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ENERGY-EFFICIENT BROADCASTING IN WIRELESS AD HOC NETWORKS: LOWER BOUNDS AND ALGORITHMS

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Energy-efficient communication is critical for increasing the life of power limited wireless ad hoc networks. Accordingly, there has been considerable interest in minimum energy broadcast. In this paper, we develop lower bounds and an algorithm for minimizing energy cost for broadcasting from any source to all other nodes in the network. Most prior work has used a simpler model for energy cost for wireless communications by accounting only for the analog radiation cost for transmission and ignoring the fixed energy cost for electronics in transmission and reception circuitry in nodes. Further, in a network, it is possible for some node pairs not be able to communicate directly even though they are in their radio ranges due to obstacles present in the terrain of the network.

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In this paper, we study the minimum energy broadcasting problem in an ad hoc network of nodes with obstacles and with a general model of energy cost for communications. We show a lower bound of $\Omega(\log N)$ on the approximation ratio for any polynomial time algorithm for this problem unless $\mathrm{P} = \mathrm{NP}$, where N is the number of nodes in the network. We present a broadcasting algorithm, called GBA, which meets this lower bound up to a constant factor. This also improves upon a recently published result for broadcasting in a network where nodes have k power levels from $O(\log^3 N)$ approximation ratio[12] to $O(\log N)$ for the symmetric cost case.

For practical network scenarios consisting of a few tens to several 100 nodes, we calculate two simple lower bounds for broadcasting (longest shortest path cost and multicasting cost to two distant destination). We show using extensive simulations that our GBA algorithm is within a small factor of these lower bounds implying that our algorithm performs well in practical networks.

Keywords: energy efficiency, lower bound, broadcasting, wireless ad hoc networks

1. Introduction

Wireless ad hoc networks are useful in any situation where temporary network connectivity is needed, such as in the battlefield and in disaster relief. In such a multi-hop wireless network, every node may be required to perform routing in order to achieve end-to-end communication among nodes. We consider networks where each node has transmit power control and an omni-directional antenna and therefore can adjust the area of coverage with its transmission.

Wireless communications consume significant amounts of battery power [11], and therefore, energy efficient operations are critical to enhance the life of such networks. Some amount of power is lost even when a node is in idle mode. A recent study [5] shows that the power consumed in transmitting and receiving packets in standard WaveLAN cards range from 800 mW to 1200 mW. During the past few years, there has been increasing interest in the design of energy efficient protocols for wireless ad hoc networks [15,19,20,22,23].

Broadcasting is an important operation in wireless ad hoc networks, where a message from a given source must reach all other nodes. Each node has local broadcast capability, that is, it can transmit a message to reach all nodes that are reachable with a certain amount of transmitted energy. There are several works on energy efficient protocols where all nodes are assumed to have fixed transmission range [11,18,20,21,22]. With power control a node can adjust its range to transmit a message to reach one or more nodes [15]. The problem of minimizing the energy cost for broadcasting from a given source to all other nodes in a network, with power control was presented in [26]. The approach taken in [26] is to build a source rooted spanning tree by adjusting transmit powers of nodes, followed by a sweep operation to remove redundant transmissions. Since finding the optimal broadcast tree is difficult, as it requires all possible spanning trees to be evaluated, the authors presented several heuristics. In [25], Wan et. al. proved that the BIP algorithm in [26] has a constant ratio to optimal solution given that there are no obstacles in the network and that the fixed energy cost for electronics is negligible. Therefore, this result will not hold when there are obstacles in the network.

1.1. *Network and Energy Cost Model*

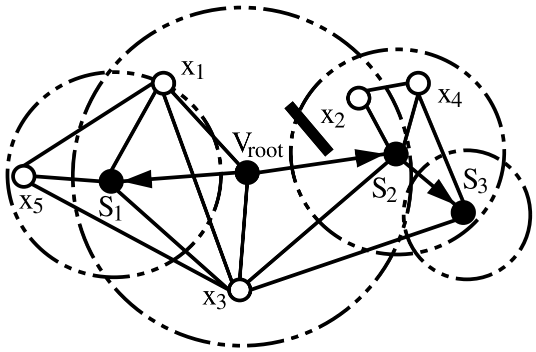

Figure 1: Broadcasting in the wireless network: there is an obstacle between V_{root} and x_2

In this paper we consider the one-to-all broadcasting problem in wireless networks, where nodes have power control and any node can be the source. A node can adjust its transmission radius, but can reach all nodes within that radius only if there are no obstacles. That is, if there is an obstacle between two nodes, then these nodes cannot communicate directly. Our network model is more realistic as many ad hoc networks operate in buildings and in the field where there are many obstacles that block radio signals. We assume that nodes are fixed and the obstacles in the field do not change.

