Markets for Public Decision-making

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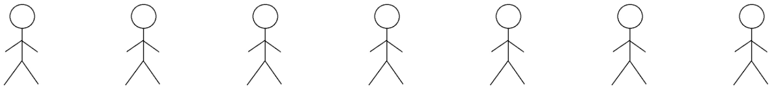
joint work with Nikhil Garg and Ben Plaut

Public decision-making

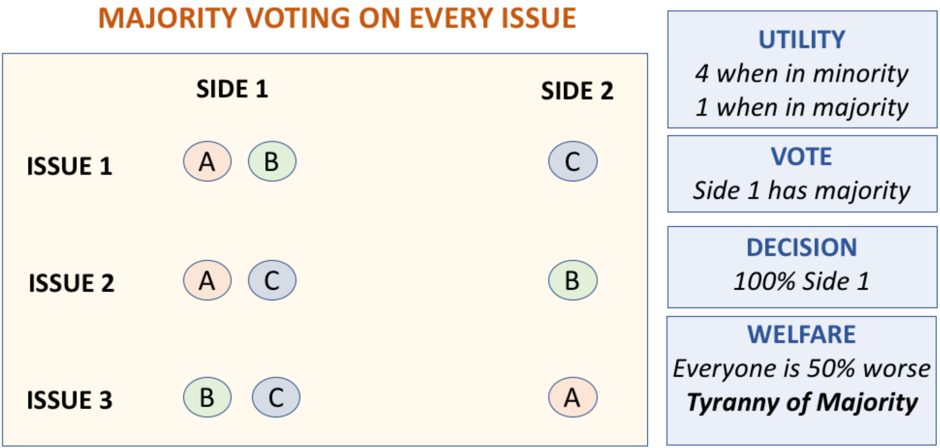


Utility Model

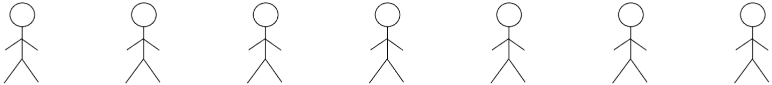
- User i has binary preferences over the issues, and a weight $w_{i\ell} > 0$ for issue ℓ. The decision z_ℓ on issue ℓ lies in $[0, 1]$.
- Utility of user i is given by $u_i(z) = \sum_\ell w_{i\ell} x_i^{(\ell)}$ where $x_i^{(\ell)} = z_\ell$ if user i prefers side 0 on issue ℓ and $1 - z_\ell$ otherwise.

“One person one vote”

- Give each person a single vote on each issue and select the outcomes which receive the most votes
- Fair in some sense
- Lacks expressiveness
- Can lead to very suboptimal outcomes


MAJORITY VOTING ON EVERY ISSUE
SIDE 1
SIDE 2
ISSUE 1
A
B
C
ISSUE 2
A
C
B
ISSUE 3
B
C
A
UTILITY
4 when in minority
1 when in majority
VOTE
Side 1 has majority
DECISION
100% Side 1
WELFARE
Everyone is 50% worse
Tyranny of Majority

Markets



Markets

- Each player has a budget they wish to spend, and has no value for leftover money
- Goods are divisible
- "Fisher market" (Irving Fisher)
- "private goods"

Market equilibrium

- Each good has a price
- Each player buys her favorite affordable bundle
- An equilibrium always exists! [Arrow and Debreu, 1954]
  - Demand meets supply
  - The equilibrium maximizes Nash welfare [Eisenberg and Gale, 1959]:

$$\sum_i \log u_i$$

where u_i is the utility for player i

Our goal

Design a mechanism for public decision-making based on private goods markets.

- More expressive than "one person one vote"
- Markets in general have nice properties
- Prices can be computed in an iterative and natural way
-

Citizens purchasing political influence?

capitalism democracy

Our goal

Design a mechanism for public decision-making based on private goods markets.

