w}_2(\lambda X) \leq \overline{w}_2(\lambda X)$ and $\overline{x}_2 \leq \lambda - \overline{\lambda}$. By Lemma 3, the solution to this problem is for player 1 to make an offer $(\lambda - \overline{\lambda} - \underline{x}_2, \underline{x}_2)$ with $r\underline{x}_2 + \alpha w_2(\overline{\lambda}XY) = \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$. The payoff player 1 gets if this offer is accepted is $\lambda - \overline{\lambda} - \underline{x}_2 + \alpha w_1(\overline{\lambda}XY)$. Using $r\underline{x}_2 + \alpha w_2(\overline{\lambda}XY) = \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$,

$$r\overline{v}_1(1, \lambda X) \leq r(\lambda - \overline{\lambda}) - \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X)) + \alpha(rw_1(\overline{\lambda}XY) + w_2(\overline{\lambda}XY))$$

which establishes (4) since $rw_1(\overline{\lambda}XY) + w_2(\overline{\lambda}XY) = W(\overline{\lambda}XY)$. □

Lemma 5. *Let $\phi > 0$ and $\lambda > \overline{\lambda}$. For each $j = 1, 2$, all SPE starting at state $(j, \lambda X)$ are payoff equivalent. Moreover, in every SPE the players reach a partial agreement, consuming a total fraction $\lambda - \overline{\lambda}$ of pie X.*

[5] To see this, note that $\overline{w}_2(\lambda X) \leq U\left(\{(x_{1t}, x_{2t})\}\right) - r\underline{w}_1(\lambda X)$ for some consumption path $\{(x_{1t}, x_{2t})\}$. By Lemma 3, $U\left(\{(x_{1t}, x_{2t})\}\right) \leq r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda}XY)$. Moreover, $\underline{w}_1(\lambda X) \geq \alpha w_1(\lambda X)$, since player 1 can get $\alpha w_1(\lambda X)$ by delaying until pie Y arrives. From these inequalities, we can show that $\overline{x}_2 < \lambda - \overline{\lambda}$.

Proof. The inequalities stated in Lemma 4 imply

$$r\overline{w}_1(\lambda X) \leq \frac{1}{2}\left(r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - \delta\left(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X)\right)\right) + \frac{1}{2} r\delta\left(pw_1(\lambda XY) + (1-p)\overline{w}_1(\lambda X)\right)$$

$$r\underline{w}_1(\lambda X) \geq \frac{1}{2}\left(r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - \delta\left(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)\right)\right) + \frac{1}{2} r\delta\left(pw_1(\lambda XY) + (1-p)\underline{w}_1(\lambda X)\right)$$

These inequalities imply $r\left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right) \leq \frac{\delta(1-p)}{2-\delta(1-p)}\left(\overline{w}_2(\lambda X) - \underline{w}_2(\lambda X)\right)$. Similarly, for player 2 we get that $\overline{w}_2(\lambda X) - \underline{w}_2(\lambda X) \leq \frac{r\delta(1-p)}{2-\delta(1-p)}\left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right)$. Combining these inequalities yields $\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X) \leq (\frac{\delta(1-p)}{2-\delta(1-p)})^2\left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right)$, which implies $\overline{w}_1(\lambda X) = \underline{w}_1(\lambda X) \equiv w_1(\lambda X)$. This in turn implies that $\overline{w}_2(\lambda X) = \underline{w}_2(\lambda X) \equiv w_2(\lambda X)$, so $\overline{v}_i(j, \lambda X) = \underline{v}_i(j, \lambda X) \equiv v_i(j, \lambda X)$ for $i, j = 1, 2$, and the SPE payoffs are unique. Using the inequalities in Lemma 4, we find that for $i, j = 1, 2, i \neq j$

$$\begin{aligned} v_i(j, \lambda X) &= \delta\left(pw_i(\lambda XY) + (1-p)w_i(\lambda X)\right) \\ rv_1(1, \lambda X) &= r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - v_2(1, \lambda X) \\ v_2(2, \lambda X) &= r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - rv_1(2, \lambda X) \end{aligned} \tag{B.6}$$

Note that $rv_1(1, \lambda X) = r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY) - (rv_1(2, \lambda X) + v_2(1, \lambda X)) + rv_1(2, \lambda X)$. Note further that $rv_1(2, \lambda X) + v_2(1, \lambda X) = \delta(pW(\lambda XY) + (1-p)(r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY)))$, since $rw_1(\lambda X) + w_2(\lambda X) = r(\lambda - \overline{\lambda}) + \alpha W(\overline{\lambda} XY)$. Therefore,

$$\begin{aligned} rv_1(1, \lambda X) &= r(\lambda - \overline{\lambda})(1 - \delta(1-p)) + \delta p(W(\overline{\lambda} XY) - W(\lambda XY)) + rv_1(2, \lambda X) \\ &= (\lambda - \overline{\lambda})\left(r(1 - \delta(1-p)) - \delta p\xi\right) + rv_1(2, \lambda X), \end{aligned} \tag{B.7}$$