We will also consider more general model for energy cost for communication. Energy consumed in radio transmissions depends on several factors including the number of bits sent, the range of transmission, and the losses in the environment [16]. The ratio of signal strength to the noise level at a receiver must be above a certain threshold for reliable detection. There are two primary components of the energy cost for communication between a pair of sensor nodes: a fixed component of energy consumed in electronic circuits when transmitting or receiving a packet, and a variable component when transmitting a packet which depends on the distance of transmission. Typically, this variable part of the energy cost in transmission is proportional to d^γ, where γ ranges from 2 to 4 [16] and this is the only cost considered in most of the previous works [26,18].

We will use the following model for transmission energy cost:

$$p = f_c + v_c \times d^2 \tag{1.1}$$

where d is the distance for transmission, and f_c is the cost for signal processing and amplification, which is needed for both reception and transmission, and v_c is the cost associated with the radiation part of transmission. We assume that the radio channel is symmetric, that is, the energy required to transmit a message from node i to node j is the same as that from node j to node i.

1.2. *The Problem*

We abstract our broadcasting problem as follows. Given a graph $G = (V, E)$, where each vertex represents a wireless node, and any edge $(u, v) \in E$ means node u and node

v can communicate with each other *directly* by using the distance of the edge (u, v) as the transmission radius. Missing an edge between a pair of nodes implies that they can not communicate directly due to obstacles. Energy cost for packet transmission is given by the above equation consisting of fixed and variable cost. For receiving a packet, there is a fixed energy cost due to the node electronics. The energy cost for one-to-all broadcasting will include all the reception costs and intermediate retransmission costs. For example, in Fig. 1, if node V_{root} is the source of a broadcast, and nodes S_1, S_2 are chosen as forwarding nodes with the transmission radius shown by the circles, then the total cost includes the transmission costs and reception cost. In general, the total energy cost for broadcasting, written as $C(s)$, in an N node network with k nodes transmitting the message is:

$$\begin{aligned} C(s) &= (N-1) \cdot f_c + \sum_{i=1}^{k} p_i \\ &= (N-1) \cdot f_c + \sum_{i=1}^{k} (f_c + v_c \times {d_i}^2) \end{aligned}$$

where k is the number of transmitting nodes (including source), p_i is the energy cost used by the ith forwarding node, and d_i is the transmission radius chosen by the ith forwarding node.

1.3. *Our Contributions*

The main contributions of this paper are the following. For the wireless ad hoc network model with obstacles we show a lower bound of $\Omega(\log N)$ on the approximation ratio of any polynomial time algorithm for this problem, unless P = NP. We then present a greedy broadcasting algorithm (GBA) which provably meets the $O(\log N)$ performance bound. Although we can prove that the worst case approximation ratio of broadcasting problem in general wireless ad hoc network is bounded by a factor $O(\log N)$, it is possible that for practical networks BIP or GBA algorithm may give much better solutions compared to optimal solution. Since we will not know the optimum solution for a given practical network, we developed lower bounds for energy cost for broadcasting for comparing the effectiveness of GBA solutions. This evaluation is performed through extensive simulations using two lower bounds (longest shortest path cost and multicasting cost to two distant nodes) for many different size networks. The results show that the GBA algorithm performs well in practical networks.

We can observe several interesting properties about our GBA algorithm:

- The $O(\log N)$ bound on the performance of GBA algorithm holds not just for the energy cost model in (1.1), but for any energy cost model for communication as long as the cost is symmetric.

- Recently, Liang [12] gave a polynomial time algorithm for broadcasting in wireless ad hoc networks where all nodes have k power levels and proved that the approxima-

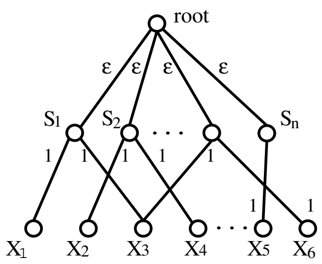

Figure 2: Network for the associated set-cover problem

tion ratio has a bound of $O(\log^3 N)$, for the symmetric case. Because GBA works for any symmetric cost structure, we improve their approximation ratio to $O(\log N)$.

- GBA produces a single shared tree which is an $O(\log N)$ approximation to the optimum for broadcasting from any source.

This paper is organized as follows. In Section 2 we present our theoretical bounds and establish the $O(\log N)$ lower bound result. In Section 3, we describe the details of our GBA algorithm. In Section 4 we develop lower bounds for energy cost for broadcasting in practical networks and present our simulation results. In this section we also compare our results with the BIP algorithm which is closest to our work. Our results show that GBA, which meets the theoretical bound for general networks, has good performance compared to BIP algorithm and for many practical networks, performs better than BIP algorithm.

2. Theoretical Lower Bound for broadcasting

In [1], it is formally proved that broadcasting problem in a wireless ad hoc network with obstacles is an NP-complete problem. Our objective is to design a practical approximation algorithm with good performance. In this section, we will prove a lower bound of $\Omega(\log N)$ on the approximation ratio of any polynomial time algorithm for this problem, unless P $=$ NP. We will prove this lower bound by reducing the set cover problem to the problem of broadcasting with obstacles in a wireless network.