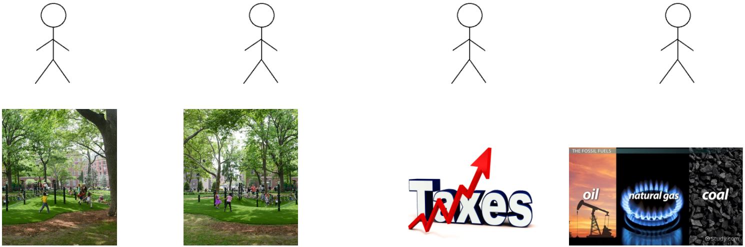
- More expressive than “one person one vote”
- Markets in general have nice properties
- Prices can be computed in an iterative and natural way
- Each person gets equal endowment of “voting Dollars”

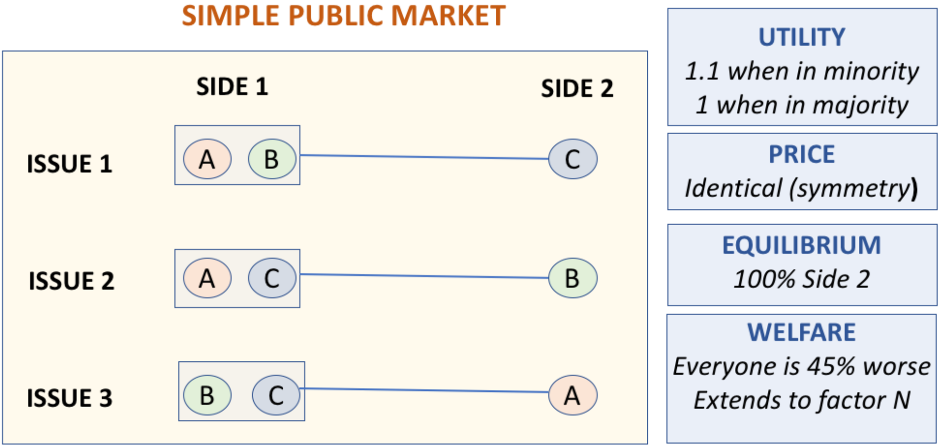
~~Citizens purchasing political influence?~~

capitalism democracy

A first attempt

- Assume issues are divisible/randomized
- Each issue has a price (this is the only thing that will change in our other model)
- Each player uses her budget to "buy probability" (ignoring supply)





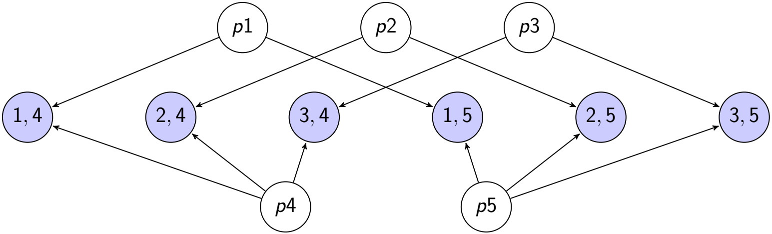
SIMPLE PUBLIC MARKET
SIDE 1
SIDE 2
ISSUE 1
A
B
C
ISSUE 2
A
C
B
ISSUE 3
B
C
A
UTILITY
1.1 when in minority
1 when in majority
PRICE
Identical (symmetry)
EQUILIBRIUM
100% Side 2
WELFARE
Everyone is 45% worse
Extends to factor N

Context on the simple market

- Similar to the “free rider” problem
- Observed in the classical literature before (e.g., [Danziger 1976])
- The same counter-example extends to several variants, e.g., Quadratic Voting [Lalley, Weyl 2014] and Trading Post [Shapley, Shubik 1977, Branzei et al 2016]
- Arbitrary per-player prices can implement the Nash-welfare solution (in fact any Pareto-optimal solution) via Lindahl equilibria [Foley 1979]
  - Lindahl prices are complex, and we would like a simple Fisher-like market, or a simple generative explanation
  - A simple market might lead to an implementable protocol

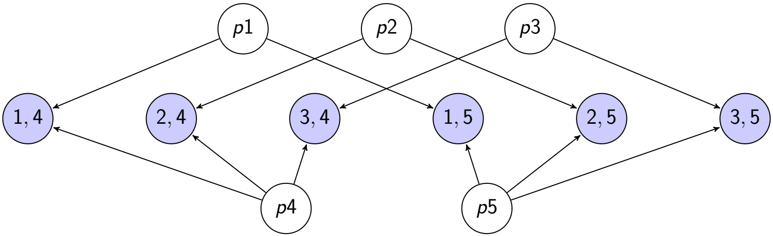
Reduction via Pairwise Expansion

- For any public decision-making instance, we create a private goods instance as follows
- Same set of players
- For each every issue, we create a good for each pair of players who disagree on that issue

