MS&E 336/CS 366: Computational Social Choice
Win 2019-20, HW 1

Submit at least 6 problems. Due Feb 10, before midnight, by email. You can do it in groups of up to 3. Please type up the answers. If you find the solution elsewhere, you can read it and digest it, but you must then type up the solution in your words without repeatedly referring to the solution.

1. For each of the following SCFs, give an example where one of the voters has an incentive to report an incorrect ranking: Instant Runoff, Borda, Plurality, Copeland.

2. Prove that there is no scoring rule that is Condorcet consistent.

3. Prove May's theorem: For two alternatives and an odd number of voters, majority rule is the unique resolute, anonymous, neutral, and monotonic SCF.

4. Prove that the Nash bargaining axioms are satisfied by the solution that maximizes the product of the additional utilities (compared to the outside option) that the two players receive.

5. Consider a two-person bargaining problem where the Pareto frontier is given by the straight line $u_A/\alpha + u_B/\beta = 1$ where u_A and u_B are the utilities that A and B receive. Assume that α, β are both positive and are known to both participants, but they are not known to you, the game designer. Devise a multi-stage game such that the only pure strategy sub-game perfect equilibrium of the game results in the Kalai-Smorodinsky bargaining solution with the outside option being $u_A = 0, u_B = 0$. This is a somewhat open-ended problem, and you can get two hints (with no penalty). Also, if you find the right game, but can only express its properties informally, you will still get full credit.

6. Give an example where the Hegselmann-Krause dynamic on N agents takes $\Omega(N)$ steps to converge.

7. Motivated by biased assimilation, we propose the following dynamic:

$$x_i(t+1) = \frac{w_{i,i}x_i(t) + x_i(t)\left(\sum_{j\neq i} w_{i,j}x_j(t)\right)}{w_{i,i} + x_i(t)\left(\sum_{j\neq i} w_{i,j}x_j(t)\right) + (1-x_i(t))\left(\sum_{j\neq i} w_{i,j}(1-x_j(t))\right)}$$

   Consider a social network with $2N$ agents. Let $x_i(t)$ denote the opinion of agent i at time t. Assume $x_i(0) = a$ for $1 \leq i \leq N$ and $x_i(0) = 1-a$ for $i > N$. Further assume that $w_{i,j} = 2/(3N)$ if i, j are both bigger than N or are both at most N, and $w_{i,j} = 1/(3N)$ otherwise. As t goes to infinity, find the limiting values for $x_i(t)$. *Hint: Examine the ratio of* $x_i(t+1)$ *to* $1 - x_i(t+1)$.

8. How would you define polarization? Your definition should be in plain English, but written with sufficient specificity that it could be converted into a measurable quantity. By historical standards, are we living in times that are more polarized than normal? Why or why not? *No more than 200 words.*