MS&E 235: Lecture 3
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Lecture 3: Keyword Auctions

This lecture builds on the Second Price Auction mechanism from the previous lecture to describe the auctions employed by search engines to sell keywords.

Motivation

The advertising industry as a whole has aggregate sales in the range of $250B-$300B, and has experienced a 3% overall decrease in revenue since 2007. In contrast, online advertising, which makes up about $23B of the aggregate sales, has experienced an increase of 10.6%. More specifically, keyword advertising sales have increased by 19.8%.

Here are a few reasons that online advertising, and especially keyword advertising (search advertising) are so popular.

- Online advertising is cost effective (efficient) compared to advertising in other media.
- Search advertising (keyword advertising) and content-based advertising are targeted
- Search advertising and content-based advertising are actionable (if you click on the ad, it takes you easily to the store)
- Search advertising captures "lookingforness" or "intentionality" (i.e. users who are searching for "watches" are often looking to buy watches)

Advertising vs. online advertising

Based on the whole advertising market, it is believed that the ceiling on online advertising sales is somewhere between $80B and $100B. This projection, however, is questionable because of the inherent differences between online advertising and traditional advertising. Traditional advertising (magazine ads, television ads, etc.) is a revenue-consuming activity for companies. The advertising department spends money, and the sales department brings in new money. In contrast, online advertising is often a profit center. This is an important distinction because if online advertising makes more money than it costs, then there is no reason for advertisers to put a cap on their online advertising spending.

Therefore, online advertising should not be thought of as a fraction of total existing advertising, but rather as a fraction of total existing *retail*, which is much larger.

Next Price Auctions

Recall from last lecture that it is too difficult to sell keyword advertising spots by posting individual prices for every available combination of keywords. There are too many, and determining the market prices would be too difficult. So, instead, auctions are used.

Keyword advertising is usually sold on a cost per click (CPC) basis. Advertisers can bid on keywords that they want their ad to be displayed with. If we try to set this up as a Second Price Auction, there are several issues that arise:

- multiple ad slots for a single keyword search
- quality (measured by click-through rate (CTR))
- truthfullness
- budgets
- uncertainty (what will the CTR be?)

Like a typical auction, advertiser i bids b_i on a specific keyword. Unlike a typical auction, the auctioneer would not only like to choose bidders who are willing to pay more, but also bidders whose ads are more likely to be clicked on. So rather than ordering the bidders by bid only, they are ordered by $b_i q_i$, where q_i is the quality, or CTR, of the advertiser.

Recall that in a Second Price Auction, the winner is the highest bidder, and the payment is the second highest bid. In a keyword auction, there are k ad spots available, where k is often greater than 1, and the bids are not considered by themselves, but are multiplied by the quality of the advertisement. So instead of the highest bidder winning, the k advertisers with the highest $b_i q_i$ values win, and they are charged the lowest price that will maintain their position in the $b_i q_i$ ordering. This modified form of a second price auction is called a "Next Price Auction" or "Generalized Second Price Auction."

Example

$N = 3$ (three advertisers bidding)
$k = 2$ (2 spots up for bid)
The bids and qualities are in the order: $b_1 q_1 > b_2 q_2 > b_3 q_3$

Advertiser 1 should get an ad spot, but how much should he be charged?
In order to maintain his place in the ordering, he must be charged m_1, where $m_1 q_1 > b_2 q_2$

$$\Rightarrow \text{ad spot 1 cpc: } \frac{b_2 q_2}{q_1}$$
$$\Rightarrow \text{ad spot 2 cpc: } \frac{b_3 q_3}{q_2}$$

Truthtelling

In this example, advertiser 1 ends up paying more for an ad than advertiser 2. This raises the question, should the advertisers be charged differently? Since it is not clear that being in the first spot is worth more than being in the second spot, truthtelling is not necessarily a Nash equilibrium in a Next Price Auction. Player 1 could lower his bid, so that $b_1 q_1 < b_2 q_2$ and, as long as he maintains $b_1 q_1 > b_3 q_3$, he will still be awarded a spot. In the next example we will see more on why this is not a Nash equilibrium. In the next lecture, we will see an auction for which truthtelling *is* a Nash equilibrium.

Position effect

Another question that arises from the results of the example is: since advertiser 1 is in the first ad spot, will his CTR be higher?

In fact, the CTRs will most likely vary depending on the order of display. Because of this, we will denote the CTR as $CTR_{ij} = q_i p_j$ for advertiser i, when in position j. Here p_j represents the "position effect." We will assume that $\{p_i\}$ is decreasing, i.e. $p_1 > p_2 > ... > p_k$. (Although in practice, p_k, the last position, is probably higher than p_{k-1}.) Since p_1 is always the largest, the convention is to set $p_1 = 1$, and then set the other p_i's as fractions of p_1.

Content-based advertising

Although this lecture has focused on keyword advertising, highly-targeted content-based advertising works largely the same way. For example, Googe Adwords places ads on websites based on content, and the advertisers bid for spots in a Next Price Auction.

Example 2

In this example, four merchants ($m_1, ..., m_4$) are bidding on three ad spots for a specific keyword. Merchant i's true valuation per click (the value he receives from one user clicking his ad) is denoted v_i
$N = 4$
$k = 3$

	b_i	q_i	v_i
m_1	\$1.20	0.01	\$1.20
m_2	\$1.50	0.02	\$1.50
m_3	\$0.50	0.03	\$0.50
m_4	\$1.00	0.05	\$1.00

Note that the merchants are bidding their true valuations. From this information, we can calculate the $b_i q_i$, the positions that the merchants are awarded, and the subsequent cpc's, as in the previous example:

	b_i	q_i	v_i	$b_i q_i$	pos.	cpc
m_1	\$1.20	0.01	\$1.20	\$0.012	–	–
m_2	\$1.50	0.02	\$1.50	\$0.03	2	\$0.75
m_3	\$0.50	0.03	\$0.50	\$0.015	3	\$0.40
m_4	\$1.00	0.05	\$1.00	\$0.05	1	\$0.60

If the position effects are: $p_1 = 1$, $p_2 = 0.95$, $p_3 = 0.90$, then $ctr_{4,1} = 0.05$ and $ctr_{4,3} = 0.045$

So Merchant 4:
Valuation per click: \$1.00
Cost per click: \$0.60
Residual value per click: \$0.40
Residual value per impression: $\$0.40 \times 0.05 = \$0.02$

Nash equilbrium test

In order for this to be a Nash equilibrium, all merchants must have no incentive to change their bids. Consider merchant 4. Can merchant 4 improve his residual value per impression by changing his bid?

→ Increasing his bid will not change his residual value, because he will still win and pay the same price.

→ Decreasing his bid? If he changes his bid to $b_4 = \$0.25$, then:
$b_4 q_4 = \$0.25$
Position: 3
$cpc = \$0.012/0.05 = \$0.24$
Residual value per click: \$0.76
$ctr = 0.045$
⇒ Residual value per impression: \$0.034

This is greater than the residual value per impression he gets from bidding truthfully ($\$.034 > \$.02$), therefore, truthtelling is *not* a Nash equilibrium. Nonetheless, the next price auction mechanism is used in practice, and maintains a stable market.