The Minimum Set Cover Problem is defined as: Given a universe U of m elements and a collection $S_1, S_2, \cdots, S_n$ of subsets of U, with cost 1 specified for each subset, the minimum set cover problem asks for a minimum cost collection of sets whose union is U.

Now we give a reduction from the Minimum Set Cover problem to our problem as follows (Fig. 2): Construct an undirected graph $G = (V, E)$, where

$$V = \{\textit{root}\} \bigcup \{X_1, X_2, \cdots, X_m\} \bigcup \{S_1, S_2, \cdots, S_n\} \tag{2.2}$$

and for each subset S_i, add an edge between node *root* and node S_i, and for each element X_j in the subset S_i, add an edge between node X_j and node S_i in graph G. And the energy cost to reach all of the S_i nodes from root is ϵ, where ϵ is a small constant greater than zero. In addition, the energy cost of each node S_i to reach all of its child nodes X_j is 1. In our construction, any missing edge implies that there is an obstacle between the corresponding nodes. To find the minimum set cover in the original problem, now becomes finding a minimum-energy broadcasting scheme from root.

Lemma 1 *In an optimum solution to the broadcasting problem specified in Fig. 2, any element node $X_i,\ i = 1, 2, \cdots, m$, will not be chosen as a forwarding node.*

Proof. Since the root node is the source of broadcasting, and all of the subset nodes $S_i, i = 1, 2, \cdots, n$ must get the broadcast message, obviously, cost ϵ must be paid to reach all the subset nodes. Since forwarding of message from any element node can only reach some subset nodes of S_is which already have the broadcast message, such transmissions are unnecessary. Therefore, in the optimum solution, element nodes $X_i, i = 1, 2, \cdots, m$ will not be chosen as forwarding nodes. □

Lemma 2 *If we can find a solution in the original minimum set cover problem with cost K, then we can find a solution in the corresponding broadcasting in wireless network with obstacles problem with cost $K + \epsilon$.*

Proof. If we could find a solution in the original minimum set cover problem with cost K, then in the corresponding broadcasting problem, we could just construct a solution by choosing these K subsets, solved from the minimum set cover problem, as the forwarding nodes, and make *root* responsible for forwarding messages to all the subset nodes. Since the union of the K subsets is the universe U, this solution is a complete broadcast scheme with energy cost $K + \epsilon$. □

Lemma 3 *If we could find a solution to the broadcasting problem with obstacles in wireless network specified in Fig. 2 with cost $K+\epsilon$, then we can find a solution in original minimum set cover problem with cost K.*

Proof. If we could find a solution to the broadcasting problem with cost $K + \epsilon$, then we could construct a solution in minimum set cover problem by choosing those K subsets whose representative nodes in Fig. 2 are chosen as forwarding Nodes. Obviously, all of the nodes in Fig. 2 get the broadcast message, which implies that the union of these K subsets is U. □

In [6] and [17], it is proved that the lower bound on the approximation ratio of any polynomial time algorithm for the minimum set cover problem is $\Omega(\log N)$. By Lemma 2 & 3, the same lower bound holds for our broadcasting problem.

Theorem 1 *The lower bound on the approximation ratio of any polynomial time algorithm for the broadcasting problem in wireless networks with obstacles is $\Omega(\log N)$ given that P $\neq$ NP.*

3. Our GBA algorithm For The broadcasting problem

From the lower bound proof, we could see the similarities between the set cover problem and the broadcasting problem in wireless networks with obstacles. There is a simple algorithm which can achieve an approximation ratio of $O(\log N)$ for the set cover problem. Therefore, it seems reasonable to try and discover a simple greedy algorithm which obtain a similar approximation ratio for our problem. In this section, we present an $O(\log N)$-approximation to our problem using a simple polynomial time algorithm that we call the Greedy Broadcasting Algorithm(GBA). From Section 2, it follows that this is the best possible (up to a constant factor) approximation ratio that can be achieved in polynomial time unless P $=$ NP.

We will assume that we are given a set of nodes $V = \{v_1, v_2, \ldots, v_N\}$. We will further assume that we are given a non-negative cost $c(v_i, v_j)$ between any pair of nodes v_i, v_j. This cost represents the amount of power required to be able to send information from v_i to v_j. The cost can be infinite for nodes which can not communicate directly (eg. because of an obstacle). We will assume that each node has a path of non-infinite cost to any other node. We will further assume that the cost function is symmetric i.e. the cost of a transmission from v_i to v_j is the same as the cost of a transmission from v_j to v_i. If a node v transmits with power δ, it can transmit to every node $x \in V$ such that $c(v, x) \leq \delta$.

3.1. *Description of GBA*

The algorithm always maintains a collection $\mathcal{C} = \{T_1, T_2, \ldots, T_J\}$ of trees. Let $V(T)$ denote the set of nodes in tree T. The algorithm ensures that the sets $V(T_1), V(T_2), \ldots, V(T_J)$ are disjoint, and that their union is the set V. The number of trees in $\mathcal{C}$ keeps decreasing as the algorithm advances. Before specifying the algorithm, we need to establish some notation.