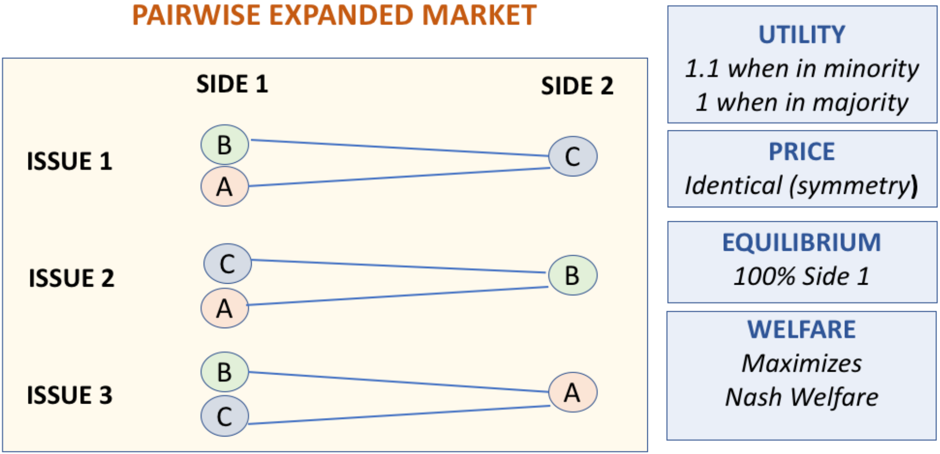


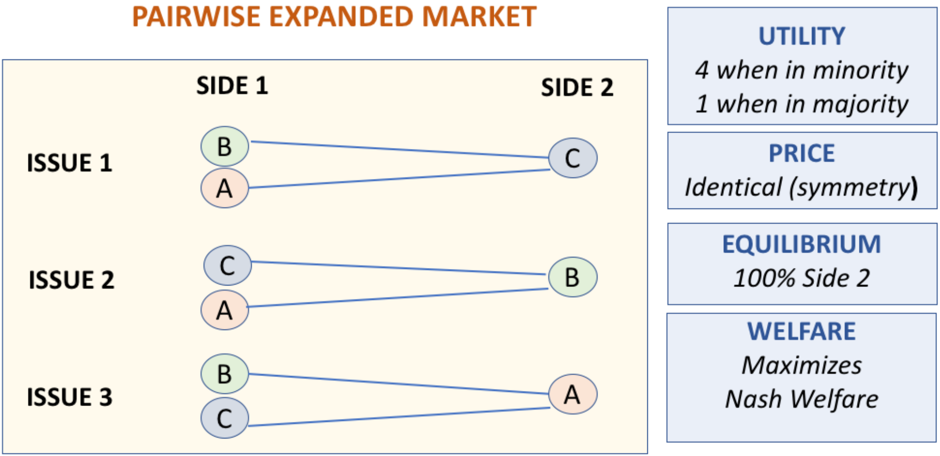
- "pairwise issue expansion"

Reduction via Pairwise Expansion



- Let u_i be the utility of player i in the private market
- One issue: x_{ij} is what player i buys of good j. Define $u_i = \min_{\text{her pairwise goods } j} x_{ij}$ (Leontief)
- Many issues: $u_i = \sum_{\text{issues } \ell} w_{i\ell} \left(\min_{\substack{\text{her pairwise goods } j \\ \text{on issue } \ell}} x_{ij}^{(\ell)} \right)$
- **Key insight:** Each player i is in direct competition with everyone she disagrees with, and with no one she agrees with


PAIRWISE EXPANDED MARKET
SIDE 1
SIDE 2
ISSUE 1
B
A
C
ISSUE 2
C
A
B
ISSUE 3
B
C
A
UTILITY
1.1 when in minority
1 when in majority
PRICE
Identical (symmetry)
EQUILIBRIUM
100% Side 1
WELFARE
Maximizes
Nash Welfare


PAIRWISE EXPANDED MARKET
SIDE 1
SIDE 2
ISSUE 1
B
A
C
ISSUE 2
C
A
B
ISSUE 3
B
C
A
UTILITY
4 when in minority
1 when in majority
PRICE
Identical (symmetry)
EQUILIBRIUM
100% Side 2
WELFARE
Maximizes
Nash Welfare

Our main result

Theorem

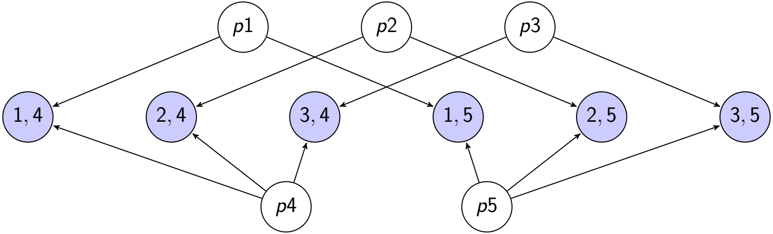
Equilibria in the constructed private goods market correspond to valid solutions in the original public decisions instance.

