Sequential Deliberation for Social Choice

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Voting in Complex Spaces

- What if:
  - The space of outcomes is large?
  - No preference structure is known a priori?
- Need not just voting, but also **negotiation** and **deliberation**

Goals

- Desiderata:

  A. The algorithm (mechanism) designer does not need to understand the decision space.

  B. We can prove guarantees on the quality of outcomes under analytical models (In particular, we should beat random dictatorship).

  C. The mechanism should restrict cognitive load on users, and encourage negotiation and deliberation

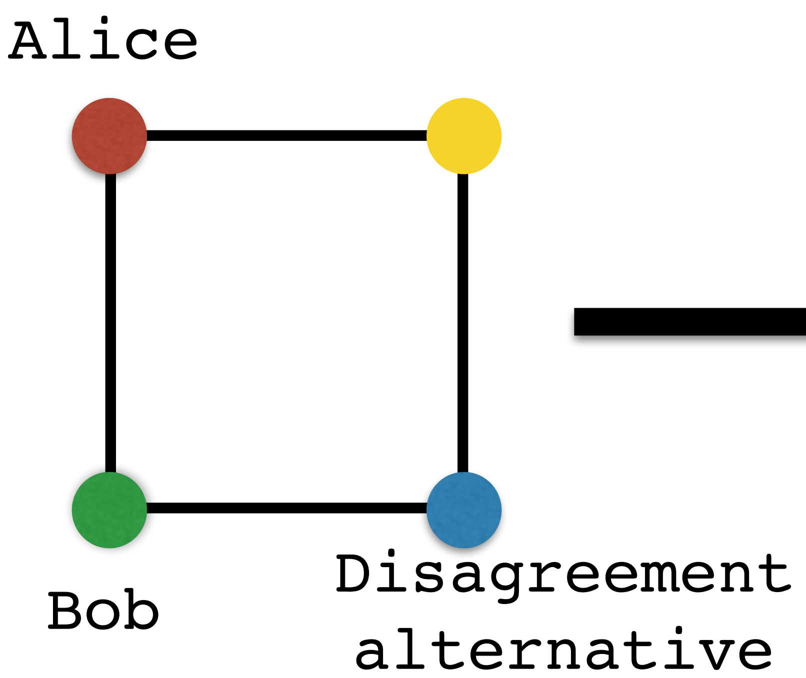
Assumptions

- Assume all users and all possible decisions lies in a common metric space d, with user v having a cost d(v,x) for decision x
- Optimum: find a decision that minimizes total cost for all users, $S(x) = \sum_v d(v,x)$
- Distortion: If x* is the optimum decision, the the **distortion** of a randomized mechanism that produces decision x is E[S(x)]/S(x*)
- Randomized dictator gives a distortion of 2, and various deterministic voting rules give good distortion (e.g., Copeland: 5 [Anshelevich et al 2015], best known 4.236 [Munagala, Wang 2019])

Sequential Deliberation

- N := set of agents.
- Start with an initial suggestion (e.g., one proposed by a random agent), and call it s^1
- For rounds from t=1 to t=T:
  - u^t, v^t ~ Two agents chosen uniformly at random
  - s^t ~ Suggestion from previous step
  - Agents u^t, v^t bargain with s^t as the outside alternative.
    - If they agree, set $s^{(t+1)}$ to their consensus
    - Else, $s^{(t+1)} = s^t$

Nash Bargaining



	Red	Yellow	Blue	Green
Dist. to Alice	0	1	2	1
Dist. to Bob	1	2	1	0

	Red	Yellow	Blue	Green
Red	(0, 1)	(2, 1)	(2, 1)	(2, 1)
Yellow	(2, 1)	(1, 2)	(2, 1)	(2, 1)
Blue	(2, 1)	(2, 1)	(2, 1)	(2, 1)
Green	(2, 1)	(2, 1)	(2, 1)	(1, 0)

Bargain({Alice,Bob}, Blue)
= Green

Results

On a class of decision problems (**median spaces**):

1. Nash bargaining between agents u and v with ideal points p_u and p_v using disagreement outcome s finds the median of p_u, p_v, s.

2. All agents bargaining by truthfully representing their ideal point is a sub-game perfect Nash equilibrium of the extensive form game defined by sequential bargaining.

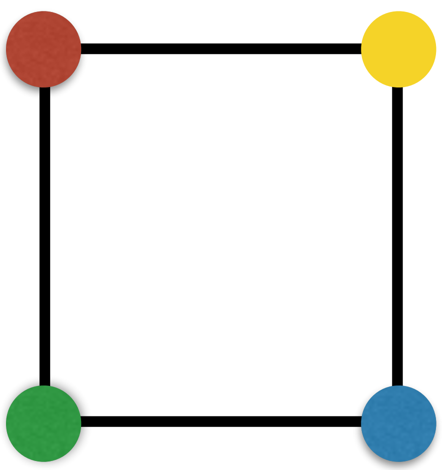
3. The chosen alternative converges to a stationary distribution in O(1) steps

But what about the distortion?

Median Spaces

For any three points, there is a unique point that lies on three pairwise shortest paths

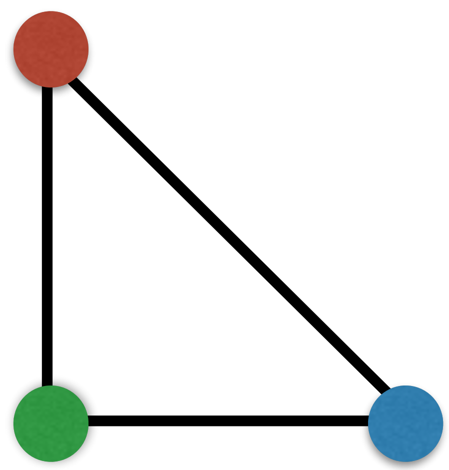
Median Space



- Trees
- Hypercubes
- Grids

Has a Condorcet winner

Not Median Space



- Triangles
- Disconnected

Distortion on Median Spaces

Cost Distortion	Random Dictatorship	Sequential Deliberation
Upper Bound	2	1.208
Lower Bound	2	1.125
Second moment	Infinite	Finite