The cost $c(v, T)$ of transmitting from a node v to a tree T is defined as $\min_{x \in V(T)} c(x, v)$. We use the term $x(v, T)$ to denote the node in $V(T)$ which achieves this minimum (if two or more nodes in $V(T)$ achieve this minimum, then we break the tie arbitrarily).

At any given time during the execution of the algorithm, define $n(v, \delta)$ as the number of trees T in the collection $\mathcal{C}$ such that $v \notin T$ and $c(v, T) \leq \delta$. Informally, $n(v, T)$ measures the number of trees which can be reached from v using a single transmission of power δ.

The profit ratio $\pi(v)$ of a node v is defined as $\min_\delta \frac{\delta}{n(v,\delta)}$. Let $\delta(v)$ refer to the transmission power which achieves this maximum; again, ties are broken arbitrarily. Intuitively, the profit ratio of a node is the best ratio of the number of trees covered to the power used.

Note there are at most N possible values of δ that are "interesting" for node v. The quantities $\pi(v), \delta(v), x(v, T)$ etc. can all be calculated in polynomial time. It is also important to note that the quantities $\pi(v)$ etc. are not static but change as the class $\mathcal{C}$ changes during the course of the algorithm.

We are now ready to describe GBA:

The Initialization Step: Let T_i be the tree consisting of a single node v_i. The class $\mathcal{C}$ consists of the trees $T_1, T_2, \ldots, T_N$. Also, initialize C to 0.

The Greedy Step:

1. If the class $\mathcal{C}$ has a single tree T then output this as the shared tree and exit, else go to step 2.

2. Find the node v with the minimum value of $\pi(v)$. Let T be the tree in $\mathcal{C}$ which contains v. Then for each tree $T' \neq T$ in $\mathcal{C}$ such that $c(v, T') \leq \delta(v)$, do the following:
   (a) Add an edge from v to $x(v, T')$. The trees T and T' have now merged into a single tree; we continue to refer to this merged tree as T.
   (b) Remove T' from $\mathcal{C}$; the merged tree T continues to remain in $\mathcal{C}$.
3. $C \leftarrow C + \delta(v)$.
4. Start another iteration of the greedy step.

One nice feature of GBA is that it returns a single tree which can be used by any source to perform a broadcast; in the next section we show that this tree is within a factor $O(\log N)$ of the optimum irrespective of the source. Notice that the quantity C is maintained by the above algorithm but is never used; this quantity will help us in our analysis.

3.2. *Analysis of GBA*

In order to use the shared tree T for a given source s, we reorient the tree so that s becomes the root. Then we start at the root, and transmit the message with sufficient power to reach all the children of the root. From each of the children, we recursively follow the same process till the message gets to all the nodes in T. Let $C(s)$ denote the total power used by the above strategy.

Lemma 4 $C(s) \leq 2C$

The proof of Lemma 4 is straightforward, and is omitted . This Lemma is very useful, as we now need to only analyze the cost C and not worry about the source.

Let $C^*(s)$ denote the minimum power needed to broadcast a message from source s to all the other nodes in the network. Observe that we do not know how to efficiently compute $C^*(s)$; we will use $C^*(s)$ only as an artifact in our analysis. Define $C^* = \min_{s \in V} C^*(s)$, and define the associated optimum broadcasting tree as T^*. Assume that GBA went through the greedy step K times. Let v_i denote the chosen vertex, let C_i denote the value of $\delta(v_i)$, and let n_i denote the value of $n(v_i, \delta(v_i))$ during the i-th iteration of the greedy step. For each node v, define $\delta^*(v)$ as its transmitting power used in T^* and define $n_i^*(v)$ as the number of distinct clusters its children belong to during the i-th iteration of the greedy step. Notice that $C^* = \sum_v \delta^*(v)$ and $\pi_i(v) \leq \frac{\delta^*(v)}{n_i^*(v)}$ due to the definition of π. Observe that the number of trees in $\mathcal{C}$ decrease by exactly n_i during the i-th iteration. $\mathcal{C}$ has N trees at the beginning of the algorithm, and only one at the end. Hence $\sum_{i=1}^{K} n_i = N - 1$. Also, $C = \sum_{i=1}^{K} C_i$. We will use the term m_i to denote the size of $\mathcal{C}$ at the beginning of the i-th iteration. Notice that $C_i/n_i = \pi(v_i) \leq \pi_i(v) \leq \frac{\delta^*(v)}{n_i^*(v)}$, and $m_i - m_{i+1} = n_i$.