- This will give us the nice private goods market equilibrium properties!
- Maximum Nash welfare

The mechanism:

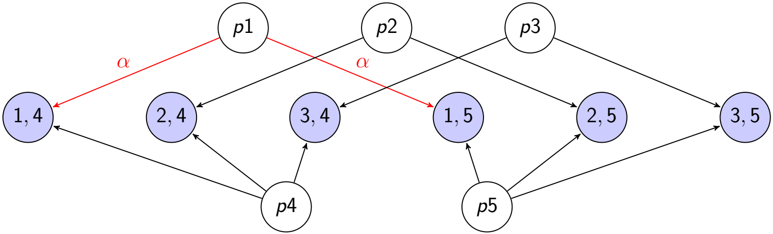
- Players never see the constructed private goods market
- Compute equilibrium prices
- Reduction turns these into per-player prices in the public decisions instance
- These per-player prices give an equilibrium in the public decisions instance that maximizes Nash welfare.

Proof sketch



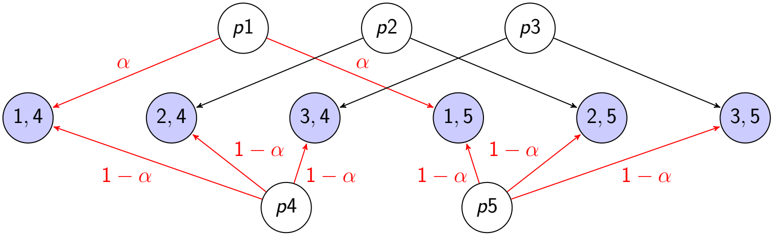
- $u_i = \min_{\text{her pairwise goods } j} x_{ij}$

Proof sketch



- $u_i = \min_{\text{her pairwise goods } j} x_{ij}$
- Say player 1 buys α of all of her pairwise goods
- Players 4 and 5 can each get at most $1 - \alpha$

Proof sketch



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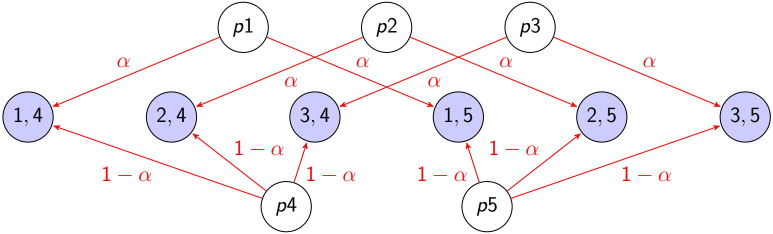
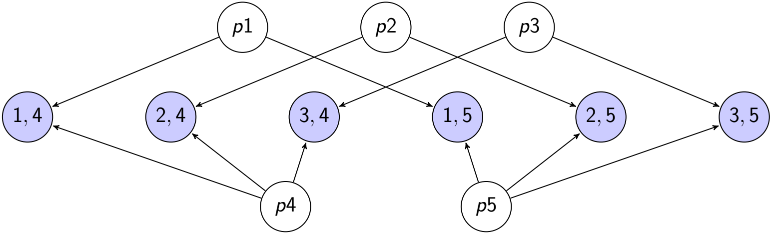


- $u_i = \min_{\text{her pairwise goods } j} x_{ij}$
- Say player 1 buys α of all of her pairwise goods
- Players 4 and 5 can each get at most $1 - \alpha$
- Players 4 and 5 will never buy more than $1 - \alpha$
- This leaves exactly α for players 2 and 3
- At equilibrium, all players on the same side of the issue buy the same amount
- That is the probability placed on that alternative in the outcome of the public decisions instance

Market recap

- Construct the private goods market



- Compute equilibrium prices in the private goods market (one shot or tâtonnement)
- This gives us one price for each pair of players who disagree on a particular issue
- Player i's price for issue j is the sum of the prices on those pairwise disagreements

Theorem

The resulting per-player prices yield an equilibrium in the public decisions instance that maximizes Nash welfare.

Conclusion

- Markets have been well-studied for private goods, lots of nice properties
- Can use these concepts to design mechanisms for public decision-making
- Theorem: Any public decisions instance can be transformed into an equivalent private goods market.
- Can lift private goods results to public decisions setting

Future work:

- More practical mechanisms (iterative? deterministic?)
- Scalability
- Applications of reduction