MS&E 235: Internet Commerce

Problem Session 1, April 16, 2009

1. Consider the following game: "Evens or Odds" (modified):

   You and your buddy simultaneously show your fingers to one another. You each decide whether to show 1 or 2 fingers. If the sum of the number of your fingers shown plus the number of his fingers shown are odd, then your friend wins and she gets a prize of 3 dollars. But, if they are even, you win – and your prize is either 2 or 4 dollars depending on the sum of the fingers shown: two dollars for two fingers and four dollars for four fingers.

   - Organize this problem into a formal game: identify the strategy sets and payoff functions of each player
   - Is there a dominant strategy for you, your friend, both, or neither? Explain.
   - Is there a pure Nash equilibrium? Explain.
   - Find a mixed Nash equilibrium. Explain why you don't have incentive to play two fingers more often.

2. Laddered Pricing. Consider three merchants AAPL, MSFT, and SNDK having a valuation-per-click of 100, 80, and 50 cents respectively bidding for two slots on a keyword. Assume that each merchant has a CTR of 0.40 in position 1 and a CTR of 0.30 in position 2.

   - How much revenue per impression should the auctioneer expect to make if a laddered pricing is chosen? Delineate the expected charges.
   - How much revenue per impression should the auctioneer expect to make if a second price pricing is chosen? Delineate the expected charges. (Assume AAPL is in the top position). If needed find a range of answers.

3. Repeat the situation as in the problem above, but now consider that AAPL, MSFT, and SNDK have ad qualities 0.3, 0.9, and 1.6 respectively. What happens when they each bid their valuation? Assume that the auctioneer allocates the positions according to the players bid*quality. Is this an equilibrium? Explain.

4. Design a two player game, where each player has a strategy set of size two, such that: each player has a strict dominant strategy – resulting in one unique dominant strategy equilibrium, but neither player is "fully satisfied" because for each player, their resulting payoff is strictly less than the amount they could get if they were allowed to choose both actions. Optional: write a set of constraints that define when this situation will occur; without loss of generality you may name both actions for both players and predetermine the equilibrium.