Lemma 5 $\sum_v n_i^*(v) \geq (m_i - 1)$

Proof. For any node v and cluster X such that v does not belong to X, define $Z^*(v, X) = 1$ if there exists a node $u \in X$ which is a child of v in T^*, and define $Z^*(v, X) = 0$ otherwise. Let $Z^*(X) = \sum_{v \notin X} Z^*(v, X)$. Let $\mathcal{C}_i$ denote the set of clusters at the beginning of step i. Now, $\sum_v n_i^*(v) = \sum_{X \in \mathcal{C}_i, v \notin X} Z^*(v, X) = \sum_{X \in \mathcal{C}_i} Z^*(X)$. Consider any cluster X

which does not contain the root of T^*. There must be a node in X whose parent does not belong to X. Hence $Z^*(X) \geq 1$. Since only one cluster in $\mathcal{C}_i$ contains the root, $\sum_{X \in \mathcal{C}_i} Z^*(X) \geq (m_i - 1)$, which proves this lemma. □

The above lemma is crucial to the following lemma:

Lemma 6

$$\exists v \textit{ s.t. } \delta^*(v)/n_i^*(v) \leq C^*/(m_i - 1) \tag{3.3}$$

Proof. Suppose that $\forall v, \delta^*(v)/n_i^*(v) > C^*/(m_i - 1)$, then we have $C^* = \sum_v \delta^*(v) > \sum_v n_i^*(v) * C^*/(m_i - 1)$, due to Lemma 6, we can get: $C^* > C^*$, which is a contradiction, thus prove the lemma. □

Since at each iteration we choose a node v with best profit ratio, we can therefore conclude that:

$$C_i/n_i = \pi(v_i) \leq \delta_i^*(v)/n_i^*(v) \leq C^*/(m_i - 1). \tag{3.4}$$

Now let, $H(N) = \sum_{j=1}^{N} \frac{1}{j}$ denote the N-th harmonic number.

Lemma 7 $C \leq C^* \cdot H(N)$.

Proof. The equation (3.4) is the crucial property of GBA which will result in the desired bound. Now,

$$\begin{aligned}
C &= \sum_{i=1}^{K} C_i \\
&\leq \sum_{i=1}^{K} n_i \cdot (C^*/(m_i - 1)) \quad \text{[Using equation 3.4]} \\
&= C^* \left(\sum_{i=1}^{K} n_i/(m_i - 1) \right) \\
&= C^* \left(\sum_{i=1}^{K} \sum_{j=1}^{n_i} 1/(m_i - 1) \right) \\
&\leq C^* \left(\sum_{i=1}^{K} \sum_{j=1}^{n_i} 1/(m_i - j) \right) \\
&= C^* \sum_{i=1}^{N-1} 1/i \quad \text{[Since } m_i - n_i = m_{i+1}\text{]} \\
&\leq C^* \cdot H(N)
\end{aligned}$$

□

The following theorem is now immediate from Lemmas 4 and 7.

Theorem 2 $C(s) \leq 2C^*(s) \cdot H(N)$

It is well known that $H(N) \leq 1 + \ln N$, which gives us an upper bound of $O(\log N)$ on the performance of GBA, irrespective of the source. It is worth observing that the constant hidden inside the O-notation is quite small.

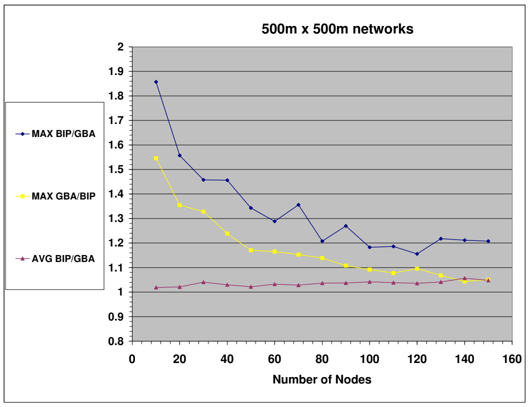

Figure 3: GBA vs. BIP on 500m x 500m practical networks

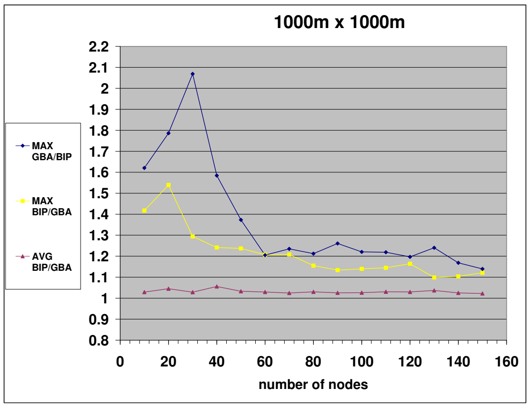

Figure 4: GBA vs. BIP on 1000m x 1000m practical networks

4. Simulation Results

We use extensive simulations to evaluate the performance of our GBA broadcasting algorithm for a number of different network scenarios. We also use simulations to compute the lower bounds for practical networks. For our simulations, we chose randomly generated undirected connected graphs as network topologies. Such a graph will represent a realistic network with obstacles, since only those pairs of nodes that can communicate directly will have edges between them. In order to see the effect of the density of nodes, we chose two different size networks: one is a network in a 500m x 500m square area; the other is a network in a 1000m x 1000m square area. We generated networks with number of nodes ranging from 10 to 150 increasing by increments of 10. Also for each size, we generated 50 different instances.[‡] So in total, we have 1500 different networks for our simulations.

