MS&E 336/CS 366: Computational Social Choice. Winter 2019-20
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8 Participatory Budgeting

The first PB process in the United States was initiated in the 49^{th} ward of Chicago in 2009. Since then PB has spread to many other cities like Vallejo, New York, Cambridge [1]. A digital PB voting platform may look like this: `https://pbstanford.org/`.

How to make a budget collaboratively or via some type of voting? We introduce a more formal model setup:

- N votes, M expense items / project;
- assume only expense items are on the ballot;
- total budget available is B. Cost of item j is c_j. Assume the all costs and B are positive integers;
- assume items can be chosen fractionally (NP hard otherwise even for one voter);
- feasible solutions: $x =< x_1, x_2, ...x_M >$ such that $x \geq 0, x \leq c, \sum_{Project j} x_j \leq B$.

The objectives of any voting election mechanism are two-fold: to elicit preferences across all alternatives, and to aggregate the elicited preferences. Before delving into each of these two topics, first we need a notion of the user utility and the cost to describe the collective preferences. For example:

- Linear Utility: voter i has utility $u_{i,j}$ for project j and
$$U_i(x) = \sum u_{i,j} x_j$$
- Overlap Utility: user i has ideal budget $z_i =< z_{i,1}, z_{i,2}, ...z_{i,M} >$ and user i's utility for budget x is
$$U_i(x) = \sum_{Project j} min\{z_{i,j}, x_j\}$$

8.1 Elicitation

8.1.1

Elicitation refers to any voting methods for which to elicit people's preferences. For example, we can ask people to rate or rank over certain alternatives. If people's utilities are linear, then one natural approach of elicitation is just directly asking what preferences are. Alternatively, if people's utilities are non-linear, we can use quadratic voting where votes are allocated by individuals to express the degree of their preferences, rather than just the direction [2].

8.1.2

As an example of quadratic voting in eliciting preferences, we introduce the following model setup:

- Current budget = X.
- If a voter can change X by $\triangle$ such that:
  1. $\triangle \in \mathbb{R}^M$
  2. $\triangle \geq 0$ (i.e. cannot decrease the budget)
  3. $\sum_{j=1}^{M} \triangle_j^2 = 1$ (to bound the change limit)
- Objective: maximize $\sum_j \triangle_j u_{ij}$

Solution to this setup is:

$$\triangle_j \propto u_{ij}$$

Two Items Case

- $\triangle_1^2 + \triangle_2^2 = 1$
- $\triangle \geq 0$

Goal:

$$maximize \quad a_1\triangle_1 + a_2\triangle_2, \qquad a_1, a_2 > 0$$

Proof:

$$a_1\triangle_1 + a_2\sqrt{1-\triangle_1^2} = y$$

$$\frac{dy}{d\triangle_1} = a_1 - a_2\triangle_1(1-\triangle_1^2)^{-\frac{1}{2}} = a_1 - a_2\frac{\triangle_1}{\triangle_2} = 0$$

$$\frac{a_1}{a_2} = \frac{\triangle_1}{\triangle_2}$$

The ratio of marginal utility is the same as the ratio of marginal cost. ■

This elicitation method is part of what we call Knapsack voting.

8.1.3

Definition 8.1 *We now introduce a notion of very weak strategy proofness. Informally, it means if a project is currently winning without i's vote, and person i likes it, then person i will vote for it.*

More formally, if:

- $u_{i,j1} > u_{i,j2}$
- j_1 *will win without i's vote*
- *voter i places* j_2 *in an optimum response"*

Then there exists an "optimum response" that contains j_1

The above is a very weak notion of strategy proofness. We observe above such as in committee election or in Knapsack voting, for which we now turn to.

8.2 Aggregation: Knapsack Voting

Aggregation procedure:

1. Assume each project is of unit cost, i.e. a collection of subprojects;
2. Count the number of votes for each subproject;
3. pick the subprojects that get the highest votes

Theorem 8.1 *Knapsack voting is (very) weakly strategy proof if we assume linear utilities. This part of elicitation will form part of Knapsack voting that is weakly strategy proof for linear utilities.*

Theorem 8.2 *Knapsack voting is strategy proof if we assume overlap voting.*

Proof Outline: Proof by contraction - if untruthful voting, then utilities will not be any higher. (Formal proof detailed in [1]) ■

Theorem 8.3 *If we assume overlap utility, assume voters vote truthfully, and assume* L_1 *costs such that*

$$C_i(x) = \sum_j |z_j - x_{i,j}|$$

then Knapsack voting will maximize total social welfare and minimize total social cost.

Theorem 8.4 *Re-interpreting Knapsack voting:*

Given vote $z_1, z_2, z_3 \ldots z_N$*, where:*

- $z_i \geq 0$
- $\forall i \sum_j z_{i,j} = B$

- $z_i \in \mathbb{R}^M$

Find $x \in \mathbb{R}^M$ *s.t.* $x \geq 0$, $\sum x_j = B$, *and* x *minimizes* $\sum_i |z_i - x|$.
This is the same as finding the ***median*** *in the multi-dimensional space.*

KS voting can be used in many different forms. For instance, a local government can ask residents to vote on proposals for how a certain fraction of their total budget should be spent. The proposals could be, in a particular city/ward, resurfacing streets, adding street lights, building playgrounds for children or renovating recreational facilities like parks [1]. Such innovation has been use in South America as well as the U.S.A at various state levels. There is also ongoing research in evaluating the effectiveness of KS voting, such as by studying the possible Electoral College performance in two-party US presidential elections [3].

References

[1] A.Goel, A. Krishnaswamy, S. Sakshuwong and T. Aitamurto. *Knapsack Voting: Voting mechanisms for Participatory Budgeting.* ACM Transactions on Economics and Computation, 2019

[2] N. Benabbou. *Possible Optimality and Preference Elicitation for Decision Making.* Algorithmic Decision Theory, 2015

[3] A.S. Belenky. *A 0-1 knapsack model for evaluating the possible Electoral College performance in two-party US presidential elections.* Mathematical and Computer Modelling, 2008