where the first equality follows from combining terms and using the definition of α, and the second equality follows from Fact 1. Using (B.6) and (B.7), $rw_1(\lambda XY) = \frac{r}{2}(v_1(1, \lambda X) + v_1(2, \lambda X))$ is equal to

$$\begin{aligned} rw_1(\lambda X) &= \frac{1}{2}(\lambda - \overline{\lambda})(r(1 - \delta(1-p)) - \delta p\xi) + r\delta(pw_1(\lambda XY) + (1-p)w_1(\lambda X)) \\ \Longrightarrow rw_1(\lambda X) &= \frac{1}{2}(\lambda - \overline{\lambda})(r - \alpha\xi) + r\alpha w_1(\lambda XY). \end{aligned} \tag{B.8}$$

Similarly, we can show that $w_2(\lambda X) = \frac{1}{2}(\lambda - \overline{\lambda})(r - \alpha\xi) + \alpha w_2(\lambda XY)$. □

Lemma 6. *Let $\phi < 0$ and $\lambda \in [0, 1]$. For each $j = 1, 2$, all SPE starting at state $(j, \lambda X)$ are payoff equivalent. Moreover, in every SPE the players reach a complete agreement.*

Proof. (Sketch) Let the state be $s = \lambda X$. Using arguments similar to those in Lemma 3, one can show that when $\phi < 0$ the proposer will find it optimal to make an offer such that the players consume all λ of pie X, regardless of whether $\lambda > \overline{\lambda}$ or $\lambda \leq \overline{\lambda}$. Because players always make offers over all λ of pie X, we can again find bounds for $\underline{v}_i(j, \lambda X)$ and $\overline{v}_i(j, \lambda X)$ as in Lemma 4, and then use an argument similar to the one in Lemma 5 to establish the uniqueness of equilibrium payoffs. □

Proof of Theorem 1. Part 1 follows from Lemma 6 and part 2 follows from Lemma 5. □

C. Equilibrium Offers in States λX with $\lambda > \overline{\lambda}(\delta, r)$ (when $\phi(p, \delta, r) > 0$)

We can use the unique SPE payoffs from Lemma 5 in Appendix B to pin down the offers that players make in states λX with $\lambda > \overline{\lambda}$. Suppose that player 2 is proposer, and note that player 1 will only accept offers giving him a payoff of at least $v_1(2, \lambda X)$. If player 1 accepts an offer (x_1^2, x_2^2) with $x_1^2+x_2^2 = \lambda-\overline{\lambda}$, his payoff is $x_1^2+\sum_{t=0}^{\infty}\delta^t(1-p)^t w_1(\overline{\lambda}XY) = x_1^2+\alpha w_1(\overline{\lambda}XY)$. Thus, player 2's offer will be such that $x_1^2+\alpha w_1(\overline{\lambda}XY) = \delta(pw_1(\lambda XY)+(1-p)w_1(\lambda X)) = \alpha w_1(\lambda XY) + \delta(1-p)\frac{(\lambda-\overline{\lambda})}{2r}(r-\alpha\xi)$, where the last equality follows from (B.8). By a symmetric argument, player 1's offer (x_1^1, x_2^1) at state λX is such that $rx_2^1 + \alpha w_2(\overline{\lambda}XY) = \alpha w_2(\lambda XY) + \delta(1-p)\frac{(\lambda-\overline{\lambda})}{2}(r-\alpha\xi)$. Using (B.1), it follows that

$$x_2^1 = (\lambda-\overline{\lambda})\left(\alpha\frac{2(1-\delta)}{4(1-\delta)+\delta^2(1-r^2)} + \frac{\delta(1-p)}{2r}(r-\alpha\xi)\right), \tag{C.1}$$

$$x_1^2 = (\lambda-\overline{\lambda})\left(\alpha\frac{2-\delta-r^2\delta}{4(1-\delta)+\delta^2(1-r^2)} + \frac{\delta(1-p)}{2r}(r-\alpha\xi)\right), \tag{C.2}$$

and $x_1^1 = \lambda-\overline{\lambda}-x_2^1$ and $x_2^2 = \lambda-\overline{\lambda}-x_1^2$. Let $x_i = \frac{1}{2}(x_i^1+x_i^2)$ denote the expected fraction of pie X that player i gets. From (C.1) and (C.2), $x_1 = (\lambda-\overline{\lambda})(\frac{1}{2}+\frac{1}{2}\frac{\alpha\delta(1-r^2)}{4(1-\delta)+\delta^2(1-r^2)})$ and $x_2 = (\lambda-\overline{\lambda})(\frac{1}{2}-\frac{1}{2}\frac{\alpha\delta(1-r^2)}{4(1-\delta)+\delta^2(1-r^2)})$. Note that $\frac{\alpha\delta(1-r^2)}{4(1-\delta)+\delta^2(1-r^2)}$ is increasing in δ and p.

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