4.1. *GBA vs BIP*

Since BIP [26] broadcasting heuristic is the most widely cited algorithm for this problem, we compare the performance of GBA and BIP for the various practical networks. We are interested in the average energy cost for broadcasting from any node. For this purpose, we iteratively chose every node as the source for broadcasting and calculated the energy cost with the GBA scheme and the BIP scheme and computed the cost ratio. For a given N, we calculated the energy cost for broadcasting with these two algorithms for 50 different instances of network topologies. The maximum cost and average costs are computed for the various networks, and for comparison purposes, we plot the maximum cost ratio of GBA over BIP and BIP over GBA as well as the ratio of the average cost. Fig. 3 and Fig. 4 show the maximum and average ratio results for networks size ranging from 10 nodes to 150 nodes. Our results clearly show that GBA performs better on the average and in many instances it performs significantly better than the BIP scheme. With increase in network

[‡]On average, the number of edges is 75% of that of the full mesh graph.

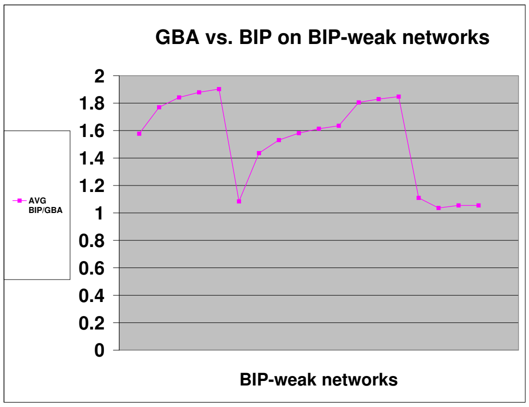

Figure 5: GBA vs. BIP on BIP-weak networks

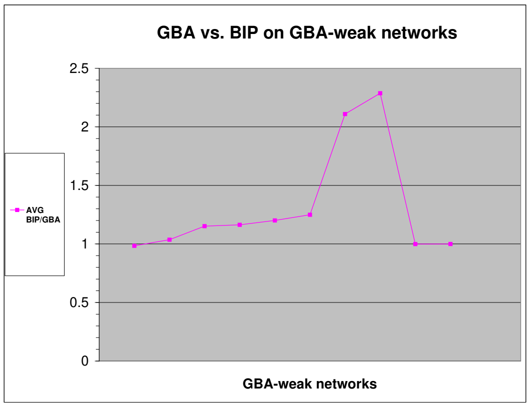

Figure 6: GBA vs. BIP on GBA-weak networks

size, the maximum energy cost ratio tends to settle down with GBA performing better than BIP by at least 20%.

In addition, we conducted simulation runs on special networks which are known to be bad for GBA or BIP schemes and computed the average energy cost for broadcasting in such networks. We evaluated the performance of both algorithms on some specially designed networks given in [25] which are weak for BIP, and Fig. 5 shows the simulation results. We also designed a family of networks that we believe gives bad performance for GBA, we call these networks *GBA-weak* (their structure is described in appendix 1), and Fig. 6 shows our results. The plots in these figures show the ratio of average energy cost for broadcasting from any source with the BIP and GBA schemes for several different BIP-weak and GBA-weak networks. Clearly, GBA performs better than BIP for both classes of weak networks in most cases.

In summary, our simulation results show that:

- GBA works better than BIP on the average.
- In some cases, GBA works significantly better than BIP.
- For both BIP-weak and GBA-wek networks, GBA outperforms BIP.
- When the density of the network becomes high, the performance of GBA and BIP converges.

4.2. *Practical Lower Bounds*

In the worst case, our algorithm (or any polynomial time algorithm) can be $\Omega(\log N)$ times more expensive than the optimum, unless P $=$ NP. However, it is possible that the performance of our algorithm may be much better on realistic scenarios. In this section, we present two efficient methods of computing a lower bound on the cost of the optimal

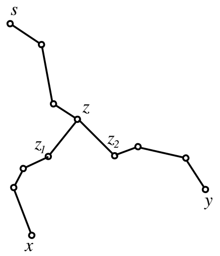

Figure 7: 2-SPT: the paths from s to z, from z_1 to x, and from z_2 to y are shortest paths. z_1 and z_2 can be reached from z by one broadcasting.

solution[§] The ratio of the cost of our algorithm to the lower bound on any given instance gives a bound on the approximation ratio of our algorithm for that instance.

a. 1-SPT[¶] $\vdash$ Find the node v such that the cost of unicasting to v from source s is the highest. Use this cost as a lower bound. This node v and the corresponding cost can easily be found using one invocation of Dijkstra's algorithm; we omit the details.

b. 2-SPT $\vdash$ Suppose we are given a source s and two nodes $x, y \in V - \{s\}$. The energy-optimum tree for multicasting from s to the set $\{x, y\}$ must have the following form: there is a node $z \in V$ such that s is connected to z via the shortest path from s to z, z is connected to z_1 and z_2, and then z_1 is connected to x via the shortest path from z_1 to x and z_2 is connected to y via the shortest path from z_2 to y, as illustrated in Fig. 7. Let $C^*(x, y)$ be the cost of this tree. Two observations can now be made:

1. $C^*(x, y)$ can be computed efficiently by guessing z and its transmission radius (by trying out all values of z and the transmission radius, for example). We omit the details.

2. $C^*(x, y)$ is a lower bound on the optimum broadcast cost.

The above observations could be used to compute $\max_{x,y \in V} C^*(x, y)$, which would be a valid lower bound. To our experiences, this turns out to be prohibitively expensive computationally for our topologies. Instead, we use a heuristic to choose values for x and y as follows: first, choose x as the node that has the highest shortest path cost from source s. Then choose y as the node such that the product of its shortest path costs from s and x is the highest. We could analogously define $C^*(w, x, y)$, $C^*(u, w, x, y)$, etc and use these as lower bounds, but the computational complexity of these larger problems prohibits these generalizations. To see how well our scheme works for practical networks, we calculated the lower bound energy cost on the 1500 randomly generated networks. We calculated the ratio of the GBA cost over the lower bound cost for practical networks and Fig. 8 and Fig. 9 show the results with respect to 1-SPT lower bound performance, and Fig. 10, Fig. 11 show the results with respect to the 2-SPT lower bound performance. Our simulation results show

[§]The through exploration of the lower bound is beyond the scope of this paper.
[¶]SPT stands for shortest path tree.

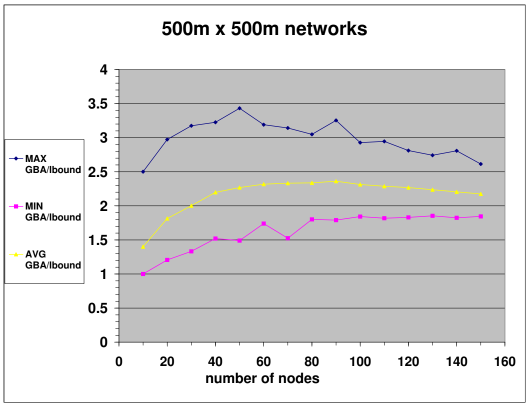

Figure 8: 1-SPT Lower Bounds for 500m x 500m practical networks

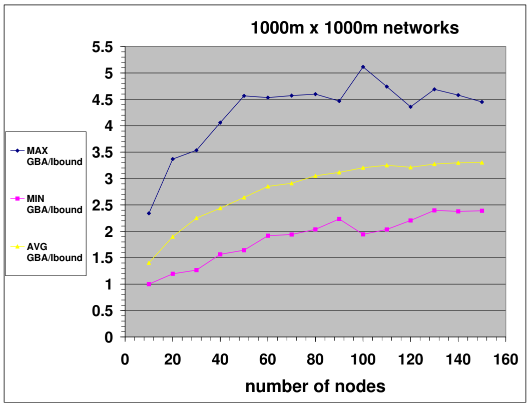

Figure 9: 1-SPT Lower Bounds for 1000m x 1000m practical networks

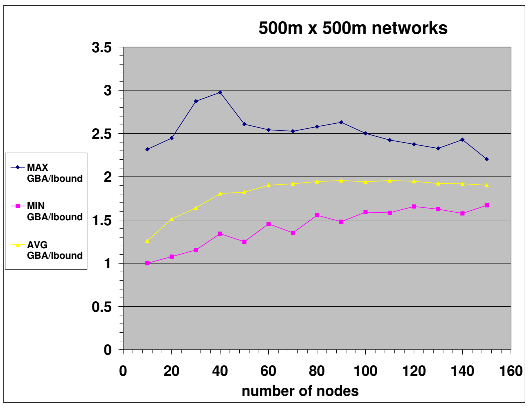

Figure 10: 2-SPT Lower Bounds for 500m x 500m practical networks

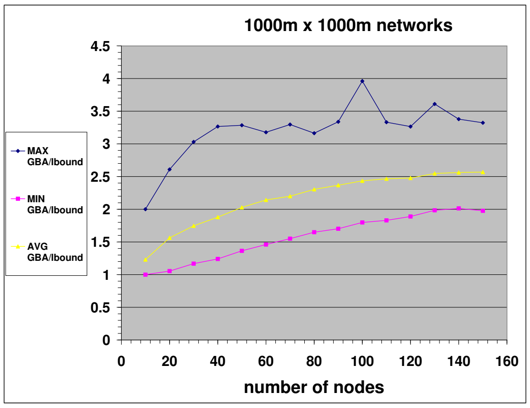

Figure 11: 2-SPT Lower Bounds for 1000m x 1000m practical networks

that 2-SPT lower bound is tighter than 1-SPT. The maximum ratio of GBA/lower bound is between 3 and 4 in different size networks. This ratio seems to converge to a small factor of 2 to 2.5 with increase in the density of nodes. We have chosen very simple lower bounds for broadcasting and our results show that the cost ratio is very small. With even tighter lower bound, we can expect this ratio to be much smaller in practical networks. Therefore, we can conclude that GBA algorithm will perform well for broadcasting in practical networks of a few hundred nodes in 1000m x 1000m area.

5. Conclusions and Future Work

In this paper, we considered realistic wireless ad hoc networks with obstacles and established theoretical lower bound on the approximation ratio of the energy cost for broadcasting. With more general network model with obstacles and energy cost model, we showed that no polynomial time algorithm can achieve an approximation ratio beter than $O(\log N)$, unless P=NP. We developed and presented a broadcasting algorithm, called GBA, and proved that this algorithm guarantees $O(\log N)$ approximation ratio performance and thus it is an optimal polynomial time approximation algorithm for energy efficient broadcasting in wireless ad hoc networks with obstacles. In practical networks, we showed that the GBA algorithm performs quite well and on the average performs better than the BIP scheme by at least 20%. Using extensive simulations we compared the performance of GBA with respect to simple lower bounds for broadcasting and showed that it is only within a factor of 4 or less with respect to this lower bound for networks up to a few hundred nodes. With this result, we established that GBA can achieve broadcasting for a given practical network which is at most 4 times the optimum cost for broadcasting. In our future work, we will develop a distributed algorithm based on GBA for broadcasting as well as consider more general energy cost models.

Appendix A

It would be useful to find some specially designed networks which are "bad" for GBA. We call these networks *GBA-weak networks*. In this appendix, we will give one construction for such a class of networks. In order to make a network bad for GBA, the construction should proceed as follows:

- Start with a network with known optimal solution.
- It has $N+1$ nodes, among which N nodes are forwarding nodes.
- The kth forwarding node will spend an energy cost of $\frac{OPT}{N-k+1}$ for the transmission.

We can see that in such a network, the energy cost of GBA produced broadcasting scheme is $O(\log N) \times OPT$, which is really bad. The network is shown in Fig. A.1.

In this network, nodes $P_1, P_2, \cdots, P_N$ are distributed on concentric circles with the same center P_0. Node P_0 can communicate with all other nodes, yet node P_i, $1 \leq i \leq N$, can only communicate with its nearest neighbors P_{i-1} and P_{i+1}. By carefully tuning the distance (r_i) between the nearest neighboring nodes and the radius (d_i) of the circles, we can make GBA to find a tree as shown in Fig. A.2. The following conditions for d_is and r_is will make GBA perform poorly on this network:

1. For each r_k , let $f_c + v_c \times r_k^2 = \frac{OPT}{N-k+1}$, where $OPT = f_c + v_c \times r_N^2$ and thus we get $r_k = \sqrt{\frac{2r_N^2 - N + k}{2 \times (N-k+1)}}$, where $r_1 = d_1$
2. For each k, $\frac{f_c + v_c \times d_k^2}{k} > \frac{f_c + v_c \times d_N^2}{N}$
3. For each $k, d_{k-1} \leq d_k \leq d_{k+1}$
4. $(d_k - d_{k-1}) \leq r_k \leq (d_k + d_{k-1})$

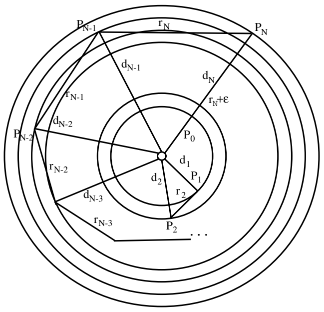

Figure A.1: A GBA-weak network with optimal cost for broadcasting: $f_c + v_c * r_n^2$

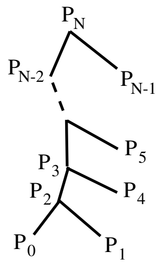

Figure A.2: Broadcast tree built by GBA for the GBA-weak network shown in Fig. A.1

We can see that all of the above conditions can hold for certain values of $d_1, d_2, \cdots, d_N$. Once condition 2) holds, we will have no chance to choose P_0 as the cluster head and any d_k as the transmission radius in GBA. Thus we have the following conclusion:

$$\frac{\frac{f_c+v_c\times r_{k+1}^2}{2}}{\frac{f_c+v_c\times r_k^2}{1}} = \frac{\frac{OPT}{2(N-k)}}{\frac{OPT}{N-k+1}} = \frac{1}{2} + \frac{1}{2(N-k)} < 1 \tag{A.1}$$

and

$$f_c + v_c \times r_1^2 = \frac{OPT}{N} < \frac{f_c + v_c \times (r_N + \epsilon)^2}{N} \tag{A.2}$$

Which in turn can lead to the following conclusion: The total energy cost required by the broadcasting scheme produced by GBA would be:

$$\sum_{k=1}^{N} \frac{1}{N-k+1} \times OPT = H(N) = O(\log N) \tag{A.3}$$

The shared broadcasting tree built by GBA is shown Fig. A.2.

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